The electromagnetic decays of $X(3823)$ as the $\psi_2(1^3D_2)$ state and its radial excited states

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1. Abstract

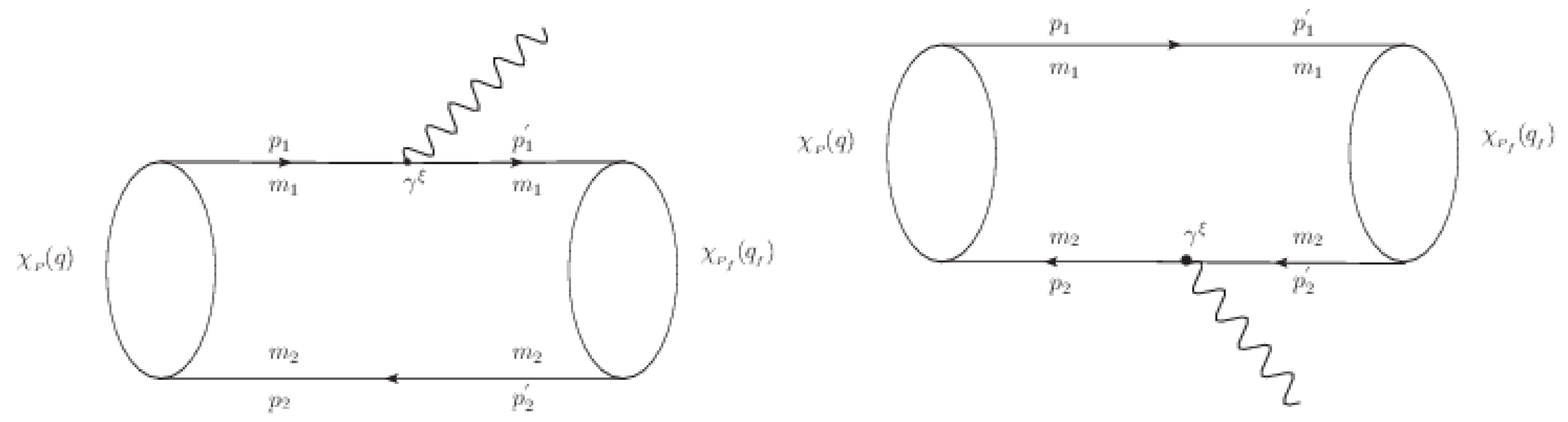
We study the electromagnetic (EM) decays of $X(3823)$ as the $\psi_2(1^3D_2)$ state by using the relativistic Bethe-Salpeter method. Our results are $\Gamma[X(3823) \to \chi_{c0}\gamma] = 1.2$ keV, $\Gamma[X(3823) \to \chi_{c1}\gamma] = 265$ keV, $\Gamma[X(3823) \to \chi_{c2}\gamma] = 57$ keV and $\Gamma[X(3823) \to \eta_c\gamma] = 1.3$ keV. The ratio $\mathcal{B}[X(3823) \to \chi_{c2}\gamma]/\mathcal{B}[X(3823) \to \chi_{c1}\gamma] = 0.22$, agrees with the experimental data. Similarly, the EM decay widths of $\psi_2(n^3D_2)$, $n = 2, 3$, are predicted, and we find the dominant decays channels are $\psi_2(n^3D_2) \to \chi_{c1}(nP)\gamma$, where $n = 1, 2, 3$. The wave function include different partial waves, which means the relativistic effects are considered. We also study the contributions of different partial waves.

2. Introduction

Recently, a new bound state $X(3823)$ has been observed, which is considered to be a good candidate for spin triplet D wave charmonium $\psi_2(1^3D_2)$. The Belle Collaboration first observed $X(3823)$ in the $B \to \chi_{c1}\gamma K$ decay with a statistical significance of 3.8. Most existing theoretical predictions of the $X(3823)$ EM decay are provided by non-relativistic methods. However, we have found the relativistic corrections are large for charmonia, especially for the higher excited states, so it is necessary to study the properties of $X(3823)$ with different methods especially relativistic one. The Bethe-Salpeter (BS) equation is a relativistic dynamic equation used to describe bound state . Salpeter equation is its instantaneous version which is suitable for the heavy meson, especially the double-heavy meson. We have solved the complete Salpeter equations for different states, as examples, and we have improved this method to calculate the transition amplitude with relativistic wave function as input, where the transition formula is also relativistic. Using this improved BS method, we can get relatively accurate theoretical results, which agree well with the experimental data.

3. Theoretical Calculation

Transition Amplitude



Invariant amplitude $\mathcal{M}^\xi$ consists of two parts, corresponding to the two subgraphs in Figure. The amplitude can be written as

$$\mathcal{M}^\xi = \int \frac{d^4q}{(2\pi)^4}\frac{d^4q_f}{(2\pi)^4} Tr[\bar{\chi}_{P_f}(q_f)Q_1 e\gamma^\xi \chi_P(q)(2\pi)^4\delta^4(p_2 - p_2')S_2^{-1}(-p_2) + \bar{\chi}_{P_f}(q_f)(2\pi)^4\delta^4(p_1 - p_1')S_1^{-1}(p_1)\chi_P(q)Q_2 e\gamma^\xi], \quad (1)$$

where $\chi_P(q)$, $\chi_{P_f}(q_f)$ are the relativistic BS wave functions for $X(3823)$ and χ_{cJ}, respectively. To simplify the calculation, the decay amplitude can be written as

$$\mathcal{M}^\xi = \int \frac{d^3q_\perp}{(2\pi)^3} Tr[Q_1 e \frac{\not{P}}{M}\bar{\varphi}_f^{++}(q_\perp + \alpha_2 P_{f_\perp})\gamma^\xi \varphi_i^{++}(q_\perp) + Q_2 e\ \bar{\varphi}_f^{++}(q_\perp - \alpha_1 P_{f_\perp})\frac{\not{P}}{M}\varphi_i^{++}(q_\perp)\gamma^\xi]. \quad (2)$$

The Relativistic Wave Function

Though the BS equation is the relativistic dynamic equation, it can not provide us the form of a relativistic wave function for a bound state. The relativistic wave function of $X(3823)$ as a 2^{--} state

$$\varphi_{2^{--}}(q_\perp) = i\epsilon_{\mu\nu\alpha\beta}\frac{P^\nu}{M}q_\perp^\alpha \epsilon^{\beta\delta} q_{\perp\delta}\gamma^\mu \left(f_1 + \frac{\not{P}}{M}f_2 + \frac{\not{P}\not{q}_\perp}{Mm_c}f_2\right), \quad (3)$$

where f_1 and f_2 are independent radial wave functions and they are function of $-q_\perp^2$. So, the positive energy wave function of $X(3823)$ as a 2^{--} state

$$\varphi_{2^{--}}^{++}(q_\perp) = i\epsilon_{\mu\nu\alpha\beta}\frac{P^\nu}{M}q_\perp^\alpha q_{\perp\delta}\epsilon^{\beta\delta}\gamma^\mu[F_1 + \frac{\not{P}}{M}F_2 + \frac{\not{P}\not{q}_\perp}{M^2}F_3], \quad (4)$$

where F_1 and F_2 terms are dominant D partial waves which will survive in the non-relativistic limit, while the relativistic term including F_3 is F partial wave.

4. Result

Decay Widths of $\psi_2(1^3D_2)$:

Considering $X(3823)$ as $\psi_2(1^3D_2)$ state, the decay EM widths are

$$\Gamma[X(3823) \to \chi_{\{c0,\ c1,\ c2\}}(1P)\gamma] = \{1.22,\ 265,\ 57\}\ \text{keV},$$
$$\Gamma[X(3823) \to \eta_c(1S,\ 2S)\gamma] = \{1.30,\ 0.069\}\ \text{keV}. \quad (5)$$

Where we can see the dominant decay channel is $X(3823) \to \chi_{c1}(1P)\gamma$, its decay width is much larger than other fours. And the ratios are

$$\frac{\mathcal{B}[X(3823)\to\chi_{c2}\gamma]}{\mathcal{B}[X(3823)\to\chi_{c1}\gamma]} = 22\ \%, \quad \frac{\mathcal{B}[X(3823)\to\chi_{c0}\gamma]}{\mathcal{B}[X(3823)\to\chi_{c1}\gamma]} = 0.46\ \%. \quad (6)$$

Contributions of different partial waves:

In a complete relativistic method, the relativistic wave function for a J^P state is not a pure wave.

TABLE VI: The EM decay width (keV) of different partial waves for $\psi_2(1D) \to \chi_{c1}(1P)\gamma$.

2^{--} \ 1^{++}	*complete*	P *wave* (C_{f_1}, C_{f_2})	D *wave* (C_{f_3})
complete	265	204	4.0
D *wave* (F_1, F_2)	209	211	4.2
F *wave* (F_3)	3.4	0.17	0.0056

From Table VI, we can see that, the main contribution of the final state come from the dominant P partial wave which provides the non-relativistic result, and the relativistic correction (D partial wave in 1^{++} state) contribute very small.

5. Discussion

The non-relativistic results of EM decay for $X(3823)$ are

$$\Gamma_0[X(3823)(1D) \to \eta_c(1S)\gamma, \chi_{\{c0,\ c1,\ c2\}}(1P)\gamma] = \{0.41,\ 0.19,\ 211,\ 44\}\ \text{keV},$$

Compared with the complete relativistic results, the relativistic effects make up 68 %, 84 %, 20 %, 23 %. So the contribution of the relativistic correction plays a leading role in the decay processes of $\psi_2(1D) \to \eta_c(1S)\gamma$ and $\psi_2(1D) \to \chi_{c0}(1P)\gamma$.

6. Conclusion

We study the EM decays of $X(3823)$ as $\psi_2(1^3D_2)$ and its radial excited states $\psi_2(n^3D_2)$ ($n = 1, 2, 3$) by using the relativistic Bethe-Salpeter method. We find for $\psi_2(n^3D_2)$, the dominant EM decay channel is $\psi_2(n^3D_2) \to \chi_{c1}(nP)\gamma$. Our results show that $\Gamma[X(3823) \to \chi_{c1}\gamma] = 265$ keV, this is the dominant decay channel. The decay ratio $\mathcal{B}[X(3823) \to \chi_{c2}\gamma]/\mathcal{B}[X(3823) \to \chi_{c1}\gamma] = 22\%$ is consistent with the observation $0.28^{+0.14}_{-0.11} \pm 0.02$, and the decay ratio $\mathcal{B}[X(3823) \to \chi_{c0}\gamma]/\mathcal{B}[X(3823) \to \chi_{c1}\gamma] \simeq 0.46\%$ is also less than experimental upper limit 0.24. In addition, we calculated the contributions of different partial waves. For the decays $X(3823) \to \eta_c(1S)\gamma$ and $X(3823) \to \chi_{c0}(1P)\gamma$, the main contribution comes from the relativistic effect, while for the $X(3823) \to \chi_{cJ}(1P)\gamma$ ($J = 1, 2$) decay, the non-relativistic contribution is the dominant one.