Light-cone distribution amplitudes of a light baryon in large-momentum effective theory

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Introduction

Light-cone distribution amplitudes (LCDAs) of a light baryon describe the momentum distributions of a quark/gluon in a baryonic system and are fundamental non-perturbative input in QCD factorization for an exclusive process with a large momentum transfer. An explicit example of this type is weak decays of bottom baryons which are valuable to extract the CKM matrix element in the standard model and to probe new physics beyond the standard model. In addition, in contrast with parton distribution functions that encode the probability density of parton momenta in hadrons, the LCDAs offer a probability amplitude description of the partonic structure of hadrons, from which one can potentially calculate various quark/gluon distributions. Thus the knowledge of LCDAs is also key to understanding the internal structure of light baryons, such as a proton.

Momentum distributions of quarks/gluons inside a light baryon in a hard exclusive process are encoded in LCDAs. In this work, we point out that the leading twist LCDAs of a light baryon can be obtained through a simulation of a quasi-distribution amplitude calculable on lattice QCD within the framework of the large-momentum effective theory. We calculate the one-loop perturbative contributions to LCDA and quasi-distribution amplitudes and explicitly demonstrate the factorization of quasi-distribution amplitudes at the one-loop level. Based on the perturbative results, we derive the matching kernel in the $\overline{\mathrm{MS}}$ scheme and regularization-invariant momentum-subtraction scheme. Our result provides a first step to obtaining the LCDA from first principle lattice QCD calculations in the future.

LCDAs for baryon Λ

The leading-twist LCDAs for baryon are made of three large components, and for Λ baryon one has the explicit form

$$\begin{aligned}&\Phi(x_1,x_2,\mu)f_\Lambda u_\Lambda(p) = \\ &\int\frac{dt_1p^+}{2\pi}\int\frac{dt_2p^+}{2\pi}e^{ix_1p^+t_1+ix_2p^+t_2} \\ &\times\epsilon_{ijk}\langle 0|U_i^T(t_1n)\Gamma D_j(t_2n)S_k(0)|\Lambda\rangle,\end{aligned} \quad (1)$$

where $\Gamma = C\gamma^5\not{n}$, $n^\mu = (1,0,0,-1)/\sqrt{2}$ and $\bar{n}^\mu = (1,0,0,1)/\sqrt{2}$, and $C = i\gamma^2\gamma^0$. f_Λ is the decay constant for Λ, and $u_\Lambda(p)$ is the Λ spinor. The light quark flight direction along to $\bar{n}$. Two pieces of gauge links are not shown in the above formulae

$$\mathcal{W}_{ij}(0,x) = \mathcal{P}\exp\left[ig_s\int_x^0 dtn_\mu A^\mu_{ij}(tn)\right]. \quad (2)$$

It is worthwhile pointing out that the above form of the Wilson line is not unique, but a gauge invariant building block, e.g. for a quark field with color i, is

$$Q_i(x) = \mathcal{W}_{ii'}(\infty,x)q_{i'}(x), \quad (3)$$

and the piece from 0 to ∞ is omitted in the above formulae since it is irrelevant to LCDA.

quasi-DA

The quasi-DA for the Λ is defined as

$$\begin{aligned}&\tilde{\Phi}(x_1,x_2,\mu)\tilde{f}_\Lambda u_\Lambda(p) = \\ &\int\frac{dt_1p^z}{2\pi}\int\frac{dt_2p^z}{2\pi}e^{ix_1p^zt_1+ix_1p^zt_2} \\ &\times\epsilon_{ijk}\langle 0|U_i^T(t_1n_z)\tilde{\Gamma}D_j(t_2n_z)S_k(0)|\Lambda\rangle,\end{aligned} \quad (4)$$

where $\tilde{f}_\Lambda$ is the quasi decay constant for Λ. $\tilde{\Gamma} = C\gamma^5\not{n}_\lambda$ ($\lambda = t$ or z) for quasi-DA. The two choices will give the same results at a leading twist. Here $n_t^\mu = (1,0,0,0)$ and $n_z^\mu = (0,0,0,-1)$.

Matching and Renormalization

1. Matching

In the large momentum $p^z \gg \Lambda_{\mathrm{QCD}}$ limit, the quasi-observables can be factorized as a convolution of a perturbatively calculable matching coefficient and the corresponding light-cone observable. The matching of quasi-DA and LCDA is given as

$$\tilde{\Phi}(x_1,x_2,\mu) = \int dy_1dy_2\mathcal{C}(x_1,x_2,y_1,y_2,\mu)\Phi(y_1,y_2,\mu) + \mathcal{O}\Big(\frac{1}{x_1p^z},\frac{1}{x_2p^z},\frac{1}{(1-x_1-x_2)p^z}\Big). \quad (5)$$

The matching kernel $\mathcal{C}$ can be extracted by perturbative theory within the large-momentum effective theory.

2. Matching Kernel

The short-distance coefficient is insensitive to the hadrons, i.e. the UV behavior of LCDAs is irrelevant to the low energy dynamics. In the calculation of LCDAs, one can replace the hadron with a partonic state with the same quantum numbers.

$$\begin{aligned}\phi S &= \int\frac{dt_1p^+}{2\pi}\int\frac{dt_2p^+}{2\pi}e^{ix_1p^+t_1+ix_1p^+t_2}\frac{\epsilon_{ijk}\epsilon_{abc}}{6} \\ &\langle 0|U_i^T(t_1n)\Gamma D_j(t_2n)S_k(0)|u_a(k_1)d_b(k_2)s_c(k_3)\rangle, \\ \tilde{\phi}\tilde{S} &= \int\frac{dt_1p^z}{2\pi}\int\frac{dt_2p^z}{2\pi}e^{ix_1p^zt_1+ix_1p^zt_2}\frac{\epsilon_{ijk}\epsilon_{abc}}{6} \\ &\langle 0|(U_i)^T(t_1n_z)\tilde{\Gamma}D_j(t_2n_z)S_k(0)|u_a(k_1)d_b(k_2)s_c(k_3)\rangle.\end{aligned}$$

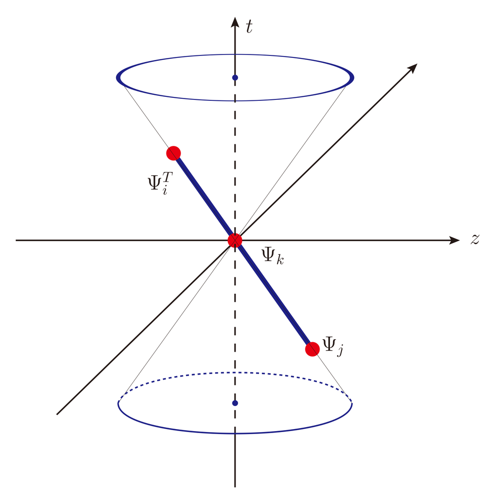

Figure 1: Operator structure for baryon LCDA.

The normalization factor S or $\tilde{S}$ can be constructed in terms of the partonic local operator matrix element respectively. Both ϕ and $\tilde{\phi}$ can be calculated by pQCD. In this work, we calculated those results at one-loop. Therefore the matching kernel can be extracted. The partonic LCDA (ϕ) and quasi-DA ($\tilde{\phi}$) satisfy the relationship in the matching formula above.

3. Renormalization:

A regularization invariant momentum subtraction method (RI/MOM) can avoid the use of lattice perturbation theory and allow a non-perturbative determination of the renormalization constants of many composite operators. We calculate the quasi-DA in the space-like $p^2 = -\rho(p^z)^2 < 0$ kinematics region. In order to eliminate this ultraviolet divergence and renormalize the lattice operators, we need a suitable renormalization of the quasi-DA $\tilde{\phi}(x_1,x_2,\mu)$. In the RI/MOM scheme, this is given as

$$\tilde{\phi}(x_1,x_2,\mu)_{\mathrm{RI/MOM}} = \tilde{\phi}(x_1,x_2,\mu)/\tilde{\phi}(x_1,x_2,\mu)_{\mathrm{OF}}.$$

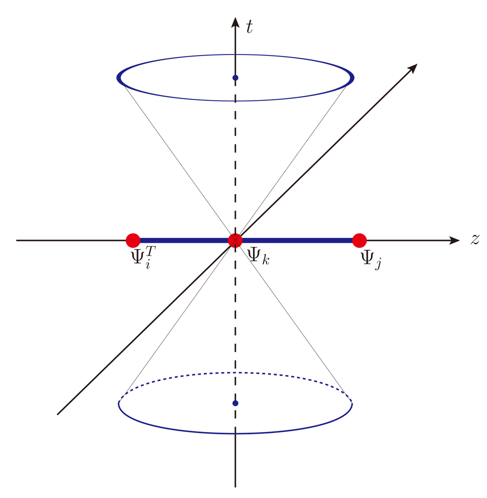

Figure 2: Operator structure for baryon quasi-DA.

In the RI/MOM scheme, the UV divergence in the quasi-DAs can be removed by the renormalization constant determined nonperturbatively.

Conclusions and Outlook

In this work, we have pointed out that LCDAs of a light baryon can be obtained through a simulation of a quasi-distribution amplitude calculable on lattice QCD under the framework of large-momentum effective theory. We have calculated the one-loop perturbative contributions to LCDA and quasi-distribution amplitudes and explicitly have demonstrated the factorization of quasi-distribution amplitudes at the one-loop level. A direct analysis using expansion by region also verifies the factorizability of quasi-DA. Based on the perturbative results, we have derived the matching kernel.

For the renormalization of quasi-distribution amplitudes, we have adopted the simplest procedure at this stage and subtracted the results with an off-shell parton state as a RI/MOM result. Our result provides a first step to obtaining the LCDA from first principle lattice QCD calculations in the future. An improved renormalization procedure might be performed in the self-renormalization or hybrid approach.