

Dark matter constraints from the observations of synchrotron emission

Ding Ran



Yuan Guan-Wen et al. arXiv:2106.05901
Yifan Chen et al. in preparation.

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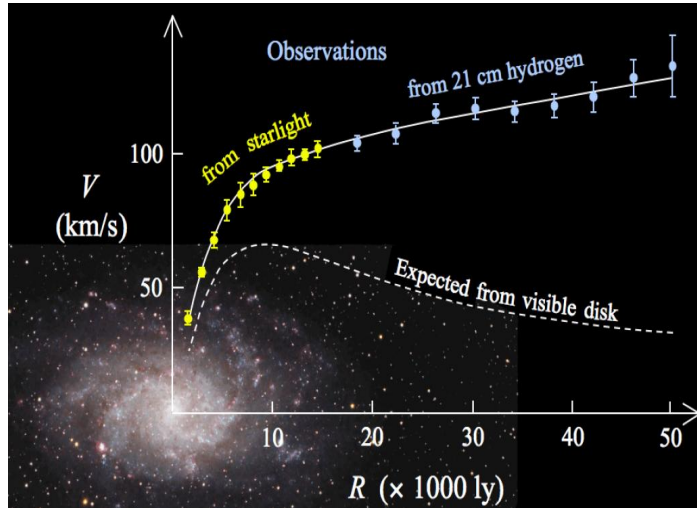
Outline

- Introduction to DM spike model
- Synchrotron emission around SMBH due to DM annihilation
- Results & Discussion

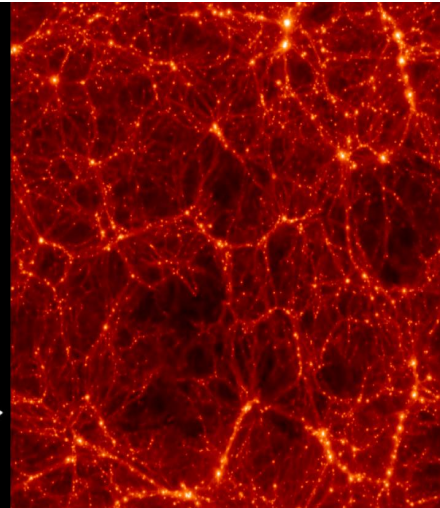
Introduction to DM spike model

● Evidences for DM

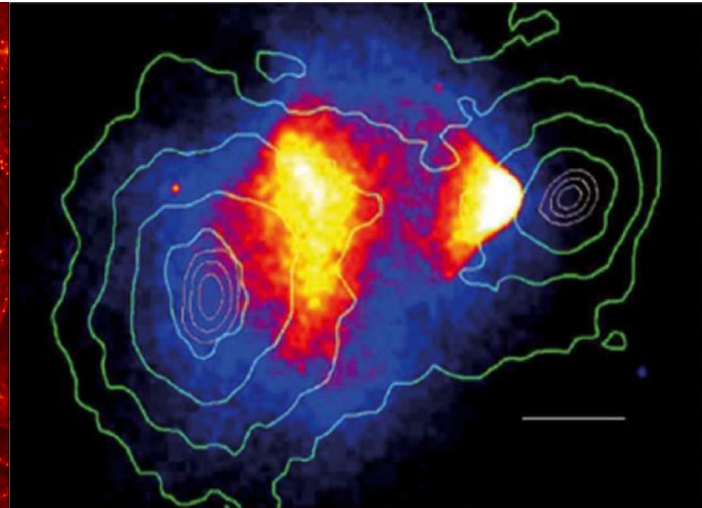
■ Rotation curves



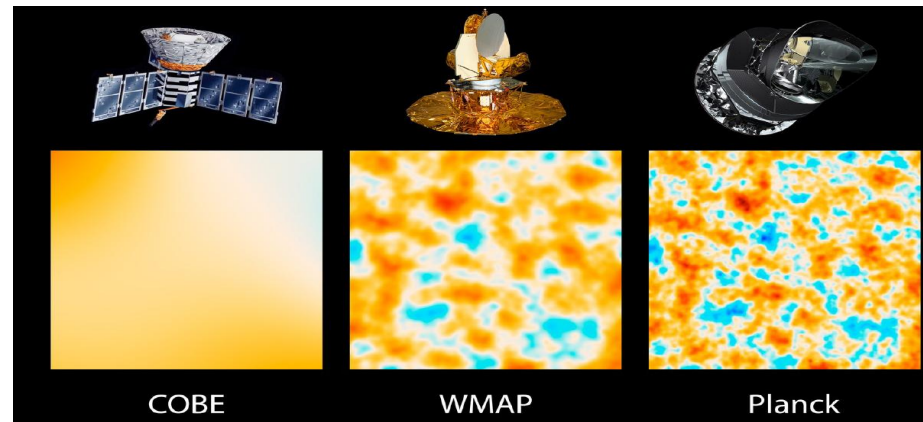
■ LSS



■ Bullet cluster



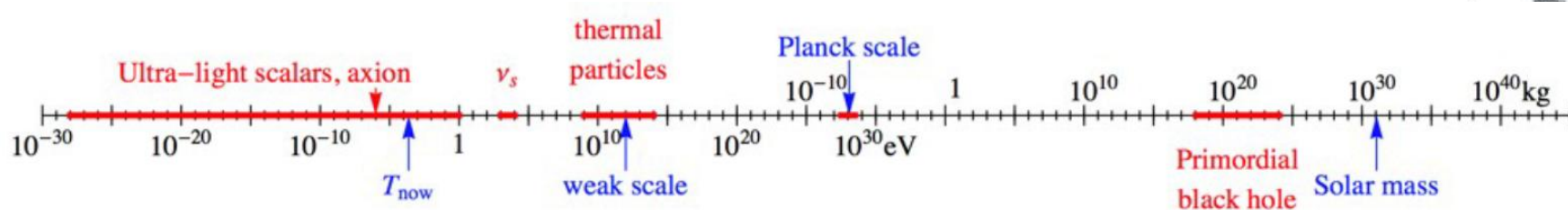
■ CMB



● DM candidates



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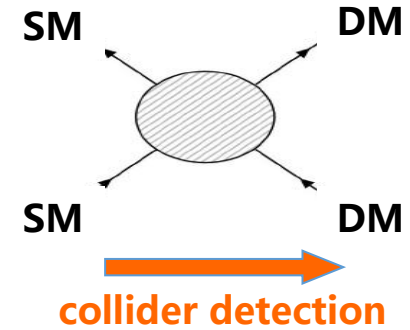
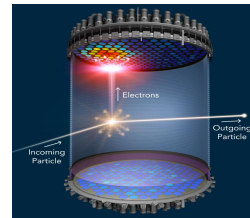
● WIMP detections

Cosmic-Ray (CR) physics



indirect detection

direct detection

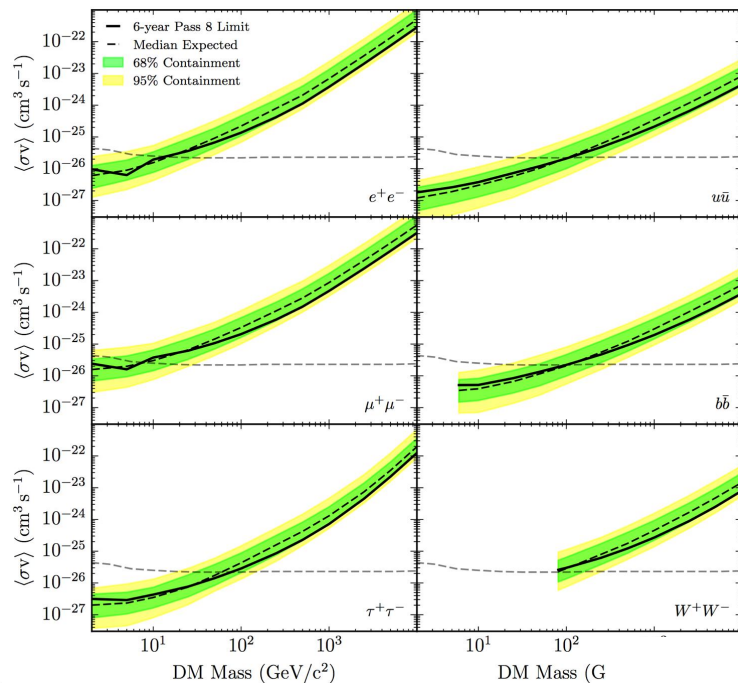
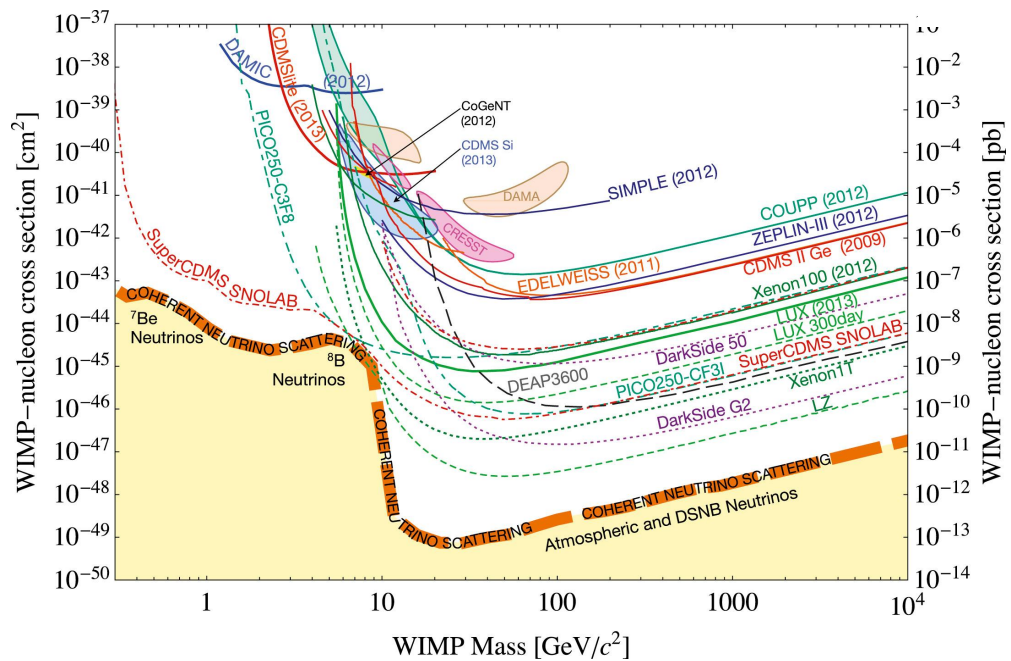


collider detection



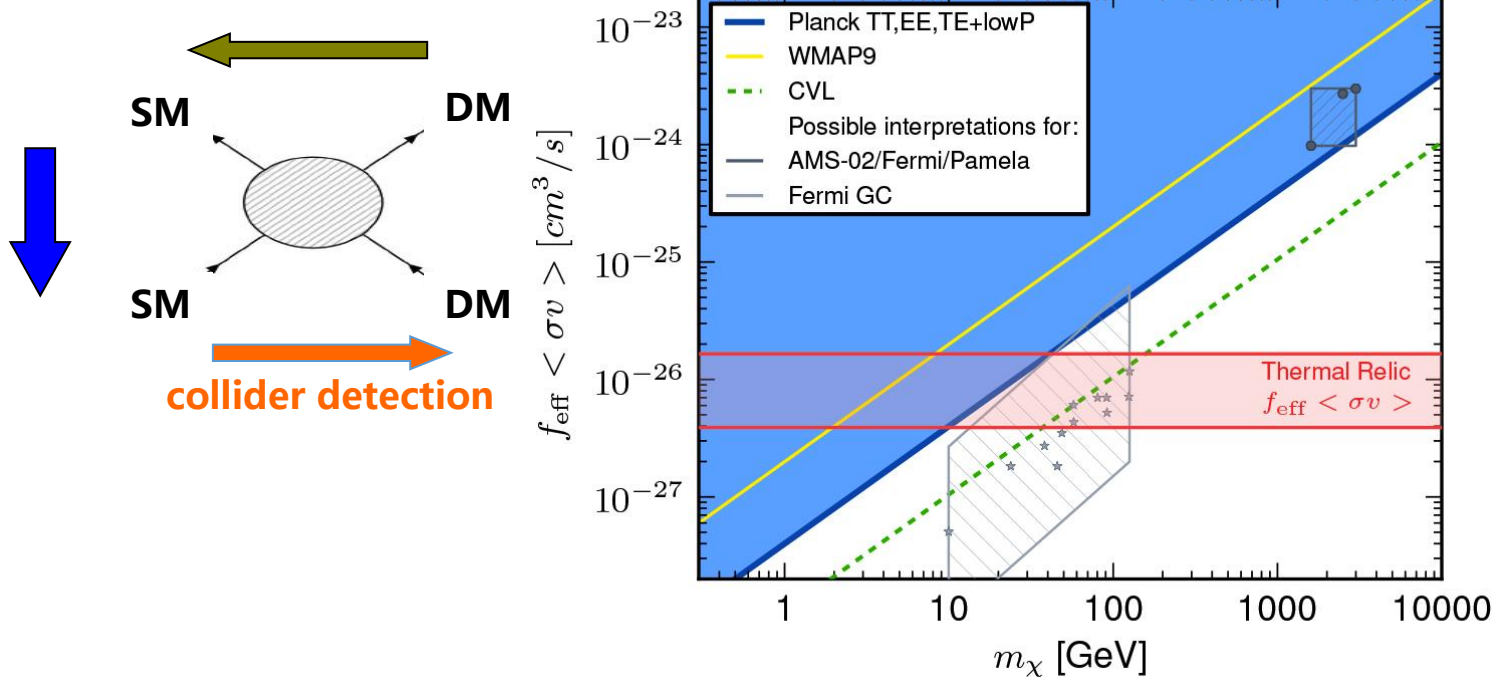
● WIMP Liimits

Direct detection



Indirect detection

CMB energy injection



● DM spike model

- Adiabatic growth of SMBH will significantly enhance the DM density and form a spike structure.

P. Dehnen, MNRAS 265,250-256 (1993)

G.D. Quinlan, Hernquist, S.sigurdsson , APJ 440;554-564250-256 (1995)

P. Gondolo & J. Silk,PRL 83(1999) 1719{1722, [astro-ph/9906391]

O. Y. Gnedin & J. R. Primack, PRL 93 (2004) 061302, [astro-ph/0308385]

P. Ullio, H. Zhao & M. Kamionkowski, PRD 64 (2001) 043504, [astro-ph/0101481]

R. Aloisio, P. Blasi & A. V. Olinto, JCAP 05 (2004) 007, [astro-ph/0402588]

T. Lacroix, M. Karami, A. E. Broderick, J. Silk & C. Boehm, PRD 96 (2017) 063008,[1611.01961]

- Scaling relation arguments

G.D. Quinlan, Hernquist, S.sigurdsson , APJ 440;554-564250-256 (1995)

Assume a power-law initial density profile $\rho_i \sim r^{-\gamma}$

Conservation of mass

$$\rho_i r_i^2 dr_i = \rho_f r_f^2 dr_f \Rightarrow r_i^{3-\gamma} \sim r_f^{3-\gamma_{\text{sp}}}$$

Conservation of angular momentum

$$r_i M_i(r) \simeq r_f M_{\text{BH}} \Rightarrow r_i^{4-\gamma} \sim r_f$$

$$\gamma_{\text{sp}} = \frac{9 - 2\gamma}{4 - \gamma}$$

- General phase space arguments

Differential energy distribution $N(E) = g(E)f(E)$

G.D. Quinlan, Hernquist, S.sigurdsson , APJ 440;554-564250-256 (1995)

Density of state

$$g_i(E_i) \sim \int_0^{\Phi_i^{-1}(E_i)} dr r^2 \sqrt{\Phi_i(r) - E_i} \sim E_i^{(8-\gamma)/2(2-\gamma)}$$

Distribution function

$$f_i(E_i) \sim [\Phi(0)_i - E_i]^{-n}$$

Invariance of radial action

$$E_i \sim E_f^{-(2-\gamma)/(4-\gamma)} \sim r^{(2-\gamma)/(4-\gamma)}$$

$$\rho_f(r) \sim r^{-2} N_f(E_f) \left(\frac{dE_f}{dr} \right) \sim r^{-4} N_i(E_i) \left(\frac{dE_i}{dE_f} \right) \sim r^{-\gamma_{sp}}$$

with $\gamma_{sp} = \frac{3}{2} + n \left(\frac{2-\gamma}{4-\gamma} \right)$

- Final density profile is determined by exact form of distribution function

P. Dehnen, MNRAS 265,250-256 (1993)

Conservation of mass

$$f(E) = \frac{1}{\sqrt{8\pi^2}} \left[\int_E^0 \frac{d^2\rho}{d\Phi^2} \frac{d\Phi}{\sqrt{\Phi - E}} - \frac{(d\rho/d\Phi)_{\Phi=0}}{\sqrt{-E}} \right]$$

$$f(E) = \frac{(3-\gamma)M}{2(2\pi^2GMa)^{3/2}} \int_0^E \frac{(1-y)^2 [\gamma + 2y + (4-\gamma)y^2]}{y^{4-\gamma} \sqrt{E-\Phi}} d\Phi \propto \begin{cases} [\Phi(0) - E]^{-1} & \gamma = 0 \\ [\Phi(0) - E]^{-(6-\gamma)/2(2-\gamma)} & 0 < \gamma < 2 \\ e^{2E} & \gamma = 2 \\ E^{(6-\gamma)/2(\gamma-2)} & 2 < \gamma < 3 \end{cases}$$

$$y = y(\Phi) = \begin{cases} [1 - (2-\gamma)\Phi]^{1/(2-\gamma)} & \gamma \neq 2 \\ e^{-\Phi} & \gamma = 2 \end{cases}$$

- Approximate analytical distribution function for NFW profile

$$f(\varepsilon) = \frac{3M_{\text{BH}}}{2(2\pi)^3 (r_0 GM_{\text{BH}})^{3/2}} \hat{f}(\hat{\varepsilon})$$

$$\hat{\varepsilon} = -\frac{2r_0}{GM_{\text{BH}}} E$$

$$\hat{f}(\hat{\varepsilon}) = \frac{1}{7 \cdot 2^{11/2} (8 - \hat{\varepsilon})(1 - \hat{\varepsilon})^2} \left[252 \frac{8 + \hat{\varepsilon}}{\sqrt{2(1 - \hat{\varepsilon})}} \arcsin \sqrt{\hat{\varepsilon}} + P_1(\hat{\varepsilon}) \sqrt{\frac{\hat{\varepsilon}}{2}} \right. \\ \left. + P_2(\hat{\varepsilon}) E_1 \left(-\frac{\hat{\varepsilon}}{8} \right) + P_3(\hat{\varepsilon}) K \left(-\frac{\hat{\varepsilon}}{8} \right) + 189(8 + \hat{\varepsilon}) \Pi \left(\hat{\varepsilon}, -\frac{\hat{\varepsilon}}{8} \right) \right]$$

complete elliptic integrals

- Final DM density profile

$$\rho_{\text{sp}}(r) = \frac{4\pi}{r^2} \int_{\Phi_f}^0 dE \int_{L_{\text{min}}}^{L_{\text{max}}} L dL \frac{f(E^i(E^f, L), L)}{\sqrt{2E - 2\Phi(r) - L^2/r^2}}$$

- Comparison of full numerical and approximate power-law

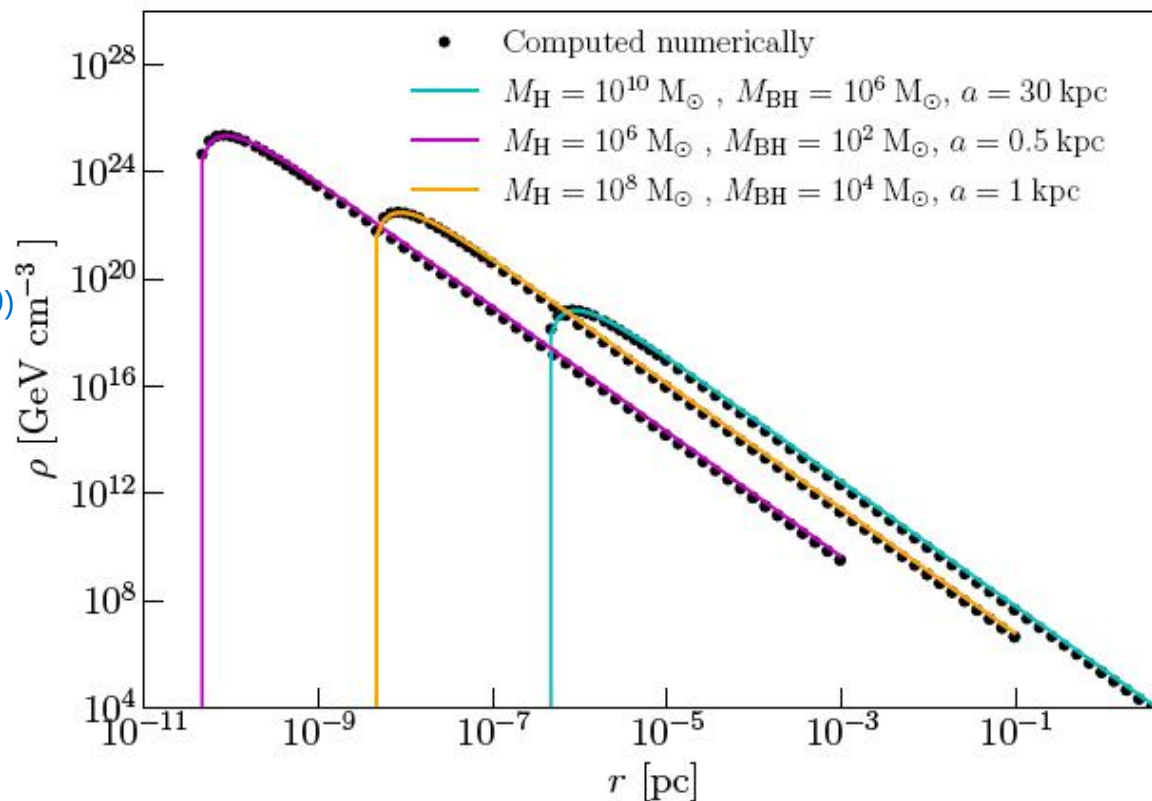
- Approximate power-law model

$$\rho_{\text{sp}}(r) = \rho_r g_\gamma(r) \left(\frac{R_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$

P. Gondolo & J. Silk, PRL 83(1999)

$$\rho_r = \rho_0 \left(\frac{R_{\text{sp}}}{r_0} \right)^{-\gamma}, \quad R_{\text{sp}} = \alpha_\gamma r_0 \left(\frac{M_{\text{BH}}}{\rho_0 r_0^3} \right)^{1/(3-\gamma)}$$

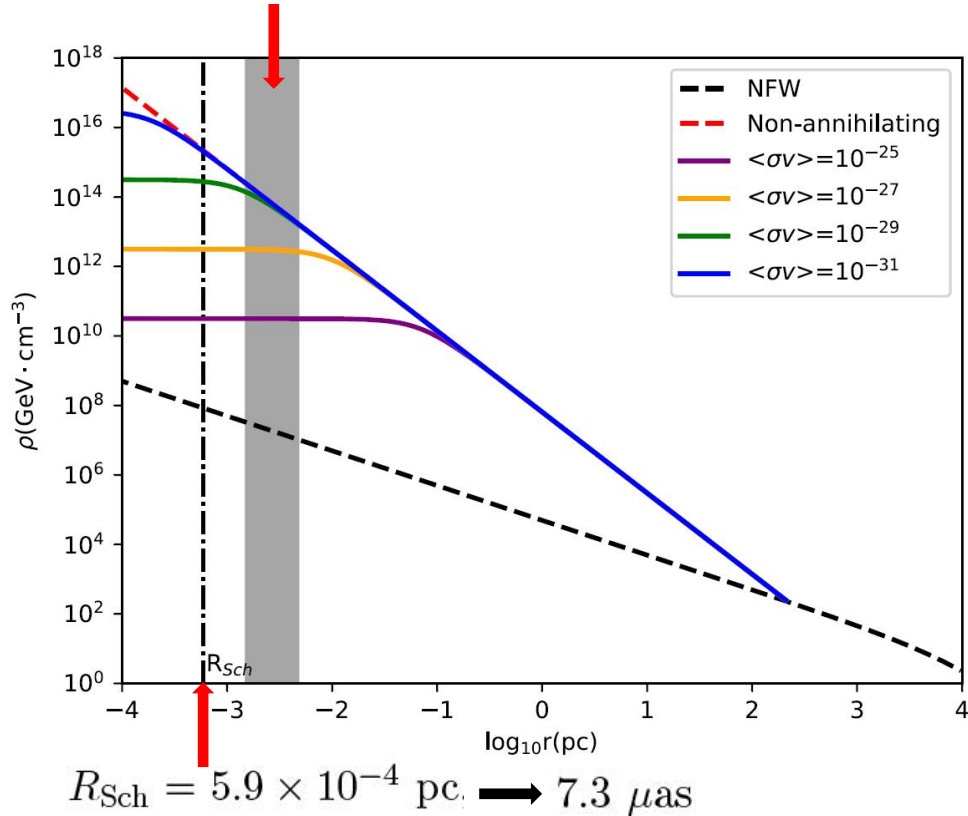
$$g_\gamma(r) = \left(1 - \frac{2R_s}{r} \right)^3, \quad \gamma_{\text{sp}} = \frac{9 - 2\gamma}{4 - \gamma}$$



- Spike with DM annihilation

T. Lacroix, M. Karami, A. E. Broderick, J. Silk & C. Boehm, PRD 96 (2017) 063008,[1611.01961]

EHT observational areas



- DM spike profile with annihilation

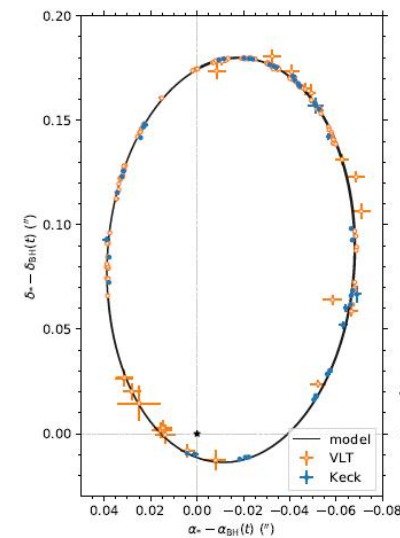
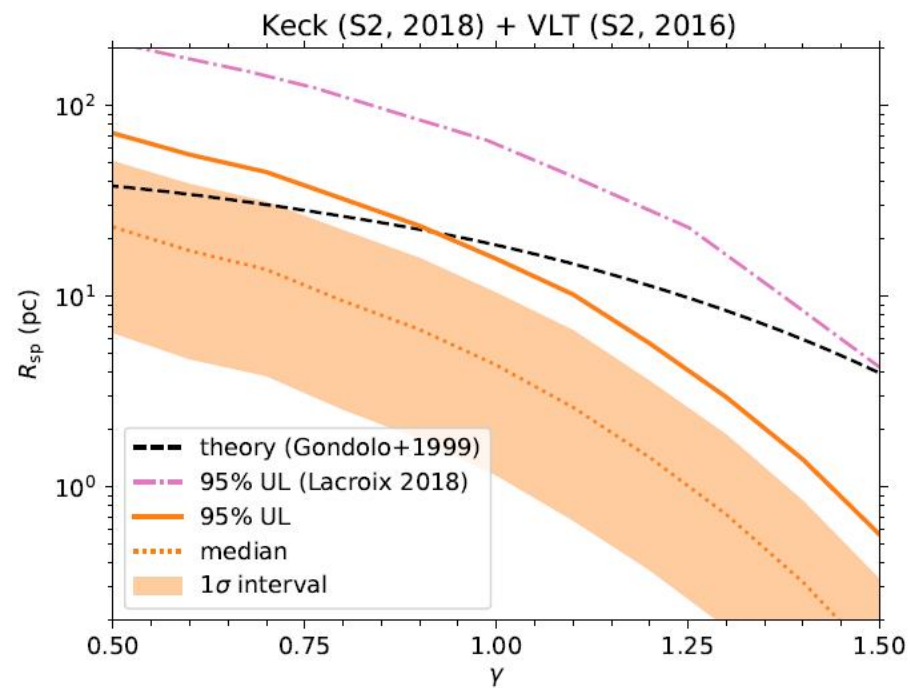
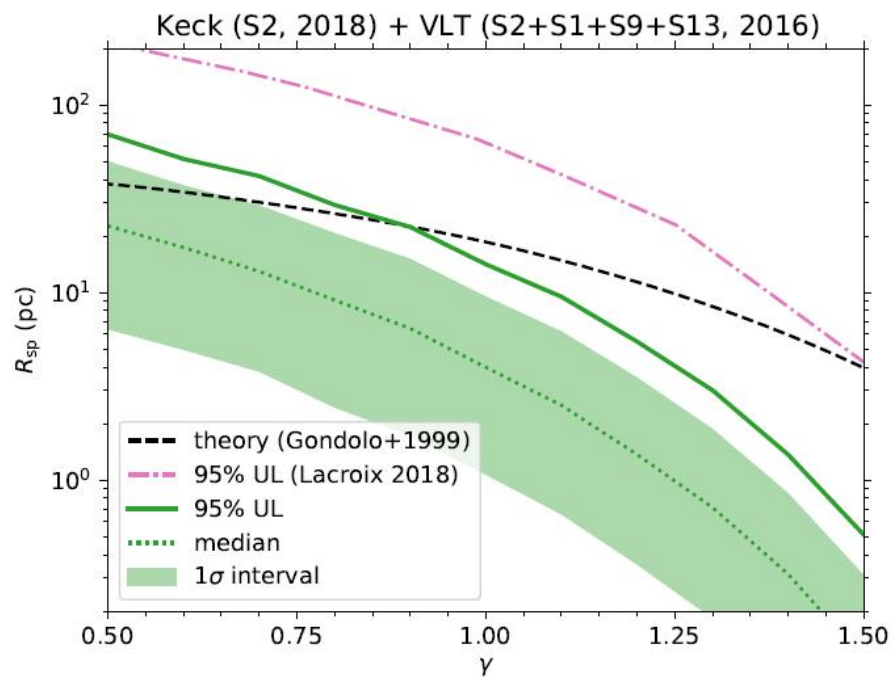
$$\rho_{\chi}(r) = \begin{cases} 0 & r < R_{Sch}, \\ \frac{\rho_{sp}(r)\rho_{sat}}{\rho_{sp}(r)+\rho_{sat}} & R_{Sch} \leq r < R_{sp}, \\ \rho_{NFW}(r) & r \geq R_{sp}. \end{cases} \quad \begin{array}{l} \text{spike radius} \\ R_{sp} \simeq 220 \text{ pc} \end{array}$$

- Saturate DM density

$$\rho_{sat} = m_{\chi} / \langle \sigma v \rangle t_{BH} \rightarrow \text{age of SMBH } t_{BH} = 10^9 \text{ yr}$$

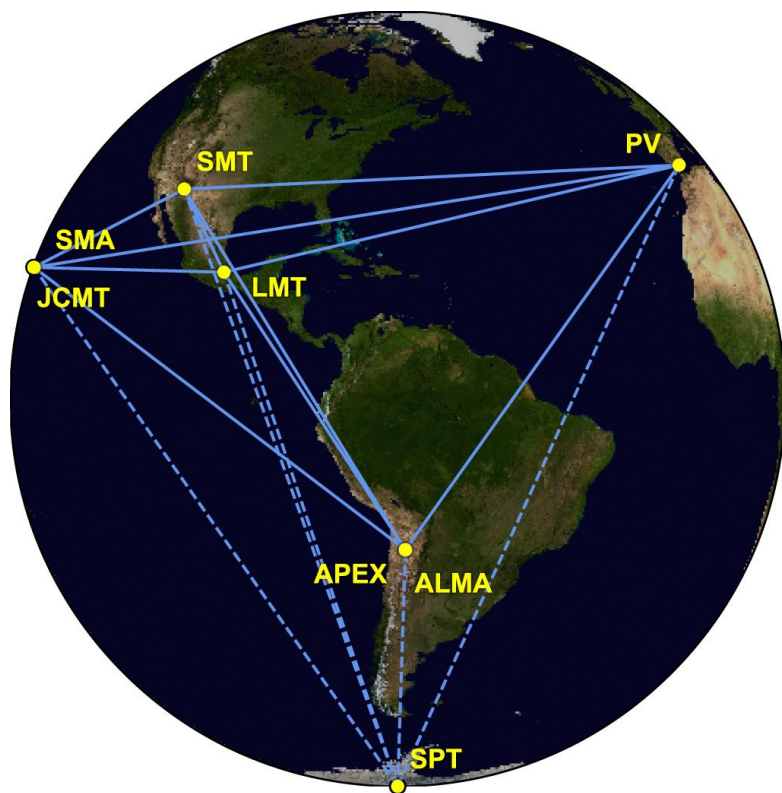
● Constraints DM spike at the GC from stellar orbits

Z-Q Shen et al., 2303.09284 [astro-ph]



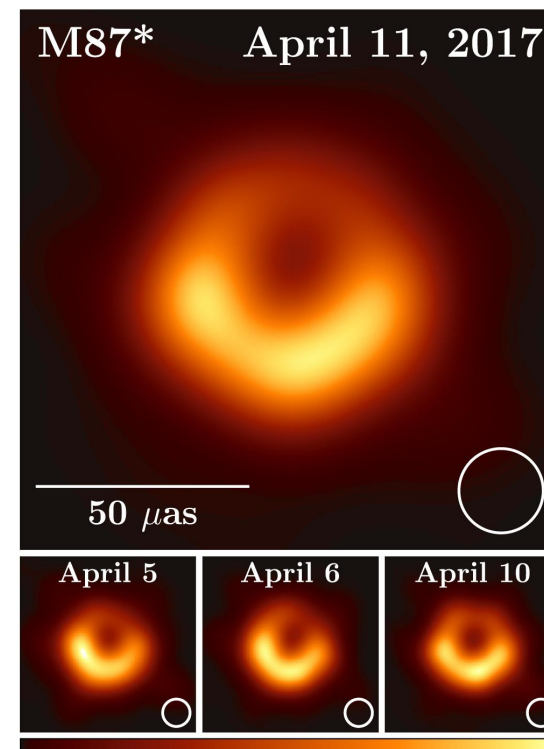
- The EHT project

- VLBI: Very Long Baseline Interferometry, an Earth-sized interferometer.
- EHT collaboration: focus on improving the capability of VLBI at short wavelengths.



- First image of a SMBH

EHT collaboration, *Astrophys. J. Lett.* 875 (2019) L1, [1906.11238]



Symbol	Value	Property
M	$6.2 \times 10^9 M_{\odot}$	Compact object mass
D	16.9 Mpc	Compact object distance
$\nu_{\text{obs},0}$	230 GHz	Observing frequency

- Synchrotron emission due to WIMP annihilations can be stringently constrained!

Calculation framework

R. Aloisio, P. Blasi & A. V. Olinto, JCAP 05 (2004) 007, [astro-ph/0402588]

T. Lacroix, M. Karami, A. E. Broderick, J. Silk & C. Boehm, PRD 96 (2017) 063008, [1611.01961]

$$S_{\text{syn}}(\nu) = \int d\Omega_{\text{obs}} \int_{l.o.s} dI_{\text{syn}}(\nu)$$

Specific intensity: radiative transfer equation

$$\frac{dI_{\text{syn}}(\nu, s)}{ds} = -\alpha(\nu, s)I_{\text{syn}}(\nu, s) + \frac{j_{\text{syn}}(\nu, s)}{4\pi}$$

relativistic Doppler effect and gravitational redshift

$$I_{\text{obs}}(\nu_{\text{obs}}) = \left(\frac{\nu_{\text{obs}}}{\nu_{\text{em}}}\right)^3 I_{\text{em}}(\nu_{\text{em}}) = g^3 I_{\text{em}}(\nu_{\text{em}})$$

emissivity

$$j_{\text{syn}}(\nu, r) = 2 \int_{m_e}^{M_\chi} dE \langle P_{\text{syn}} \rangle(\nu, E_e, B) n_e(r, E_e)$$

$$P_{\text{syn}}(\nu, E_e, B, \theta_p) = \frac{\sqrt{3}e^3 B \sin \theta_p}{m_e c^2} F(\nu/\nu_c)$$

$B \sim 1 - 30 \text{ G}$

power of synchrotron emission

electron and positron energy spectrum

$$n_e(r, E) = \frac{4\pi p^2}{c} f_e(r, p)$$

outside accretion radius : propagation equation

$$-\frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 D \frac{\partial f_e}{\partial r} \right] + v \frac{\partial f_e}{\partial r} - \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 v) p \frac{\partial f_e}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} (\dot{p} p^2 f_e) = q(r, p)$$

source function

$$q(r, p) = \frac{c}{4\pi p^2} Q(r, E) = \frac{c}{4\pi p^2} \frac{\langle \sigma v \rangle \rho_\chi^2(r)}{2m_\chi^2} \sum_i \text{BR}_i \frac{dN_{e^\pm}^{\text{inj}}}{dE}(E)$$

annihilation rate

injection spectrum

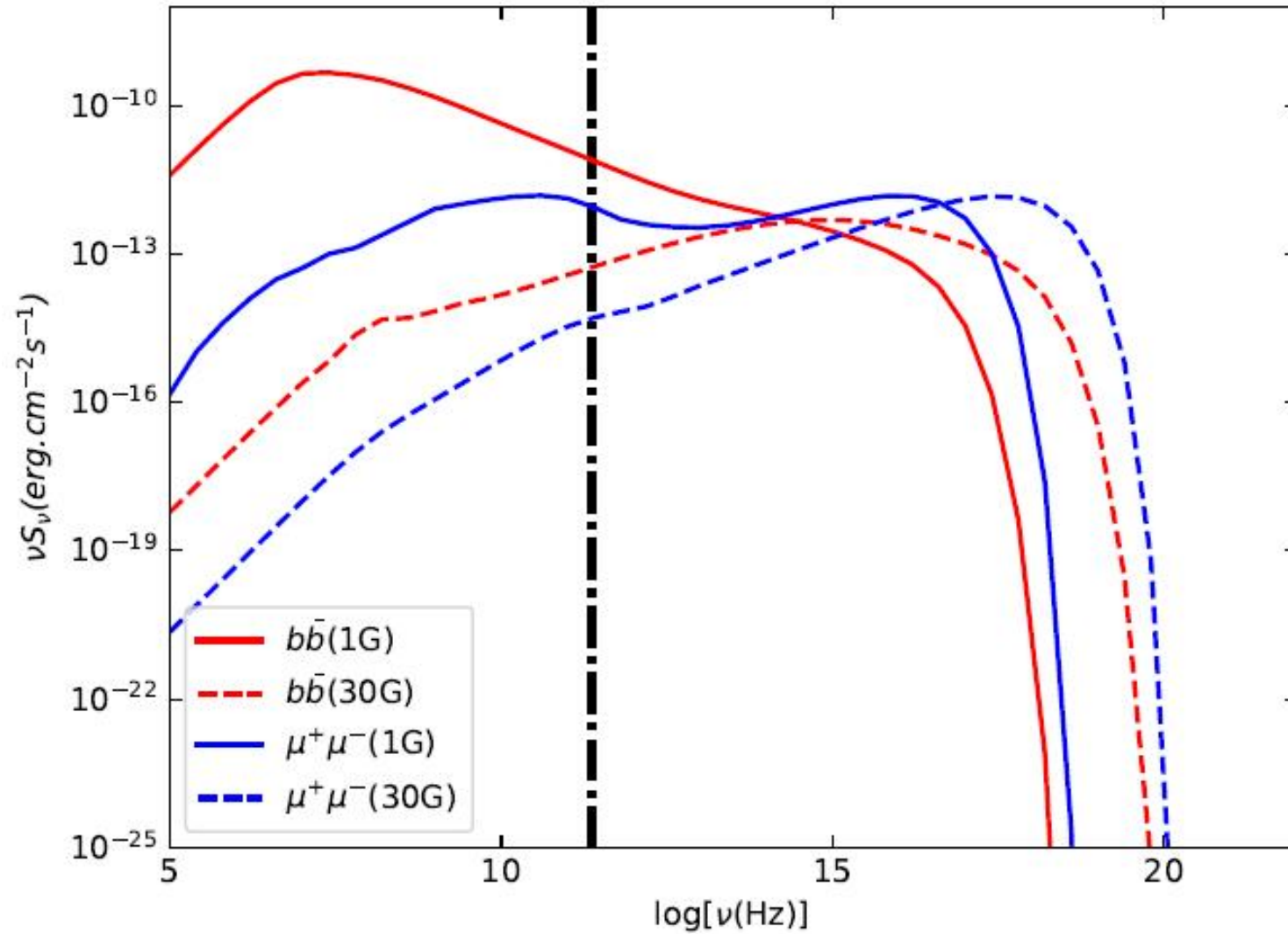
inside accretion radius: integral analytic solution

$$f_e(r, p) = \int_r^{r_{\text{acc}}} \frac{Q_i(R_{\text{inj}}, p_{\text{inj}})}{v(R_{\text{inj}})} \left(\frac{R_{\text{inj}}}{R_{\text{Sch}}}\right)^{\frac{5}{2}} \left(\frac{p_{\text{inj}}}{p}\right)^4 dR_{\text{inj}}$$

injection momentum

$$p_{\text{inj}}(R_{\text{inj}}; r, p) = p \left[\frac{k_0 R_{\text{Sch}}^{-\frac{1}{2}}}{c} R_{\text{inj}}^3 p \left(\frac{r}{R_{\text{inj}}} - 1\right) + \left(\frac{R_{\text{inj}}}{r}\right)^{\frac{1}{2}} \right]^{-1}$$

- Spectra of synchrotron emission from DM annihilation

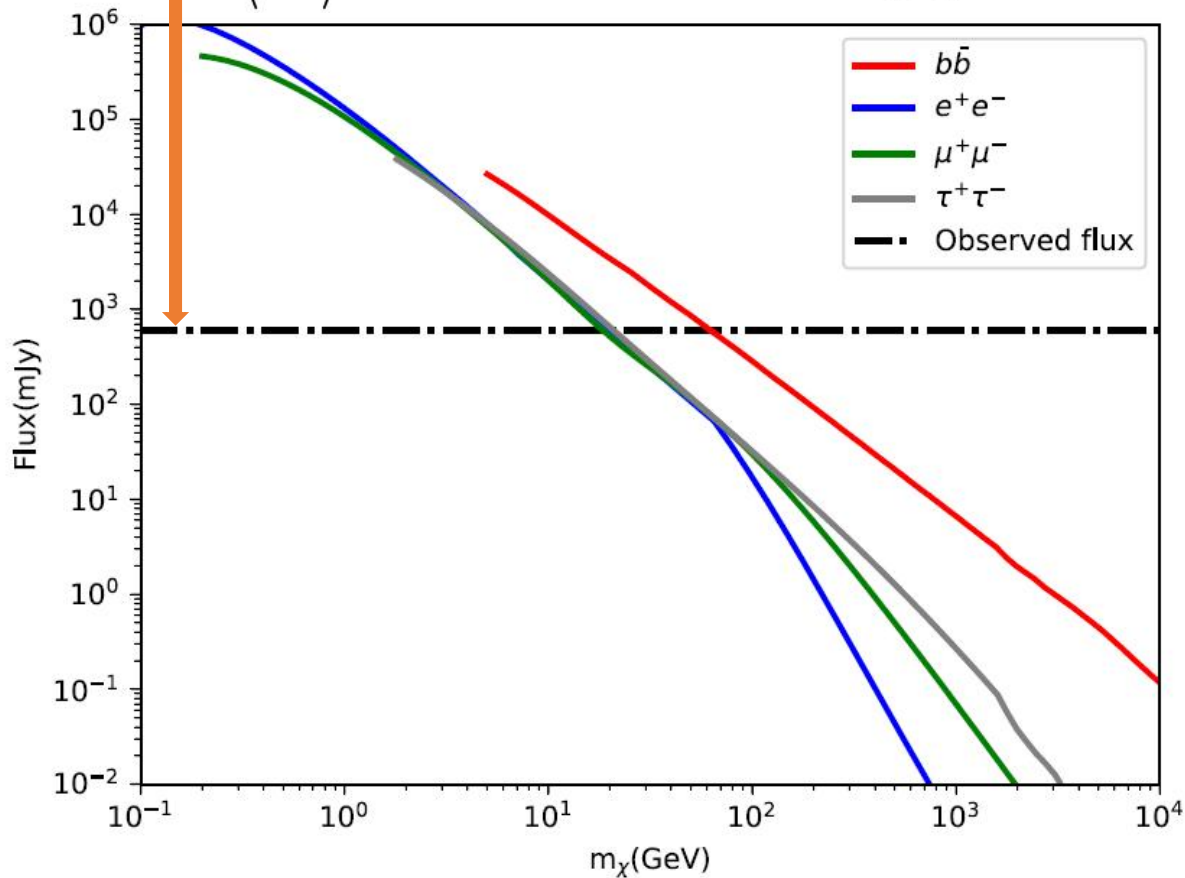


● Benchmark flux for four annihilation channels

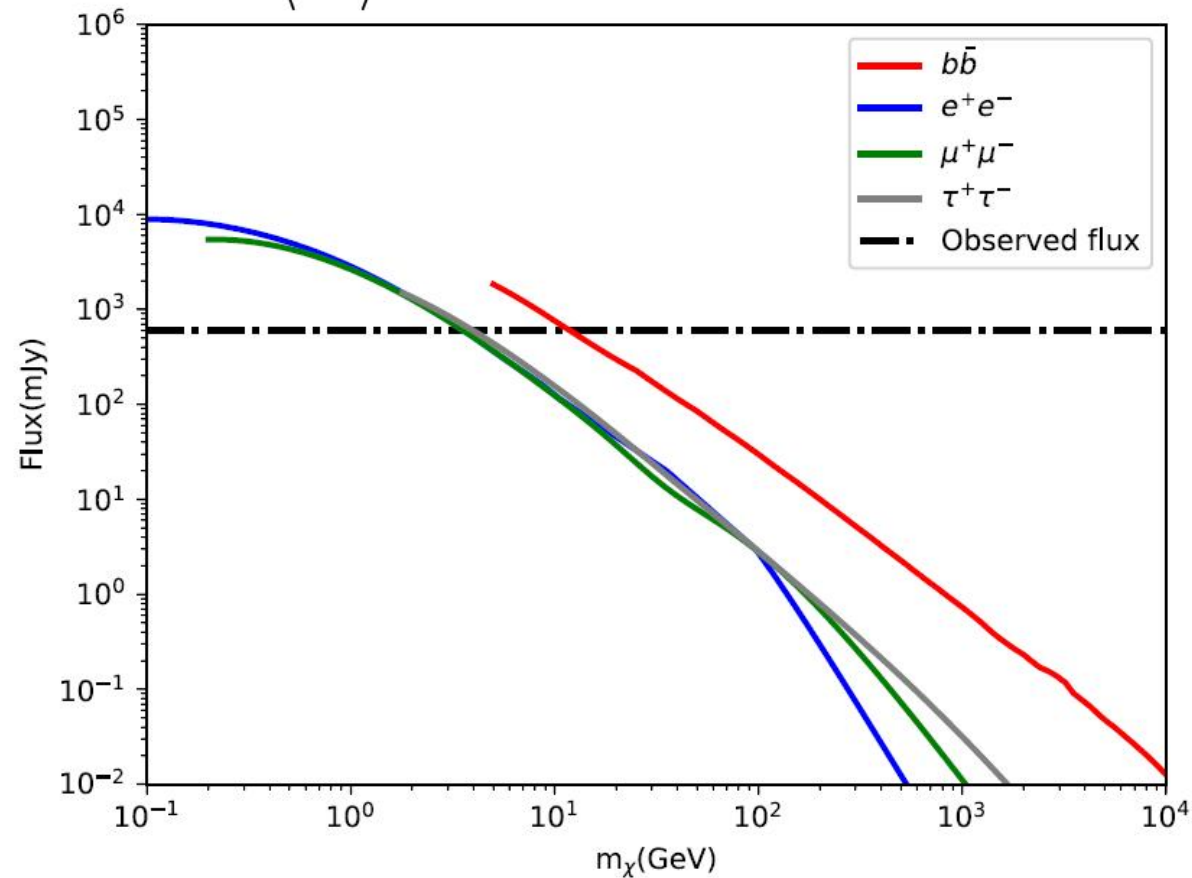
EHT collaboration, *Astrophys. J. Lett.* 910 (2021) L12, [2105.01169]

EHT collaboration, *Astrophys. J. Lett.* 910 (2021) L13, [2105.01173]

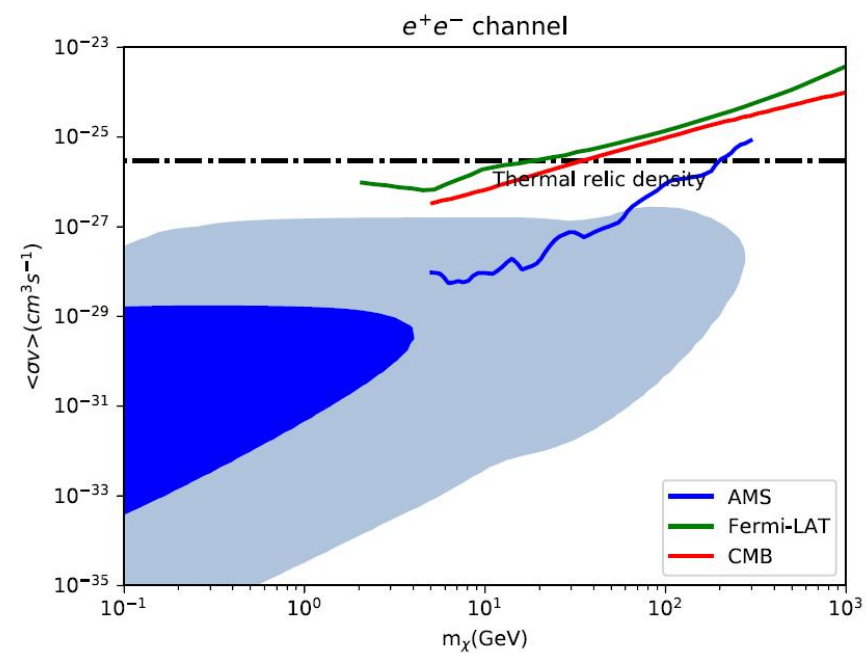
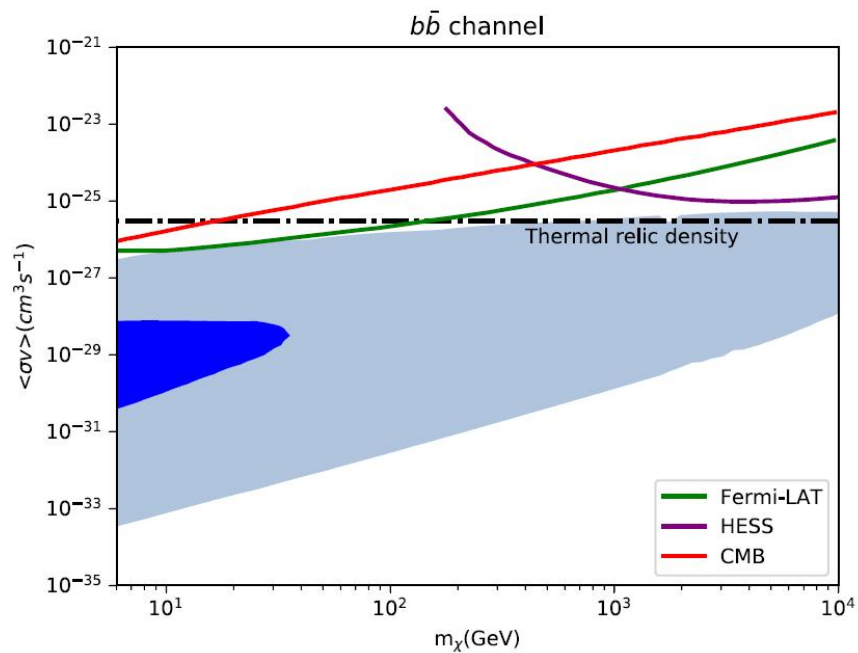
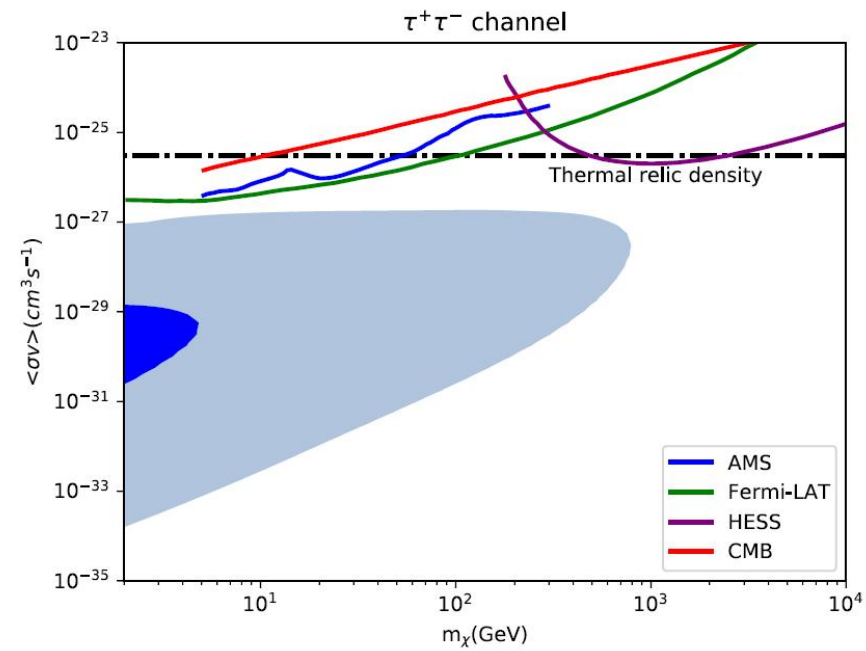
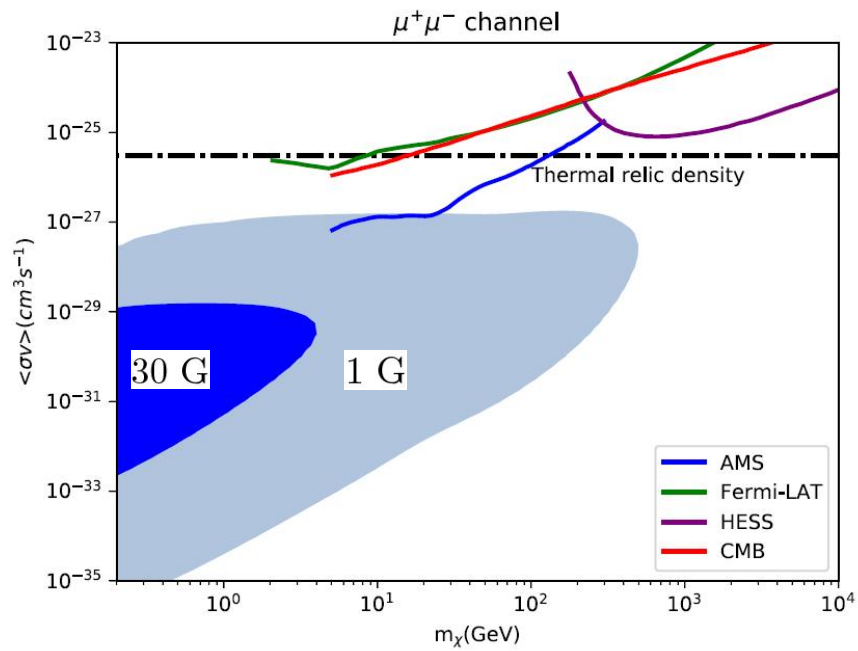
$\langle\sigma v\rangle = 10^{-32} \text{ cm}^3 \text{ s}^{-1}$ 1 G



$\langle\sigma v\rangle = 10^{-30} \text{ cm}^3 \text{ s}^{-1}$ 30 G



● Limits on WIMP annihilation cross sections



Future work for improvement

Yifan Chen et al. in preparation

- Realistic magnetic field model from GRMHD
- Characteristic electron trajectory configuration captured by the magnetic lines
- GR effect on electron injection momentum
- The effect of the accretion disk

Thanks for your attention

