$U(1)_{Y'}$ Universal Seesaw

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1. Motivation and Background

‡ The discovery of neutrino oscillations in the atmospheric, solar, accelerator and reactor neutrino experiments means three flavors of neutrinos should be massive and mixed (PDG 2022).

$$\text{Normal hierarchy}: \quad \Delta m^2_{32} = (2.424 \pm 0.003)10^{-3}\,\text{eV}^2\,, \quad \Delta m^2_{12} = 7.55^{+0.20}_{-0.16} \times 10^{-5}\,\text{eV}^2\,.$$

$$\text{Inverted hierarcht}: \quad \Delta m^2_{32} = -2.50^{+0.04}_{-0.03} \times 10^{-3}\,\text{eV}^2\,, \quad \Delta m^2_{12} = 7.55^{+0.20}_{-0.16} \times 10^{-5}\,\text{eV}^2\,.$$

‡ The kinematic search for neutrino mass in tritium decay sets an upper limit on the electron neutrino mass (KATRIN 2022).

$$m_{\nu_e} < 0.8\,\text{eV}\,.$$

‡ The cosmological observation requires a stringent bond on the summer of three neutrino masses (PDG 2022).

$$\sum m_\nu < 0.26\,\text{eV}\,.$$

We need extend the standard model to naturally generate the tiny neutrino masses!

‡ All particles should come in particle-antiparticle pairs.

‡ No primordial antimatter significantly exists in the present universe.

‡ An initial matter-antimatter asymmetry cannot survive after inflation.

The matter-antimatter asymmetry is as same as a baryon asymmetry, which has been precisely measured by the cosmological observations (PDG 2022),

$$\eta_B = \frac{n_B}{s} \sim 10^{-10}\,.$$

We need a dynamical baryogenesis mechanism!

If CPT (C – charge conjugation, P – parity, T – time reversal.) is invariant, any successful baryogenesis mechanisms should satisfy the Sakharov conditions (Sakharov '67):

‡ baryon number nonconservation,

‡ C and CP violation,

‡ departure from equilibrium.

$$\left.\begin{array}{l} B \xrightarrow{C} -B \ \text{ for } \ q_{L(R)} \xrightarrow{C} q^c_{L(R)} \\ B \xrightarrow{CP} -B \ \text{ for } \ q_L \xrightarrow{CP} q^c_R \end{array}\right\} \Longrightarrow n_B \equiv n_b - n_{\bar{b}} = \frac{1}{3}(n_{q_L} - n_{\bar{q}_L} + n_{q_R} - n_{\bar{q}_R}) \xrightarrow{C,CP} 0\,.$$

$$\langle B\rangle = \text{Tr}(e^{-\frac{H}{T}}B) = \text{Tr}[e^{-\frac{H}{T}}(CPT)^{-1}B(CPT)] = \text{Tr}[e^{-\frac{H}{T}}(-B)] = -\langle B\rangle \Rightarrow \langle B\rangle = 0\,.$$

Both of the baryon (B) and lepton (L) numbers are violated by quantum effects in the standard model ('t Hooft, '76.). The transition of the baryon and lepton numbers from one vacuum to the next vacuum is

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = N_f \frac{g_2^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\,(W^{\mu\nu}W^{\rho\sigma}) \Rightarrow \Delta B = \Delta L = N_f = 3\,, \;\; \Delta(B-L) = 0\,.$$

At zero temperature, the baryon and lepton number violating processes via a tunneling between the different vacua are highly suppressed and hence are unimportant today. However, such processes can have a sphaleron solution during the temperatures near and above the electroweak phase transition (Kuzmin, Rubakov, Shaposhnikov, '85.),

$$100\,\mathrm{GeV} < T < 10^{12}\,\mathrm{GeV}\,.$$

In the standard model, the sphaleron processes, the CKM phase and the electroweak phase transition can fulfill all of the three Sakharov conditions to realize an electroweak baryogenesis scenario.

Unfortunately, the baryon asymmetry induced by the electroweak baryogenesis in the standard model is too small to explain the observed value.

‡ The electroweak phase transition should be strongly first-order to avoid the washout of the induced baryon asymmetry. This requires the Higgs boson lighter than about $m_H < 40$ GeV, which is much lower than the experimental value $m_H = 125$ GeV.

‡ Even if the electroweak phase transition is strongly first-order, the induced baryon asymmetry can only arrive at the order of $\eta_B = \mathcal{O}(10^{-20})$.

We need extend the standard model to successfully generate the observed baryon asymmetry!

The extension of the standard model may be also motivated by other phenomena, such as the mass hierarchy among the charged fermion masses.

$$m_e \simeq 0.511\,\mathrm{MeV}\,, \qquad m_\mu \simeq 106\,\mathrm{MeV}\,, \qquad m_\tau \simeq 1.77 GeV\,,$$

$$m_d \simeq 4.67\,\mathrm{GeV}\,, \qquad m_s \simeq 93.4\,\mathrm{MeV}\,, \qquad m_b \simeq 4.18\,\mathrm{GeV}\,,$$

$$m_u \simeq 2.16\,\mathrm{GeV}\,, \qquad m_c \simeq 1.27\,\mathrm{GeV}\,, \qquad m_t \simeq 173\,\mathrm{GeV}\,.$$

2. Majorana seesaw and leptogenesis

Based on the standard model $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge groups, the leptogenesis (Fukugita, Yanagida, '86.) mechanism within the so-called type-I, II and III seesaw models (Minkowski, '77; Yanagida, '79; Gell-Mann, Ramond, Slansky, '79; Glashow, '80; Mohapatra, Senjanović, '80; Magg, Wetterich, '80; Schechter, Valle, '80; Cheng, Li, '80; Lazarides, Shafi, Wetterich, '81; Mohapatra, Senjanović, '81; Foot, Lew, He, '89.) or their combinations can simultaneously explain the small neutrino masses and the observed baryon asymmetry.

$$\text{Type-I}: \quad \mathcal{L} \supset -y_N \bar{l}_L \phi N_R - \frac{1}{2} M_N \bar{N}_R^c N_R + \text{H.c.}\,. \qquad N_R(1,1,0)$$

$$\text{Type-II}: \quad \mathcal{L} \supset -\frac{1}{2} f_\Delta \bar{l}_L^c i\tau_2 \Delta l_L - \mu \phi^T i\tau_2 \Delta \phi + \text{H.c.} - M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta)\,. \qquad \Delta(1,3,1) = \begin{bmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{bmatrix}$$

$$\text{Type-III}: \quad \mathcal{L} \supset -y_T \bar{l}_L^c i\tau_2 T_L \tilde{\phi} - \frac{1}{2} M_T \text{Tr}(i\tau_2 \bar{T}_L^c i\tau_2 T_L) + \text{H.c.}\,. \qquad T_L(1,3,0) = \begin{bmatrix} \frac{1}{\sqrt{2}}T_L^0 & T_L^+ \\ T_L^- & -\frac{1}{\sqrt{2}}T_L^0 \end{bmatrix}$$

$$l_L(1,2,-\tfrac{1}{2}) = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \qquad \phi(1,2,-\tfrac{1}{2}) = \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix}$$

In the most popular type-I seesaw model, the neutrino masses can be given by

$$\mathcal{L} \supset -\frac{1}{2} m_\nu \bar{\nu}_L \nu_L^c + \text{H.c. with}$$

$$m_\nu \simeq -y_N \frac{\langle\phi\rangle^2}{M_N} y_N^T = U\text{diag}\{m_{\nu_1}, m_{\nu_2}, m_{\nu_2}\}U^T = U\widehat{m}_\nu U^T\,.$$

In the base where the Majorana mass term of the right-handed neutrinos N_R are diagonal and real, we can define the heavy Majorana neutrinos, i.e.

$$M_N = \text{diag}\{M_{N_1}, M_{N_2}, M_{N_3}\}\,, \qquad N_i = N_{Ri} + N_{Ri}^c = N_i^c\,.$$

As long as the CP is not conserved, the out-of-equilibrium decays of the heavy Majorana neutrinos N_i can generate a lepton asymmetry η_L between the standard model leptons l_L and the antileptons l_L^c,

$$\eta_L \propto \varepsilon_{N_i} = \frac{\Gamma(N_i \to l_L \phi^*) - \Gamma(N_i \to l_L^c \phi)}{\Gamma(N_i \to l_L \phi^*) + \Gamma(N_i \to l_L^c \phi)}\,.$$

Due to the sphaleron processes, the produced lepton asymmetry η_L will be partially converted to a baryon asymmetry η_B,

$$\eta_B = C\eta_{B-L} = -C\eta_L\,.$$

The CP asymmetry ε_{N_i} can arrive at a nonzero value if and only if the Yukawa couplings y_N are complex,

$$\varepsilon_{N_i} = \frac{1}{8\pi}\sum_{j\neq i}\frac{\mathrm{Im}\{[(y_N^\dagger y_N)_{ij}]^2\}}{(y_N^\dagger y_N)_{ii}}\left[S\left(\frac{M_{N_j}^2}{M_{N_i}^2}\right)+V\left(\frac{M_{N_j}^2}{M_{N_i}^2}\right)\right] \quad \text{with}$$

$$S(x) = \frac{\sqrt{x}}{1-x} \quad \text{(self-energy correction (Flanz, Paschos, Sarkar, '95.))},$$

$$V(x) = \sqrt{x}\left[1-(1+x)\ln\left(\frac{1+x}{x}\right)\right] \quad \text{(vertex correction (Fukugita, Yanagida, '86.))}.$$

Note the CP phases in the Yukawa couplings y_N may be irrelevant to the CP phases in the neutrino mass matrix,

$$y_N = i\frac{\sqrt{M_N}}{\langle\phi\rangle}U\sqrt{\widehat{m}_\nu}O \ \text{ with } \ OO^T = O^T O = 1\,.$$

The orthogonal matrix O is arbitrary and its CP phases can provide the required CP violation even if the neutrino mass matrix doesn't contain any CP phases (Davidson, Ibarra, '01.).

The lepton-to-baryon conversation factor C can be calculated by analysing the chemical potentials. The excess of a particle i over its antiparticle can be described by the net number density (e.g. Kolb, Turner, *The Early Universe.*),

$$n_i = n_i^+ - n_i^- = \begin{cases} \frac{1}{6} g_i T^3 \left(\frac{\mu_i}{T}\right) & \text{for fermion}\,, \\ \frac{1}{3} g_i T^3 \left(\frac{\mu_i}{T}\right) & \text{for boson}\,, \end{cases}$$

$$n_B = \frac{1}{3}\sum_{i=1}^{3}(n_{q_i} + n_{u_i} + n_{d_i})\,,$$

$$n_L = \sum_{i=1}^{3}(n_{l_i} + n_{e_i})\,.$$

A $B-L$ asymmetry can be partially converted into a baryon asymmetry (Kuzmin, Rubakov, Shaposhnikov, '85.).

$$\left.\begin{array}{ll}\text{Yukawa}: & -\mu_q+\mu_u+\mu_\phi=0\,,\ \ -\mu_q+\mu_d-\mu_\phi=0\,,\ \ -\mu_l+\mu_e-\mu_\phi=0\,,\\[2ex] \text{Hypercharge}: & 3(\mu_q+2\mu_u-\mu_d-\mu_l-\mu_e)-2\mu_\phi=0\,,\\[2ex] \text{Sphalerons}: & 3(3\mu_q+\mu_l)=0\,.\end{array}\right\}$$

$$\Longrightarrow\left\{\begin{array}{l} n_B=\frac{1}{6}T^2\sum_{i=1}^{3}(2\mu_q+\mu_u+\mu_d)=\frac{2}{3}\mu_l T^2\\[3ex] n_L=\frac{1}{6}T^2\sum_{i=1}^{3}(2\mu_l+\mu_e)=-\frac{17}{14}\mu_l T^2\end{array}\right\}\Rightarrow n_B=C(n_B-n_L)\ \text{ with }\ C=\frac{28}{79}\,.$$

3. Dirac seesaw and neutrinogenesis

The theoretical assumption of the lepton number violation and then the Majorana neutrinos has not been confirmed by any experiments yet.

In analogy to the usual seesaw models for the Majorana neutrino mass generation, we can construct some Dirac seesaw models for the Dirac neutrino mass generation (Roncadelli, Wyler, '83; Roy, Shanker, '84; Murayama, Pierce, '01; PHG, He, '06; PHG, Sarkar, '07; PHG, '12; PHG, '15;...)

The interactions of Dirac seesaw can induce a lepton asymmetry stored in the standard model left-handed leptons and an opposite lepton asymmetry stored in the right-handed neutrinos although the total lepton number is exactly zero.

The right-handed neutrinos will go into equilibrium with the left-handed neutrinos at a very low temperature, where the sphalerons have already stopped working. Therefore, the sphalerons will never affect the right-handed neutrino asymmetry, but it can still transfer the standard model lepton asymmetry.

This type of leptogenesis is named as the neutrinogenesis mechanism (Dick, Lindner, Ratz, Wright, '99.) and has been studied in many literatures.

$$\text{Type-I}: \quad \mathcal{L} \supset -y_L \bar{l}_L \phi N_R - y_R \bar{N}_L \nu_R \xi - M_N \bar{N}_R N_L + \text{H.c. with}$$

$$N_{L,R}(1,1,0)\,.$$ (Roncadelli, Wyler, '83; Roy, Shanker, '84.)

$$\text{Type-II}: \quad \mathcal{L} \supset -\frac{1}{2} f_\eta \bar{l}_L \eta \nu_R - \mu \eta \xi \eta^\dagger \phi + \text{H.c.} - M_\eta^2 \eta^\dagger \eta \text{ with}$$

$$\eta(1,2,-\frac{1}{2}) = \begin{bmatrix} \eta^0 \\ \eta^- \end{bmatrix}.$$ (PHG, He, '06.)

$$\text{Type-III}: \quad \mathcal{L} \supset -y_L \bar{l}_L i\tau_2 \Psi'^c_L \xi - y_R \bar{\Psi}_L \phi \nu_R - M_\Psi \bar{\Psi}'^c_L i\tau_2 \Psi_L + \text{H.c. with}$$

$$\Psi_L(1,2,-\frac{1}{2}) = \begin{bmatrix} \Psi^0_L \\ \Psi^-_L \end{bmatrix}, \quad \Psi'_L(1,2,+\frac{1}{2}) = \begin{bmatrix} \Psi'^+_L \\ \Psi'^0_L \end{bmatrix}.$$ (Murayama, Pierce, '01.)

$$\mathcal{L} \supset -m_\nu^{I,II,III} \bar{\nu}_L \nu_R + \text{H.c. with}$$

$$m_\nu^I = -y_L \frac{\langle\phi\rangle\langle\xi\rangle}{M_N} y_R\,, \quad m_\nu^{II} = -f_\eta \frac{\mu\langle\phi\rangle\langle\xi\rangle}{M_\eta^2}\,, \quad m_\nu^{III} = -y_L \frac{\langle\phi\rangle\langle\xi\rangle}{M_\Psi} y_R\,.$$

In order to realize a Dirac seesaw, we need introduce certain symmetries to forbid the Yukawa couplings of the right-handed neutrinos to the standard model lepton and Higgs doublets, i.e.

$$\mathcal{L} \not\supset - y_\nu \bar{l}_L \phi \nu_R + \text{H.c.}\,.$$

Such symmetries could be a discrete symmetry, a global symmetry or a gauge symmetry and could be inspired by other interesting topics, such as the Peccei-Quinn symmetry (PHG, '15.), the mirror world (PHG, '12.), the gauged baryon and lepton numbers (Montero, Pleitez, '09; Ma, Srivastava, '15; PHG, '19.), the weakly interacting massive particle (PHG, Sarkar, 07; PHG, '19.) and so on.

4. Universal seesaw from $U(1)_{Y'}$ gauge symmetry

Needless to say, the neutrino mass generation is so different from the charged fermion mass generation in the framework of the standard model plus its seesaw extension.

This somewhat mysterious feature may be addressed in a universal seesaw scenario (Berezhiani, '83; Rajpoot, '87; Davidson, Wali'87) where not only the neutrinos but also the charged fermions acquire their masses through the seesaw mechanism. An early attempt was to consider the $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ left-right symmetric model (Pati, Salam, '74; Mohapatra, Pati, '75; Senjanović, Mohapatra, '75.) with minimal Higgs sector including a left-handed doublet and its right-handed partner (Berezhiani, '83; Rajpoot, '87; Davidson, Wali, '87).

In this unconventional left-right symmetric scenario, the standard model left-handed fermions are still placed in the $SU(2)_L$ doublets, while the standard model right-handed charged fermions and the right-handed neutrinos are placed in the $SU(2)_R$ doublets. Then the left-handed and right-handed fermion and Higgs doublets can construct some dimension-5 operators mediated by additionally heavy fermion singlets with appropriate electric charges.

$Fermion\ \&\ Higgs$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Y'}$
$q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}$	3	2	$+\frac{1}{6}$	$-\frac{1}{4}$
d_R	3	1	$-\frac{1}{3}$	$-\frac{3}{4}$
u_R	3	1	$+\frac{2}{3}$	$+\frac{1}{4}$
$l_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$	1	2	$-\frac{1}{2}$	$+\frac{3}{4}$
e_R	1	1	-1	$+\frac{1}{4}$
ν_R	1	1	0	$+\frac{5}{4}$
$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}$	1	1	$+\frac{1}{2}$	$+\frac{1}{2}$
ξ	1	1	0	$+\frac{5}{2}$

The standard model fermions and the right-handed neutrinos have the Yukawa couplings,

$$\mathcal{L} \supset -y_d \bar{q}_L \phi d_R - y_u \bar{q}_L \tilde{\phi} u_R - y_e \bar{l}_L \phi e_R - y_\nu \bar{l}_L \tilde{\phi} \nu_R - \frac{1}{2} f \xi \bar{\nu}_R \nu_R^c + \text{H.c.}\,.$$

After the new Higgs singlet ξ develops its vacuum expectation value to spontaneously break the $U(1)_{Y'}$ gauge symmetry, we can obtain the type-I seesaw. This is the most popular realization of the $U(1)_{Y'}$ gauge symmetry.

‡ Clearly, the Y' charge is defined by

$$Y' = Y - \frac{5}{4}(B - L)\,.$$

‡ However, the standard model Higgs doublet definitely need not obey the relation $Y' = Y - \frac{5}{4}(B - L)$ since only the standard model quarks and leptons have a specific definition of baryon and lepton number (Chen, PHG, '22.).

$Fermion\ \&\ Higgs$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Y'}$
$q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}$	3	2	$+\frac{1}{6}$	$-\frac{1}{4}$
d_R	3	1	$-\frac{1}{3}$	$-\frac{3}{4}$
u_R	3	1	$+\frac{2}{3}$	$+\frac{1}{4}$
$l_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$	1	2	$-\frac{1}{2}$	$+\frac{3}{4}$
e_R	1	1	-1	$+\frac{1}{4}$
ν_R	1	1	0	$+\frac{5}{4}$
$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}$	1	1	$+\frac{1}{2}$	0
ξ	1	1	0	$+\frac{1}{2}$

Do not assign any Y' charges to the standard model Higgs doublet.

Not only the right-handed neutrinos but also the standard model fermions can not have the dimension-4 Yukawa couplings at all.

The fermion doublets and singlets can construct the following dimension-5 operators with the standard model Higgs doublet and the new Higgs singlet (Chen, PHG, '22.),

$$\mathcal{L} \supset -\frac{c_d}{\Lambda_d}\bar{q}_L\phi d_R\xi - \frac{c_u}{\Lambda_u}\bar{q}_L\tilde{\phi}u_R\xi^* - \frac{c_e}{\Lambda_e}\bar{l}_L\phi e_R\xi - \frac{c_\nu}{\Lambda_\nu}\bar{l}_L\tilde{\phi}\nu_R\xi^* + \text{H.c.}\,.$$

The famous Weinberg dimension-5 operator for generating the Majorana neutrino masses should be absent from the present effective theory,

$$\mathcal{L} \not\supset -\frac{c'_\nu}{2\Lambda'_\nu}\bar{l}_L\tilde{\phi}\tilde{\phi}^T l^c_L + \text{H.c.} \;\Rightarrow\; m'_\nu = \frac{c'_\nu v_\phi^2}{2\Lambda_\nu}\,.$$

When the new and electroweak symmetries are broken spontaneously, the Higgs singlet ξ and the Higgs doublet ϕ can be expressed by

$$\xi = \frac{1}{\sqrt{2}}\left(v_\xi + h_\xi\right) \, , \;\; \phi = \frac{1}{\sqrt{2}}\left[\begin{array}{c} 0 \\ v_\phi + h_\phi \end{array}\right] .$$

The fermions then can acquire their Dirac masses,

$$m_d = \frac{c_d v_\xi v_\phi}{2\Lambda_d} \, , \;\; m_u = \frac{c_u v_\xi v_\phi}{2\Lambda_u} \, , \;\; m_e = \frac{c_e v_\xi v_\phi}{2\Lambda_e} \, , \;\; m_\nu = \frac{c_\nu v_\xi v_\phi}{2\Lambda_\nu} \, .$$

Once the new vacuum expectation value v_ξ is fixed, the hierarchy among the above fermion masses can be elegantly understood by choosing either large cutoff $\Lambda_{d,u,e,\nu}$ or small couplings $c_{d,u,e,\nu}$.

Fermion singlets	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Y'}$
$D_{L,R}$	3	1	$-\frac{1}{3}$	$-\frac{1}{4}$
$U_{L,R}$	3	1	$+\frac{2}{3}$	$-\frac{1}{4}$
$E_{L,R}$	1	1	-1	$+\frac{3}{4}$
$N_{L,R}$	1	1	0	$+\frac{3}{4}$

$$\begin{aligned}\mathcal{L} \supset\ & -y_D\bar{q}_L\phi D_R - f_D\bar{D}_L d_R\xi - \widehat{M}_D\bar{D}_L D_R \\ & -y_U\bar{q}_L\tilde{\phi}U_R - f_U\bar{U}_L u_R\xi^* - \widehat{M}_U\bar{U}_L U_R \\ & -y_E\bar{l}_L\phi E_R - f_E\bar{E}_L e_R\xi - \widehat{M}_E\bar{E}_L E_R \\ & -y_N\bar{l}_L\tilde{\phi}N_R - f_N\bar{N}_L\nu_R\xi^* - \widehat{M}_N\bar{N}_L N_R + \text{H.c.}\,.\end{aligned}$$

‡ The electric neutral fermion singlets $N_{L,R}$ are forbidden to have any Majorana masses because of their Y' charge. This is different from the left-right symmetric models.

If the vector-like fermion singlets are heavy enough, they can be integrated out and then can mediate the dimension-5 operators,

$$\mathcal{L} \supset \left(y_D \frac{1}{\widehat{M}_D} f_D\right) \bar{q}_L \phi d_R \xi + \left(y_U \frac{1}{\widehat{M}_U} f_U\right) \bar{q}_L \tilde{\phi} u_R \xi^* + \left(y_E \frac{1}{\widehat{M}_E} f_E\right) \bar{l}_L \phi e_R \xi + \left(y_N \frac{1}{\widehat{M}_N} f_N\right) \bar{l}_L \tilde{\phi} \nu_R \xi^* + \text{H.c.}\,.$$

Accordingly, the dimension-4 Yukawa couplings are determined by

$$y_d = -y_D \frac{v_\xi}{\sqrt{2}\widehat{M}_D} f_D\,,\ \ y_u = -y_U \frac{v_\xi}{\sqrt{2}\widehat{M}_U} f_U\,,\ \ y_e = -y_E \frac{v_\xi}{\sqrt{2}\widehat{M}_E} f_E\,,\ \ y_\nu = -y_N \frac{v_\xi}{\sqrt{2}\widehat{M}_N} f_N\,.$$

These Yukawa couplings can be suppressed either by the products of two Yukawa couplings or by the ratio between vacuum expectation value and mass. As a result, the Yukawa coupling parameters can have ”more natural” values compared to their values in the standard model. This is most noticeable for the Dirac neutrinos, whereas due to seesaw property, which have $y_\nu \sim 10^{-12}$ we need $y_N \sim f_N \sim 10^{-6}$ for $\widehat{M}_N \sim v_\xi$.

Scalar doublets	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Y'}$
$\eta = \begin{bmatrix} \eta^+ \\ \eta^0 \end{bmatrix}$	1	2	$+\frac{1}{2}$	$+\frac{1}{2}$

$$\mathcal{L} \supset -\widehat{M}_\eta^2 \eta^\dagger \eta - \mu_\eta \xi \eta^\dagger \phi - f_d \bar{q}_L \eta d_R - f_u \bar{q}_L \tilde{\eta} u_R - f_e \bar{l}_L \eta e_R - f_\nu \bar{l}_L \tilde{\eta} \nu_R + \text{H.c.}\,.$$

$$\begin{aligned}\mathcal{L} \supset & \left(f_d \frac{1}{\widehat{M}_\eta^2} \mu_\eta\right) \bar{q}_L \phi d_R \xi + \left(f_u \frac{1}{\widehat{M}_\eta^2} \mu_\eta\right) \bar{q}_L \tilde{\phi} u_R \xi^* + \left(f_e \frac{1}{\widehat{M}_\eta^2} \mu_\eta\right) \bar{l}_L \phi e_R \xi \\ & + \left(f_\nu \frac{1}{\widehat{M}_\eta^2} \mu_\eta\right) \bar{l}_L \tilde{\phi} \nu_R \xi^* + \text{H.c.}\,.\end{aligned}$$

$$y_d = -f_d \frac{v_\xi}{\sqrt{2}\widehat{M}_\eta^2} \mu_\eta\,, \;\; y_u = -f_u \frac{v_\xi}{\sqrt{2}\widehat{M}_\eta^2} \mu_\eta\,, \;\; y_e = -f_e \frac{v_\xi}{\sqrt{2}\widehat{M}_\eta^2} \mu_\eta\,, \;\; y_\nu = -f_\nu \frac{v_\xi}{\sqrt{2}\widehat{M}_\eta^2} \mu_\eta\,.$$

It is straightforward to see

$$y_d : y_u : y_e : y_\nu = f_d : f_u : f_e : f_\nu \,.$$

This means we should take $f_\nu \ll f_{d,u,e}$ for $m_\nu \ll m_{d,u,e}$.

This definitely is not a natural explanation for the large hierarchy between the charged fermions and the neutral neutrinos.

The heavy scalar doublets can not help us to realize a universal seesaw.

Fermion doublets	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Y'}$
$\Omega_L = \begin{bmatrix} \Omega_L^{+\frac{2}{3}} \\ \Omega_L^{-\frac{1}{3}} \end{bmatrix}$	3	2	$+\frac{1}{6}$	$-\frac{3}{4}$
$\Omega_R = \begin{bmatrix} \Omega_R^{+\frac{2}{3}} \\ \Omega_R^{-\frac{1}{3}} \end{bmatrix} = i\tau_2 \Omega_L'^c$	3	2	$+\frac{1}{6}$	$-\frac{3}{4}$
$\Psi_L = \begin{bmatrix} \Psi_L^{+\frac{2}{3}} \\ \Psi_L^{-\frac{1}{3}} \end{bmatrix}$	3	2	$+\frac{1}{6}$	$+\frac{1}{4}$
$\Psi_R = \begin{bmatrix} \Psi_R^{+\frac{2}{3}} \\ \Psi_R^{-\frac{1}{3}} \end{bmatrix} = i\tau_2 \Psi_L'^c$	3	2	$+\frac{1}{6}$	$+\frac{1}{4}$
$\Sigma_L = \begin{bmatrix} \Sigma_L^0 \\ \Sigma_L^- \end{bmatrix}$	1	2	$-\frac{1}{2}$	$+\frac{1}{4}$
$\Sigma_R = \begin{bmatrix} \Sigma_R^0 \\ \Sigma_R^- \end{bmatrix} = i\tau_2 \Sigma_L'^c$	1	2	$-\frac{1}{2}$	$+\frac{1}{4}$
$\Delta_L = \begin{bmatrix} \Delta_L^0 \\ \Delta_L^- \end{bmatrix}$	1	2	$-\frac{1}{2}$	$+\frac{5}{4}$
$\Delta_R = \begin{bmatrix} \Delta_R^0 \\ \Delta_R^- \end{bmatrix} = i\tau_2 \Delta_L'^c$	1	2	$-\frac{1}{2}$	$+\frac{5}{4}$

$$
\begin{aligned}
\mathcal{L} \supset\ & -y_\Omega \bar{q}_L \Omega_R \xi - f_\Omega \bar{\Omega}_L \phi d_R - \widehat{M}_\Omega \bar{\Omega}_L \Omega_R \\
& -y_\Psi \bar{q}_L \Psi_R \xi^* - f_\Psi \bar{\Psi}_L \tilde{\phi} u_R - \widehat{M}_\Omega \bar{\Psi}_L \Psi_R \\
& -y_\Sigma \bar{l}_L \Sigma_R \xi - f_\Sigma \bar{\Sigma}_L \phi e_R - \widehat{M}_\Sigma \bar{\Sigma}_L \Sigma_R \\
& -y_\triangle \bar{l}_L \triangle_R \xi^* - f_\triangle \bar{\triangle}_L \tilde{\phi} \nu_R - \widehat{M}_\triangle \bar{\triangle}_L \triangle_R + \text{H.c.}\,.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L} \supset\ & \left(y_\Omega \frac{1}{\widehat{M}_\Omega} f_\Omega\right) \bar{q}_L \phi d_R \xi + \left(y_\Psi \frac{1}{\widehat{M}_\Psi} f_\Psi\right) \bar{q}_L \tilde{\phi} u_R \xi^* + \left(y_\Sigma \frac{1}{\widehat{M}_\Sigma} f_\Sigma\right) \bar{l}_L \phi e_R \xi \\
& + \left(y_\triangle \frac{1}{\widehat{M}_\triangle} f_\triangle\right) \bar{l}_L \tilde{\phi} \nu_R \xi^* + \text{H.c.}\,.
\end{aligned}
$$

$$
y_d = -y_\Omega \frac{v_\xi}{\sqrt{2}\widehat{M}_\Omega} f_\Omega\,, \;\; y_u = -y_\Psi \frac{v_\xi}{\sqrt{2}\widehat{M}_\Psi} f_\Psi\,, \;\; y_e = -y_\Sigma \frac{v_\xi}{\sqrt{2}\widehat{M}_\Sigma} f_\Sigma\,, \;\; y_\nu = -y_\triangle \frac{v_\xi}{\sqrt{2}\widehat{M}_\triangle} f_\triangle\,.
$$

We should keep in mind that the universal seesaw probably is not suitable for the top quark. This is because the seesaw condition would result in a too small mass to the top quark, i.e. $\hat{M}_U \gg y_U v_\phi/\sqrt{2}\,, f_U v_\xi/\sqrt{2}$ in the heavy fermion singlet case and $\hat{M}_\Psi \gg f_U v_\phi/\sqrt{2}\,, y_U v_\xi/\sqrt{2}$ in the heavy fermion doublet case.

We could consider a weaker seesaw condition for the top quark, i.e. $\hat{M}_U \sim f_U v_\xi/\sqrt{2} \gg y_U v_\phi/\sqrt{2}$ and $\hat{M}_\Psi \sim y_U v_\xi/\sqrt{2} \gg f_U v_\phi/\sqrt{2}$.

In this case, while the up component u_{L3} of the left-handed doublet q_{L3} rather than the left-handed singlets U_{L3} dominates the standard model left-handed top quark t_L, the two right-handed singlets u_{R3} and U_{R3} have a sizable mixing and then one of their two linear combinations become the standard model right-handed top quark t_R. Such arrangement should be acceptable as long as the fermion singlet U_3 is heavy enough to suppress its correction to the CKM matrix.

Similarly, the up components $\Psi_{L3}^{+\frac{2}{3}}$ and u_{L3} of the left-handed doublets Ψ_{L3} and q_{L3} have a sizable mixing to form the standard model left-handed top quark t_L.

The heavy vector-like fermion singlets have two decay modes,

$$N_i \to l_L + \phi\,, \quad N_i \to \nu_R + \xi^*\,.$$

$$\varepsilon_{N_i} = \frac{\Gamma(N_i \to l_L + \phi) - \Gamma(N_i^c \to l_L^c + \phi^*)}{\Gamma_{N_i}} \equiv \frac{\Gamma(N_i^c \to \nu_R^c + \xi) - \Gamma(N_i \to \nu_R + \xi^*)}{\Gamma_{N_i}} \neq 0\,.$$

$$\Gamma_{N_i} = \Gamma(N_i \to l_L + \phi) + \Gamma(N_i \to \nu_R + \xi^*) \equiv \Gamma(N_i^c \to l_L^c + \phi^*) + \Gamma(N_i^c \to \nu_R^c + \xi)\,.$$

$$\Gamma_{N_i} = \frac{1}{32\pi}\left[2\left(y_N^\dagger y_N\right)_{ii} + \left(f_N f_N^\dagger\right)_{ii}\right] M_{N_i}\,. \qquad m_\nu = -y_N \frac{v_\xi v_\phi}{2\widehat{M}_N} f_N - y_\Delta \frac{v_\xi v_\phi}{2\widehat{M}_\Delta} f_\Delta\,.$$

$$\begin{aligned}
\varepsilon_{N_i} &= \frac{\mathrm{Im}\left\{\sum_{j\neq i}\left[\left(y_N^\dagger y_N\right)_{ij}\left(f_N f_N^\dagger\right)_{ji}\right]\right\}}{4\pi\left[2\left(y_N^\dagger y_N\right)_{ii} + \left(f_N f_N^\dagger\right)_{ii}\right]} \frac{M_{N_i} M_{N_j}}{M_{N_i}^2 - M_{N_j}^2} \\
&\quad + \frac{\mathrm{Im}\left\{\sum_{k}\left[\left(y_N^\dagger y_\Delta\right)_{ik}\left(f_\Delta f_N^\dagger\right)_{ki}\right]\right\}}{2\pi\left[2\left(y_N^\dagger y_N\right)_{ii} + \left(f_N f_N^\dagger\right)_{ii}\right]} \frac{M_{\Delta_k}}{M_{N_i}}\left[1 - \left(1 + \frac{M_{\Delta_k}^2}{M_{N_i}^2}\right)\ln\left(1 + \frac{M_{N_i}^2}{M_{\Delta_k}^2}\right)\right]. \\
\varepsilon_{N_i} &\simeq \frac{1}{2\pi} \frac{M_{N_i}\mathrm{Im}\left[\left(y_N^\dagger m_\nu f_N^\dagger\right)_{ii}\right]}{v_\xi v_\phi\left[2\left(y_N^\dagger y_N\right)_{ii} + \left(f_N f_N^\dagger\right)_{ii}\right]} \\
&\leq \frac{1}{2\pi} \frac{M_{N_i}\mathrm{Im}\left[\left(y_N^\dagger m_\nu f_N^\dagger\right)_{ii}\right]}{2 v_\xi v_\phi \sqrt{2\left(y_N^\dagger y_N\right)_{ii}\left(f_N f_N^\dagger\right)_{ii}}} < \frac{1}{4\sqrt{2}\pi} \frac{M_{N_i} m_\nu^{\max}}{v_\xi v_\phi} \quad \text{for} \quad M_{N_i}^2 \ll M_{N_j}^2, M_{\Delta_k}^2\,.
\end{aligned}$$

The heavy vector-like fermion doublets also have two decay modes,

$$\Delta_i \to l_L + \xi\,, \;\; \Delta_i \to \nu_R + \phi^*\,.$$

$$\varepsilon_{\Delta_i} = \frac{\Gamma(\Delta_i \to l_L + \xi) - \Gamma(\Delta_i^c \to l_L^c + \xi^*)}{\Gamma_{\Delta_i}} \equiv \frac{\Gamma(\Delta_i^c \to \nu_R^c + \phi) - \Gamma(\Delta_i \to \nu_R + \phi^*)}{\Gamma_{\Delta_i}} \neq 0\,.$$

$$\Gamma_{\Delta_i} = \Gamma(\Delta_i \to l_L + \xi) + \Gamma(\Delta_i \to \nu_R + \phi^*) \equiv \Gamma(\Delta_i^c \to l_L^c + \xi^*) + \Gamma(\Delta_i^c \to \nu_R^c + \phi)\,.$$

$$\Gamma_{\Delta_i} = \frac{1}{32\pi}\left[\left(y^\dagger_\Delta y_\Delta\right)_{ii} + \left(f_\Delta f^\dagger_\Delta\right)_{ii}\right] M_{\Delta_i}\,.\qquad m_\nu = -y_N \frac{v_\xi v_\phi}{2\widehat{M}_N} f_N - y_\Delta \frac{v_\xi v_\phi}{2\widehat{M}_\Delta} f_\Delta\,.$$

$$\begin{aligned}
\varepsilon_{\Delta_i} &= \frac{\mathrm{Im}\left\{\sum_{j\neq i}\left[\left(y^\dagger_\Delta y_\Delta\right)_{ij}\left(f_\Delta f^\dagger_\Delta\right)_{ji}\right]\right\}}{8\pi\left[\left(y^\dagger_\Delta y_\Delta\right)_{ii} + \left(f_\Delta f^\dagger_\Delta\right)_{ii}\right]}\frac{M_{\Delta_i}M_{\Delta_j}}{M^2_{\Delta_i} - M^2_{\Delta_j}} \\
&\quad + \frac{\mathrm{Im}\left\{\sum_k\left[\left(y^\dagger_\Delta y_N\right)_{ik}\left(f_N f^\dagger_\Delta\right)_{ki}\right]\right\}}{4\pi\left[\left(y^\dagger_\Delta y_\Delta\right)_{ii} + \left(f_\Delta f^\dagger_\Delta\right)_{ii}\right]}\frac{M_{N_k}}{M_{\Delta_i}}\left[1 - \left(1 + \frac{M^2_{N_k}}{M^2_{\Delta_i}}\right)\ln\left(1 + \frac{M^2_{\Delta_i}}{M^2_{N_k}}\right)\right].
\end{aligned}$$

$$\begin{aligned}
\varepsilon_{\Delta_i} &\simeq \frac{1}{4\pi}\frac{M_{\Delta_i}\mathrm{Im}\left[\left(y^\dagger_\Delta m_\nu f^\dagger_\Delta\right)_{ii}\right]}{v_\xi v_\phi\left[\left(y^\dagger_\Delta y_\Delta\right)_{ii} + \left(f_\Delta f^\dagger_\Delta\right)_{ii}\right]} \\
&\leq \frac{1}{8\pi}\frac{M_{\Delta_i}\mathrm{Im}\left[\left(y^\dagger_\Delta m_\nu f^\dagger_\Delta\right)_{ii}\right]}{v_\xi v_\phi\sqrt{\left(y^\dagger_\Delta y_\Delta\right)_{ii}\left(f_\Delta f^\dagger_\Delta\right)_{ii}}} < \frac{1}{8\pi}\frac{M_{\Delta_i} m^{\max}_\nu}{v_\xi v_\phi} \quad \text{for } M^2_{\Delta_i} \ll M^2_{\Delta_j}, M^2_{N_k}\,.
\end{aligned}$$

After the heavy vector-like fermion singlets N_i or the heavy vector-like fermion doublets Δ_i go out of equilibrium, their decays can generate a lepton number $L_{l_L} = L$ stored in the left-handed lepton doublets l_L and an opposite lepton number $L_{\nu_R} = -L$ stored in the right-handed neutrinos ν_R.

When the heavy decaying particles have a hierarchical spectrum, i.e. $M^2_{N_1} \ll M^2_{N_{2,...}}, M^2_{\Delta_{2,...}}$, or $M^2_{\Delta_1} \ll M^2_{\Delta_{2,...}}, M^2_{N_{1,...}}$, the decays of the lightest N_1/Δ_1 should dominate the final L, i.e.

$$L = \varepsilon_{N_1/\Delta_1} \left(\frac{n^{eq}_{N_1/\Delta_1}}{s} \right) \Big|_{T=T_D} .$$

Here the symbols $n^{eq}_{N_1/\Delta_1}$ and T_D respectively are the equilibrium number density and the decoupled temperature of the heavy decaying fermions N_1/Δ_1, while the character s is the entropy density of the universe (e.g. Kolb, Turner, *The Early Universe.*).

The sphaleron processes eventually will partially transfer this lepton number to a baryon asymmetry,

$$B = -\frac{28}{79} L = -\frac{28}{79} \varepsilon_{N_1/\Delta_1} \left(\frac{n^{eq}_{N_1/\Delta_1}}{s} \right) \Big|_{T=T_D} .$$

The right-handed neutrinos will affect the effective neutrino number which is stringently constrained by the BBN (e.g. Kolb, Turner, *The Early Universe.*).

The right-handed neutrinos should decouple above the QCD scale and hence give a negligible contribution to the effective neutrino number (e.g. Kolb, Turner, *The Early Universe.*).

We need check the annihilations of the right-handed neutrinos into the other light species,

$$\sigma_{\nu_R} = \sum_{f=d,u,s,e,\mu,\nu_L} \sigma(\nu_R + \bar{\nu}_R \to f + \bar{f}) = \frac{2825}{3\times 2^{12}\pi}\frac{g_{Y'}^4}{M_{Z'}^4}s = \frac{2825}{3\times 2^{8}\pi}\frac{s}{v_\xi^4}\,.$$

$$(s = \text{Mandelstam variable}\,, \quad M_{Z'}^2 = \frac{1}{4}g_{Y'}^2 v_\xi^2\,.)$$

The interaction rate then should be (Giudice, Notari, Raidal, Riotto, Strumia, '03.)

$$\Gamma_{\nu_R} = \frac{\frac{T}{32\pi^4}\int_0^\infty s^{3/2} K_1\left(\frac{\sqrt{s}}{T}\right)\sigma_{\nu_R} ds}{\frac{2}{\pi^2}T^3} = \frac{2825}{64\pi^3}\frac{T^5}{v_\xi^4} . \quad (K_1 = \text{Bessel function}) .$$

Comparing the above interaction rate with the Hubble constant,

$$H(T) = \left[\frac{8\pi^3 g_*(T)}{90}\right]^{\frac{1}{2}} \frac{T^2}{M_{\text{Pl}}} ,$$

we can find

$$\Gamma_{\nu_R} < H(T)\left.\right|_{T\simeq 300\,\text{MeV}} \Longrightarrow v_\xi > 14\,\text{TeV} .$$

Here $M_{\text{Pl}} \simeq 1.22 \times 10^{19}\,\text{GeV}$ is the Planck mass and $g_*(T) \simeq 61.75$ is the relativistic degrees of freedom at $T \simeq 300\,\text{MeV}$ (e.g. Kolb, Turner, *The Early Universe.*).

Summary

- In the conventional seesaw models, the neutrinos should be the Majorana particles. These seesaw models can accommodate a lepton-number-violation leptogenesis mechanism to explain the matter-antimatter asymmetry in the present universe.

- Currently, no experimental results require the neutrinos to be the Majorana particles rather than the Dirac particles. On the theoretical hand, the Dirac neutrinos can nicely acquire their tiny masses in some Dirac seesaw models where a lepton-number-conservation neutrinogenesis mechanism is available to explain the cosmological matter-antimatter asymmetry.
- Under a $U(1)_{Y'}$ gauge symmetry, both the neutral neutrinos and the charged fermions can obtain their Dirac masses through the seesaw mechanism. The heavy fermion singlets or doublets for neutrino mass generation can allow a successful neutrinogenesis to explain the matter-antimatter asymmetry in the universe.

Thank you very much for your attention!