Inflation correlators at tree and loop levels [1-4]

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Background

Inflation correlators encode physics at the inflation scale, $H \sim 10^{14}$GeV. During the inflation, particles with masses comparable to H can be spontaneously produced. These massive particles can couple to inflaton and then leave oscillatory shapes in the inflation correlators, which can be used to detect heavy particles. This program is known as Cosmological Collider (CC) Physics, and such CC signals can be searched for in the near future.

However, the calculation of inflation correlators is extremely difficult, mostly because the background spacetime is (nearly) dS. Recently, we develop some useful methods for analytic calculation, including the partial Mellin-Barnes (MB) representation, and an improved cosmological bootstrap, with which we have made progress in the calculation of inflation correlators at both tree and loop levels.

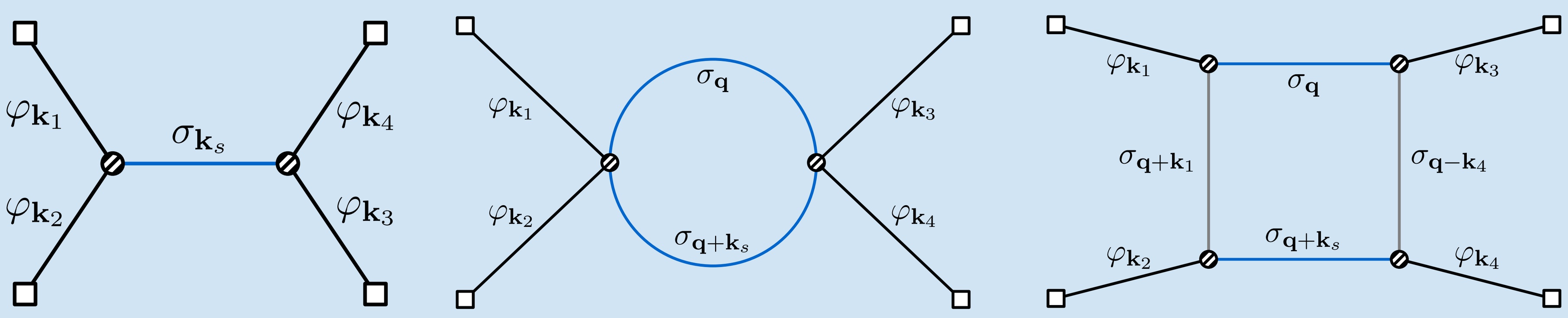


Figure 1. Typical tree-level and loop-level CC processes.

Methods

Partial MB representation [1,2]: We apply the inverse Mellin transform to all the internal modes. For example, the opposite-sign propagators for massive scalar are:

$$D_{\pm\mp}(k;\tau_1,\tau_2)=\frac{1}{4\pi}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\,e^{\mp\mathrm{i}\pi(s_1-s_2)}\left(\frac{k}{2}\right)^{-2s_{12}}(-\tau_1)^{-2s_1+3/2}(-\tau_2)^{-2s_2+3/2}$$
$$\times\,\Gamma\left[s_1-\frac{\mathrm{i}\widetilde{\nu}}{2},s_1+\frac{\mathrm{i}\widetilde{\nu}}{2},s_2-\frac{\mathrm{i}\widetilde{\nu}}{2},s_2+\frac{\mathrm{i}\widetilde{\nu}}{2}\right].$$

As a result, the integrand as a function of conformal times and loop momenta is greatly simplified and typically calculable. Then we can integrate all the Mellin variables $\mathrm{s_i}$, using the residue theorem.

Bootstrap equation [2,3]: For 4pt tree-level processes, the equation of motion of the internal massive mode:

$$(\tau_1^2\partial_{\tau_1}^2-2\tau_1\partial_{\tau_1}+k_s^2\tau_1^2+m^2)D_{\mathsf{ab}}(k_s;\tau_1,\tau_2)=0\text{ or a contact term},$$

implies ordinary differential equations for the full correlator:

$$\mathcal{D}_{r_1}^{p_1}\mathcal{I}^{p_1,p_2}(r_1,r_2)=\text{coefficient}\times\left(\frac{r_1r_2}{r_1+r_2}\right)^{5+p_1+p_2},\quad r_1\equiv\frac{k_s}{k_1+k_2},\quad r_2\equiv\frac{k_s}{k_3+k_4},$$
$$\mathcal{D}_r^p\equiv(r^2-r^4)\partial_r^2-\left[(4+2p)r+2r^3\right]\partial_r+\widetilde{\nu}^2+\frac{(5+2p)^2}{4}.$$

With BC in the squeezed limit, we can solve for the full 4pt correlator. Furthermore, with a clever change of variables $\mathrm{u_i}=2\mathrm{r_i}/(1+\mathrm{r_i})$, we can easily deal with the spurious folded divergence and derive closed-form expressions for 3pt and 2pt inflation correlators [3].

Tree-Level Result

We obtain full analytical expressions for typical tree-level 4pt, 3pt, and 2pt correlators with massive exchanges by both partial MB and bootstrap [2, 3]. We emphasize that our methods apply to nontrivial dS-boost-breaking cases, including models with helical chemical potential (Fig. 2).

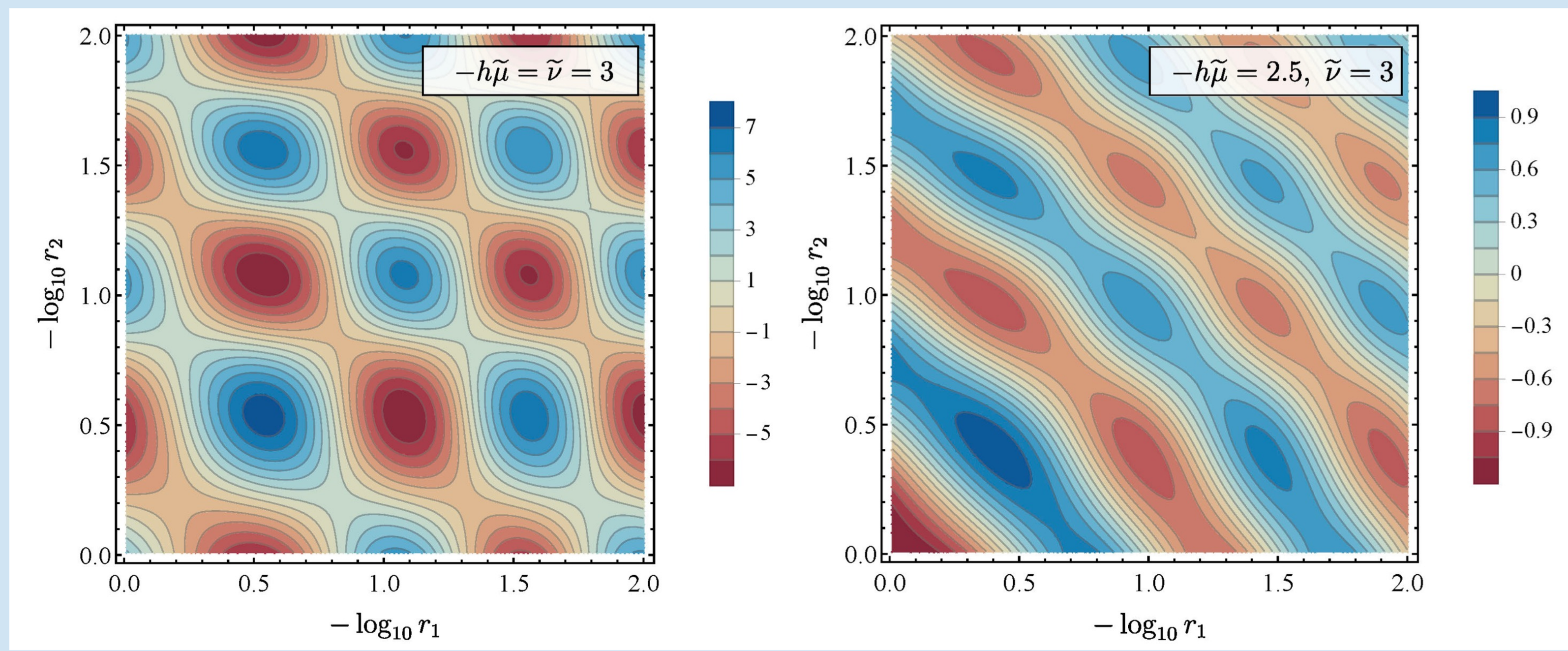
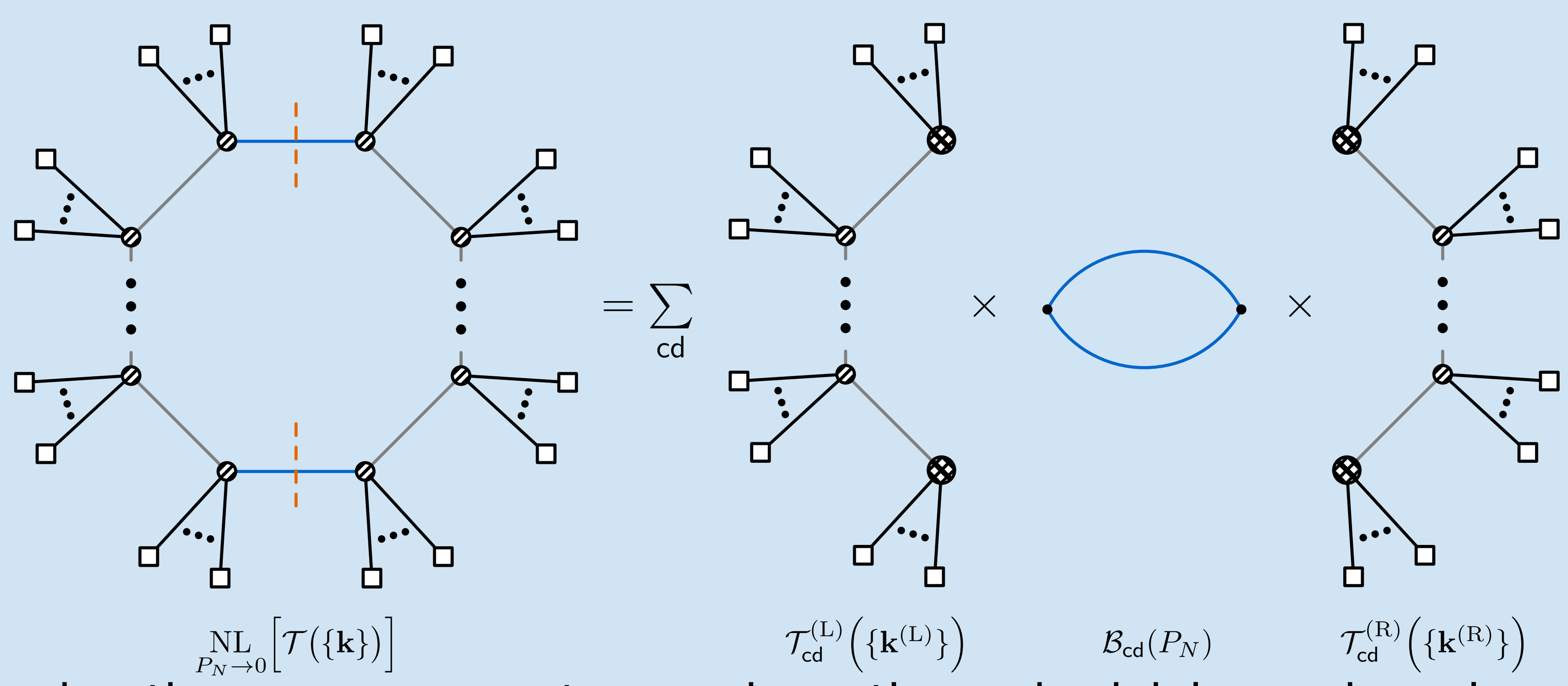


Fig2. 4pt tree correlators from a massive vector with chemical potential.

Loop-Level Result

In a recent work [4], we identify all possible nonlocal signals for arbitrary one-particle-irreducible (1PI) graphs, with the help of partial MB representation.

We proved a factorization theorem and the cutting rule which show that, for an arbitrary 1PI graph, the nonlocal signal associated with the momentum transfer $\mathrm{P_N}$ is factorized into three parts:



and the propagators in the bubble signal are automatically cut.

With this factorization theorem, we are able to calculate the nonlocal signals in the bubble, triangle, and box diagrams. The results meet the EFT intuition: The uncut internal line in the triangle (or box) diagram can be pinched as an effective operator, whose coupling scales as $1/\mathrm{m}^2$ for large m but has nontrivial deviation for not-so-large masses.

[1] "Phase information in cosmological collider signals," JHEP 10 (2022) 192, arXiv:2205.01692 [hep-th];

[2] "Helical inflation correlators: partial Mellin-Barnes and bootstrap equations," JHEP 04 (2023) 059, arXiv:2208.13790 [hep-th].

[3] "Closed-Form Formulae for Inflation Correlators," arXiv:2301.07047 [hep-th].

[4] "Inflation Correlators at the One-Loop Order: Nonanalyticity, Factorization, Cutting Rule, and OPE," arXiv:2304.13295 [hep-th].

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