$\Xi_c - \Xi_c'$ mixing From Lattice QCD

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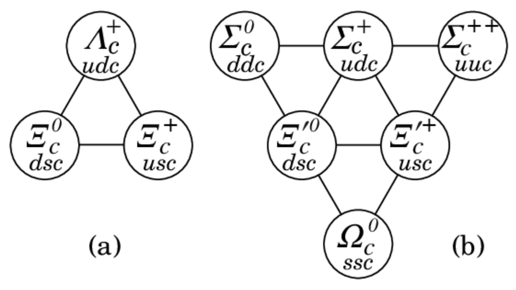
Introduction

Ground states of baryons with one heavy quark can be categorized into two sets: if the light quark system is spinless, three baryons form an SU(3) triplet;
if the spin of the light quark system is one, then six charmed baryons form a sextet.
In reality, since the charm quark mass is finite and the strange quark is heavier than the u/d quark, charm baryons in the triplet and sextet are mixing.



$\Xi_c - \Xi_c'$ mixing in Heavy Quark Effective Theory

We can obtain the baryonic currents of $SU(3)_F$ eigenstate as

$$O^{\bar{3}} = \epsilon^{abc}\left(\ell^{Ta}C\gamma_5 s^b\right)P_+\tilde{c}^c$$
$$O^{6} = \epsilon^{abc}\left(\ell^{Ta}C\vec{\gamma}s^b\right)\cdot\vec{\gamma}\gamma_5 P_+\tilde{c}^c$$

the two Ξ_c/Ξ_c' mass eigenstates mixing together

$$\begin{pmatrix}|\Xi_c\rangle\\|\Xi_c'\rangle\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}|\Xi_c^{\bar{3}}\rangle\\|\Xi_c^{6}\rangle\end{pmatrix} \qquad H_{\rm QCD}|\Xi_c\rangle = m_{\Xi_c}|\Xi_c\rangle,\quad H_{\rm QCD}|\Xi_c'\rangle = m_{\Xi_c'}|\Xi_c'\rangle$$

Two-point correlation function matrix From Lattice QCD

We consider the 2×2 correlation function matrix

$$\mathcal{C}(t,t_0) = \sum_{\vec{x}}\begin{pmatrix}\langle O_p^{\bar{3}}(\vec{x},t)\bar{O}_w^{\bar{3}}(\vec{0},t_0)\rangle & \langle O_p^{\bar{3}}(\vec{x},t)\bar{O}_w^{6}(\vec{0},t_0)\rangle\\ \langle O_p^{6}(\vec{x},t)\bar{O}_w^{\bar{3}}(\vec{0},t_0)\rangle & \langle O_p^{6}(\vec{x},t)\bar{O}_w^{6}(\vec{0},t_0)\rangle\end{pmatrix}$$

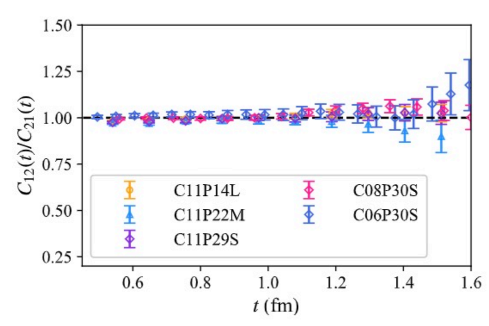


Fig1.Ratios of C_{12}/C_{21} as function of t.

Two-point functions can be constructed by the quark propagators. Insert the mass eigenstates, the eigenstates will be related to the mixing angle:

$$C_{11}(t,t_0) = A_pA_w^\dagger\left[\frac{\cos^2\theta}{2m_{\Xi_c}}e^{-m_{\Xi_c}(t-t_0)} + \frac{\sin^2\theta}{2m_{\Xi_c'}}e^{-m_{\Xi_c'}(t-t_0)}\right]$$
$$C_{12}(t,t_0) = \sum_{\vec{x}}\langle O_p^{\bar{3}}(\vec{x},t)\bar{O}_w^{6}(\vec{0},t_0)\rangle = A_pB_w^\dagger\left[\frac{\cos\theta\sin\theta}{2m_{\Xi_c}}e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos\theta\sin\theta}{2m_{\Xi_c'}}e^{-m_{\Xi_c'}(t-t_0)}\right]$$
$$C_{21}(t,t_0) = \sum_{\vec{x}}\langle O_p^{6}(\vec{x},t)\bar{O}_w^{\bar{3}}(\vec{0},t_0)\rangle = B_pA_w^\dagger\left[\frac{\cos\theta\sin\theta}{2m_{\Xi_c}}e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos\theta\sin\theta}{2m_{\Xi_c'}}e^{-m_{\Xi_c'}(t-t_0)}\right]$$
$$C_{22}(t,t_0) = \sum_{\vec{x}}\langle O_p^{6}(\vec{x},t)\bar{O}_w^{6}(\vec{0},t_0)\rangle = B_pB_w^\dagger\left[\frac{\sin^2\theta}{2m_{\Xi_c}}e^{-m_{\Xi_c}(t-t_0)} + \frac{\cos^2\theta}{2m_{\Xi_c'}}e^{-m_{\Xi_c'}(t-t_0)}\right]$$

Mixing Angle from Joint fit

To extract the masses and mixing angle, we apply a joint fit based on the parametrization formula of the correlation function matrix with parameters $A, B, m_{\Xi_c}, m_{\Xi_c'}$ and θ.

	m_{Ξ_c} (GeV)	$m_{\Xi_c'}$ (GeV)	θ (°)	χ^2/d.o.f	fit range (fm)
C11P14L	2.4256(19)	2.5196(22)	1.083(30)	0.96	1.19 – 2.81
C11P22M	2.4380(27)	2.5351(30)	0.988(49)	1.0	1.19 – 2.92
C11P29S	2.4587(27)	2.5536(29)	1.002(50)	1.1	1.19 – 3.24
C08P30S	2.4753(21)	2.5809(26)	1.080(42)	0.95	1.20 – 2.40
C06P30S	2.4695(37)	2.5815(48)	1.021(67)	1.2	1.32 – 2.40
Extrapolated	$2.4380(68)_{\rm stat}(403)_{\rm syst}$	$2.5562(74)_{\rm stat}(422)_{\rm syst}$	$1.20(9)_{\rm stat}(2)_{\rm syst}$		
Exp. data [19]	$2.46794^{+0.00017}_{-0.00020}$	2.5784 ± 0.0005	—		

Tab1. Results of masses and mixing angles from correlated matrix fits on different ensembles

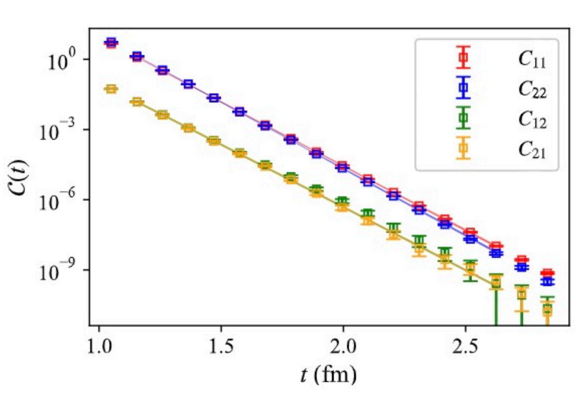


Fig2. Joint fit of the correlation function matrix

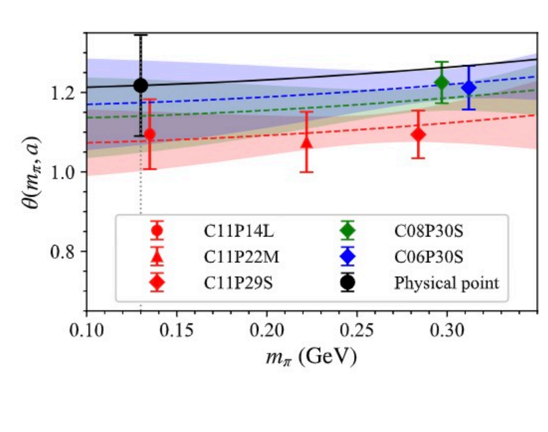


Fig3. Chiral and continuum extrapolations of θ as a function of m_π

the chiral and continuum extrapolation through the following ansatz:

$$\theta(m_\pi,a) = \theta_{\rm phy} + c_1\left(m_\pi^2 - m_{\pi,{\rm phy}}^2\right) + c_2a^2,$$
$$m_n(m_\pi,a) = m_{n,{\rm phy}} + c_1\left(m_\pi^2 - m_{\pi,{\rm phy}}^2\right) + c_2a^2$$

With the extrapolation uncertainties taken into account, we find the mixing angle is:

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

Mixing angle from generalized eigenvalue problem

Besides the correlated fit, the mixing angle can be also extracted by solving the generalized eigenvalue problem (GEVP):

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$

Orthogonality condition allows us to extract the spectrum of Ξ_c and Ξ_c', the fit function can be expressed as:

$$\lambda_n(t) = c_0e^{-m_n(t-t_r)}\left(1 + c_1e^{-\Delta E(t-t_r)}\right)$$

the mixing effects will be collected into the GEVP eigenvectors

$$v_1 = \sqrt{1 + \frac{A_p^2\cot^2\theta}{B_p^2}}\begin{pmatrix}\frac{A_p}{B_p}\cot\theta\\ 1\end{pmatrix}$$
$$v_2 = \sqrt{1 + \frac{A_p^2\tan^2\theta}{B_p^2}}\begin{pmatrix}-\frac{A_p}{B_p}\tan\theta\\ 1\end{pmatrix}$$

Masses and the mixing angle can be extracted

	m_{Ξ_c} (GeV)	$m_{\Xi_c'}$ (GeV)	θ (°)
C11P14L	2.4274(33)	2.5201(20)	1.095(88)
C11P22M	2.4322(39)	2.5321(44)	1.075(76)
C11P29S	2.4622(59)	2.5603(79)	1.094(60)
C08P30S	2.4763(11)	2.5868(13)	1.225(52)
C06P30S	2.4682(38)	2.5905(39)	1.212(55)
Extrapolated	$2.428(11)_{\rm fit}(44)_{\rm ext.}$	$2.547(12)_{\rm fit}(36)_{\rm ext.}$	$1.22(13)_{\rm fit}(1)_{\rm ext.}$
Exp. data [19]	$2.46794^{+0.00017}_{-0.00020}$	2.5784 ± 0.0005	—

Tab2. Results of masses and mixing angles from fitting GEVP eigenvalues and eigenvectors with the model averaging approach

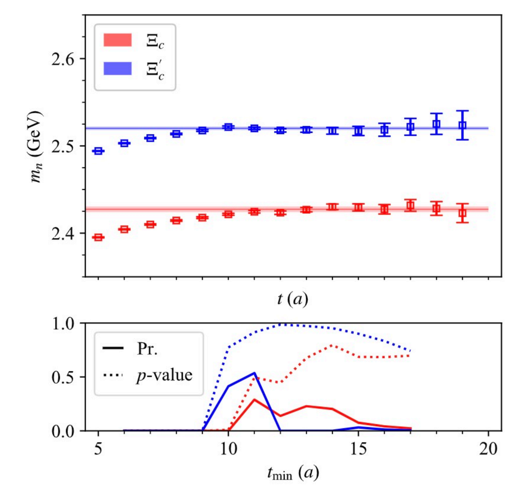


Fig4. Upper panel: effective masses from eigenvalues; Lower panel: the model weight factors (solid lines) and standard p-values (dashed lines).

Charm quark mass dependence

In HQET, the mixing between Ξ_c and Ξ_c' would vanish in the heavy-quark limit. In reality, the mixing occurs through a finite quark mass correction and thus is proportional to $1/m_c$. We calculate at several bare charm quark masses around the physical one

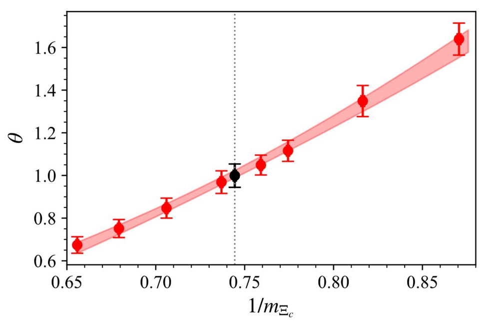
$m_c^{\rm b}$	0.2	0.3	0.4	0.44	**0.478**	0.5	0.6	0.7	0.8
m_{Ξ_c} (GeV)	2.0987(25)	2.2380(28)	2.3594(26)	2.4069(26)	**2.4587(27)**	2.4793(29)	2.5878(30)	2.6898(30)	2.7859(31)
$m_{\Xi_c'}$ (GeV)	2.1834(24)	2.3249(29)	2.4514(24)	2.4999(24)	**2.5536(29)**	2.5718(29)	2.6823(29)	2.7859(30)	2.8835(30)
θ (°)	1.639(75)	1.349(73)	1.116(49)	1.049(46)	**1.002(50)**	0.969(53)	0.847(47)	0.751(42)	0.674(39)

Tab3. Results of m_{Ξ_c},$m_{\Xi_c'}$ and the mixing angle at different bare charm quark masses.

we employ a roughly fit ansatz

$$\theta = \frac{B_1}{m_{\Xi_c}} + \frac{B_2}{m_{\Xi_c}^2}$$

obtain $B_1 = -2.78(52)$ GeV, $B_2 = 12.9(1.3)$ GeV^2 with $\chi^2/{\rm d.o.f} = 0.11$.



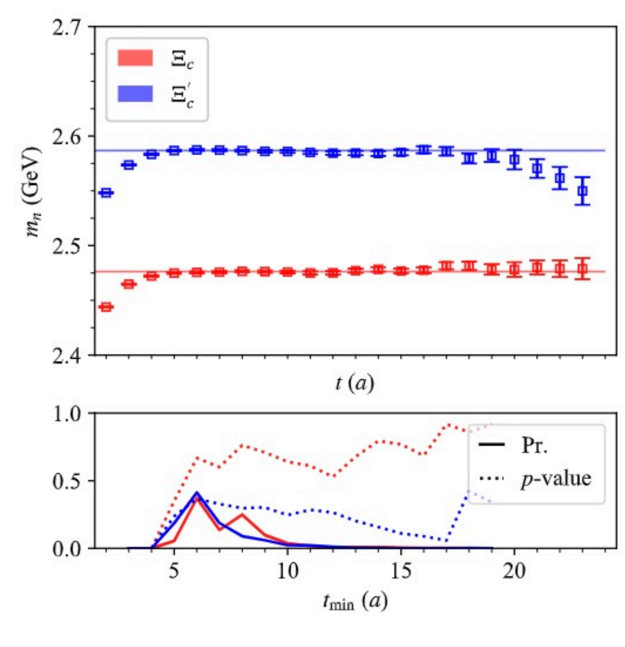
Statistical modeling plays a crucial role in calculations. Multiple model may exist for the same lattice data. To account for the uncertainties with model selection, we employ model averaging approach. Taking a probability-weighted average of all the model variations.

1. a set of models $\{M\}$
2. the weight factor
   $${\rm Pr}(M|D) \approx \exp\left[-\frac{1}{2}\left(\chi^2_{\rm aug}(\mathbf{a}^*) + 2k + 2N_{\rm cut}\right)\right]$$
3. normalized the weight factor
4. The model-averaged values can be determined

$$\langle a\rangle = \sum_M \langle a\rangle_M {\rm Pr}(M|D)$$

and its error can be estimated

$$\sigma = \sqrt{\sum_M\left[\sigma_M^2 + \left(\langle a\rangle_M - \langle a\rangle\right)^2\right]{\rm Pr}(M|D)}$$



(c) C08P30S

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