First principles determination of bubble wall velocity and LTE approximation.

Benoit Laurent

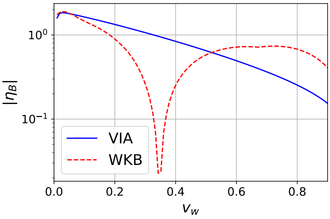
Based on
J. Cline and BL (2204.13120)
W.-Y. Ai, BL and J. van de Vis (2303.10171)

SPCS 2023: Phase Transitions, Gravitational Waves, and Colliders
Tsung Dao Lee Institute, Shanghai
September 23, 2023

Why is ξ_w important?

Baryogenesis:

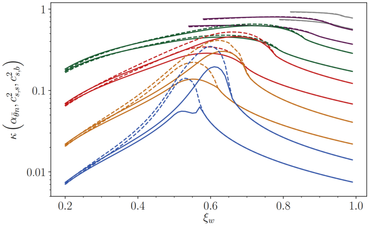
- FOPTs offer a mechanism to explain the observed baryon asymmetry of the Universe (e.g. electroweak baryogenesis),
- Baryogenesis requires a departure from equilibrium, which can be provided by the bubble wall.



Cline and Laurent (2108.04249)

Gravitational waves:

- In the main mechanism, the energy in the wall is converted into sound waves which are then dissipated as GWs,
- These GWs could be detected by future space-based experiments such as LISA.



Giese, Konstandin, Schmitz and van de Vis (2010.09744)

Outline

- Wall velocity: General framework
- Total pressure and application to xSM
- Local thermal equilibrium approximation

Wall velocity: General framework

Dynamics of a FOPT

A FOPT is triggered by the variation of a scalar field ϕ's VEV:

$$\phi(x^\mu):\ \phi_1 \longrightarrow \phi_2.$$

This can be described by the **EOM**:

$$\partial^2\phi + \frac{\partial V_0(\phi)}{\partial\phi} + \sum_i \frac{\partial m_i^2(\phi)}{\partial\phi}\int\frac{d^3p}{(2\pi)^3 2E_i} f_i(p,x) = 0.$$

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The set of f_i is the distribution function of each species in the plasma. They follow the **Boltzmann equations**:

$$\left(p^\mu\partial_\mu + \frac{1}{2}\partial^\mu(m_i^2)\partial_{p_\mu}\right) f_i(p,x) = -\mathcal{C}_i[f].$$

Background fluid

To simplify the analysis, f_i is split into a "background" part (common to all species) and an "out-of-equilibrium" part:

$$f_i = f_{bg} + \delta f_i,$$

with

$$f_{bg} \equiv f_{\text{eq}} = \left[\exp\left(\frac{1}{T(\mathbf{x})} p_\mu u^\mu_{\text{pl}}(\mathbf{x})\right) \pm 1\right]^{-1}.$$

δf_i is assumed to be nonzero only for a few species that undergo a large mass variation (e.g. top quark).

Fluid equations

The evolution of $T(\mathbf{x})$ and $v_{\rm pl}(\mathbf{x})$ is derived from the conservation of the energy-momentum tensor

$$0 = \partial_\mu T^{\mu\nu} = \partial_\mu (T_\phi^{\mu\nu} + T_{\rm plasma}^{\mu\nu}),$$

where

$$\begin{aligned} T_\phi^{\mu\nu} &= \partial^\mu\phi\,\partial^\nu\phi - \eta^{\mu\nu}\left[\frac{1}{2}\partial_\alpha\phi\,\partial^\alpha\phi - V_0(\phi)\right], \\ T_{\rm plasma}^{\mu\nu} &= \omega u_{\rm pl}^\mu u_{\rm pl}^\nu - p_T\eta^{\mu\nu} + \sum_i\int\frac{d^3p}{(2\pi)^3E_i}p^\mu p^\nu\delta f_i(\mathbf{x},\mathbf{p}), \end{aligned}$$

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In the wall frame, these equations can be integrated directly:

$$T^{30} = c_1(\xi_w) \quad \text{and} \quad T^{33} = c_2(\xi_w),$$

with $c_{1,2}$ independent of z.
These equations can be solved for T and $v_{\rm pl}$.

(More details in Cline and Laurent 2204.13120)

3 types of solutions

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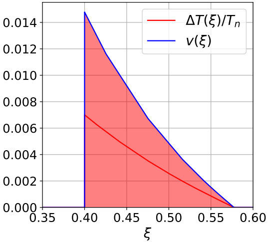
Deflagration:

- Subsonic: $\xi_w < c_s$
- $v_- = \xi_w$ and $T_+^{\mathrm{sw}} = T_n$
- Shock wave



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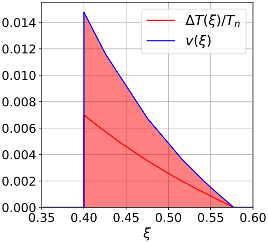

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Detonation:

- Supersonic: $\xi_w \geq \xi_J > c_s$
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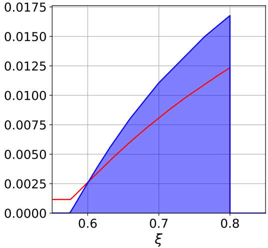
- Rarefaction wave



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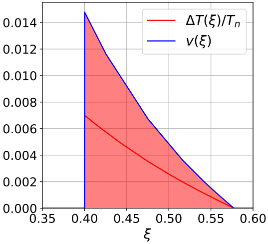

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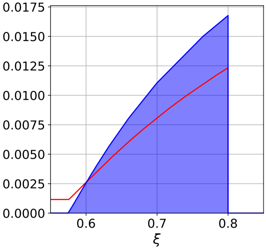

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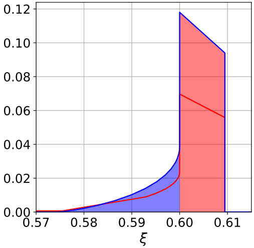
Hybrid:

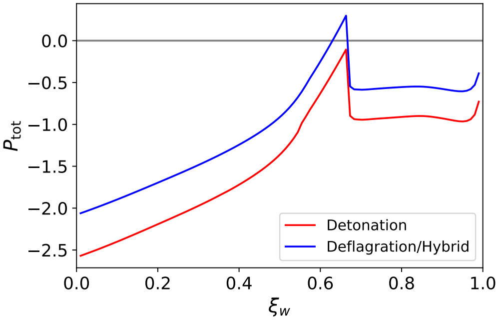
- $c_s < \xi_w < \xi_J$
- $v_- = c_s$ and $T_+^{\rm sw} = T_n$
- Rarefaction and shock waves



$\xi_w = 0.6$

Total pressure and application to xSM

Pressure on the wall



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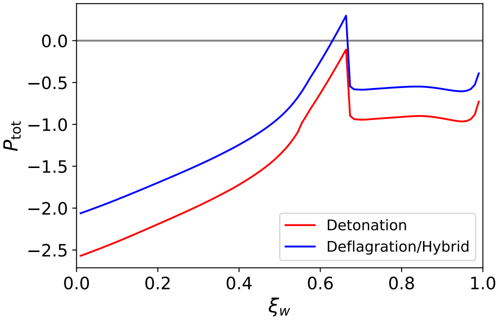

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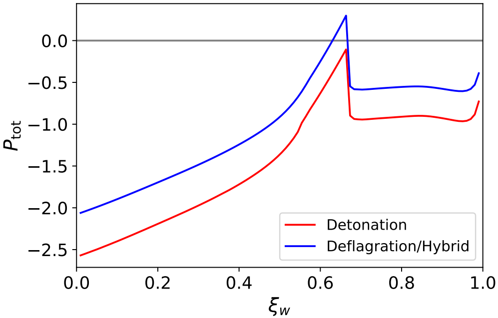

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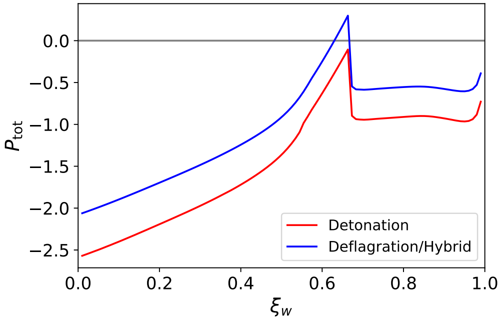

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- The pressure is maximized at ξ_J, so if there is no deflagration/hybrid solution, the wall runs away (or is stopped at $\gamma_w \gg 1$),
- Solutions can be classified in **2 categories**: deflagrations/hybrids ($\xi_w \sim c_s$) and ultrarelativistic detonations ($\gamma_w \gg 1$).

Example: Singlet scalar extension

We consider the singlet scalar extension with a Z_2 symmetry:

$$V_0(h,s) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_s^2}{2}s^2 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2 s^2.$$

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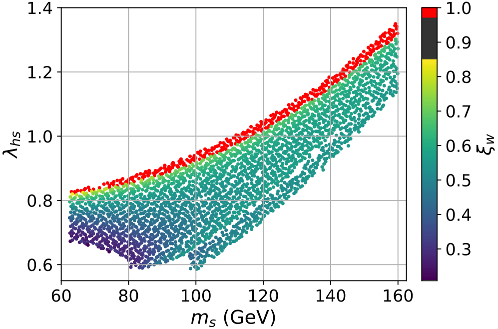
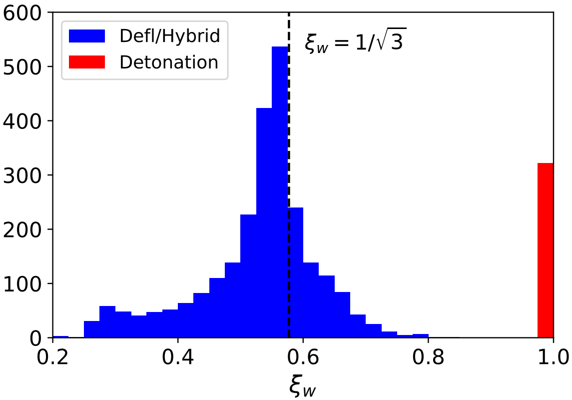
Scan with $\lambda_s = 1$:



Velocity distribution



- No models below 0.2,
- No models with $\xi_J < \xi_w \ll 1$,
- $\xi_w > 0.5$ for 75% of the models.

Local thermal equilibrium approximation

Matching equations in LTE

Far away from the wall, the fluid equations become:

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Now, let's assume $\delta f_i = 0$:
We can contract the divergence of $T^{\mu\nu}$ with u^ν to get

$$0 = u_\nu \partial_\mu T^{\mu\nu} = u^\nu \partial_\nu \phi \underbrace{\left[\partial^2 \phi + \frac{\partial V_T}{\partial \phi}\right]}_{\text{EOM}=0} + T\partial_\nu S^\nu,$$

with the entropy flux:

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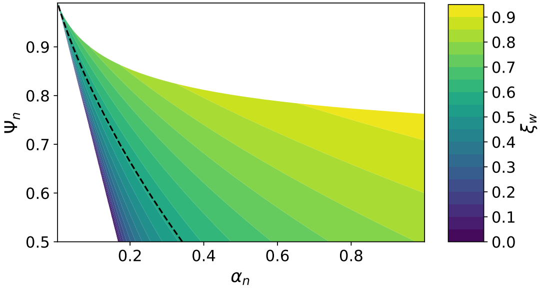
This new matching equation replaces the EOM!

By fitting to the template model (constant sound speed), ξ_w only depends on 4 quantities:

$$\alpha_n \text{ (strength of PT)}, \quad \Psi_n = \frac{\omega_-}{\omega_+}, \quad \nu = 1 + 1/c_{s,-}^2, \quad \mu = 1 + 1/c_{s,+}^2.$$

See Ai, Laurent and van de Vis (2303.10171) for a Code snippet to compute ξ_w.



Summary

- The wall velocity can be calculated from a set of fluid equations (EOM, Boltzmann equation and conservation of EMT),
- These equations can be significantly simplified with the LTE approximation,
- Using the template model, the fluid equations can be written in a model-independent way which only depends on 4 free parameters,
- Solutions can be classified as deflagrations/hybrids ($\xi_w \sim c_s$) or ultrarelativistic detonations ($\gamma_w \gg 1$),
- For $\Psi_n \lesssim 0.75$, hybrid walls can be arbitrarily strong, so baryogenesis and GW production could be highly efficient.

Thank you!