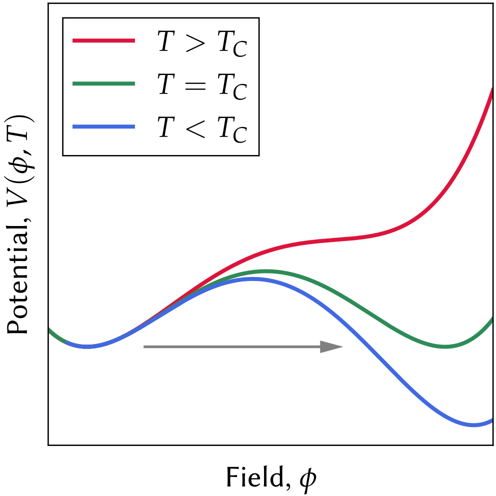
From first-order phase transitions to gravitational waves

Andrew Fowlie and Peter Athron

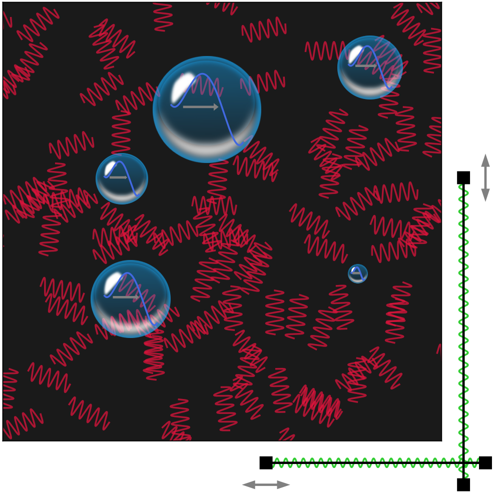
SPCS 2023

Xi'an Jiaotong Liverpool University
西交利物浦大学

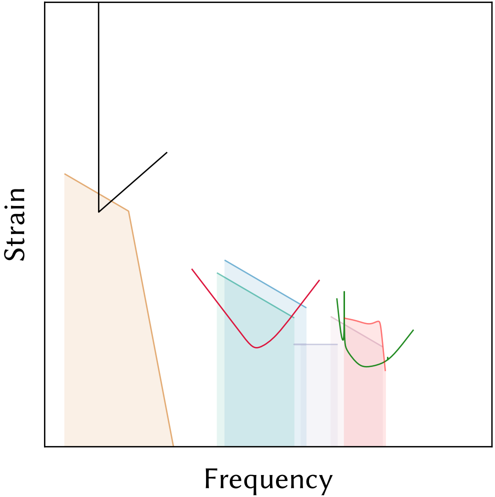
1. First-Order Phase Transition



2. Bubbles of new phase



3. Observable gravitational waves



Introduction

In this first part, I will discuss

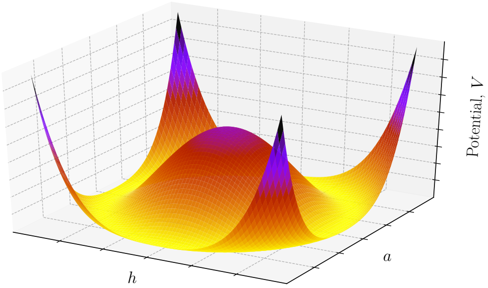
- Constructing an effective potential for a particle physics model
- Identifying vacuum states as the Universe cools — the phases and phase history
- Computing the transition rate between vacuum states

I will emphasise outstanding issues and computer codes along the way

Building effective potential

Tree-level potential

- Tree-level was enough to understand SSB
- Won't be enough here
- After all, we're interested in transitions as Universe cools
- Finite-temperature effects are one-loop



One-loop

The one-loop zero-temperature potential takes the well-known Coleman-Weinberg form

$$V_{\text{eff}}(\phi_{\text{cl}}) \sim m^4 \left[\log\left(\frac{m^2(\phi,\xi)}{Q^2}\right) - k\right]$$

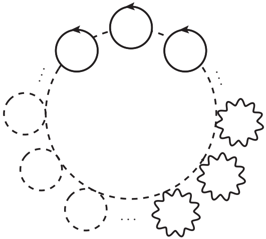
and finite-temperature corrections take the form

$$V_{\text{eff}}(\phi, T) \sim T^4\, J\left(\frac{m^2(\phi)}{T^2}\right)$$

- Renormalization dependent — scale Q and scheme impacts k
- Gauge dependent! Depends on ξ through tree-level masses and more broadly on class of gauge beyond R_ξ

Daisy resummation

- Perturbation theory breaks down when $T \ggg m$
- Furthermore, IR divergences at 4-loop in effective potential
- We must resum the most dangerous diagrams



Daisy resummation

- Perturbation theory breaks down when $T \ggg m$
- Furthermore, IR divergences at 4-loop in effective potential
- We must resum the most dangerous diagrams

1. Parwani method — replace scalar and longitudinal vector boson masses with Debye masses

$$V_{\text{eff}} \to V_{\text{eff}}\big|_{m^2 \to m^2 + \Delta m^2}$$

2. Arnold-Espinosa — add daisy correction term

$$V_{\text{eff}} \to V_{\text{eff}} + V_{\text{daisy}}$$

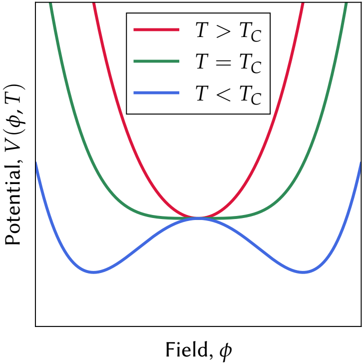
Three-dimensional effective field theory

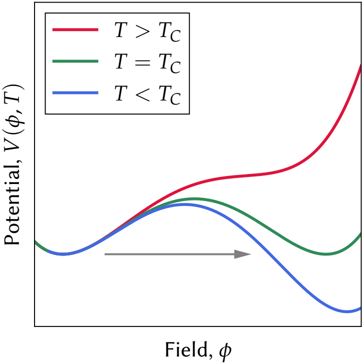
- 4d theory at finite temperature $\Leftrightarrow$ 3d Euclidean theory that has an infinite number of Matsubara modes
- Integrate out the modes — obtain 3d EFT for the high-temperature behaviour of the original theory
- Perform matching at $Q \approx T$ — temperature dependence enters through matching
- Resum dangerous contributions from $T \ggg m$, including large logs
- Better than daisy, but could be hard work; though see DRAlgo (Ekstedt, Schicho, and Tenkanen, 2023)

Tracing phases

Tracing phases

If there are distinct overlapping phases, there could be a FOPT



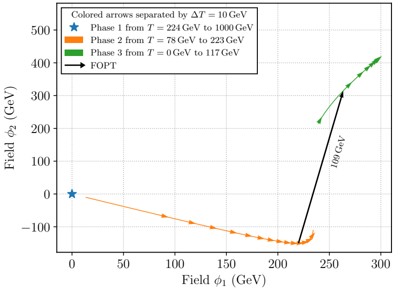


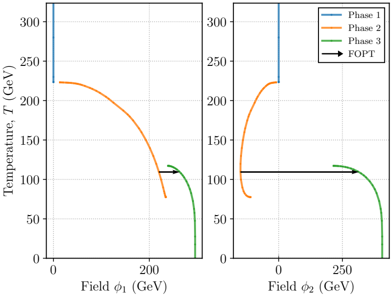
Tracing phases

- Effective potentials are complicated objects — functions of several fields and the temperature
- We want to trace minima of potential as Universe cools
- Minima could appear and disappear as we change temperature
- Typically, spontaneously broken symmetries are restored at high temperature
- Cannot be tackled analytically; efficient numerical methods required
- Good solutions available; see CosmoTransitions (Wainwright, 2012) and PhaseTracer (Athron et al., 2020)

Two-dimensional example

Simple example in which phases appear, disappear, and first- and second-order transitions could occur

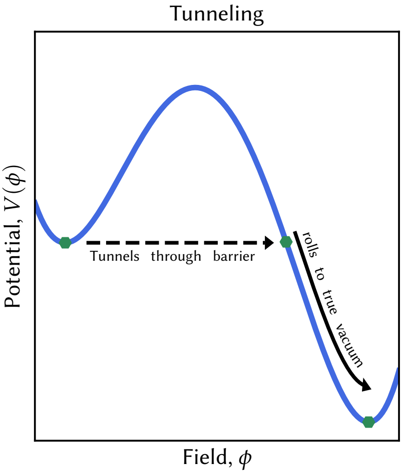




Computing transition probabilities

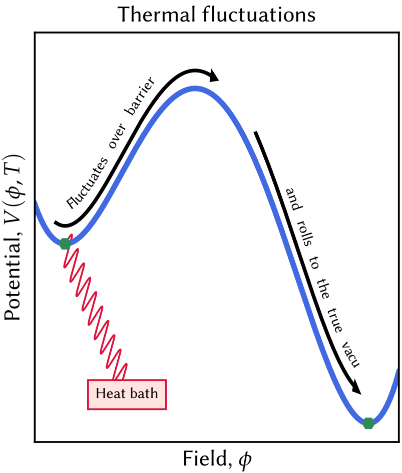
Phase transitions

The field could tunnel through the barrier



Phase transitions

The field could fluctuate over the barrier



Tunnelling

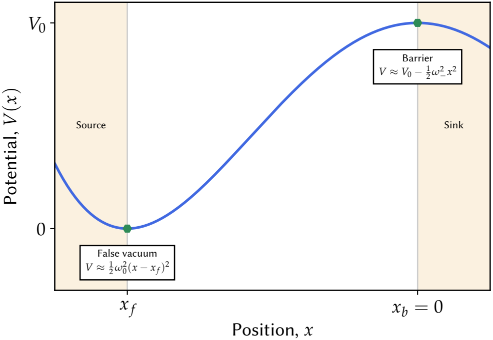
- No classical solution for tunnelling — trapped by conservation of energy
- Wick rotation — upturn potential by looking at Euclideanized equation of motion

$$V_{\text{eff}} \to -V_{\text{eff}}$$

- Semi-classical analogue for tunnelling; no longer trapped by energy conservation
- We may now find semi-classical contribution

Fluctuations

Fluctuation rate as equilibrium rate at which particles go round this system



Wick rotation leads to Boltzmann suppression factor for fluctuating over barrier

The bounce

In each case, the transition probability is connected to the bounce

- A non-trivial solution to the Euclideanized equation of motion

$$\ddot{\phi} + \frac{d-1}{\rho}\dot{\phi} = +V'_{\text{eff}}$$

  in which the particle rolls from at rest and ends exactly at the true vacuum

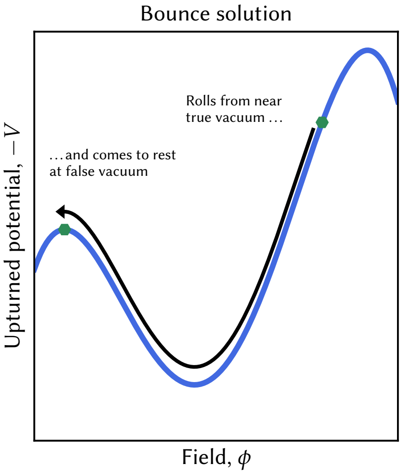
$$\dot{\phi}(0) = 0 \quad \text{and} \quad \lim_{\rho\to\infty} \phi(\rho) = \phi_t$$

- This extremizes the Euclidean action at B
- Transition probability per unit time per unit volume

$$\Gamma = Ae^{-B}$$

The bounce

In each case, the transition probability is connected to the bounce



Computational methods

- Quite a lot activity in solving bounce equation reliably and quickly
- Unstable, fine-tuned problem
- New methods appearing regularly — from math and physics
- I expect situation to stabilise soon, though unclear which method will win
- See e.g., AnyBubble (Masoumi, Olum, and Wachter, 2017), BubbleProfiler (Athron et al., 2019), FindBounce (Guada, Nemevšek, and Pintar, 2020; Guada, 2021), CosmoTransitions (Wainwright, 2012), and SimpleBounce (Sato, 2020; Sato, 2021), and the OptiBounce algorithm (Bardsley, 2022)

Symmetric bounces

Usually assumed that bounces for quantum tunnelling are $O(4)$-symmetric, while bounces for thermal fluctuations are $O(3)$-symmetric

- Coleman mentioned a forthcoming proof, but tempered statements in an erratum
- Later proven that bounces are $O(d)$ symmetric (Coleman, Glaser, and Martin, 1978) for $d > 2$ for a single field under certain regularity conditions for the potential
- Progress towards a proof for multi-field cases is discussed in Blum et al., 2017

Double counting

The tree-level potential might not even possess more than one minimum, so we cannot begin the computation from it alone (Weinberg, 1993); how about

$$\exp\left\{i\int \mathrm{d}^4x\,(T+V)\right\} \to \exp\left\{i\int \mathrm{d}^4x\,(T+\mathrm{Re}\,V_{\mathrm{eff}})\right\}$$

No theoretical justification for this replacement (Croon et al., 2021; Gould and Hirvonen, 2021)

- Double counts the fluctuations — first to create the effective potential and second to consider fluctuations about the bounce
- Throw away a mysterious imaginary part of the effective potential
- Introduces scale and gauge dependence as discussed

Prefactors

- We usually focus on the bounce, B
- What about prefactor, A? Bah, B determines the order of magnitude,

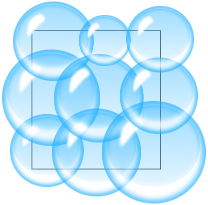
$$\log \Gamma = \log A - B$$

- Hang on, what if A exponential! E.g., $A = e^C$. See Ekstedt, Gould, and Hirvonen, 2023 and BubbleDet
- Correction from A could be just as important as any other loop correction versus leading order

Back to reality

Transforming back to real time, the bounce solution corresponds to a Lorentz invariant bubble

Does the transition complete? Peter's part of the talk.



NNU · 南京师范大学
NANJING NORMAL UNIVERSITY

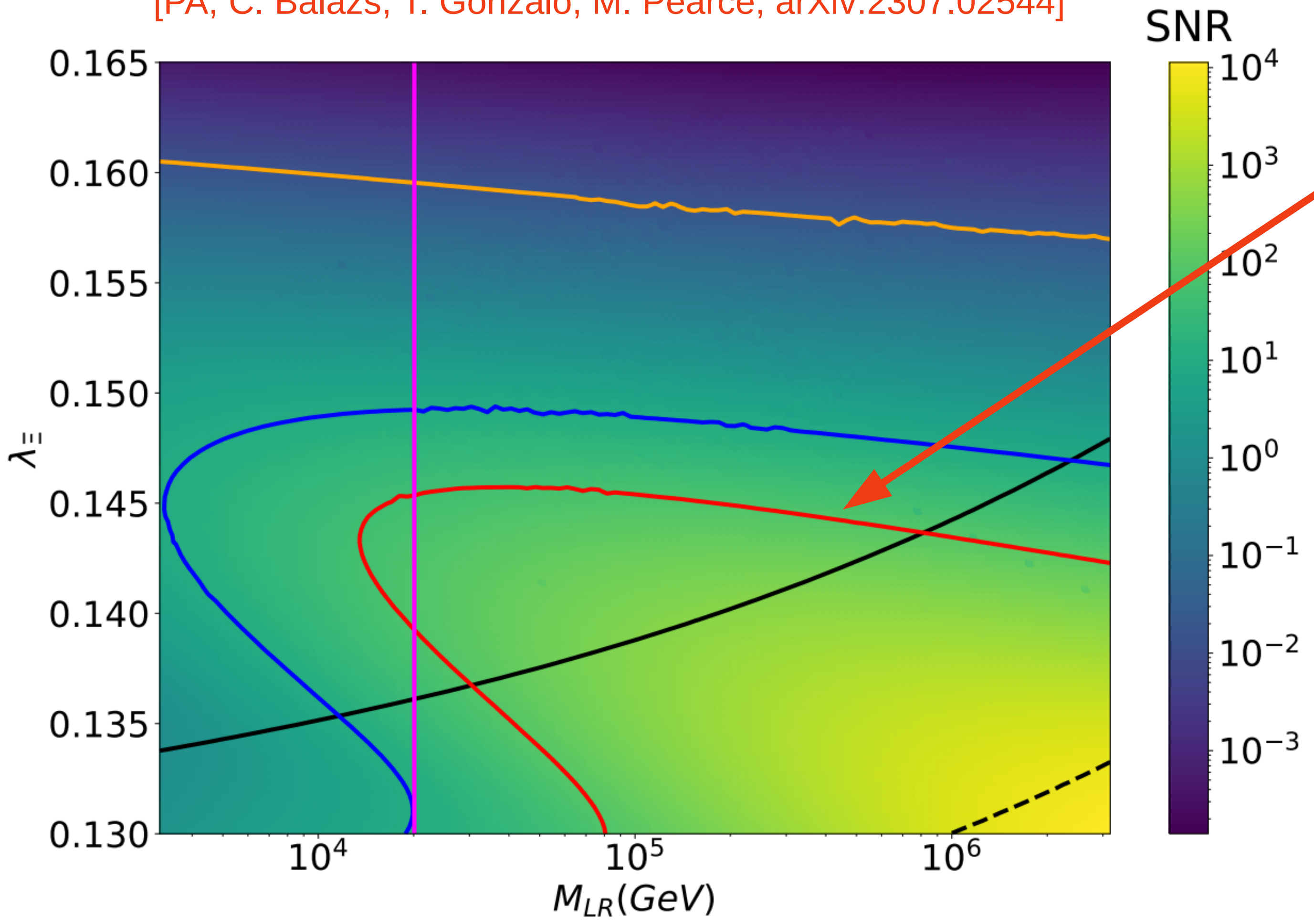
Peter Athron
and Andrew Fowlie

We are entering an era
where
precise GWs predictions matter

Precise GWs predictions matter

LIGO data already constrains well motivated Pati-Salam GUT models

[PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544]

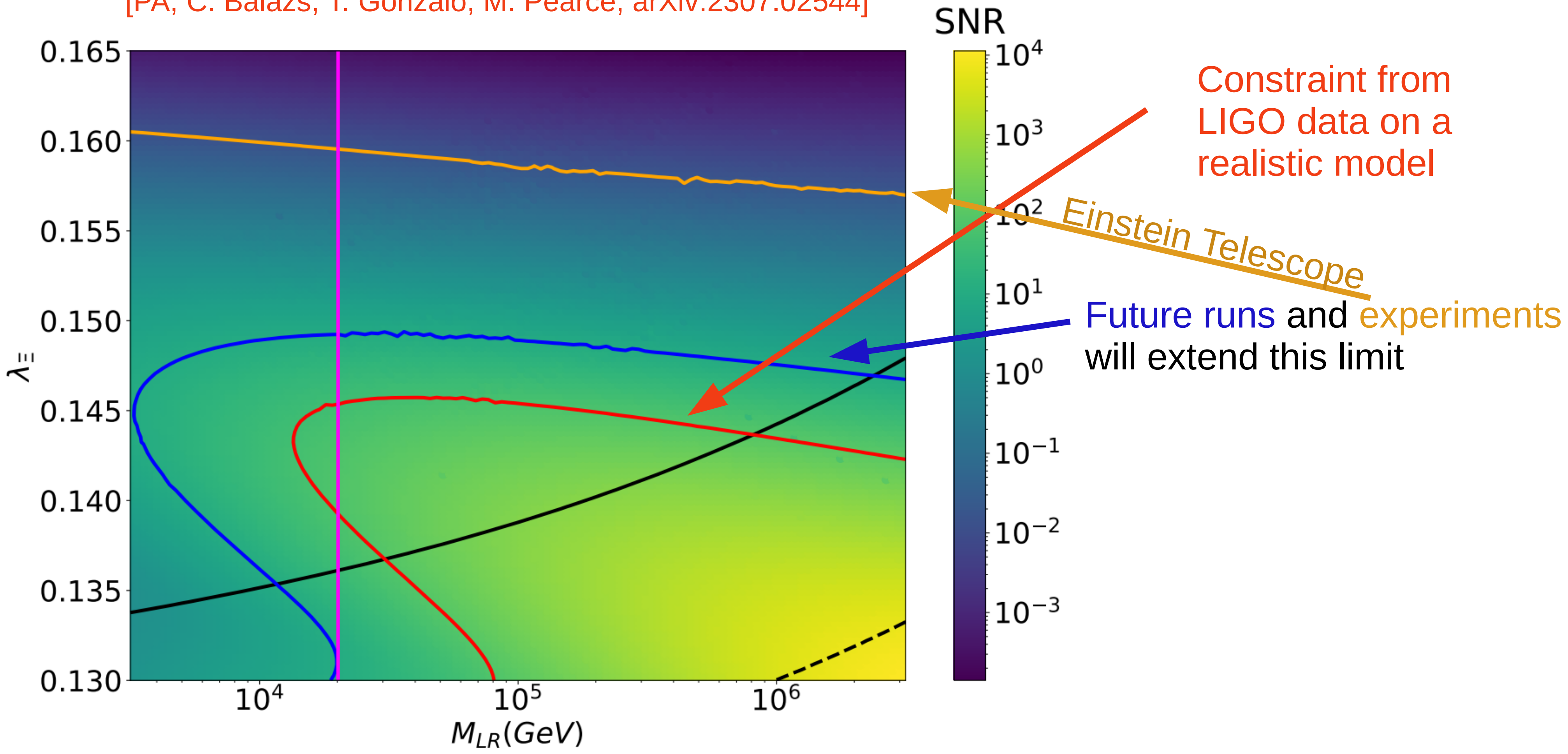


Constraint from LIGO data on a realistic model

Precise GWs predictions matter

LIGO data already constrains well motivated Pati-Salam GUT models

[PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544]



Big news last month:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments

IOP Publishing

physicsworld

Magazine | Latest | People | Impact |

ASTRONOMY AND SPACE | RESEARCH UPDATE

Pulsar timing irregularities reveals hidden gravitational-wave background

29 Jun 2023

Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

Big news last month:

A stochastic gravitational wave background has been observed

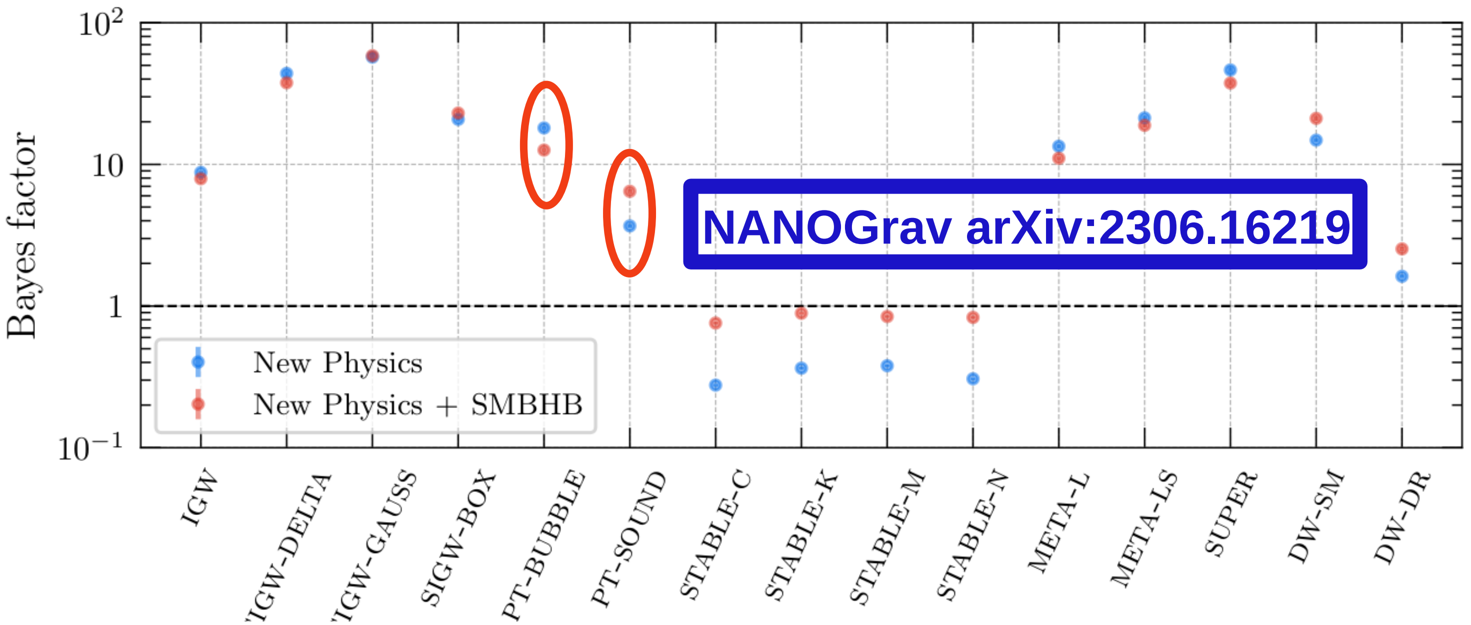
by multiple Pulsar Timing Arrays experiments

More excitement:

first order phase transitions

fit the data better (slightly)

than super massive black hole binaries



WARNING

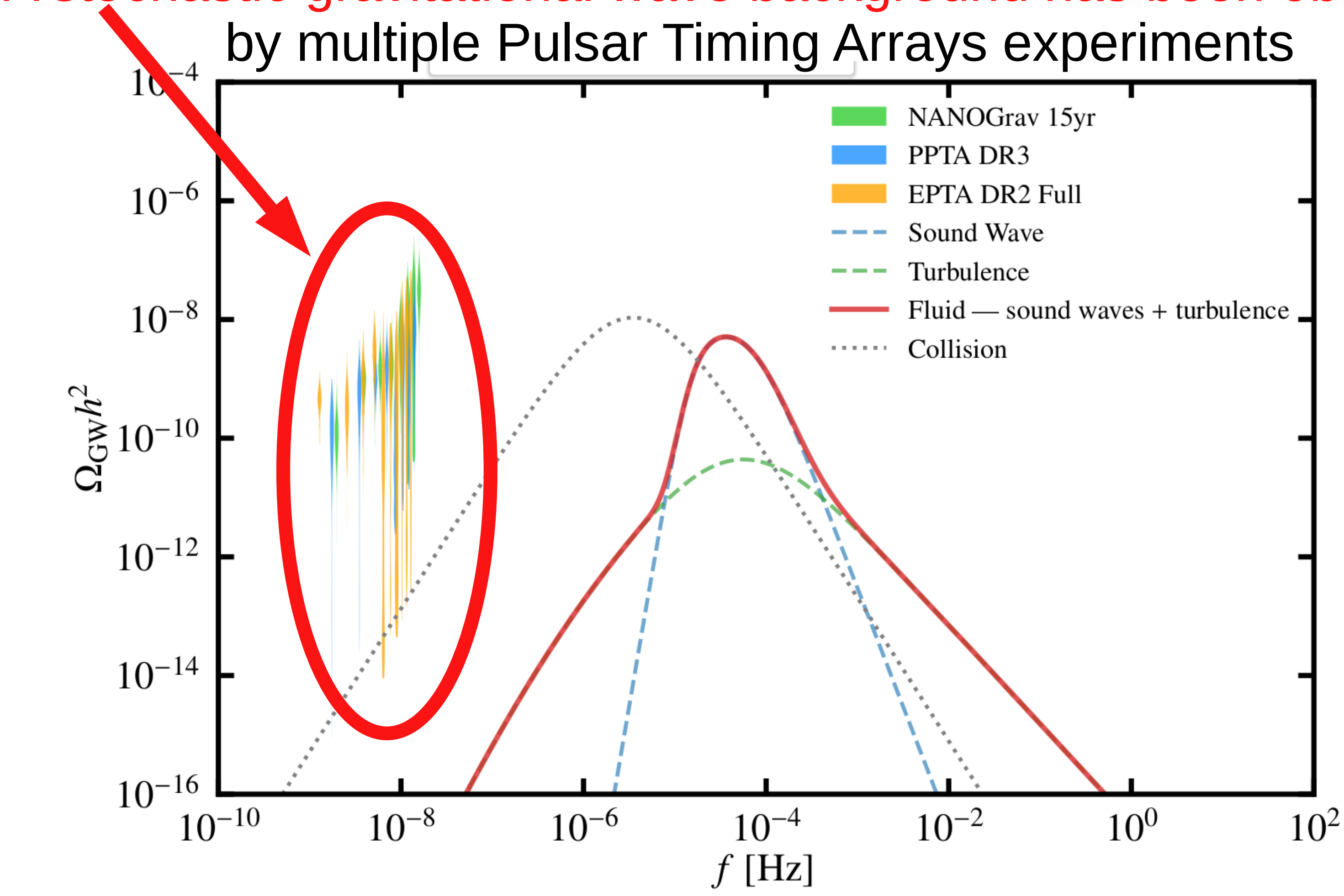
However for specific models these predictions require great care!

We looked at one model prominantly cited as able to to fit nHz signals from PTAs...

Big news last month:

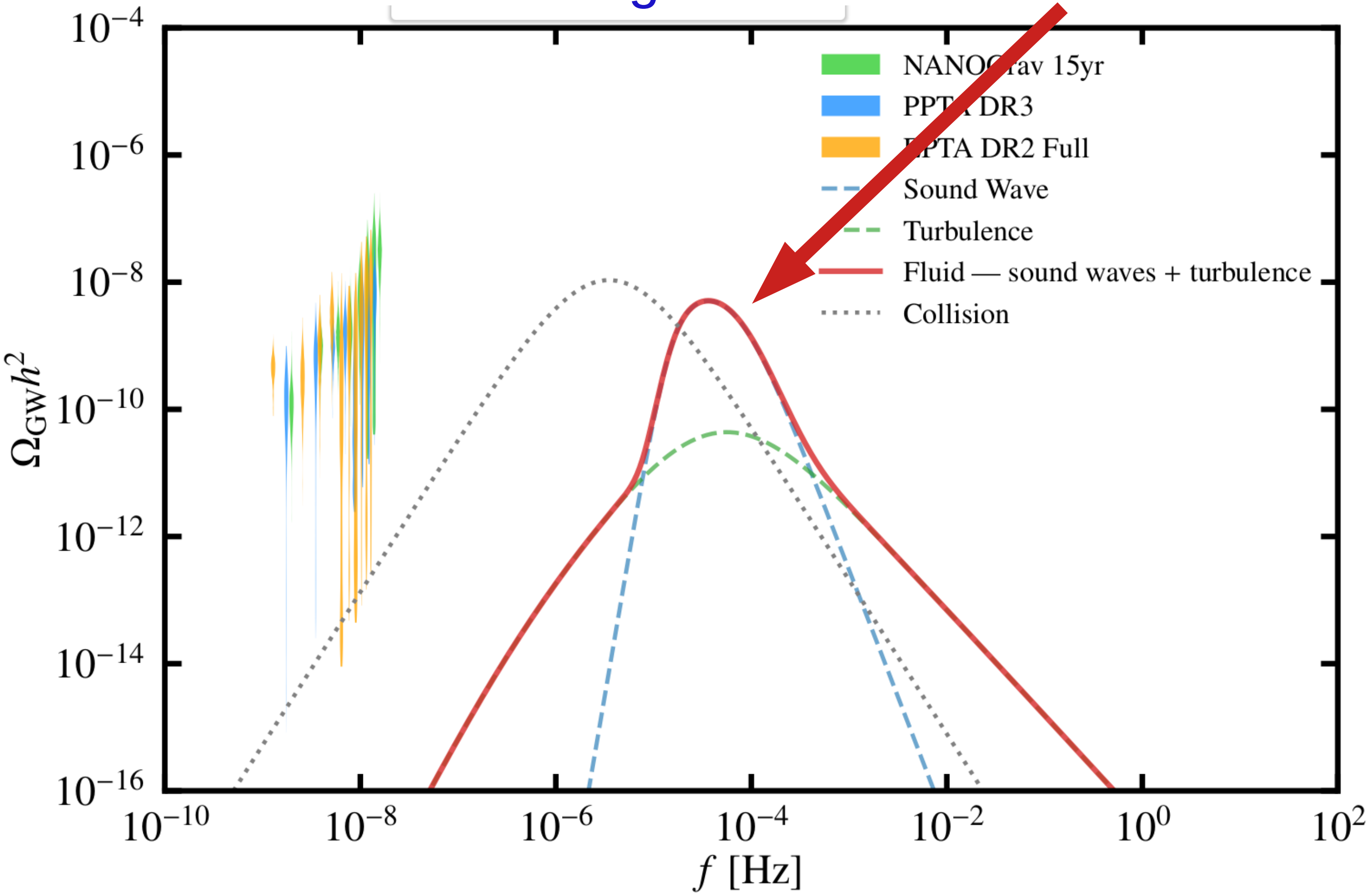
A stochastic gravitational wave background has been observed

by multiple Pulsar Timing Arrays experiments



But for the protypical model of supercooled PTs cited by NANOgrav as a possible explanation:

GWs can’t fit the signal with careful calculation



[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239]

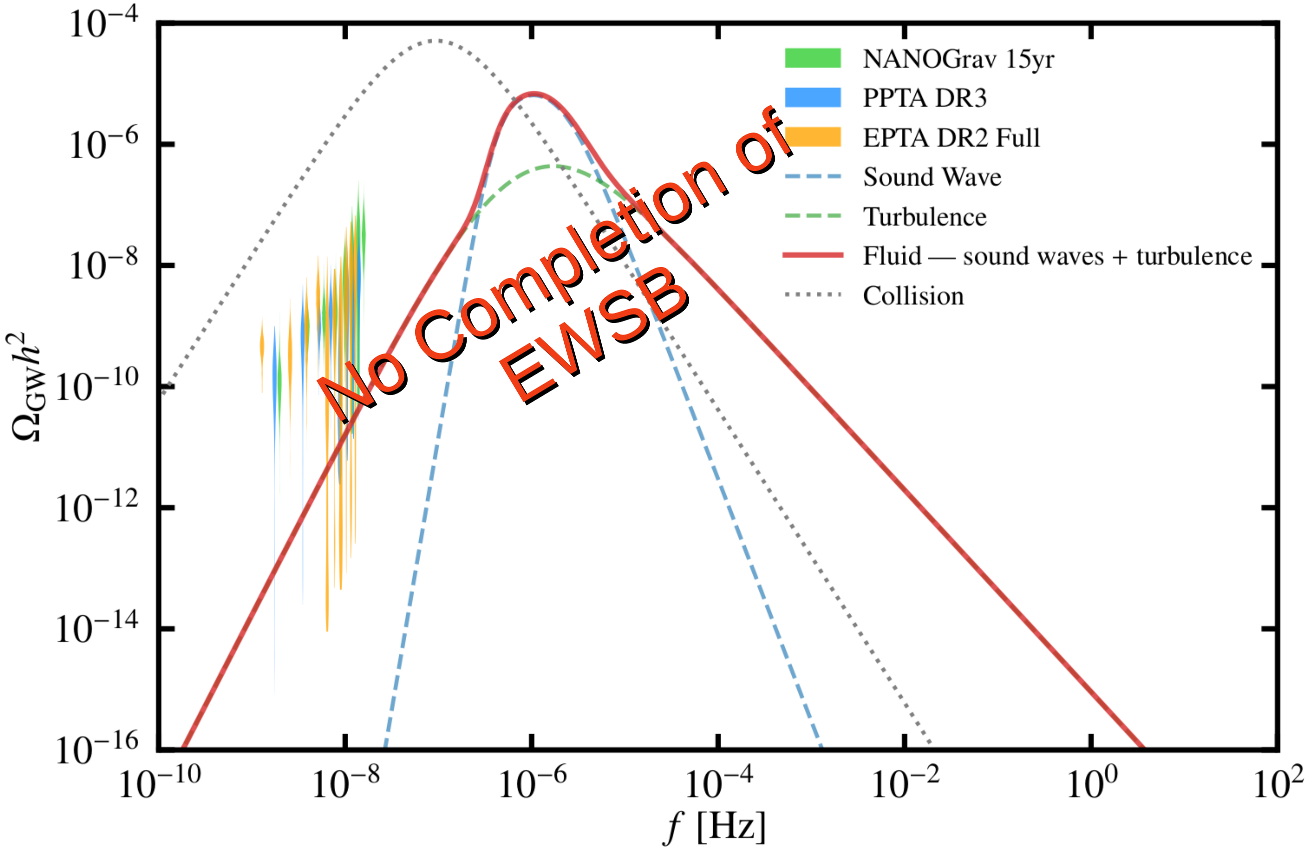
Big news last month:

A stochastic gravitational wave background has been observed

by multiple Pulsar Timing Arrays experiments

Larger signals are ruled out in this model because the PT does not complete

This is the first of the subtle effects I will discuss today!



Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

More careful studies checked that bubbles nucleate (one per Hubble volume)

$$N(T) = \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')}$$

$$N(T_n) = 1$$

Nucleation rate is computed from the bounce action, obtained from a bounce solver (e.g. BubbleProfiler, CosmoTransitions)

Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

More careful studies checked that bubbles nucleate (one per Hubble volume)

Assumption is that once they nucleate they will grow and fill the entire space

Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

More careful studies checked that bubbles nucleate (one per Hubble volume)

Assumption is that once they nucleate they will grow and fill the entire space

This is far from certain, slow transitions may never complete

Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

More careful studies checked that bubbles nucleate (one per Hubble volume)

Assumption is that once they nucleate they will grow and fill the entire space

This is far from certain, slow transitions may never complete

Much better to calculate the false vacuum fraction

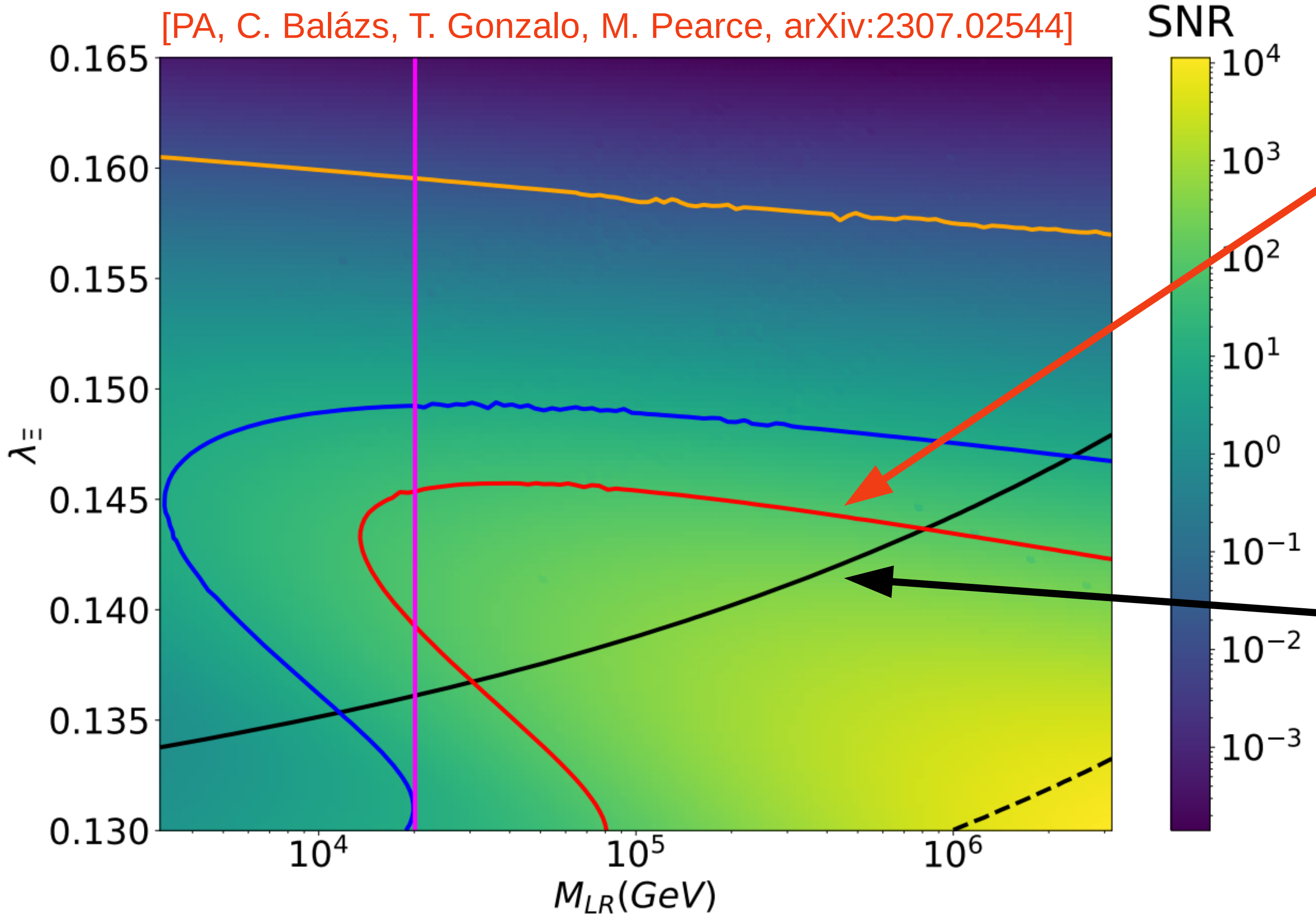
$$P_f(T) = \exp\left[-\frac{4\pi}{3} v_w^3 \int_T^{T_c} \frac{\Gamma(T')dT'}{T'^4 H(T')} \left(\int_T^{T'} \frac{dT''}{H(T'')}\right)^3\right]$$

Check this can be reduced to a sufficiently small value, e.g. $P_f(T_f) < 0.01$

LIGO data already constrains well motivated Pati-Salam GUT models

But checking completion is essential here too!

[PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544]



Constraint from LIGO data on a realistic model

Constraint from completion competes with LIGO constraint

Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

More careful studies checked that bubbles nucleate (one per Hubble volume)

Assumption is that once they nucleate they will grow and fill the entire space

This is far from certain, slow transitions may never complete

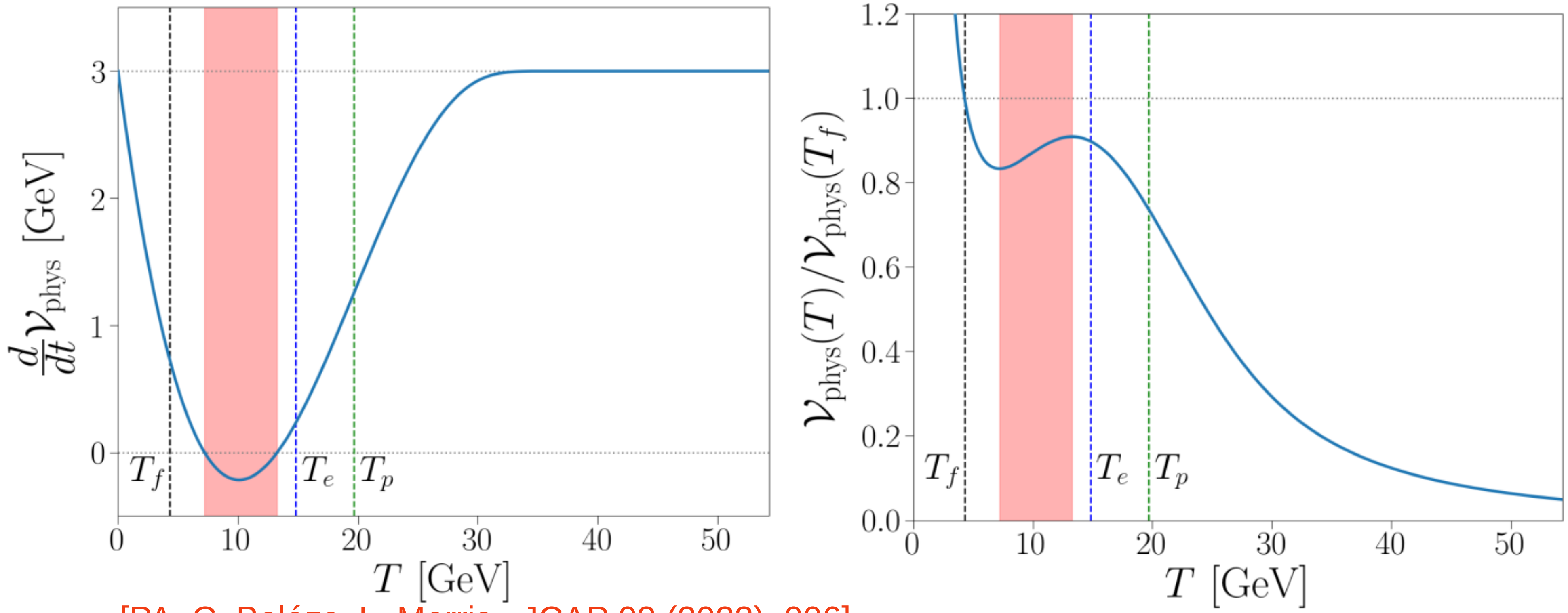
Much better to calculate the false vacuum fraction

$$P_f(T) = \exp\left[-\frac{4\pi}{3} v_w^3 \int_T^{T_c} \frac{\Gamma(T')dT'}{T'^4 H(T')} \left(\int_T^{T'} \frac{dT''}{H(T'')}\right)^3\right]$$

Check this can be reduced to a sufficiently small value, e.g. $P_f(T_f) < 0.01$

Warning: even this may not be enough to guarantee completion since space between the bubbles is also growing.

Tricky Effects from expansion of space



[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

To ensure it really completes also require: $$3 + T_p \left.\frac{\mathrm{d}\mathcal{V}_t^{\rm ext}}{\mathrm{d}T}\right|_{T_p} < 0,$$

Percolation temperature: T_p: $P_f(T_p) = 0.71$

Gravitational wave amplitude and frequency

Each component of the amplitude $h^2\Omega_{\text{GW-tot}} = h^2\Omega_{\text{coll}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{turb}}$

is defined in terms of the energy density ρ via $\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_{\text{tot}}}\frac{d\rho_{\text{GW}}}{d\ln f}$

$$\Omega_{\text{GW}}(f) \propto R_\Omega\, K^n L^m$$

redshift factor

Kinetic energy fraction

Length scale related to duration

Redshift factor to account for redshifting from the transition time to today

Kinetic energy fraction is the energy that can be available to source GWs

Length scale that is sensitive to the lifetime of the source

Implicit dependence of the transition temperature and the velocity the bubble walls expand also influences things

Powers depend on the source and the modelling, coefficients found in simulation/calculations

Temperature dependence

Many studies have nucleation temperature as the reference temperature

But the nucleation temperature is not really connceted to bubble collisions

Percolation is directly defined in terms of contact between bubbles

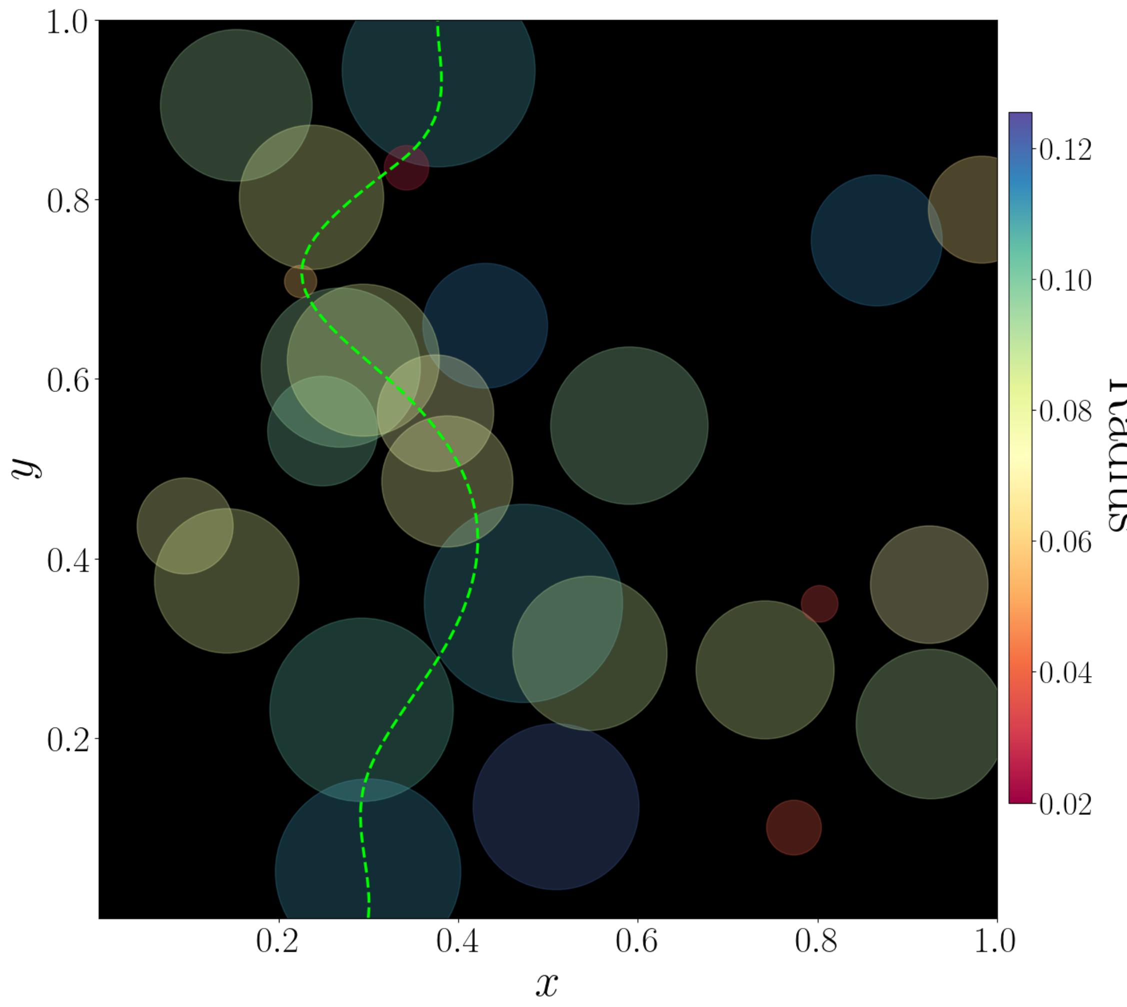
Percolation temperature

T_p: $P_f(T_p) = 0.71$

- Percolation is when there is a connected path between bubbles across the space
- Strongly linked to bubble collisions
- Good choice for a temperature at which to evaluate the GWs spectrum

Example from simple simulation →

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

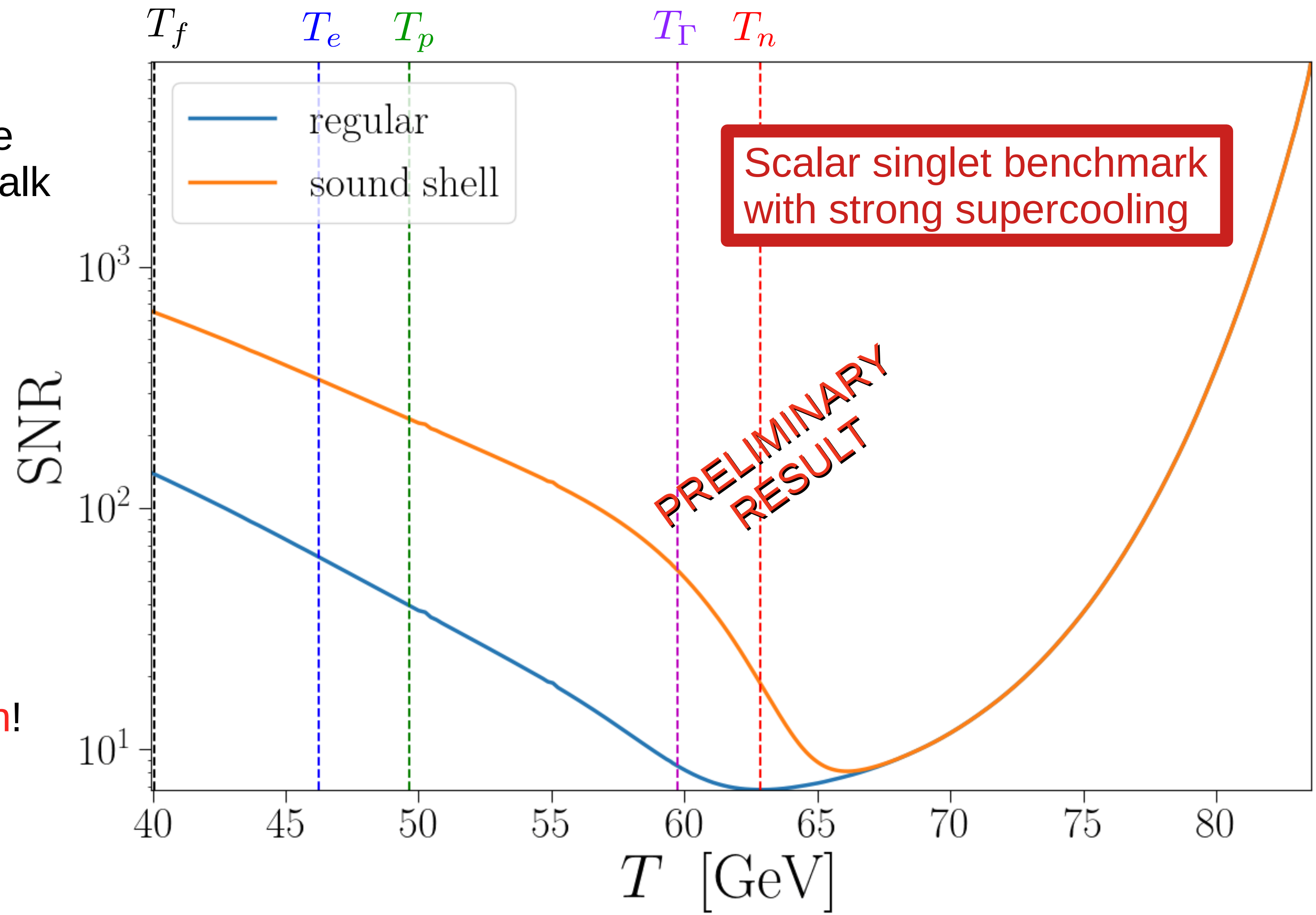


Temperature dependence

Point taken from same paper but I just made the GW predictions for this talk

Slow transition, reaches nucleation well before percolation, and then completion

Significant difference between SNR at percolation vs nucleation!



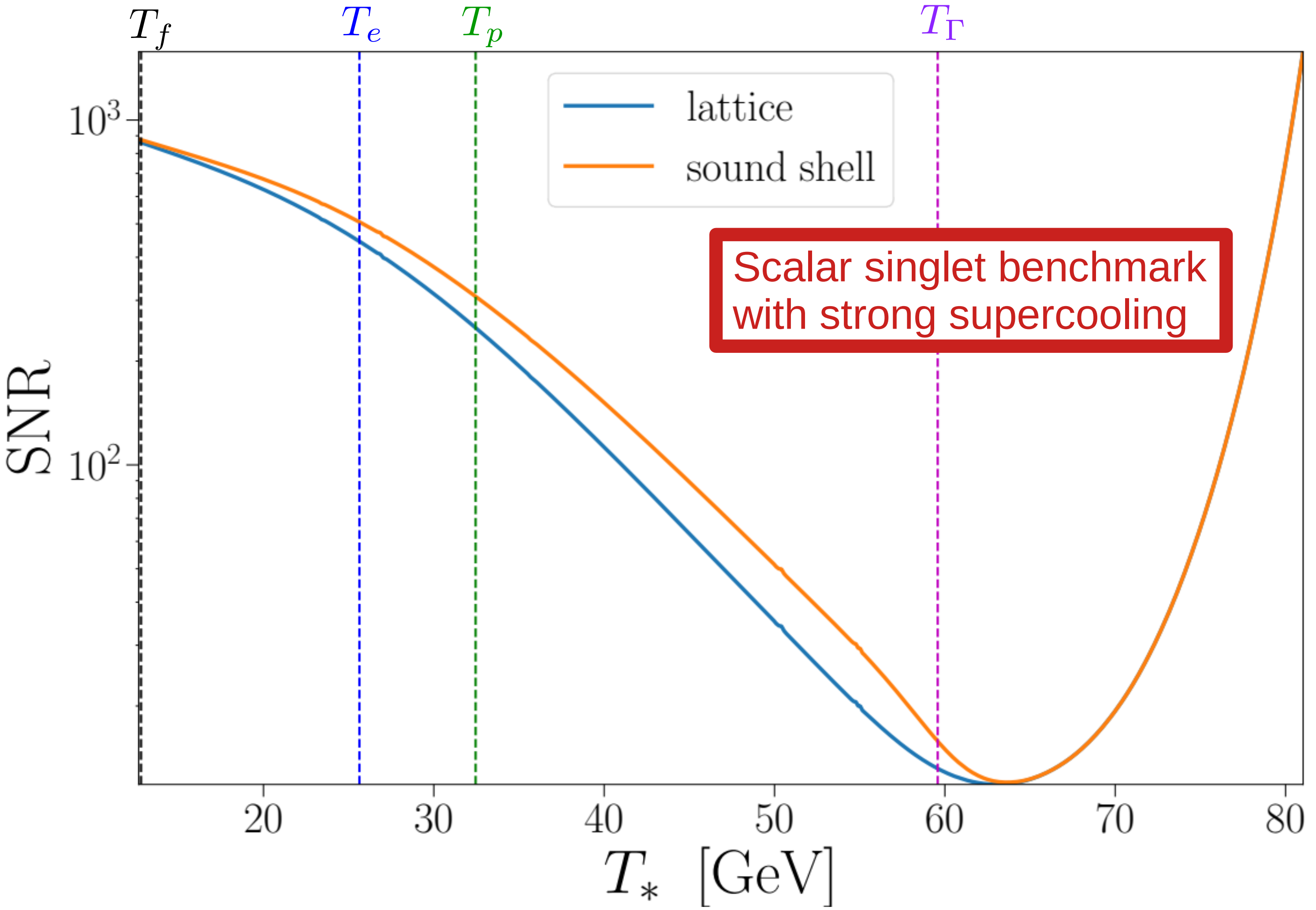
Temperature dependence

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Point taken from new paper on GW Uncertainties & approximation (BP2, plot simplified)

Another slow transition but percolates and completes before nucleation

Again SNR varies by more than an order of magnitude!



Temperature dependence

Many studies have nucleation temperature as the reference temperature

But the nucleation temperature is not really connceted to bubble collisions

Percolation is directly defined in terms of contact between bubbles

We recommend use of percolation tempearture based on this reasonable argument

But its still not established what the “right” reference temperature is or what the uncertainty using any single reference temperature is.

These issues could be probed with a hydrodynamic simulation perfomed alongside an analysis of the false vacuum fraction and thermal parameters.

This could help bridge the gap between lattice and non-lattice studies.

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = H T_* \left.\frac{{\rm d}S}{{\rm d}T}\right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\text{sep}}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\text{sep}} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = H T_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\text{sep}}(T_p)$

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = H T_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\rm sep}(T_p)$

Kinetic energy fraction (K):

Common approach $K = \frac{\kappa \alpha_i}{1+\alpha_i}$

Efficiency coefficient

~ energy released by PT

Time scales / length scales:

$$n_b(T) = T^3\int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}}\frac{v_w}{\beta} \qquad \beta = HT_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\rm sep}(T_p)$

Kinetic energy fraction (K):

Common approach $K = \frac{\kappa\alpha_i}{1+\alpha_i}$

Efficiency coefficient

~ energy released by PT

$$\alpha_\rho = \frac{\Delta\rho}{\rho_R}$$

"Latent heat"

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = HT_* \left. \frac{\mathrm{d}S}{\mathrm{d}T} \right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\rm sep}(T_p)$

Kinetic energy fraction (K):

Common approach $K = \frac{\kappa \alpha_i}{1+\alpha_i}$

Efficiency coefficient

~ energy released by PT

$$\alpha_\rho = \frac{\Delta\rho}{\rho_R}$$

"Latent heat"

$$\alpha_p = \frac{\Delta p}{\rho_R}$$

pressure

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = H T_* \left. \frac{\mathrm{d}S}{\mathrm{d}T} \right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\rm sep}(T_p)$

Kinetic energy fraction (K):

Common approach $K = \frac{\kappa \alpha_i}{1+\alpha_i}$

Efficiency coefficient

~ energy released by PT

overestimate

$$\alpha_\rho = \frac{\Delta\rho}{\rho_R}$$

"Latent heat"

$$\alpha_p = \frac{\Delta p}{\rho_R}$$

underestimate

pressure

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = HT_* \left. \frac{{\rm d}S}{{\rm d}T} \right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\rm sep}(T_p)$

Kinetic energy fraction (K):

Common approach $K = \frac{\kappa \alpha_i}{1+\alpha_i}$

Efficiency coefficient

~ energy released by PT

overestimate

$$\alpha_\rho = \frac{\Delta \rho}{\rho_R} \quad > \quad \alpha_\theta = \frac{\Delta(V - \frac{1}{4} T \frac{\partial V}{\partial T})}{\rho_R} \quad > \quad \alpha_p = \frac{\Delta p}{\rho_R}$$

underestimate

"Latent heat"

Trace anomaly $\theta = \frac{1}{4}(\rho - 3p)$

pressure

Time scales / length scales:

$$n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

Lattice simulations use the mean bubble separation $R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}}$

But very frequently GW spectra are rewritten assuming exponential nucleation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta} \qquad \beta = H T_* \left. \frac{{\rm d}S}{{\rm d}T} \right|_{T=T_*}$$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error compared to using $R_{\rm sep}(T_p)$

Kinetic energy fraction (K):

Common approach $K = \frac{\kappa \alpha_i}{1+\alpha_i}$ ~ energy released by PT

Efficiency coefficient

overestimate

underestimate

$$\alpha_\rho = \frac{\Delta \rho}{\rho_R} \quad > \quad \alpha_\theta = \frac{\Delta(V - \frac{1}{4} T \frac{\partial V}{\partial T})}{\rho_R} \quad > \quad \alpha_p = \frac{\Delta p}{\rho_R}$$

"Latent heat" Trace anomaly $\theta = \frac{1}{4}(\rho - 3p)$ pressure

Very new improvement $K = \frac{\bar{\theta}_f(T_*) - \bar{\theta}_t(T_*)}{\rho_{\rm tot}(T_*)} \kappa_{\bar{\theta}}(\alpha_{\bar{\theta}}(T_*), c_{s,f}(T_*)).$ $\bar{\theta} = \frac{1}{4}(\rho - \frac{p}{c_{s,t}^2})$

[F. Giese, T. Konstandin and J. van de Vis, JCAP 07 (2020) 057,(+K. Schmitz), JCAP 01 (2021) 072]

Numerical Packages

The good news is many of these issues can be avoided with careful numerical implementations

We are developing a set of numerical packages for PhaseTransitions:
PhaseTracer, BubbleProfiler and...

TransitionSolver is designed to treat nucleation and Gws as well as can feasiby be done in BSM studies

TransitionSolver finds possible FOPTs, checks they complete, computes thermal parameters and gravitational wave specra as well as we are able.

→ v1 Release is imminent, ETA by end of ~~summer~~ winter 2023

→ Future releases (v2) will automate effective potential,
Combine with PhaseTracer 2 / BubbleProfiler 2
link to DRalgo for best feasible handing of effective potential as well!

Conclusions

- Very exciting recent results indicate we have entered an era where GW experiments have sensitivity to SGBG from BSM physics
- Now it’s very important to do calculations as carefully as possible – Many issues:
  - * Effective potential – IR divergences, scale & gauge dependence
  - * Vacuum Decay – bounce, double counting, and prefactor
  - * Completion of the Phase Transition
  - * Reference Temperature dependence of GW predictions.
  - * Thermal parameters - kinetic energy & length scales (& bubble wall vlocity)
- It’s very important that the theory community takes this seriously and BSM predictions are done as well as possible, as well as improving methods and understanding of uncertainties.
- We hope our review helps:

  PA, Csaba Balazs, AF, Lachlan Morris, Lei Wu, https://arxiv.org/abs/2305.02357, Invited review for Progress in Particle and Nuclear Physics

The END

Thanks for listening!

Comparison of predictions for a weakly supercooled point

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Variation	$h^2\Omega_{\mathrm{sw}}^{\mathrm{lat}}$ $(\times10^{-13})$	$h^2\Omega_{\mathrm{sw}}^{\mathrm{ss}}$ $(\times10^{-14})$	$f_{\mathrm{sw}}^{\mathrm{lat}}$ $(\times10^{-5})$	$f_{\mathrm{sw}}^{\mathrm{ss}}$ $(\times10^{-4})$	$h^2\Omega_{\mathrm{turb}}$ $(\times10^{-16})$	f_{turb} $(\times10^{-5})$	$\mathrm{SNR}_{\mathrm{lat}}$	$\mathrm{SNR}_{\mathrm{ss}}$	α $(\times10^{-2})$	κ	K $(\times10^{-3})$
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\mathrm{sep}}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in K: trace anomaly approximation is quite good in this case

Variation	$h^2\Omega_{\mathrm{sw}}^{\mathrm{lat}}$ ($\times 10^{-13}$)	$h^2\Omega_{\mathrm{sw}}^{\mathrm{ss}}$ ($\times 10^{-14}$)	$f_{\mathrm{sw}}^{\mathrm{lat}}$ ($\times 10^{-5}$)	$f_{\mathrm{sw}}^{\mathrm{ss}}$ ($\times 10^{-4}$)	$h^2\Omega_{\mathrm{turb}}$ ($\times 10^{-16}$)	f_{turb} ($\times 10^{-5}$)	$\mathrm{SNR}_{\mathrm{lat}}$	$\mathrm{SNR}_{\mathrm{ss}}$	α ($\times 10^{-2}$)	κ	K ($\times 10^{-3}$)
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\mathrm{sep}}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in sound wave amplitude (sound shell): latent heat (and pressure) variants give substanial differences

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ ($\times 10^{-13}$)	$h^2\Omega_{\rm sw}^{\rm ss}$ ($\times 10^{-14}$)	$f_{\rm sw}^{\rm lat}$ ($\times 10^{-5}$)	$f_{\rm sw}^{\rm ss}$ ($\times 10^{-4}$)	$h^2\Omega_{\rm turb}$ ($\times 10^{-16}$)	$f_{\rm turb}$ ($\times 10^{-5}$)	$\mathrm{SNR}_{\rm lat}$	$\mathrm{SNR}_{\rm ss}$	α ($\times 10^{-2}$)	κ	K ($\times 10^{-3}$)
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\rm sep}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in sound wave SNR: latent heat (and pressure) variants give substanial differences

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ $(\times 10^{-13})$	$h^2\Omega_{\rm sw}^{\rm ss}$ $(\times 10^{-14})$	$f_{\rm sw}^{\rm lat}$ $(\times 10^{-5})$	$f_{\rm sw}^{\rm ss}$ $(\times 10^{-4})$	$h^2\Omega_{\rm turb}$ $(\times 10^{-16})$	$f_{\rm turb}$ $(\times 10^{-5})$	$\rm SNR_{lat}$	$\rm SNR_{ss}$	α $(\times 10^{-2})$	κ	K $(\times 10^{-3})$
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\rm sep}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a strongly supercooled point

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

However the variation in K estimates is much smaller for strongly supercooled scenarios

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ $(\times 10^{-7})$	$h^2\Omega_{\rm sw}^{\rm ss}$ $(\times 10^{-8})$	$f_{\rm sw}^{\rm lat}$ $(\times 10^{-6})$	$f_{\rm sw}^{\rm ss}$ $(\times 10^{-6})$	$h^2\Omega_{\rm turb}$ $(\times 10^{-10})$	$f_{\rm turb}$ $(\times 10^{-6})$	$\mathrm{SNR_{lat}}$	$\mathrm{SNR_{ss}}$	α	κ	K
None	1.861	3.748	9.345	23.48	6.348	20.70	249.6	307.7	1.651	0.7175	0.4536
$T_* = T_e$	4.318	8.872	7.908	19.12	14.74	17.52	443.7	498.2	4.257	0.8422	0.6950
$T_* = T_f$	17.04	35.42	4.111	9.722	81.84	9.106	864.5	876.4	71.06	0.9831	0.9803
$R_{\rm sep}(\beta_V)$	1.193	2.402	12.80	32.17	3.394	28.36	222.6	356.9			
$K(\alpha(\theta))$	1.819	3.663			6.227		244.9	301.5	1.605	0.7269	0.4478
$K(\alpha(p))$	1.768	3.560			6.083		239.2	294.2	1.564	0.7269	0.4409
$K(\alpha(\rho))$	1.967	3.962			6.646		261.4	323.0	1.728	0.7383	0.4677
ϵ_2					17.95		700.0	742.2			
ϵ_3					0		18.36	130.9			
ϵ_4					288.4		11210	11230			

The duration affects the of the source of gravitational waves affects the GW signal a lot

This depends on the particle physics model

The duration can be related to a length scale and in hydrodynamical simulations of sound waves contributions the mean bubble separation is used:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatement

This can also be estimated by taylor expanding the bounce action

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots ,$$

1st order → explonential nucleation rate $\Gamma(t) = \Gamma(t_*)\exp(\beta(t-t_*)),$

$$\beta = -\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} = HT_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Widely used to replace mean bubble separation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta}$$

Rough approximation

The duration affects the of the source of gravitational waves affects the GW signal a lot

This depends on the particle physics model

The duration can be related to a length scale and in hydrodynamical simulations of sound waves contributions the mean bubble separation is used:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatement

This can also be estimated by taylor expanding the bounce action

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} (t - t_*) + \frac{1}{2} \left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*} (t - t_*)^2 + \cdots ,$$

1st order ⟶ explonential nucleation rate $\Gamma(t) = \Gamma(t_*) \exp(\beta(t - t_*)),$

$$\beta = -\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} = H T_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Sometimes can't even use β :
If Γ reaches a maxiumum
$\Rightarrow \beta < 0$ after or tiny close to maximum

The duration affects the of the source of gravitational waves affects the GW signal a lot

This depends on the particle physics model

The duration can be related to a length scale and in hydrodynamical simulations of sound waves contributions the mean bubble separation is used:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatement

This can also be estimated by taylor expanding the bounce action

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots\,,$$

2nd order → Gaussian nucleation rate $\Gamma(t) = \Gamma(t_*) \exp\left(\frac{\beta_{\rm V}^2}{2}(t-t_*)^2\right),$

$$\beta_{\rm V} = \sqrt{\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_\Gamma}}$$

Can be used to replace mean bubble separation

$$R_{\rm sep} = \left(\sqrt{2\pi}\frac{\Gamma(T_\Gamma)}{\beta_{\rm V}}\right)^{-\frac{1}{3}}$$

Rough approximation

The mean bubble separation varies a lot with temperature

Should not be used until $T \approx T_p$

For fast transitions

Estimating this with $\beta(T_p)$ GW amp. falls by factor 2 (larger variation in SNR)
Worse if using $\beta(T_n)$ as is standard practise

Mean bubble radius is more stable and $\beta(T)$ tracks this better.

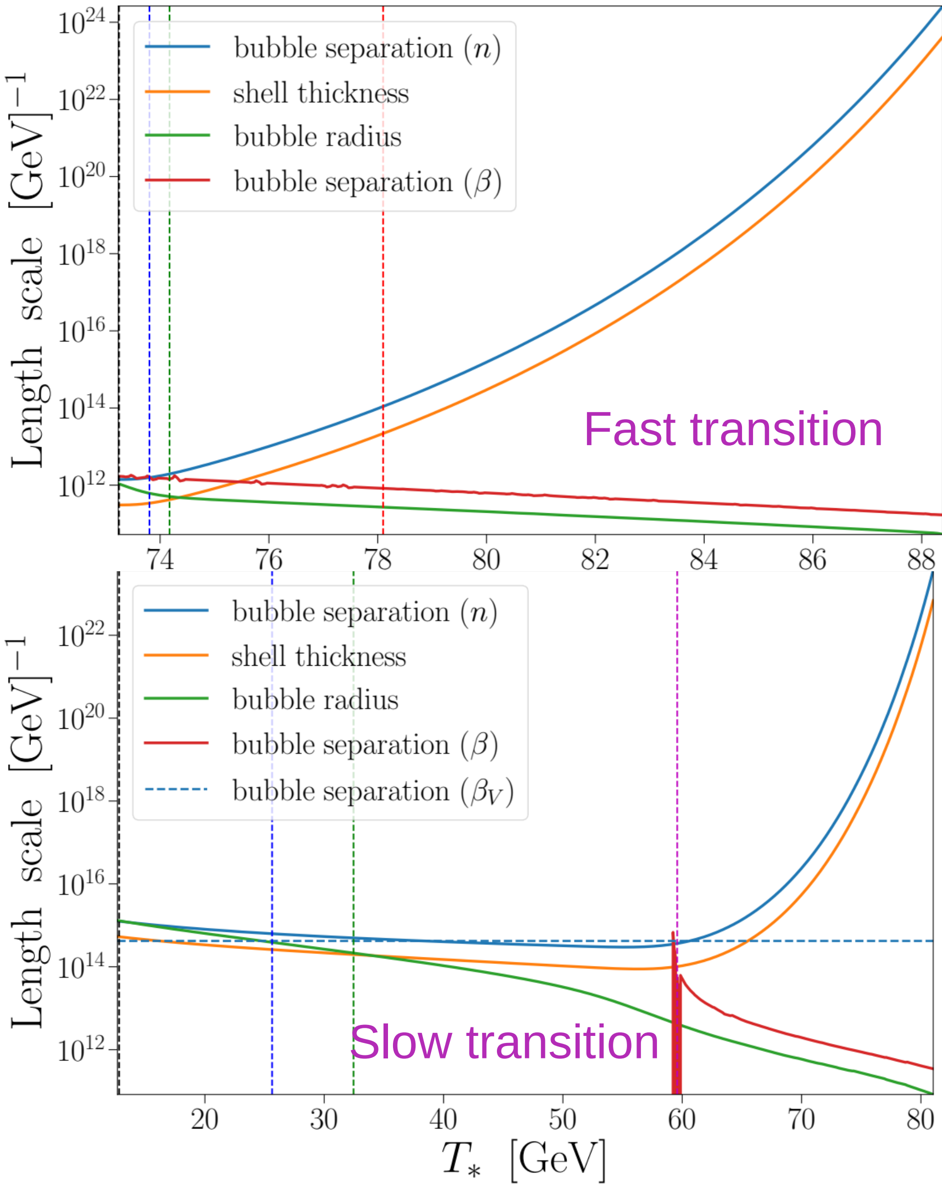
For slow transitions

Mean bubble radius varies more as bubbles have longer to grow.

Using $\beta(T_p)$ makes no sense below T_Γ orders of magnitude errors above

β_V gives a factor 1.5 drop in GW amplitide

[PA, L. Morris, Z. Xu, arXiv:2309.05474]



The mean bubble separation varies a lot with temperature

Should not be used until $T \approx T_p$

For fast transitions

Estimating this with $\beta(T_p)$ GW amp. falls by factor 2 (larger variation in SNR)
Worse if using $\beta(T_n)$ as is standard practise

Mean bubble radius is more stable and $\beta(T)$ tracks this better.

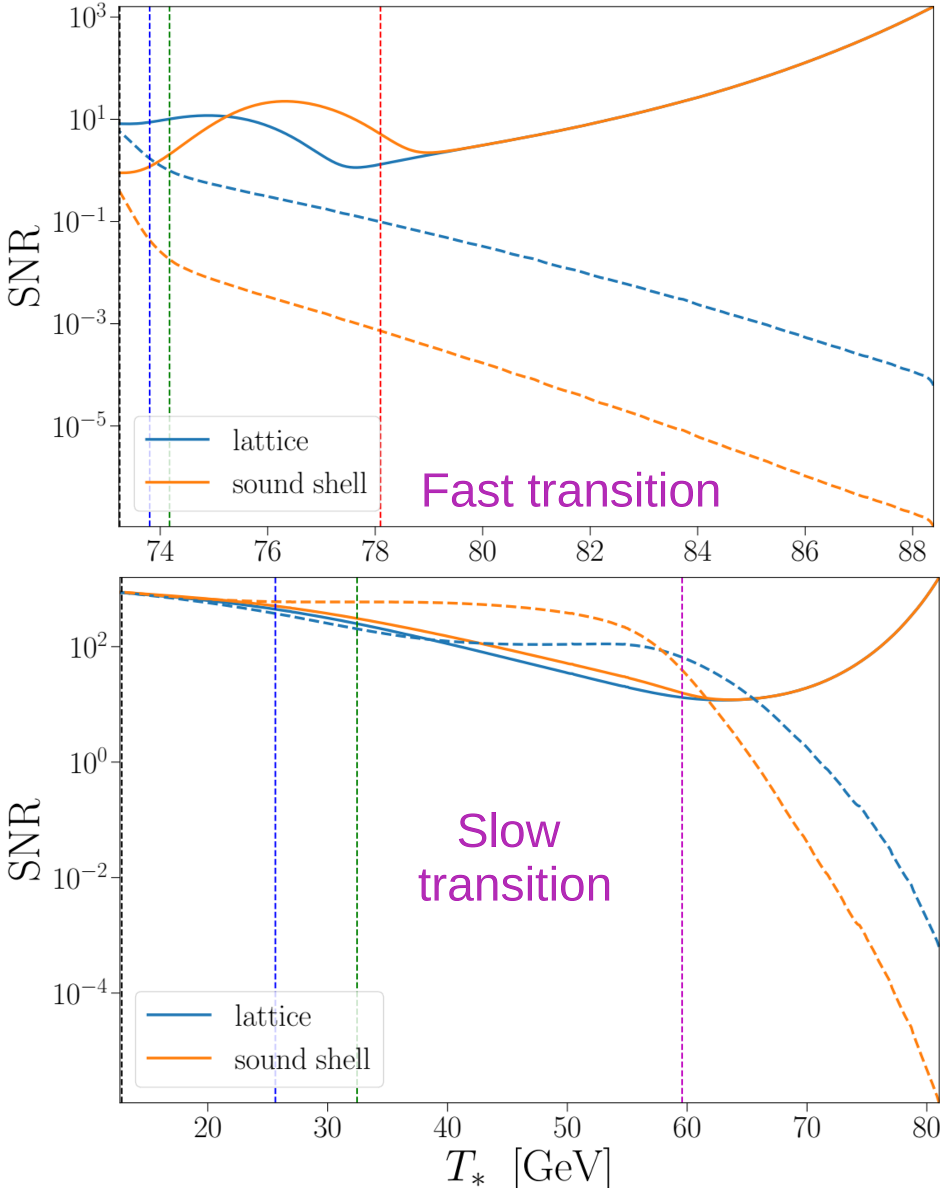
For slow transitions

Mean bubble radius varies more as bubbles have longer to grow.

Using $\beta(T_p)$ makes no sense below T_Γ orders of magnitude errors above

β_V gives a factor 1.5 drop in GW amplitide

[PA, L. Morris, Z. Xu, arXiv:2309.05474]



False vacuum fraction → several important milestone temperatures

Completion temperature: $T_f\!: \; P_f(T_f) = 0.01$

Percolation temperature: $T_p\!: \; P_f(T_p) = 0.71$

e-folding temperature: $T_e\!: \; P_f(T_e) = 1/e$

The nucleation temperature is instead given by $N(T_n) = 1$

$$N(T) = \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')}$$

The nucleation temperature is frequently used for evaluating GW signals

but it may not exist…

and for slow transitions is decouples from the other the other temperatures

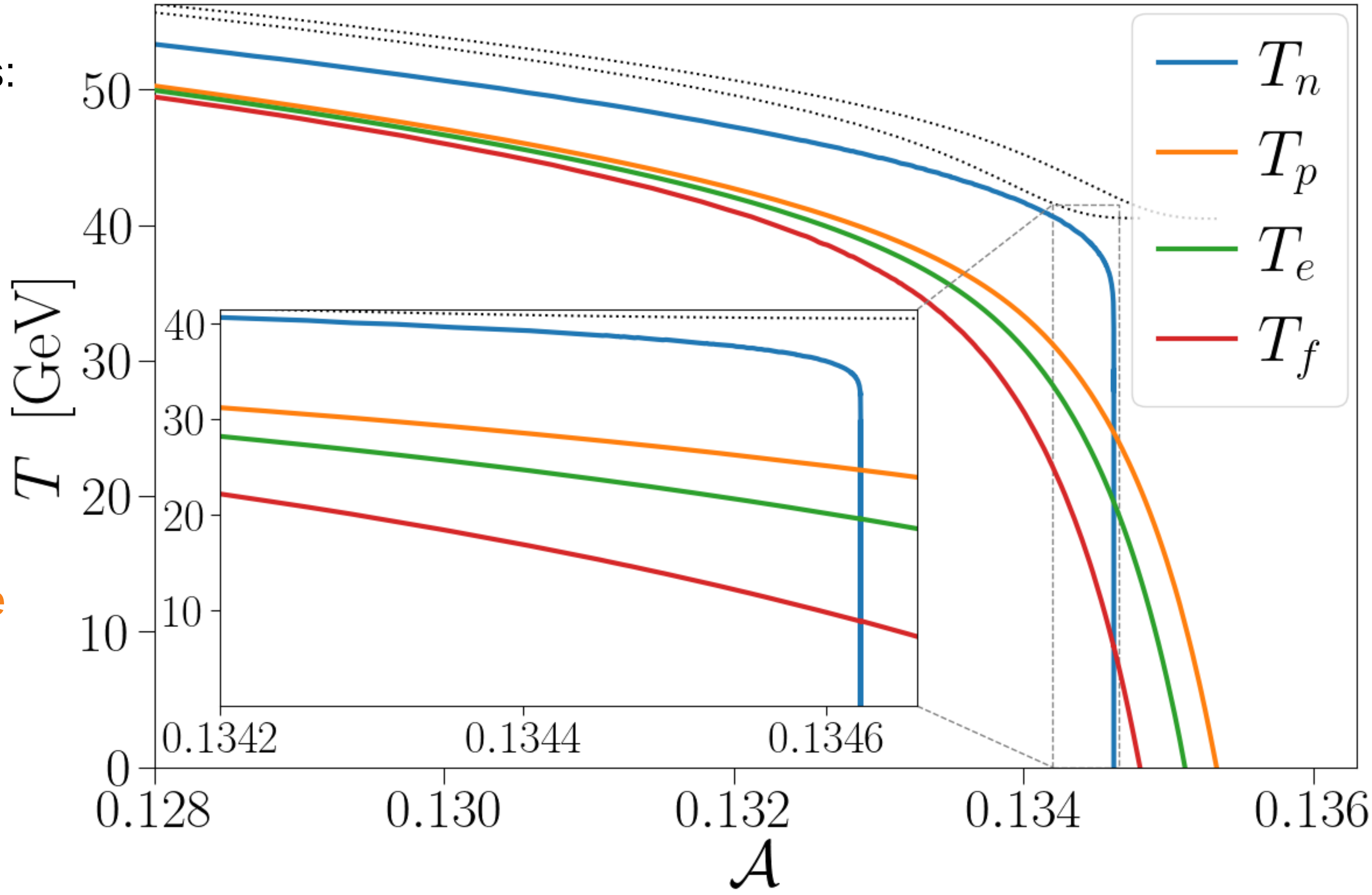
Milestone temperatures

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

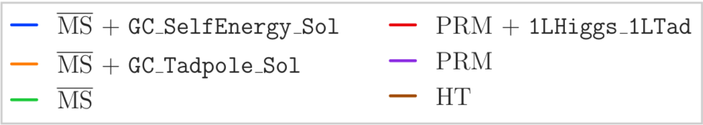
Nucleation temperature is:

- Not related to bubble collisions
- Not related to other temperatures
- May not even exist

Percolation temperature is a better choice for GWs

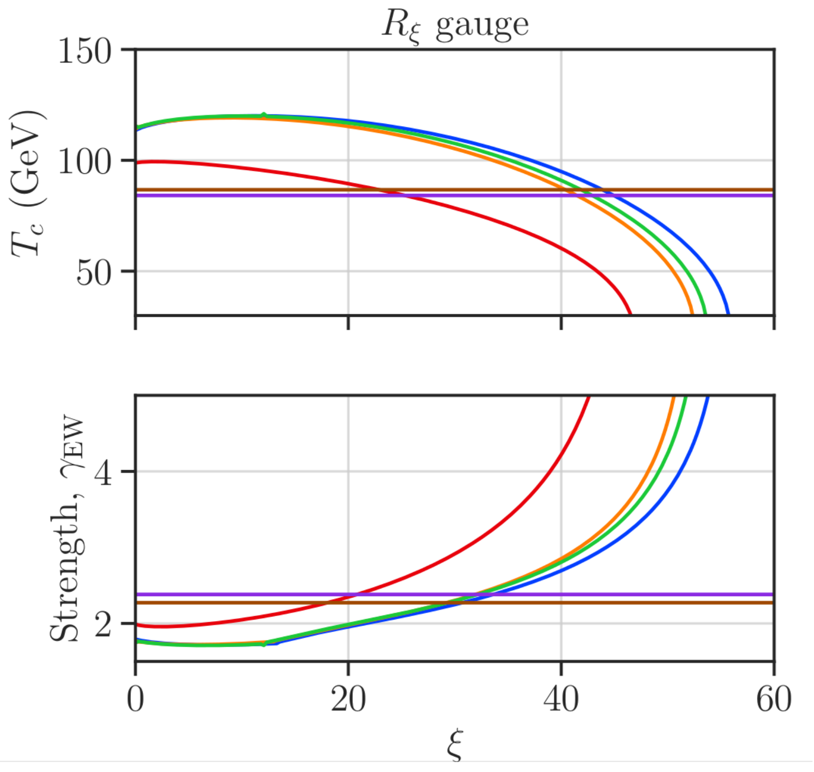


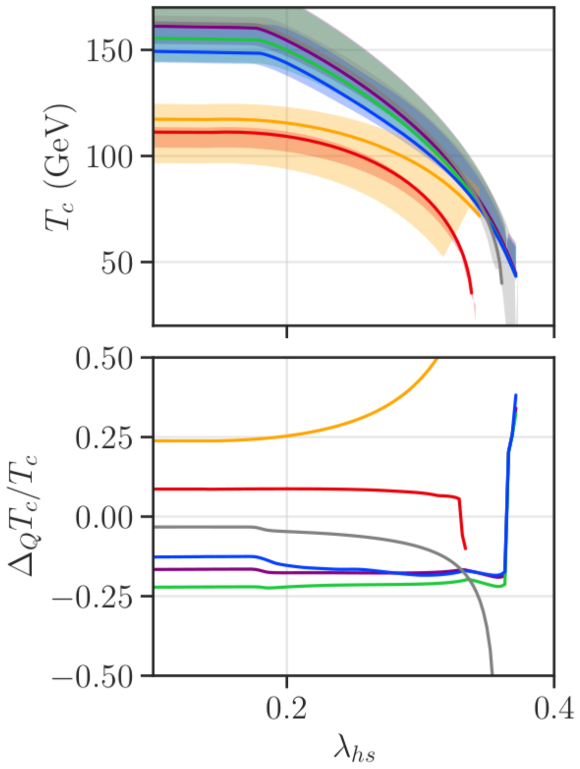
Effective Potential



Perturbative estimates of the effective potential can be tricky

Significant variance from gauge and renormalisation scale





Effective Potential

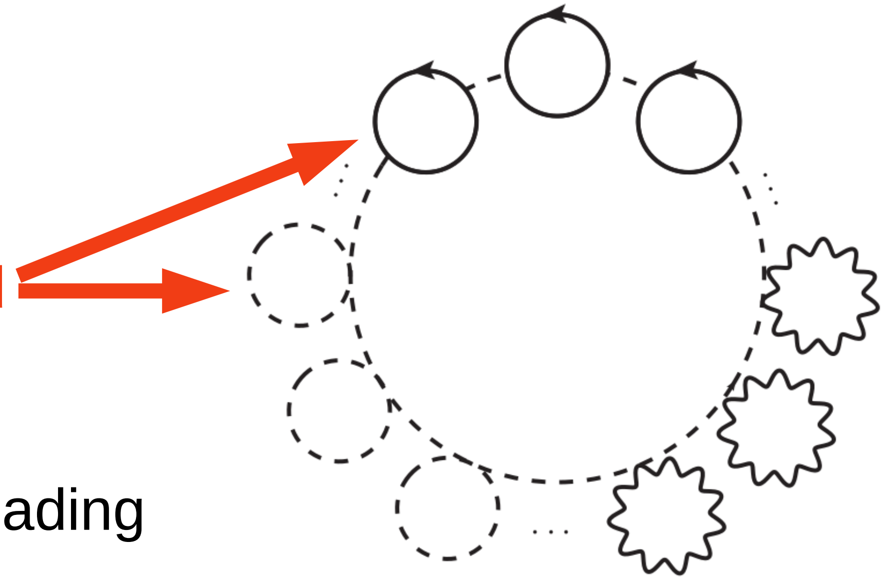
Perturbative estimates of the effective potential can be tricky

Significant variance from gauge and renormalisation scale

Resummation needed to to deal with high temperatures spoiling perturbativity

Daisy diagram with N-loops

Individual petals are inserted one-loop corrections



Resum daisy diagrams for leading order $\frac{T^2}{m^2}$

Effective Potential

- Better resummation by constructing a 3DEFT often called Dimensional Reductions (see e.g. D.Croon, O.Gould, P.Schicho, T.Tenkanen and G.White JHEP 04 (2021) 055)

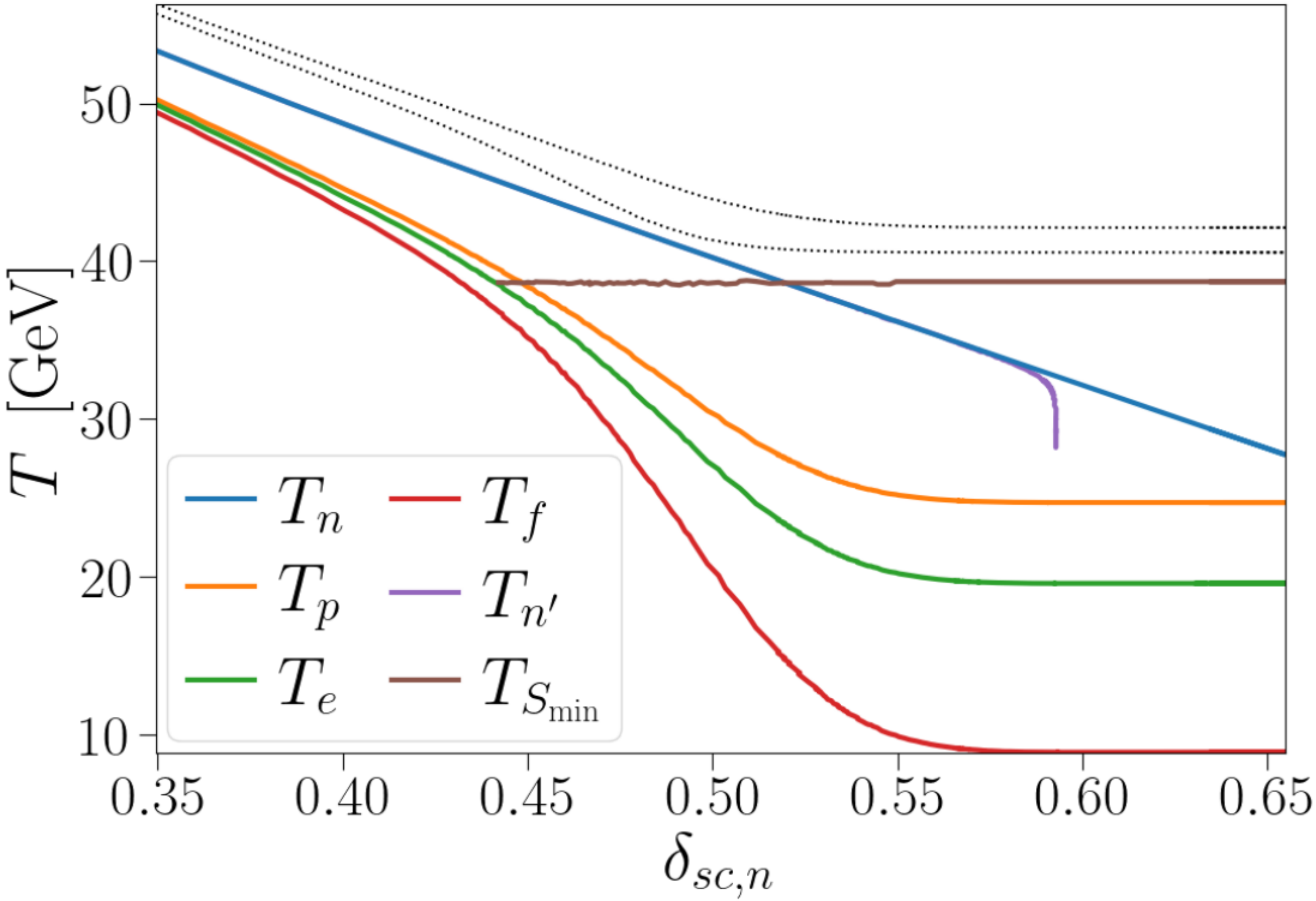
- This can be done via automation of DRalgo for best feasible handing of effective potential as well!

- Gold standard is really to do things non-peturbatively on the lattice

- Not really feasible for scans in BSM models with many parameters

- But can be done for most exciting cases.

Temperature dependence



GWs from First Order Phase Transitions

There are many subtleties and challenges in calculating GW spectra from cosmological PTs

For example this makes it easy to mistakenly predict a given model explains the data, get the wrong projections for future experiments or miss correlated features/constraints

I will discuss some subtle issues from JCAP 03 (2023), 006 and our review arxiv:2305.02357

- Nucleation is not enough – check PT completes
- Temperature dependence is very important, using most relevant temperatures really matters
- Hidden assumptions and approximations in thermal parameters and fits to calculations or simulations of gravitatonal wave spectra
- Resummation and gauge invariance in the effective potential treatment

There are many other details I can't cover, see original papers for details

NNU · 南京师范大学

NANJING NORMAL UNIVERSITY

Gravitational waves from first order phase transitions

Peter Athron and Andrew Fowlie

(Nanjing Normal University)