Magnetic Monopole Theory and Experiment

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Intriduction and motivation

Maxwell equation

$$\nabla \cdot \vec{E} = \rho_{\mathrm{E}}$$

$$\nabla \cdot \vec{B} = \rho_{\mathrm{M}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{J}_{\mathrm{M}}$$

$$\nabla \times \vec{B} = -\frac{\partial \vec{E}}{\partial t} + \vec{J}_{\mathrm{E}}$$

$$\partial_\mu F^{\mu\nu} = j_e^\nu, \qquad \partial_\mu * F^{\mu\nu} = j_m^\nu$$

Does magnetic charge g exits? Replacement or Duality

$$e \rightarrow g \text{ or } 1/g$$

Dirac monopole

If magnetic monopole exits with charge g, the magnetic field can be shown as a form which is very similar to the electric field:

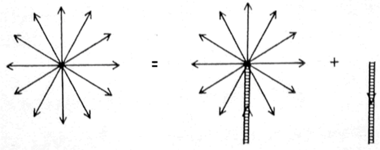
$$\vec{B} = \frac{g\,\hat{r}}{4\pi r^2}$$

but

$$\nabla \cdot \vec{B} \neq 0 \Rightarrow \vec{B} \neq \nabla \times \vec{A}$$

Solution: Dirac string

Add an infinetely small, infinitely extended solenoid field along z-axis



And choose proper vector potential at different region

$$\vec{A}_N = \frac{g}{4\pi r}\frac{(1-\cos\theta)}{\sin\theta}\hat{e}_\phi$$
$$\vec{A}_S = -\frac{g}{4\pi r}\frac{(1+\cos\theta)}{\sin\theta}\hat{e}_\phi$$
$$\vec{A}_N - \vec{A}_S = \nabla f \quad \Rightarrow f = \frac{g}{2\pi}\phi$$

$\Rightarrow \vec{B} = \nabla \times \vec{A}_N = \nabla \times \vec{A}_S$
Consider a particle (+e) is moving around the solenoid, and the particle interference fails to detect the solenoid if

$$\exp(i\mathcal{G}) = \exp\left(-ie\oint \vec{A}\cdot \mathrm{d}\vec{r}\right) = 1$$

$\Rightarrow$ Dirac quatization condition

$$eg = 2\pi N, N \in \mathbb{Z}$$

where N is the winding number

't Hooft-Polyakov Monopole (1974)

- Based on Georgi-Glashow model

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}D^\mu\Phi^a D_\mu\Phi^a - \frac{\lambda}{4}\left(\Phi^a\Phi^a - v^2\right)^2$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc}A^b_\mu A^c_\nu,\ D_\mu\Phi^a = \partial_\mu\Phi^a + e\epsilon^{abc}A^b_\mu\Phi^c$$

a=1,2,3 adjoint representatin of SO(3)

- After symmetry breaking $SO(3) \rightarrow U(1)$

$$\Phi^a = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \rightarrow \begin{cases} 2 & M_W = ev \\ 1 & M_H = \sqrt{2\lambda}v \end{cases}$$

- Energy

$$E = \int \mathrm{d}^3x \left[\frac{1}{2}\left(\vec{E^a}\cdot\vec{E^a} + \vec{B^a}\cdot\vec{B^a} + \Pi^a\Pi^a + \vec{D}\Phi^a\cdot\vec{D}\Phi^a\right) + V(\Phi)\right]$$

should be finite at r$\rightarrow \infty$

Solutions

Homotopy structure

$$\Phi : S^2_\infty \to S^2 = M_H = \{\Phi : V(\Phi) = 0\}$$

2nd homotopy group of $S^2 \Rightarrow \Pi_2(S^2) \simeq \mathbb{Z}$ We can find the solution (time-independent)

$$\Phi^a \xrightarrow[r\to\infty]{} v\frac{r^a}{r},\ A^a_\mu \xrightarrow[r\to\infty]{} -\frac{1}{e}\varepsilon^{\mu ab}\frac{r^b}{r^2},\ F_{\mu\nu} \equiv \frac{F^a_{\mu\nu}\Phi^a}{|\Phi|} - \frac{\varepsilon^{abc}\Phi^a\,(D_\mu\Phi)^b\,(D_\nu\Phi)^c}{e|\Phi|^3}$$

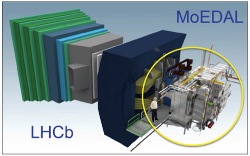
$$\Rightarrow F_{\mu\nu} = -\frac{1}{er^3}\epsilon_{\mu\nu a}r^a\ \mu,\nu \neq 0$$

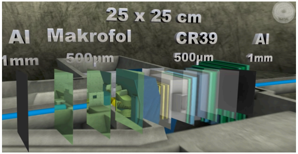
$$\vec{B} = \frac{\vec{r}}{er^3} \Rightarrow eg = 4\pi N?$$

In the fundamental 2 representation of SU(2). These would carry electric charge $\pm$e/2 for W.

MoEDAL Experiment

- Monopole and Exotics Detector at the LHC
- The 7th experiment at the Large Hadron Collider (LHC), shares LHC intersection point 8 with LHCb
- The most important motivation for the MoEDAL experiment is to pursue the quest for magnetic monopoles and dyons at LHC energies.
- Like a huge camera





MoEDAL Experiment

Nuclear Track Detectors(NTD)

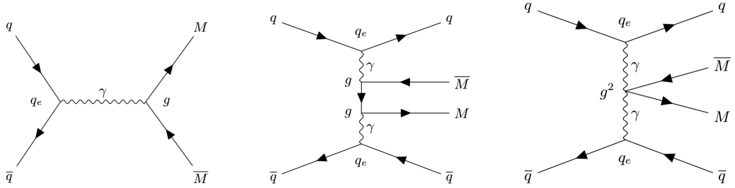
- Stacks of 9 plastic sheets (CR39, Makrofol and Lexan)
- Total surface area $\sim 100m^2$
- Removed and etched, analysed for particle tracks with optical microscope
- Ionisation threshold $Z/\beta \sim 5$. SM particles $Z/\beta \sim 1$. In principle no background
- When a charged particle crosses a plastic nuclear track detector it produces damages at the level of polymeric bounds in a small cylindrical region around its trajectory forming the so-called latent track

Magnetic Monopole Trapper(MMT)

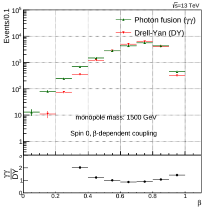
- Consists of roughly 1 tonne of aluminium (Al) paramagnetic volumes placed at three points around IP8
- Capture electrically- and magnetically-charged highly-ionising particles

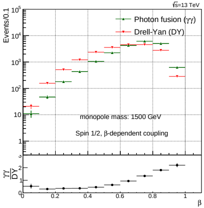
Production process

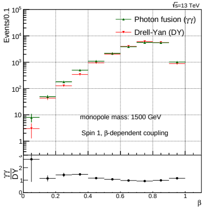
- Collider searches generally assume tree-level $g = 2\pi/e \approx 20.7 \gg 1$
- $g \to g\beta^{\xi}$ Monopole production described from this EFT is relevant only if these particles are produced at threshold where $\beta \ll 1$
- M spin: 0, $\frac{1}{2}$, 1
- Photon Fusion(PF) & Drell-Yan(DY)



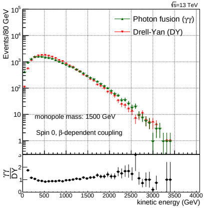
Results $\sqrt{s} = 13\,TeV$, NNPDF23-DY & LUXqed-PF [1]

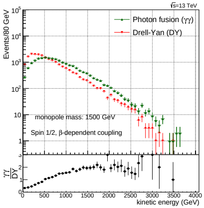


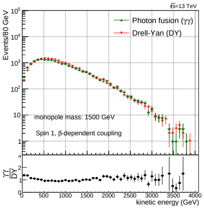




low β Spin-0: PF $>$ DY, Spin-1/2: PF $<$ DY, Spin-0: PF $\sim$ DY.

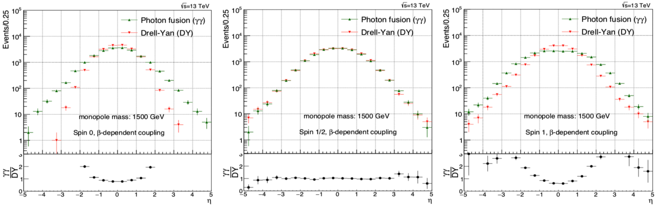




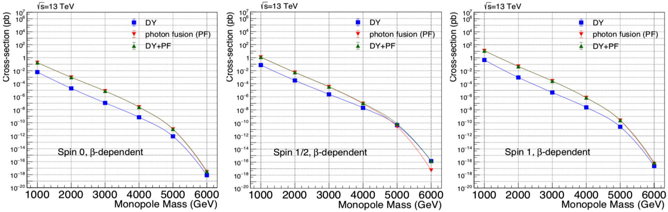


Spin-0,1: PF $<$ DY, Spin-1/2: PF $>$ DY

Results $\sqrt{s} = 13\,TeV$, NNPDF23-DY & LUXqed-PF [1]



The spin-0 & and spin-1 yield a more central production for DY than PF, whilst for spin-1/2 the spectra are practically the same.



Spin-0/1: PF: 1-6 TeV; Spin 1/2: PF:$\sim$ 5 TeV, DY: M $\geq$ 5 TeV

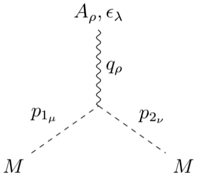
Reproduction and validation

- SQED model [1]

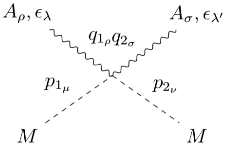
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^{\mu}\Phi)^{\dagger}(D_{\mu}\Phi) - M^2\Phi^{\dagger}\Phi$$

where $D_\mu = \partial_\mu - ig_\beta A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Φ is the scalar monopole field with mass M.

- Two types of vertex



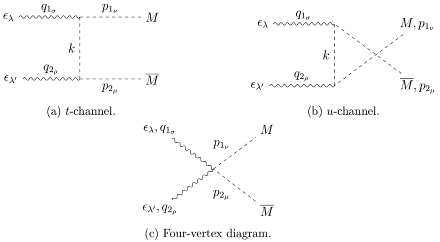
$\propto -ig_\beta(p_1 + p_2)_\mu$



$\propto 2ig_\beta^2 g_{\mu\nu}$

Reproduction and validation

- Three channels



- Cross section for PF:

$$\sigma_{PF} = \frac{4\pi\alpha_g^2}{s_{\gamma\gamma}}\left[\left(\beta^4 - 1\right)\tanh^{-1}(\beta) - \beta\left(\beta^2 - 2\right)\right]$$

- Cross section for DY:

$$\sigma_{DY} = \frac{5\pi\alpha_g\alpha_e\beta^3}{27 s_{q\bar{q}}}$$

- All the calculations are ahieved by using FeynRules and FeynArts and can be here: [Link]

Cross section

MadGraph implementaion

- Create the model with usage of FeynRules and generate the UFO files. The original model file can be found here: [Link]
- In MadGraph command prompt, import that model and generate the process
- Fix the centre-of-mass energy, colliding particles, PDF and particle mass in the run card and parameter card

Run	Collider	Banner	Cross section (pb)	Events
run_01	a a 6500.0 x 6500.0 GeV	tag_1	0.08176 ± 9.9e-05	10000
run_02	a a 6500.0 x 6500.0 GeV	tag_1	0.06495 ± 6.1e-05	10000
run_03	a a 6500.0 x 6500.0 GeV	tag_1	0.04949 ± 5.8e-05	10000
run_04	a a 6500.0 x 6500.0 GeV	tag_1	0.03959 ± 6.7e-05	10000
run_05	a a 6500.0 x 6500.0 GeV	tag_1	0.0353 ± 6.8e-05	10000
run_06	a a 6500.0 x 6500.0 GeV	tag_1	0.02897 ± 6.7e-05	10000

Run	Collider	Banner	Cross section (pb)
run_01	u u~ 6500.0 x 6500.0 GeV	tag_1	0.0002145 ± 2.6e-07
run_02	u u~ 6500.0 x 6500.0 GeV	tag_1	0.0001917 ± 1.7e-07
run_03	u u~ 6500.0 x 6500.0 GeV	tag_1	0.0001552 ± 1.7e-07
run_04	u u~ 6500.0 x 6500.0 GeV	tag_1	0.000109 ± 2.7e-07
run_05	u u~ 6500.0 x 6500.0 GeV	tag_1	5.807e-05 ± 1.7e-07
run_06	u u~ 6500.0 x 6500.0 GeV	tag_1	1.266e-05 ± 4.3e-08

Run	Collider	Banner	Cross section (pb)
run_01	d d~ 6500.0 x 6500.0 GeV	tag_1	5.362e-05 ± 6.4e-08
run_02	d d~ 6500.0 x 6500.0 GeV	tag_1	4.792e-05 ± 4.3e-08
run_03	d d~ 6500.0 x 6500.0 GeV	tag_1	3.88e-05 ± 4.2e-08
run_04	d d~ 6500.0 x 6500.0 GeV	tag_1	2.725e-05 ± 6.7e-08
run_05	d d~ 6500.0 x 6500.0 GeV	tag_1	1.452e-05 ± 4.2e-08
run_06	d d~ 6500.0 x 6500.0 GeV	tag_1	3.164e-06 ± 1.1e-08

times scale factor $g^4\beta^4 = (137\pi \times (1 - \frac{4M}{s}))^2$

Summary & Problem

- Introduction & motivation
- Dirac & 't Hooft solution
- MoEDAL Experiment
- Reproduce

- Combine SM.fr with Mon.fr for convenience
- Do not generate it's own .gen file (GenericFile -> False)

- No complete theory (spin, gβ)
- Model parameter (LHC, quark energy)

Reference

[1] Baines, S.; Mavromatos, N.E.; Mitsou, V.A.; Pinfold, J.L.; Santra, A. Monopole production via photon fusion and Drell- Yan processes: Madgraph implementation and perturbativity via velocity-dependent coupling and magnetic moment as novel features. Eur. Phys. J. C 2018, 78, 966, doi:10.1140/epjc/s10052-018-6440-6.
[2] Dirac, P.A.M. The Theory of Magnetic Poles. Phys. Rev. 1948, 74, 817.
[3] G. 't Hooft, "Magnetic Monopoles In Unified Gauge Theories," Nucl. Phys. B 79, 276 (1974).
[4] A. M. Polyakov, "Particle Spectrum In Quantum Field Theory," JETP Lett. 20 (1974) 194 [Pisma Zh. Eksp. Teor. Fiz. 20 (1974) 430]

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