Electroweak precision measurements from LEP/SLC to future e+e- colliders

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review topics prepared for discussion of CEPC working group

for INPAC & TDLI at SJTU

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✦ References:

1. Precision Electroweak Measurements on the Z Resonance [2005, hep-ex/0509008]
2. Standard Model Theory for the FCC-ee Tera-Z stage [2018, 1809.01830]
3. Theory Requirements and Possibilities for the FCC-ee and other Future High Energy and Precision Frontier Lepton Colliders [2019, 1901.02648]
4. QED challenges at FCC-ee precision measurements [2019, 1903.09895]
5. Theory for the FCC-ee [2019, 1905.05078]
6. Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee [2019, 1906.05379]

✦ Claims:

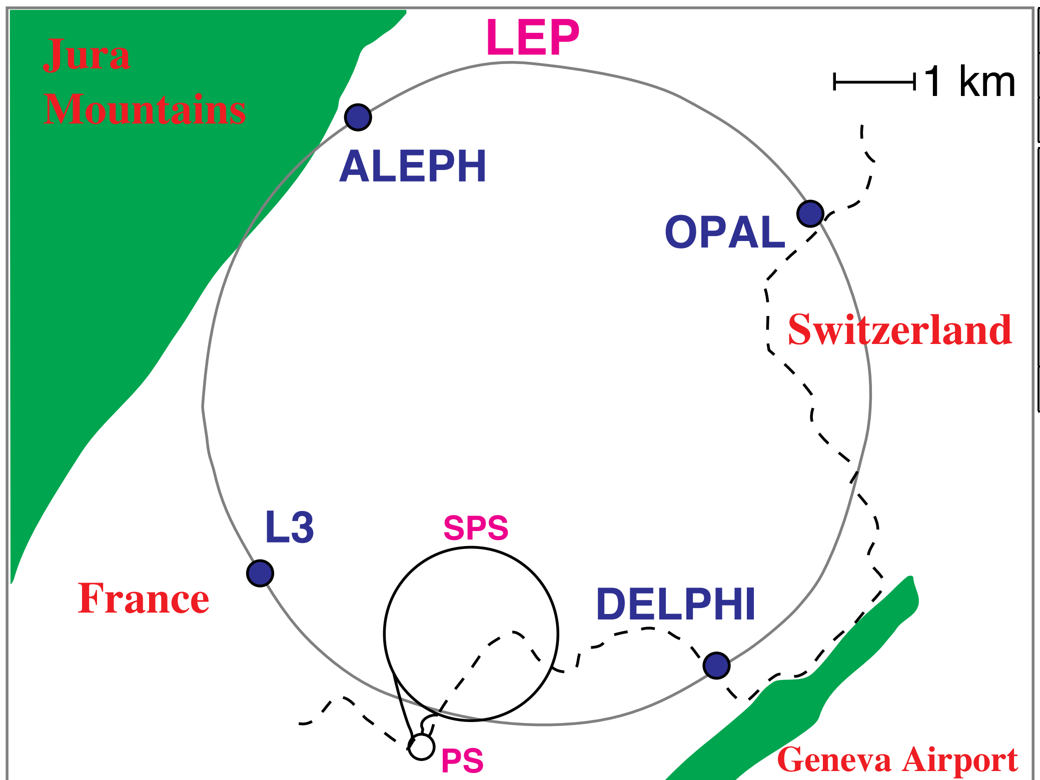
1. focus on **theoretical framework/calculations** related to EW precision measurement
2. rather self-learning materials; more on triggering discussions
3. loops and legs are not entertaining or even tedious, but crucial building blocks

✦ Outline

1. Precision EW measurements at Z resonance from LEP and SLC
2. Theory challenges for future EW precision measurements at Z-pole
3. Precision measurements around threshold of W boson pair
4. Precision measurements at Higgs factory
5. Precision measurements around threshold of top-quark pair

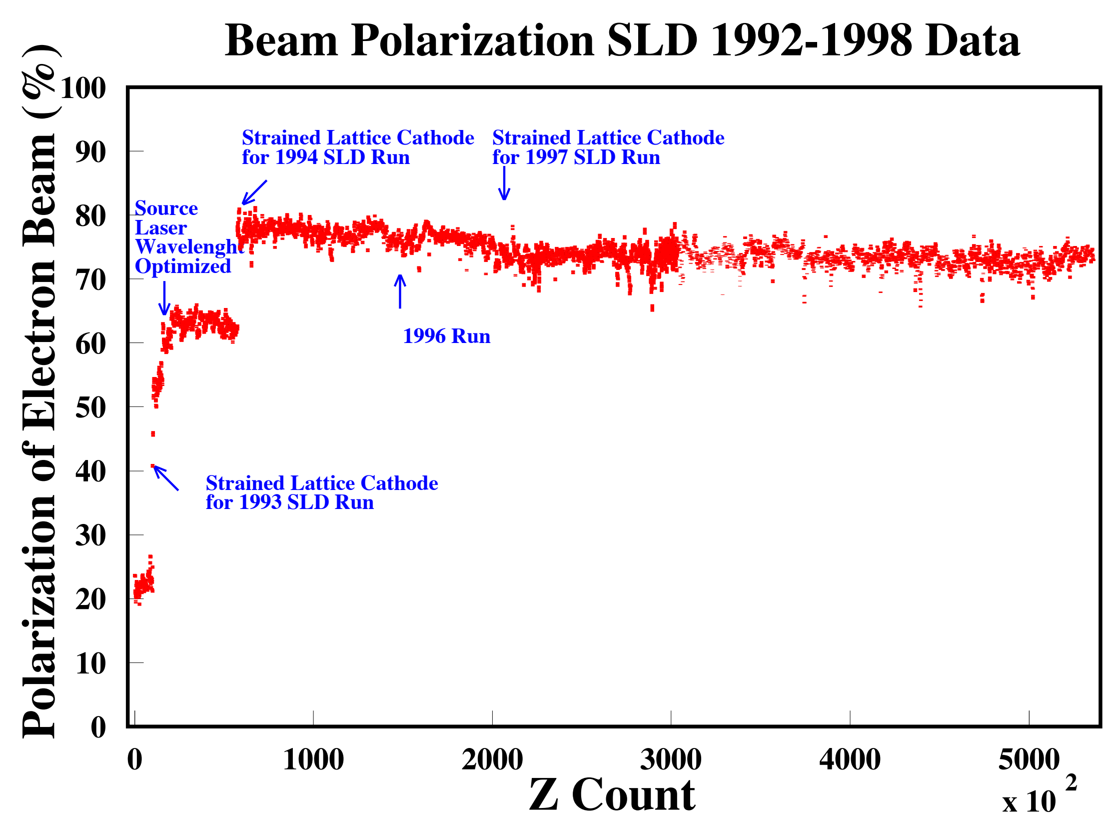
LEP and SLC data at Z resonance

- About 17 million Z boson events recorded at LEP with 4 detectors; 0.6 million Z boson events recorded by SLD at SLC with an average electron polarization of ~70%



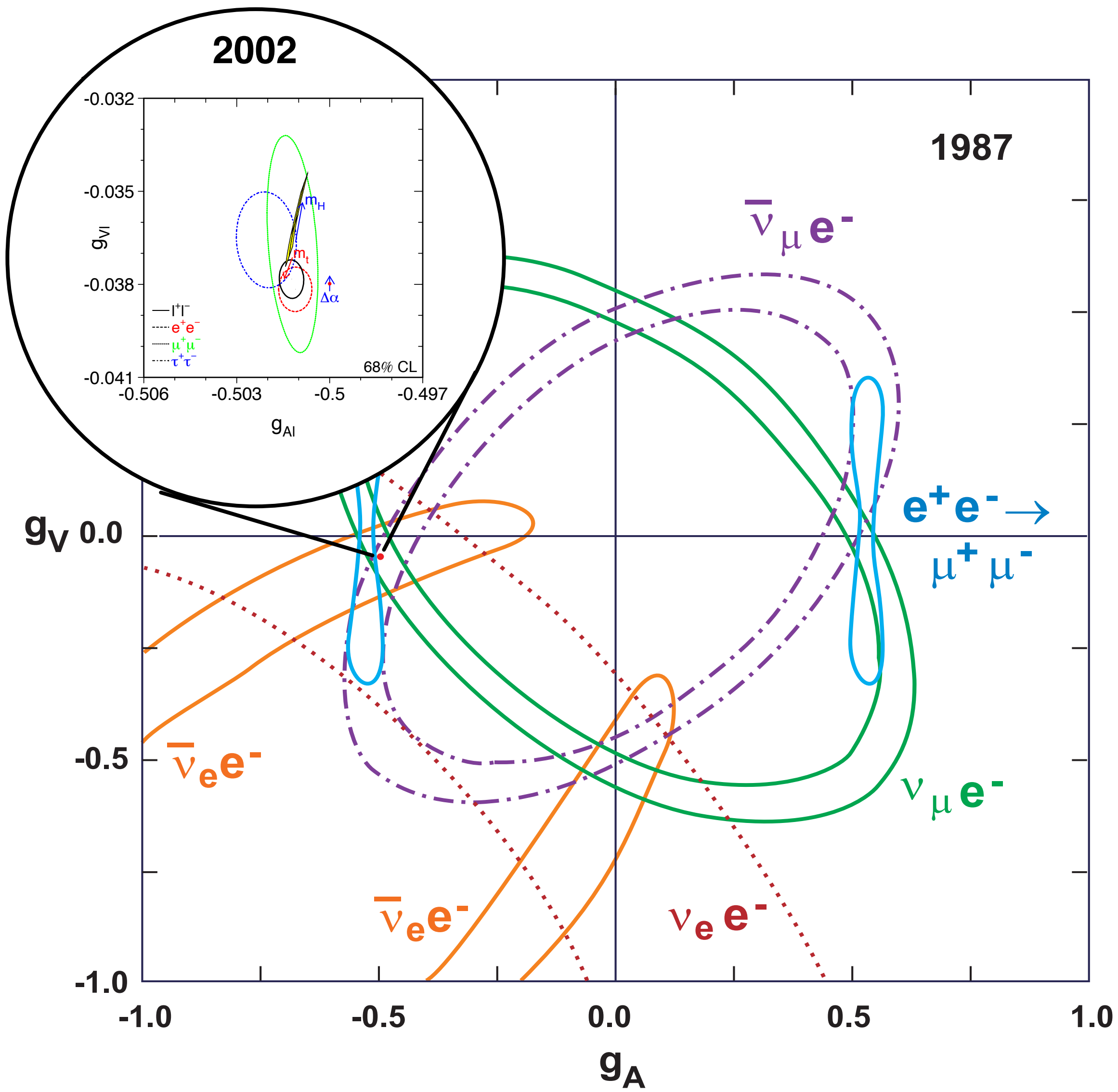
Number of Events in unit of 1000										
	$Z \to q\bar{q}$					$Z \to \ell^+\ell^-$				
Year	A	D	L	O	LEP	A	D	L	O	LEP
1990/91	433	357	416	454	1660	53	36	39	58	186
1992	633	697	678	733	2741	77	70	59	88	294
1993	630	682	646	649	2607	78	75	64	79	296
1994	1640	1310	1359	1601	5910	202	137	127	191	657
1995	735	659	526	659	2579	90	66	54	81	291
Total	4071	3705	3625	4096	15497	500	384	343	497	1724

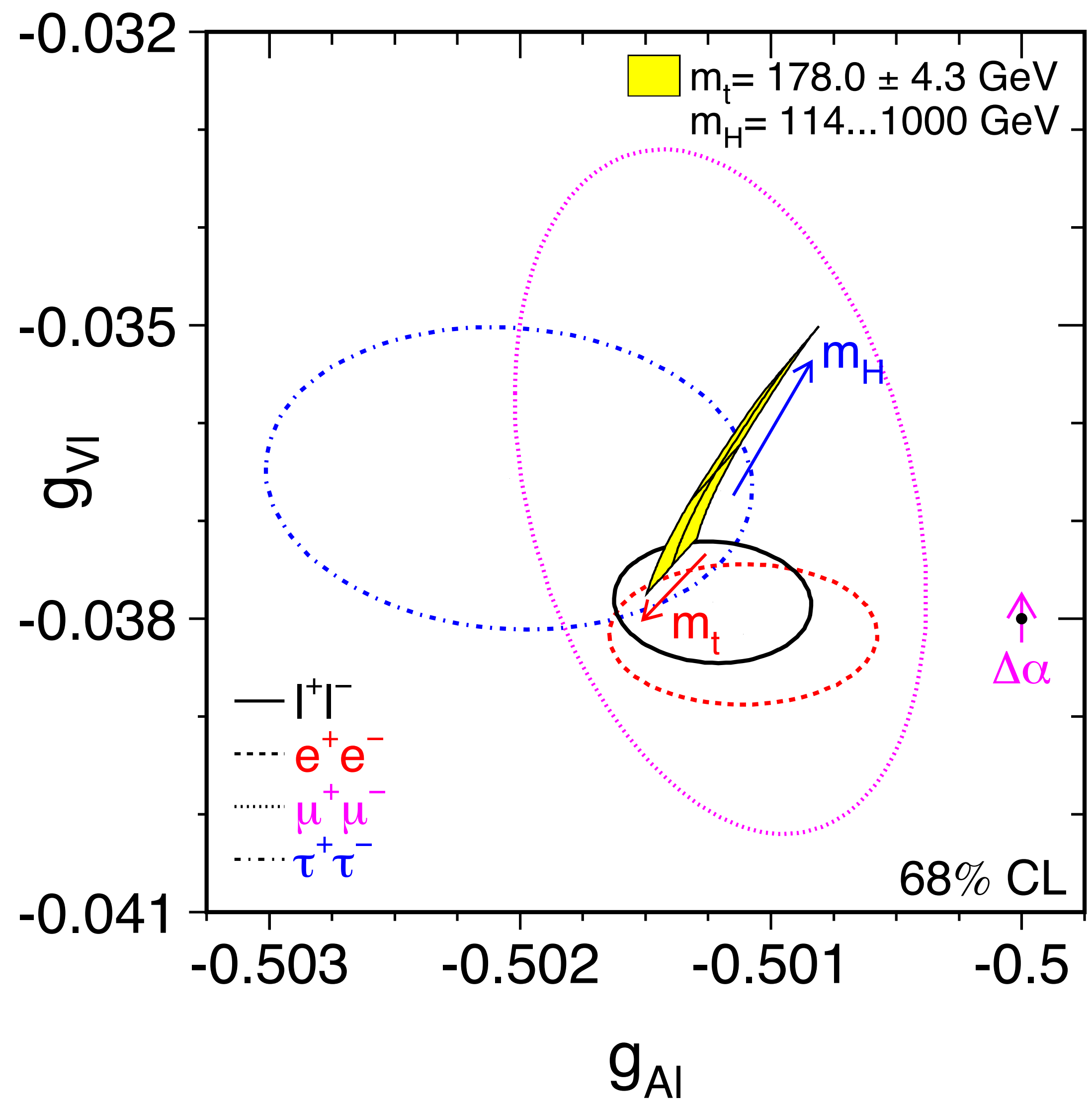
Year	Centre-of-mass energy range [GeV]	Integrated luminosity [pb^{-1}]
1989	88.2 – 94.2	1.7
1990	88.2 – 94.2	8.6
1991	88.5 – 93.7	18.9
1992	91.3	28.6
1993	89.4, 91.2, 93.0	40.0
1994	91.2	64.5
1995	89.4, 91.3, 93.0	39.8



Precision test of SM

- Determine the Z boson parameters with high precision: its mass, its partial and total widths, and its couplings to fermion pairs. These results are compared to the predictions of the SM and found to be in good agreement





effective couplings of charged leptons, gV vs. gA

Electroweak pseudo-/precision observables

- LEP data were analyzed/reported in terms of EWPOs, including for the following interpretations of SM or its extensions

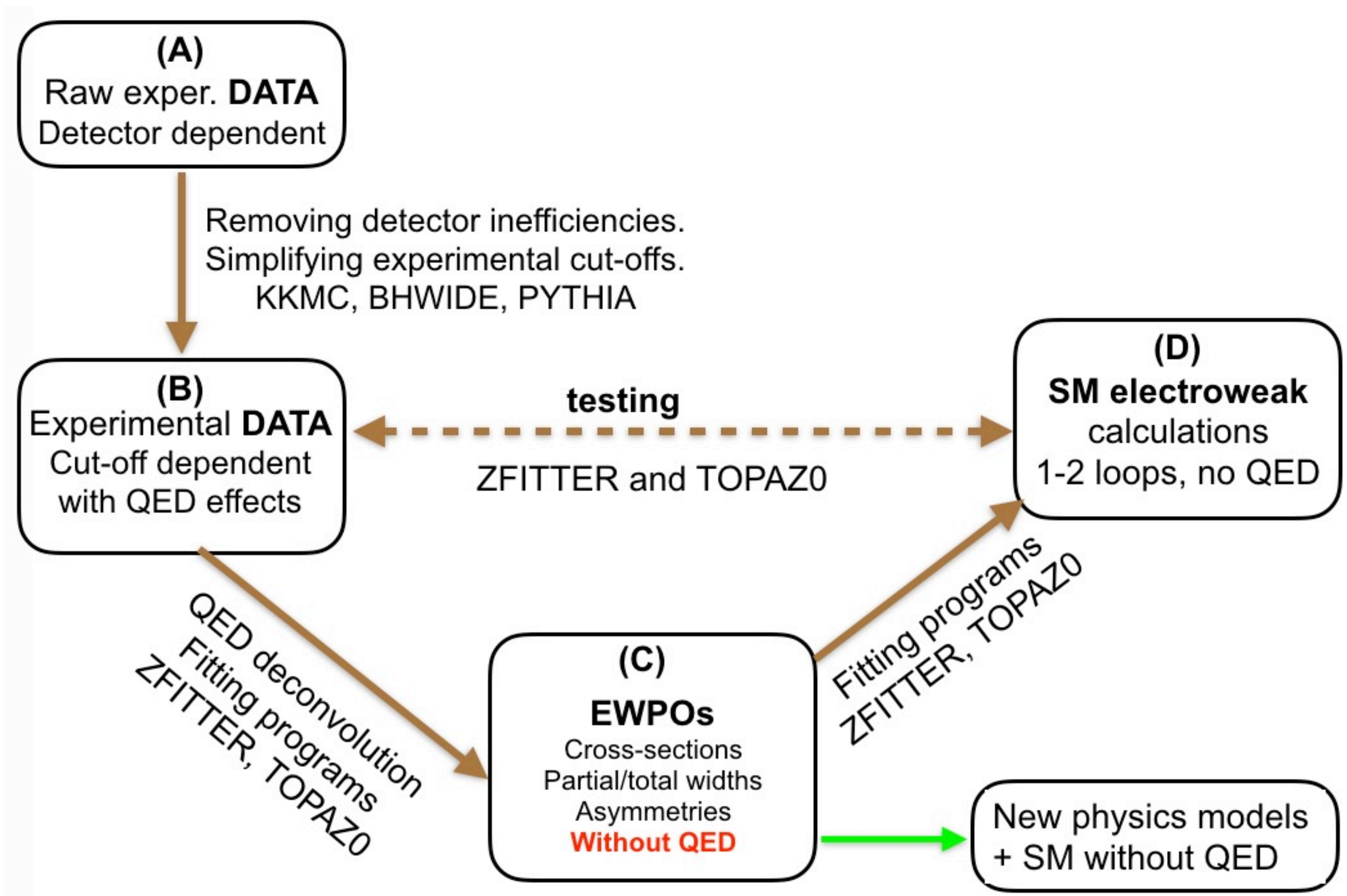


Fig. C.1: Construction of EWPOs in data analysis of the LEP

Electroweak pseudo-/precision observables

- LEP data were analyzed/reported in terms of EWPOs, including for the following interpretations of SM or its extensions

The quantities listed in Table A.1.2 are called *electroweak precision observables* (EWPO) and encapsulate experimental data after extraction of well-known and controllable QED and QCD effects, $\boxed{\text{in a model-independent manner.}}$ They provide a convenient bridge between real data and the predictions of the SM, or of the SM plus new physics. Contrary to raw experimental data (like differential cross-sections), $\boxed{\text{EWPOs are also well-suited for archiving and long-term}}$ use. Archived EWPOs can be exploited over long periods of time for comparisons with steadily

improving theoretical calculations of the SM predictions, and for validations of the new physics models beyond the SM. They are also useful $\boxed{\text{for the comparison and combination of results}}$ from different experiments. However, removing trivial but sizeable QED or QCD effects from EWPOs $\boxed{\text{might induce additional sources of uncertainty.}}$

Z lineshape and leptonic F-B asymmetries

- 9 EWPOs including forward backward asymmetry for each of 3 charged leptons provide a complete (hadron-inclusive) description of the Z resonance

closely related to direct experiment observables

- the mass of the Z, m_{Z};
- the Z total width, Γ_{Z};
- the "hadronic pole cross-section",

$$\sigma^0_{\mathrm{had}} \equiv \frac{12\pi}{m_{\mathrm{Z}}^2}\frac{\Gamma_{\mathrm{ee}}\Gamma_{\mathrm{had}}}{\Gamma_{\mathrm{Z}}^2};$$

- the three ratios

$$R^0_{\mathrm{e}} \equiv \Gamma_{\mathrm{had}}/\Gamma_{\mathrm{ee}},\ R^0_{\mu} \equiv \Gamma_{\mathrm{had}}/\Gamma_{\mu\mu} \text{ and } R^0_{\tau} \equiv \Gamma_{\mathrm{had}}/\Gamma_{\tau\tau}.$$

- the forward-backward asymmetries $A^{0,\,\mathrm{e}}_{\mathrm{FB}},\ A^{0,\,\mu}_{\mathrm{FB}},\ A^{0,\,\tau}_{\mathrm{FB}}$

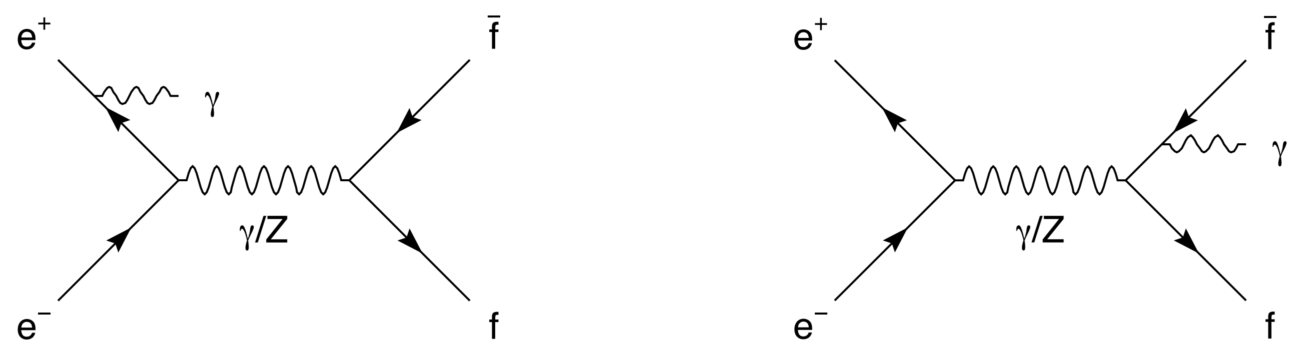
$$\frac{\mathrm{d}\sigma_{\mathrm{f\bar{f}}}}{\mathrm{d}\cos\theta} = \frac{3}{8}\sigma^{\mathrm{tot}}_{\mathrm{f\bar{f}}}\left[(1-\mathcal{P}_{\mathrm{e}}\mathcal{A}_{\mathrm{e}})(1+\cos^2\theta)+2(\mathcal{A}_{\mathrm{e}}-\mathcal{P}_{\mathrm{e}})\mathcal{A}_{\mathrm{f}}\cos\theta\right].$$

$$A^{0,\,\mathrm{f}}_{\mathrm{FB}} = \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{f}}$$

QED deconvolution

- So-called "deconvolution" of QED effects is the procedure of removing universal (process independent) QED effects from experimental data for the total and angular differential cross sections

$$\mathrm{e^+e^-} \to \mathrm{f^+f^-} + \text{invisible}\,(n\,\gamma + \mathrm{e^+e^-}\text{pairs} + \cdots)$$



ISR

$$\sigma^{\mathrm{e^+e^-\to f^+f^-+\cdots}}(s) = \int \mathrm{d}x\; f(x)\; \sigma^{\mathrm{e^+e^-\to f^+f^-}}(s')\;\delta(x - s'/s)$$

ISR+FSR+IFI

$$\begin{aligned}\sigma_{\mathrm{A}}^{\mathrm{real}}(s) &= \sigma_{\mathrm{A}}^{(0)}(s) + \sigma_{\mathrm{A}}^{\mathrm{real,ini}}(s) + \sigma_{\mathrm{A}}^{\mathrm{real,fin}}(s) + \sigma_{\mathrm{A}}^{\mathrm{real,int}}(s)\\ &= \sigma_{\mathrm{A}}^{(0)}(s) + \int \mathrm{d}R\sigma_{\mathrm{A}}^{(0)}(s')\rho_{\mathrm{A}}^{\mathrm{ini}}(R) + \sigma_{\mathrm{A}}^{(0)}(s)\int \mathrm{d}R\rho_{\mathrm{A}}^{\mathrm{fin}}(R)\\ &\quad + \int \mathrm{d}R \sum_{V_i,V_j=\gamma,\mathrm{Z}} \sigma_{\mathrm{A}}^{(0)}(s,s',i,j)\rho_{\mathrm{A}}^{\mathrm{int}}(R,i,j).\end{aligned}$$

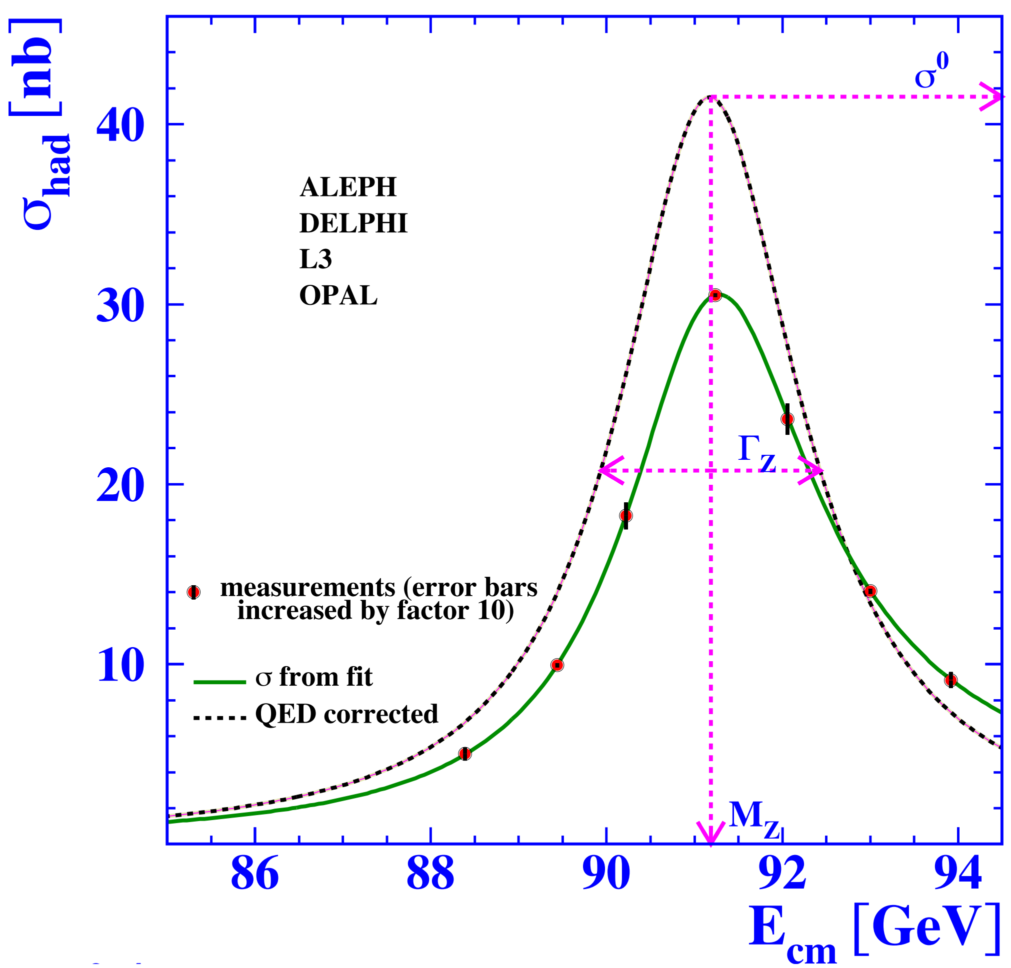
one-loop + soft photon resummation

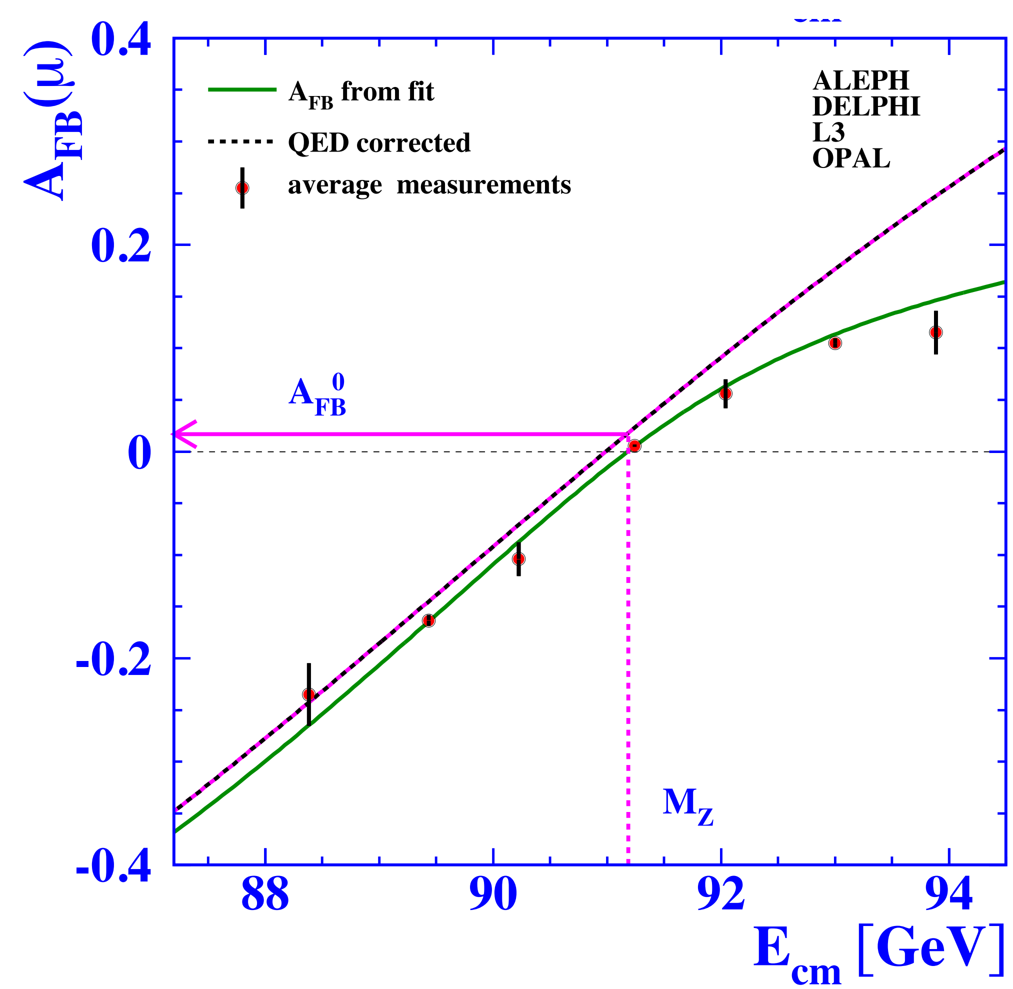
$$\sigma_{\mathrm{tot}}^{(0)}(s) + \sigma_{\mathrm{tot}}^{\mathrm{real,ini}}(s) \to \int \mathrm{d}R\;\sigma_{\mathrm{tot}}^{(0)}(s')\;\rho_{\mathrm{tot}}^{\mathrm{ini}}(R),$$

$$\begin{aligned}\rho_{\mathrm{tot}}^{\mathrm{ini}}(R) &= \left(1+\bar{S}\right)\beta(1-R)^{\beta-1} + \bar{H}_{\mathrm{tot}}^{\mathrm{ini}}(R),\\ \bar{S} &= \frac{3}{4}\beta + \frac{\alpha}{\pi}Q_{\mathrm{e}}^2\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \text{h.o.},\\ \beta &= \frac{2\alpha}{\pi}Q_{\mathrm{e}}^2 L_{\mathrm{e}},\\ L_{\mathrm{e}} &= \left(\ln\frac{s}{m_{\mathrm{e}}^2} - 1\right),\\ \bar{H}_{\mathrm{T}}^{\mathrm{ini}}(R) &= \left[H_{\mathrm{BM}}(R) - \frac{\beta}{1-R}\right] + \text{h.o.},\end{aligned}$$

QED deconvolution

- So-called "deconvolution" of QED effects is the procedure of removing universal (process independent) QED effects from experimental data for the total and angular differential cross sections

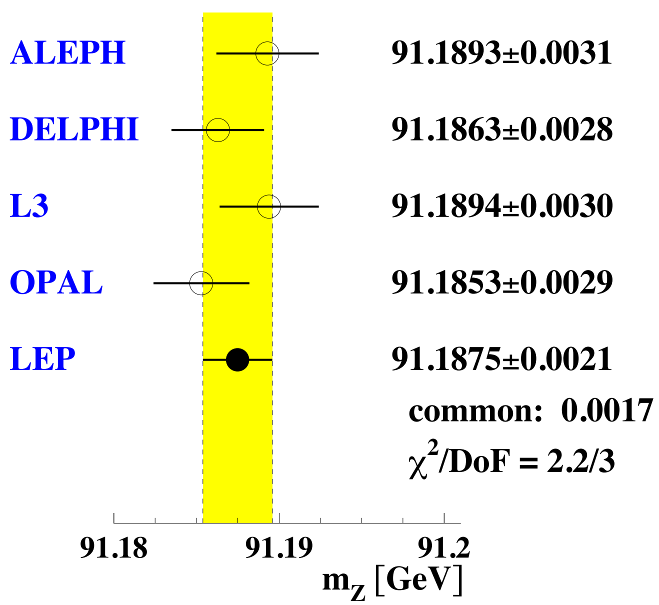


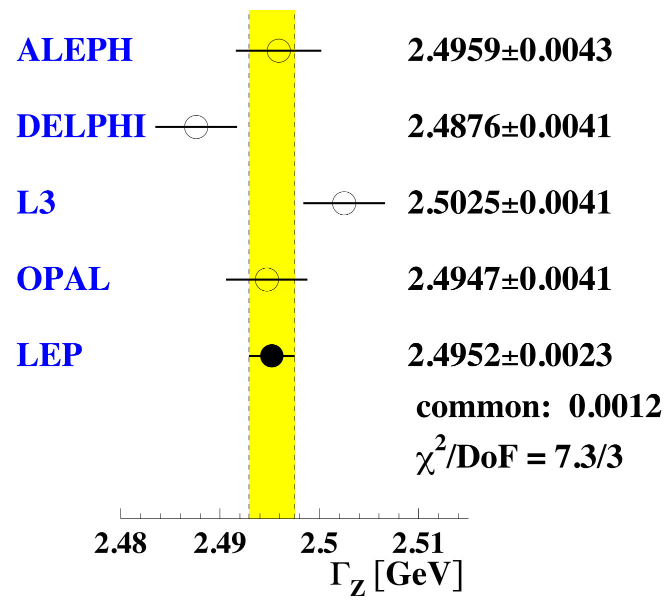


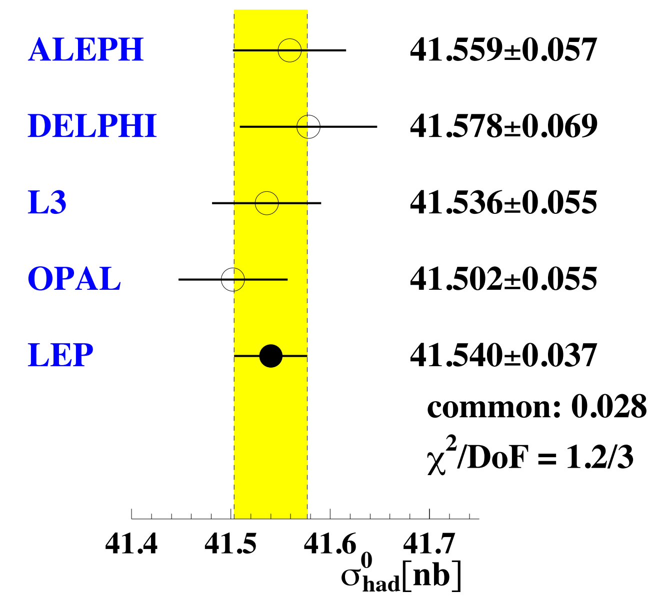
large corrections from QED deconvolution

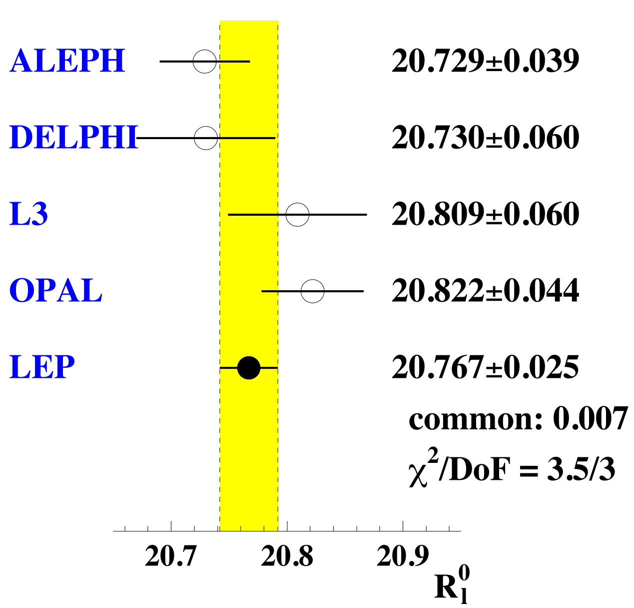
LEP combination on EWPOs

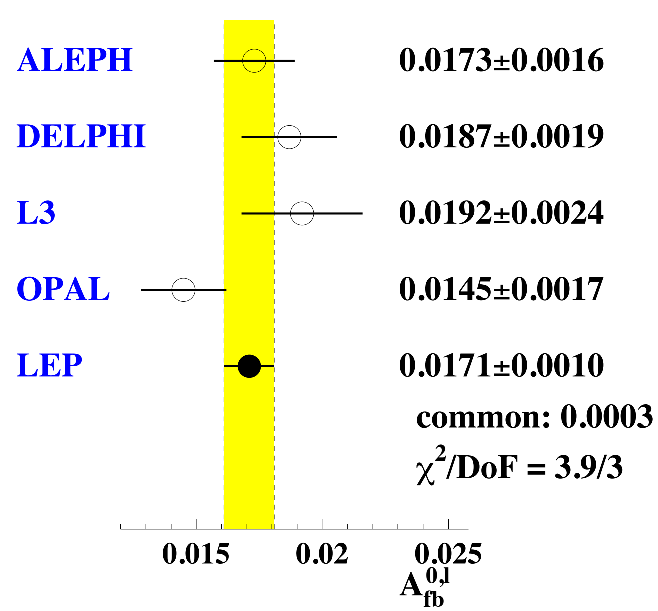
- The value of chi2/d.o.f of 36.5/31 for the combination of the four LEP sets of nine pseudo-observables with five parameters (assuming lepton universality)











Theory for EWPOs

- Deconvoluted cross sections/EWPOs around the Z resonance are matched onto a born-type structure using the complex-valued effective coupling constants

$$
\begin{aligned}
\mathcal{G}_{\rm Vf} &= \sqrt{\mathcal{R}_{\rm f}}\,(T_3^{\rm f} - 2Q_{\rm f}\mathcal{K}_{\rm f}\sin^2\theta_{\rm W}) \\
\mathcal{G}_{\rm Af} &= \sqrt{\mathcal{R}_{\rm f}}\,T_3^{\rm f}\,.
\end{aligned}
$$

effective neutral current couplings

$$
\frac{2s}{\pi}\frac{1}{N_c^{\rm f}}\frac{d\sigma_{\rm ew}}{d\cos\theta}({\rm e^+e^-}\to{\rm f\bar{f}}) =
$$

$$
\underbrace{|\alpha(s)Q_{\rm f}|^2\,(1+\cos^2\theta)}_{\sigma^{\gamma}}
$$

further subtracted

$$
\underbrace{-8\Re\left\{\alpha^*(s)Q_{\rm f}\chi(s)\left[\mathcal{G}_{\rm Ve}\mathcal{G}_{\rm Vf}(1+\cos^2\theta)+2\mathcal{G}_{\rm Ae}\mathcal{G}_{\rm Af}\cos\theta\right]\right\}}_{\gamma\text{–Z interference}}
$$

vanish at Z pole

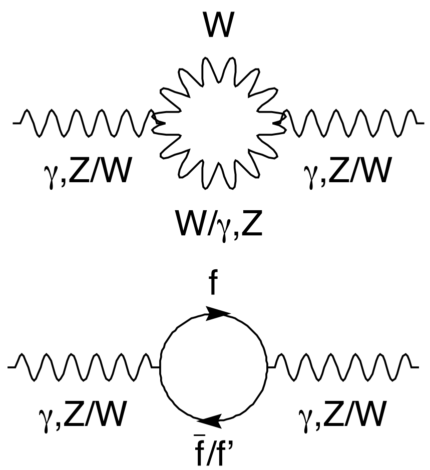
$$
\underbrace{\begin{array}{c}+16|\chi(s)|^2\left[(|\mathcal{G}_{\rm Ve}|^2+|\mathcal{G}_{\rm Ae}|^2)(|\mathcal{G}_{\rm Vf}|^2+|\mathcal{G}_{\rm Af}|^2)(1+\cos^2\theta)\right. \\ \left.+8\Re\left\{\mathcal{G}_{\rm Ve}\mathcal{G}_{\rm Ae}{}^*\right\}\Re\left\{\mathcal{G}_{\rm Vf}\mathcal{G}_{\rm Af}{}^*\right\}\cos\theta\right]\end{array}}_{\sigma^{\rm Z}}
$$

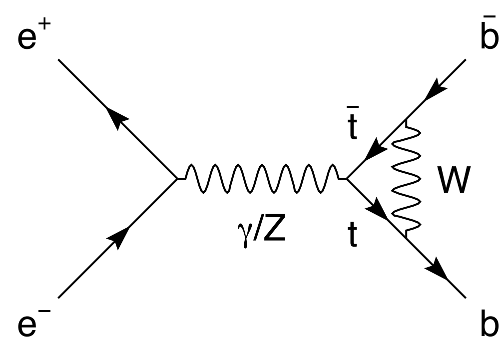
$\longrightarrow$ **EWPOs**

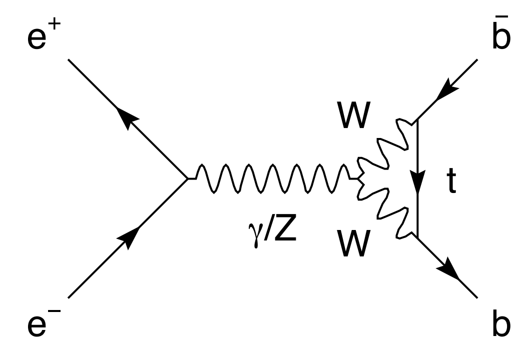
$$
\chi(s) = \frac{G_{\rm F}m_{\rm Z}^2}{8\pi\sqrt{2}}\frac{s}{s-m_{\rm Z}^2+is\Gamma_{\rm Z}/m_{\rm Z}}\,,
$$

Electroweak radiative corrections

- Further electroweak corrections are needed to relate SM theory/input parameters to EWPOs/effective couplings at high accuracy







$$\begin{aligned} \mathcal{G}_{\mathrm{Vf}} &= \sqrt{\mathcal{R}_{\mathrm{f}}}\,(T_3^{\mathrm{f}} - 2Q_{\mathrm{f}}\mathcal{K}_{\mathrm{f}}\sin^2\theta_{\mathrm{W}}) \\ \mathcal{G}_{\mathrm{Af}} &= \sqrt{\mathcal{R}_{\mathrm{f}}}\,T_3^{\mathrm{f}}\,. \end{aligned}$$

$$\begin{aligned} \rho_{\mathrm{f}} &\equiv \Re(\mathcal{R}_{\mathrm{f}}) = 1 + \Delta\rho_{\mathrm{se}} + \Delta\rho_{\mathrm{f}} \\ \kappa_{\mathrm{f}} &\equiv \Re(\mathcal{K}_{\mathrm{f}}) = 1 + \Delta\kappa_{\mathrm{se}} + \Delta\kappa_{\mathrm{f}} \end{aligned}$$

$$\begin{aligned} \sin^2\theta_{\mathrm{eff}}^{\mathrm{f}} &\equiv \kappa_{\mathrm{f}}\sin^2\theta_{\mathrm{W}} \\ g_{\mathrm{Vf}} &\equiv \sqrt{\rho_{\mathrm{f}}}\,(T_3^{\mathrm{f}} - 2Q_{\mathrm{f}}\sin^2\theta_{\mathrm{eff}}^{\mathrm{f}}) \\ g_{\mathrm{Af}} &\equiv \sqrt{\rho_{\mathrm{f}}}\,T_3^{\mathrm{f}}\,, \end{aligned}$$

one-loop self-energy correction

$$\begin{aligned} \Delta\rho_{\mathrm{se}} &= \frac{3G_{\mathrm{F}}m_{\mathrm{W}}^2}{8\sqrt{2}\pi^2}\left[\frac{m_{\mathrm{t}}^2}{m_{\mathrm{W}}^2} - \frac{\sin^2\theta_{\mathrm{W}}}{\cos^2\theta_{\mathrm{W}}}\left(\ln\frac{m_{\mathrm{H}}^2}{m_{\mathrm{W}}^2} - \frac{5}{6}\right) + \cdots\right] \\ \Delta\kappa_{\mathrm{se}} &= \frac{3G_{\mathrm{F}}m_{\mathrm{W}}^2}{8\sqrt{2}\pi^2}\left[\frac{m_{\mathrm{t}}^2}{m_{\mathrm{W}}^2}\frac{\cos^2\theta_{\mathrm{W}}}{\sin^2\theta_{\mathrm{W}}} - \frac{10}{9}\left(\ln\frac{m_{\mathrm{H}}^2}{m_{\mathrm{W}}^2} - \frac{5}{6}\right) + \cdots\right] \end{aligned}$$

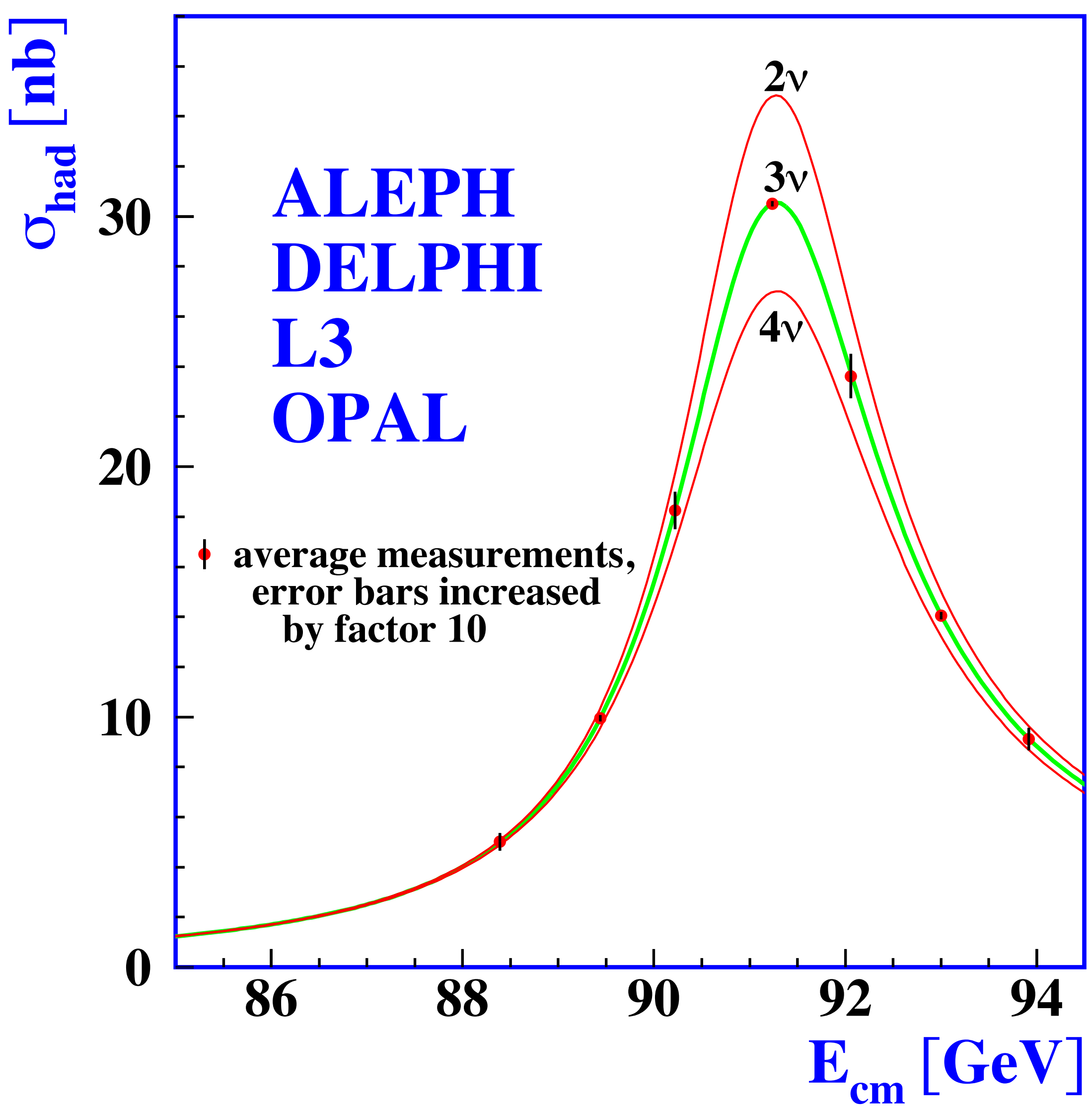
$$\mathcal{A}_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2 - g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2 + g_{\mathrm{Rf}}^2} = \frac{2g_{\mathrm{Vf}}g_{\mathrm{Af}}}{g_{\mathrm{Vf}}^2 + g_{\mathrm{Af}}^2} = 2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1 + (g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}$$

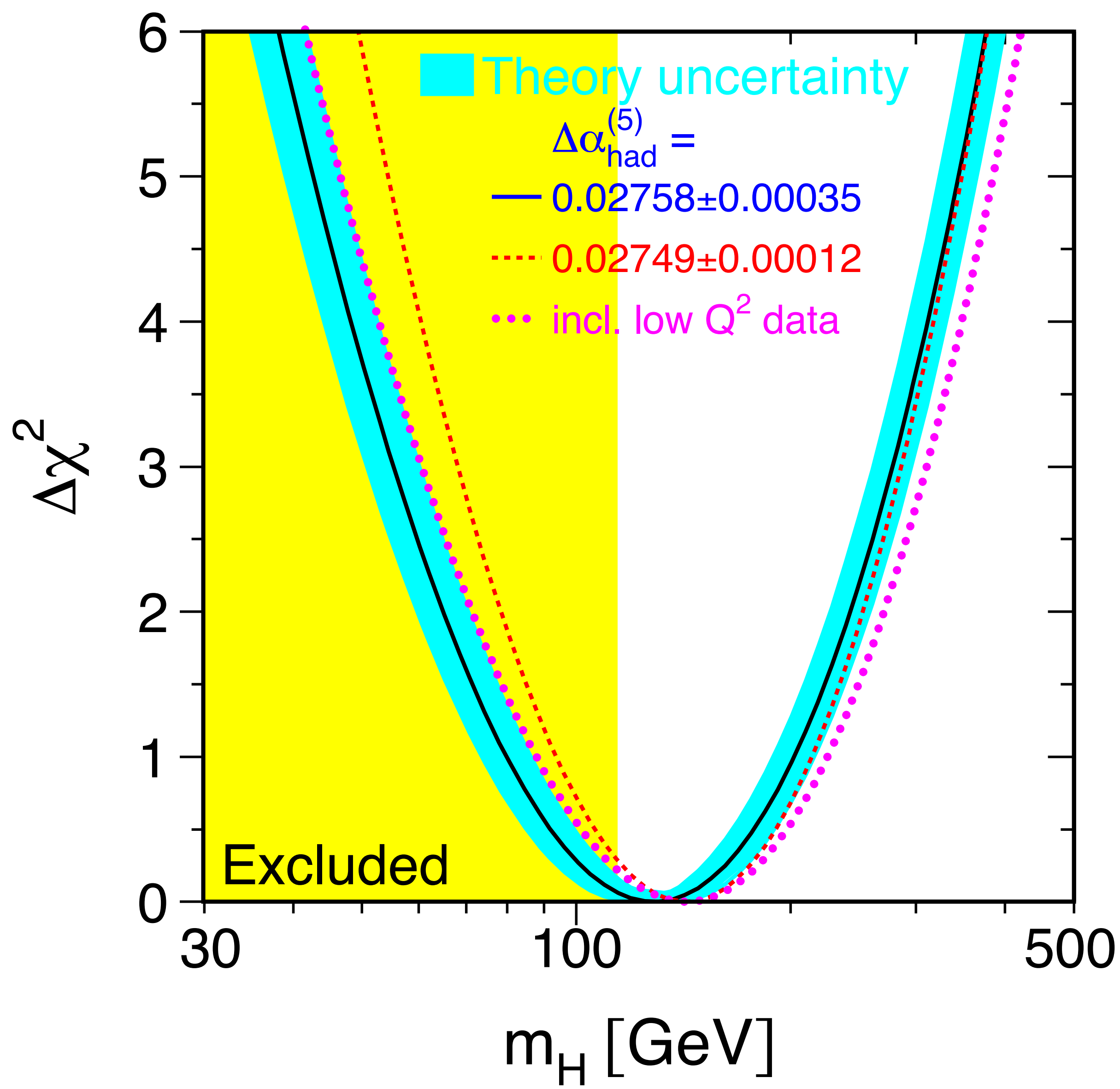
$$\frac{g_{\mathrm{Vf}}}{g_{\mathrm{Af}}} = \Re\left(\frac{\mathcal{G}_{\mathrm{Vf}}}{\mathcal{G}_{\mathrm{Af}}}\right) = 1 - 4|Q_{\mathrm{f}}|\sin^2\theta_{\mathrm{eff}}^{\mathrm{f}}$$

F-B asymmetry and effective mixing angle

Precision test of SM

- The combination of EWPOs and SM predictions at loop-level provides firmly test of the SM and further constraints on the unknown parameter, e.g., the Higgs boson mass





invisible decay width/neutrino species, 2.9840 ± 0.0082

Higgs boson mass from a EW global fit

Future e+e- machines

- Future e+e- machines will have dedicated run as Z factory with high statistics, a few orders higher than at LEP1, e.g., FCC-ee-Z, CEPC

Table A.1.1: Run plan for FCC-ee in its baseline configuration with two experiments. The WW event numbers are given for the entirety of the FCC-ee running at and above the WW threshold.

Phase	Run duration (years)	Centre-of-mass energies (GeV)	Integrated luminosity (ab^{-1})	Event statistics
FCC-ee-Z	4	88–95	150	3×10^{12} visible Z decays
FCC-ee-W	2	158–162	12	10^8 WW events
FCC-ee-H	3	240	5	10^6 ZH events
FCC-ee-tt	5	345–365	1.7	10^6 $t\bar{t}$ events

Future e+e- machines

- Unprecedented experimental precision (dominated by systematics) expected on measurements of EWPOs or its alternative forms

Observable	Current value	± Error	FCC-ee stat.	FCC-ee syst.	Comment, dominant experimental error
m_Z (keV)	91186700	± 2200	4	100	From Z line shape scan, beam energy calibration
Γ_Z (keV)	2495200	± 2300	7	100	From Z line shape scan, beam energy calibration
R_ℓ^Z ($\times 10^3$)	20767	± 25	0.06	0.2–1	Ratio of hadrons to leptons, acceptance for leptons
$\alpha_s(m_Z)$ ($\times 10^4$)	1196	± 30	0.1	0.4–1.6	From R_ℓ^Z
R_b ($\times 10^6$)	216290	± 660	0.3	<60	Ratio of $b\bar{b}$ to hadrons, stat. extrapolated from SLD
σ_{had}^0 ($\times 10^3$) (nb)	41541	± 37	0.1	4	Peak hadronic cross-section, luminosity measurement
$N_\nu(\times 10^3)$	2991	± 7	0.005	1	Z peak cross-sections, luminosity measurement
$\sin^2\theta_W^{eff}(\times 10^6)$	231480	± 160	3	2–5	From $A_{FB}^{\mu\mu}$ from $A_{FB}^{\mu\mu}$ at Z peak, beam energy calibration
$1/\alpha_{QED}(m_Z)(\times 10^3)$	128952	± 14	4	Small	From $A_{FB}^{\mu\mu}$ off peak

here no theoretical uncertainties included for Fcc-ee sys. projection

Future e+e- machines

- Unprecedented experimental precision (dominated by systematics) expected on measurements of EWPOs or **its alternative forms**

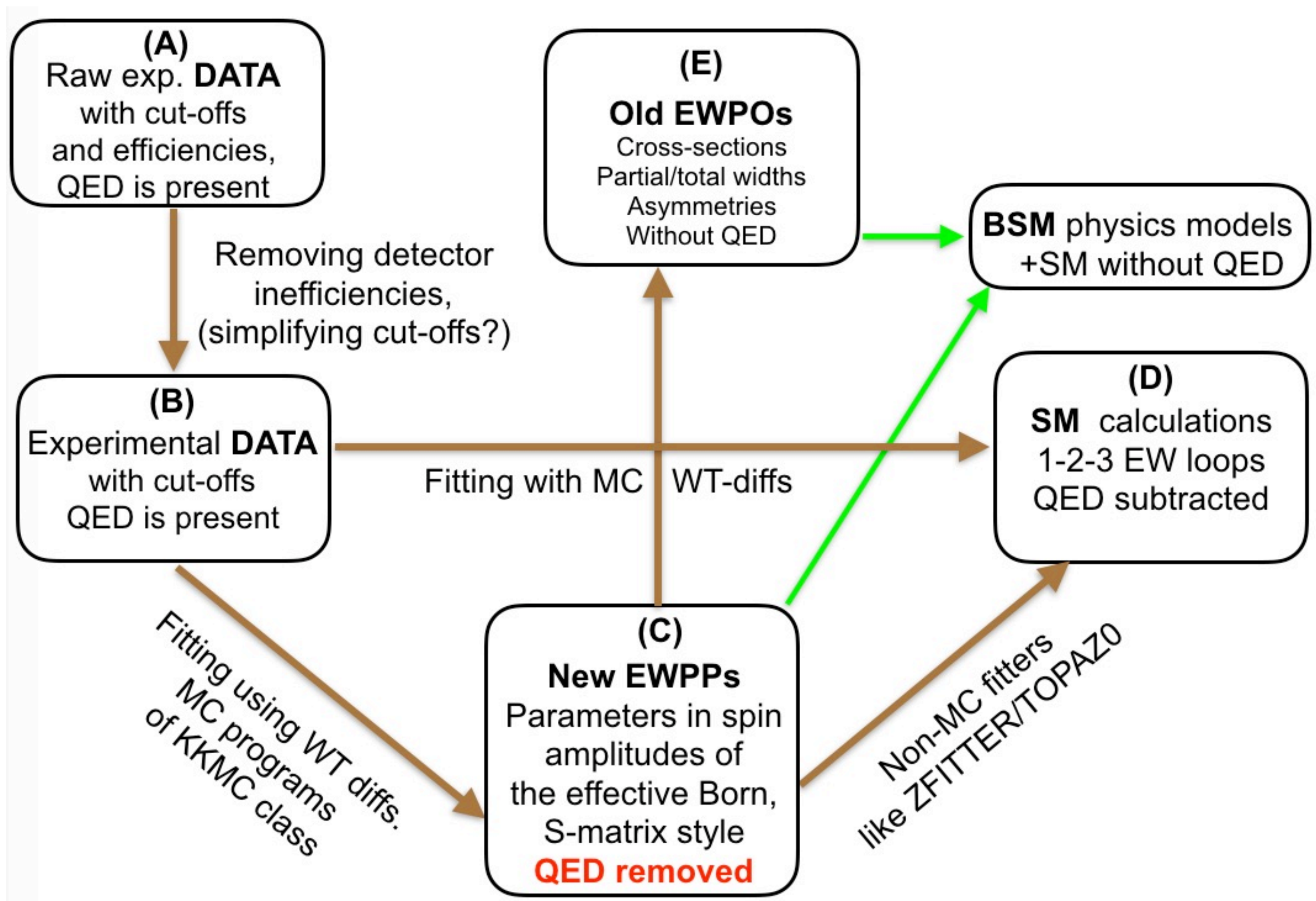


Fig. C.2: Possible scheme of construction of EWPPs in data analysis of the FCC-ee

Challenge on theory side

- The huge advance in projected experimental precision naturally leads to concerns on whether the theory uncertainties can match up or even stay well below the precision goal as at LEP

Theory Requirements and Possibilities for the FCC-ee and other Future High Energy and Precision Frontier Lepton Colliders*

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18 December 2018

Abstract

The future lepton colliders proposed for the High Energy and Precision Frontier set stringent demands on theory. The most ambitious, broad-reaching and demanding project is the FCC-ee. We consider here the present status and requirements on precision calculations, possible ways forward and novel methods, to match the experimental accuracies expected at the FCC-ee. We conclude that the challenge can be tackled by a distributed collaborative effort in academic institutions around the world, provided sufficient support, which is estimated to about 500 man-years over the next 20 years.

Challenge on theory side

- The huge advance in projected experimental precision naturally leads to concerns on whether the theory uncertainties can match up or even stay well below the precision goal as at LEP

1. To adjust the precision of theory predictions to the experimental demands from the FCC-ee, an update of existing software and the development of new, independent software will be needed. This should include, in the first instance, solutions to the following issues:

   (a) factorisation to infinite order of multiphoton soft-virtual QED contributions;

   (b) resummations in Monte Carlo generators;

   (c) disentangling of QED and EW corrections beyond one loop, with soft-photon factorisation or resummation;

   (d) proper implementation of higher-loop effects, such as Laurent series around the Z peak;

   (e) further progress in methods and tools for multiloop calculations and Monte Carlo generators.

2. To meet the experimental precision of the FCC-ee Tera-Z for electroweak precision observables (EWPOs), even three-loop EW calculations of the $\mathrm{Zf\bar{f}}$ vertex will be needed, comprising the loop orders $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^2), \mathcal{O}(N_{\mathrm{f}}\alpha^2\alpha_{\mathrm{s}}), \mathcal{O}(N_{\mathrm{f}}^2\alpha^3)$, and also the corresponding QCD four-loop terms. This was mainly a subject of the 2018 report [1].

Challenge on theory side

- The huge advance in projected experimental precision naturally leads to concerns on whether the theory uncertainties can match up or even stay well below the precision goal as at LEP

3. To decrease the α_{QED} uncertainty by a factor of five to ten, to the level $(3–5) \times 10^{-5}$, will require improvements in low-energy experiments. Alongside this, the perturbative QCD (pQCD) prediction of the Adler function must be improved by a factor of two, accomplished with better uncertainty estimates for m_{c} and m_{b}. The next mandatory improvements required are:

   (a) four-loop massive pQCD calculation of the Adler function;

   (b) improved α_{s} in the low Q^2 region above the τ mass;

   (c) a better control and understanding of $\Delta\alpha_{\mathrm{had}}^{(5)}(M_{\mathrm{Z}}^2)$, in terms of R data;

   (d) different methods for directly accessing $\alpha(M_{\mathrm{Z}}^2)$, e.g., the muon forward–backward asymmetry, or for calculating α_{QED}, either based on a radiative return experiment, e.g., at the FCC-ee Tera-Z, or using lattice QCD methods.

4. FCC-ee precision measurements require many improvements on the theoretical QCD side. These include: (i) higher-order pQCD fixed-order calculations; (ii) higher-order logarithmic resummations; (iii) per-mille-precision extractions of the α_{s} coupling; and (iv) an accurate control of non-perturbative QCD effects (such as, e.g., colour reconnection, hadronization), both analytically and as implemented in the Monte Carlo generators. These issues are discussed in Chapter B.

QED deconvolution revisit

- QED corrections at fixed-order and with soft/collinear resummations are implemented in various Monte Carlo/semi-analytical generators; fully differential MC generator with high QED precision is non-trivial

e.g. for inclusive cross section

(a) 0.6%

1				
$\alpha\mathrm{L}$	α			
$\alpha^2\mathrm{L}^2$	$\alpha^2\mathrm{L}$	α^2		
$\alpha^3\mathrm{L}^3$	$\alpha^3\mathrm{L}^2$	$\alpha^3\mathrm{L}$	α^3	
$\alpha^4\mathrm{L}^4$	$\alpha^4\mathrm{L}^3$	$\alpha^4\mathrm{L}^2$	$\alpha^4\mathrm{L}^1$	α^4
$\vdots$	$\vdots$	$\vdots$		

(b) 0.03%

1				
$\alpha\mathrm{L}$	α			
$\alpha^2\mathrm{L}^2$	$\alpha^2\mathrm{L}$	α^2		
$\alpha^3\mathrm{L}^3$	$\alpha^3\mathrm{L}^2$	$\alpha^3\mathrm{L}$	α^3	
$\alpha^4\mathrm{L}^4$	$\alpha^4\mathrm{L}^3$	$\alpha^4\mathrm{L}^2$	$\alpha^4\mathrm{L}^1$	α^4
$\vdots$	$\vdots$	$\vdots$		

(c) 0.001%

1				
$\alpha\mathrm{L}$	α			
$\alpha^2\mathrm{L}^2$	$\alpha^2\mathrm{L}$	α^2		
$\alpha^3\mathrm{L}^3$	$\alpha^3\mathrm{L}^2$	$\alpha^3\mathrm{L}$	α^3	
$\alpha^4\mathrm{L}^4$	$\alpha^4\mathrm{L}^3$	$\alpha^4\mathrm{L}^2$	$\alpha^4\mathrm{L}^1$	α^4
$\alpha^5\mathrm{L}^5$	$\vdots$	$\vdots$	$\vdots$	

$$\gamma_{nr} = \left(\frac{\alpha}{\pi}\right)^n \left(2\ln\frac{M_Z^2}{m_f^2}\right)^r, \; 0 \le r \le n,$$

other enhancement may occur, need resummation

$$S^{(n)} \sim \left(\frac{\alpha}{\pi}\, 2L_f \, \ln\frac{E_{\max}}{E_{beam}}\right)^n \qquad S^{(n)} \sim \left(\frac{\alpha}{\pi}\, \ln\frac{\Gamma_R}{M_R}\right)^n$$

QED deconvolution revisit

- Current state-of-art generators are not much different wrt. those used in LEP analysis 20 years ago; large gaps wrt. FCC-ee precision

Observable	Source LEP	Err.{QED} LEP	Stat[Syst] FCC-ee	$\frac{\text{LEP}}{\text{FCC-ee}}$	main development to be done
M_Z [MeV]	Z linesh.	2.1{0.3}	0.005[0.1]	$3{\times}3^{\star}$	light fermion pairs
Γ_Z [MeV]	Z linesh.	2.1{0.2}	0.008[0.1]	$2{\times}3^{\star}$	fermion pairs
$R_l^Z \times 10^3$	$\sigma(M_Z)$	25{12}	0.06[1.0]	$12{\times}3^{\star\star}$	better FSR
$\sigma^0_{\rm had}$ [pb]	$\sigma^0_{\rm had}$	37{25}	0.1[4.0]	$6{\times}3^{\star}$	better lumi MC
$N_\nu \times 10^3$	$\sigma(M_Z)$	8{6}	0.005[1.0]	$6{\times}3^{\star\star}$	CEEX in lumi MC
$N_\nu \times 10^3$	$Z\gamma$	150{60}	0.8[<1]	$60{\times}3^{\star\star}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$	53{28}	0.3[0.5]	$55{\times}3^{\star\star}$	h.o. and EWPOs
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle, A_{\rm FB}^{pol,\tau}$	41{12}	0.6[<0.6]	$20{\times}3^{\star\star}$	better τ decay MC
M_W [MeV]	mass rec.	33{6}	0.5[0.3]	$12{\times}3^{\star\star\star}$	QED at threshold
$A_{FB,\mu}^{M_Z\pm3.5\text{GeV}} \times 10^5$	$\frac{d\sigma}{d\cos\theta}$	2000{100}	1.0[0.3]	$100{\times}3^{\star\star\star}$	improved IFI

Table 2: Comparing experimental and theoretical errors at LEP and FCC-ee as in Table 1. 3rd column shows LEP experimental error together with uncertainty induced by QED and 4th column shows anticipated FCC-ee experimental statistical [systematic] errors. Additional factor $\times 3$ in the 5-th column (4th in Table 1) reflects what is needed for QED effects to be *subdominant*. Rating from * to *** marks whether the needed improvement is relatively straightforward, difficult or very difficult to achieve.

Theoretical uncertainties on EWPOs

- Theoretical uncertainties on EWPOS can be divided as intrinsic errors due to missing EW radiative corrections and parametric uncertainties due to SM inputs

intrinsic error

Quantity	FCC-ee	Current intrinsic error		Projected intrinsic error
M_W [MeV]	0.5–1 ‡	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b [10^{-5}]	6	11	$(\alpha^3, \alpha^2\alpha_s)$	5
R_l [10^{-3}]	1	6	$(\alpha^3, \alpha^2\alpha_s)$	1.5

‡The pure experimental precision on M_W is ~ 0.5 MeV [1, 2], see Sec. 4.2.2 for more details.

parametric error

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$0.5 - 1$	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	0.6	2 (1)	$\delta(\Delta\alpha)$
Γ_Z [MeV]	0.1	0.1 (0.06)	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_ℓ [10^{-3}]	1	1.3 (0.7)	$\delta\alpha_s$

the most important SM parameters at the FCC-ee:

$$\delta m_t = 50 \text{ MeV}, \quad \delta m_b = 13 \text{ MeV}, \quad \delta M_Z = 0.1 \text{ MeV}, \quad \delta\alpha_s = 0.0002\ (0.0001),$$
$$\delta(\Delta\alpha) = 5\times 10^{-5}\ (3\times 10^{-5}).$$

Theoretical uncertainties on EWPOs

- Projection of intrinsic errors for FCC-ee EWPOs with different assumptions on theoretical inputs, i.e. available loop calculations

Table B.7: Comparison of experimental FCC-ee precision goals for selected EWPOs (EXP2, from Table B.1) with various scenarios for theoretical error estimations. TH1-new, current theoretical error based on extrapolations through geometric series; TH2, estimated theoretical error (using prefactor scalings), assuming that electroweak three-loop corrections are known; TH3, a scenario where the dominant four-loop corrections are also available. Since reliable quantitative estimates of TH3 are not possible at this point, only conservative upper bounds of the theoretical error are given.

	FCC-ee-Z EWPO error estimates			
	$\delta\Gamma_Z$ (MeV)	δR_ℓ (10^{-4})	δR_b (10^{-5})	$\delta \sin^2\theta^\ell_{\rm eff}$ (10^{-6})
EXP2 [46]	0.1	10	$2 \div 6$	6
TH1-new	0.4	60	10	45
TH2	0.15	15	5	15
TH3	<0.07	<7	<3	<7

State of art calculations

- The last pieces (bosonic contributions) of full two-loop electroweak corrections on Z pole observables are completed in 2018

Table B.4: Loop contributions to the partial and total Z widths with fixed M_W as input parameter. Here N_f and N_f^2 refer to corrections with one and two closed fermion loops, respectively, whereas α_bos^2 denotes contributions without closed fermion loops. Furthermore, $\alpha_\mathrm{t} = y_\mathrm{t}^2/(4\pi)$, where y_t is the top Yukawa coupling. Table taken from Ref. [21] (Creative Commons Attribution Licence, CC BY).

Γ_i (MeV)	Γ_e	Γ_ν	Γ_d	Γ_u	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_\mathrm{s})$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(\alpha_\mathrm{t}\alpha_\mathrm{s}^2,\ \alpha_\mathrm{t}\alpha_\mathrm{s}^3,\ \alpha_\mathrm{t}^2\alpha_\mathrm{s},\ \alpha_\mathrm{t}^3)$	0.038	0.059	0.191	0.170	0.190	1.20
$\mathcal{O}(N_\mathrm{f}^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_\mathrm{f}\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_\mathrm{bos}^2)$	0.017	0.019	0.059	0.058	0.167	0.51

1. Determination of relevant prefactors of a class of higher-order corrections, such as couplings, group factors, particle multiplicities, mass ratios, etc., and assuming the remainder of the loop amplitude to be order $\mathcal{O}(1)$.
2. Extrapolation under the assumption that higher-order radiative corrections can be approximated by a geometric series.
3. Testing the scale-dependence of a given fixed-order result obtained using the $\overline{\mathrm{MS}}$ renormalization scheme, in order to estimate the size of the missing higher orders; this is used more often in QCD.
4. Comparing results obtained using the on-shell and $\overline{\mathrm{MS}}$ schemes, where the differences are of the next order in the perturbative expansion.

how to estimate intrinsic errors?

State of art calculations

- The last pieces (bosonic contributions) of full two-loop electroweak corrections on Z pole observables are completed in 2018

Table B.5: Intrinsic theoretical error estimates (TH1) for Γ_Z [45, 75], updates taking into account the newly completed $\mathcal{O}(\alpha^2_{\rm bos})$) corrections (TH1-new) [21] and a projection into the future, assuming $\delta_{2,3}$ and the fermionic parts of δ_1 to be known (TH2).

δ_1	δ_2	δ_3	δ_4	δ_5	$\delta\Gamma_Z$ (MeV)
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_{\rm s})$	$\mathcal{O}(\alpha\alpha_{\rm s}^2)$	$\mathcal{O}(\alpha\alpha_{\rm s}^3)$	$\mathcal{O}(\alpha^2_{\rm bos})$	$=\sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from geometric series of perturbation)					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from geometric series of perturbation)					
0.2	0.21	0.23	0.035	$< 10^{-4}$	0.4
δ_1'	δ_2'	δ_3'	δ_4		$\delta\Gamma_Z$ (MeV)
$\mathcal{O}(N_{\rm f}^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_{\rm s})$	$\mathcal{O}(\alpha^2\alpha_{\rm s}^2)$	$\mathcal{O}(\alpha\alpha_{\rm s}^3)$		$=\sqrt{\delta_1'^2+\delta_2'^2+\delta_2'^3+\delta_4^2}$
TH2 (extrapolation through prefactor scaling)					
0.04	0.1	0.1	0.035	10^{-4}	0.15

method of geometric series

$$\mathcal{O}(\alpha^3)-\mathcal{O}(\alpha_{\rm t}^3)\sim\frac{\mathcal{O}(\alpha^2_{\rm ferm})-\mathcal{O}(\alpha_{\rm t}^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2_{\rm ferm})$$

$$\mathcal{O}(\alpha^2\alpha_{\rm s})-\mathcal{O}(\alpha_{\rm t}^2\alpha_{\rm s})\sim\frac{\mathcal{O}(\alpha^2_{\rm ferm})-\mathcal{O}(\alpha_{\rm t}^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_{\rm s}),$$

$$\mathcal{O}(\alpha\alpha_{\rm s}^2)-\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)\sim\frac{\mathcal{O}(\alpha\alpha_{\rm s})-\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s})}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_{\rm s}),$$

Three-loop calculations

- Ingredients for 3-loop calculations of Z decay; challenges due to both large number of diagram/integrals, multi-mass scales, as well as high numerical precision required

Table B.6: Number of topologies and diagrams for $Z \to f\bar{f}$ decays in the Feynman gauge. Statistics for planarity, QCD, and EW-type diagrams are also given. Label 'A' denotes statistics after elimination of tadpoles and wavefunction corrections, and label 'B' denotes statistics after elimination of topological symmetries of diagrams.

$Z \to b\bar{b}$	1 loop	2 loops	3 loops
Number of topologies	1	$14 \overset{(A)}{\to} 7 \overset{(B)}{\to} 5$	$211 \overset{(A)}{\to} 84 \overset{(B)}{\to} 51$
Number of diagrams	15	$2383 \overset{(A,B)}{\to} 1074$	$490\,387 \overset{(A,B)}{\to} 120\,472$
Fermionic loops	0	150	17 580
Bosonic loops	15	924	102 892
Planar / non-planar	15 / 0	981/133	84 059/36 413
QCD / EW	1 / 14	98 / 1016	10 386/110 086
$Z \to e^+e^-, \ldots$			
Number of topologies	1	$14 \overset{(A)}{\to} 7 \overset{(B)}{\to} 5$	$211 \overset{(A)}{\to} 84 \overset{(B)}{\to} 51$
Number of diagrams	14	$2012 \overset{(A,B)}{\to} 880$	$397\,690 \overset{(A,B)}{\to} 91\,472$
Fermionic loops	0	114	13104
Bosonic loops	14	766	78 368
Planar / non-planar	14 / 0	782/98	65 487/25 985
QCD / EW	0 / 14	0 / 880	144/91 328

$\mathcal{O}(10^3) - \mathcal{O}(10^4)$ distinct three-loop Feynman integrals before a reduction

assume that the values TH1-new in Table B.5 are representative of the actual size of the currently unknown three-loop corrections. Second, the achievement of at least two digits intrinsic net numerical precision for the three-loop electroweak corrections will probably require the evaluation of single Feynman integrals with much greater precision than in the two-loop case, since the larger number of diagrams leads to more numerical cancellations, and each new diagram topology poses new challenges for the numerical convergence.