An $\mathrm{SU}(8)$ theory of the SM quarks and leptons

an endeavor to the SM flavor puzzle

Ning Chen

School of Physics, Nankai University

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南開大學
Nankai University

Contents

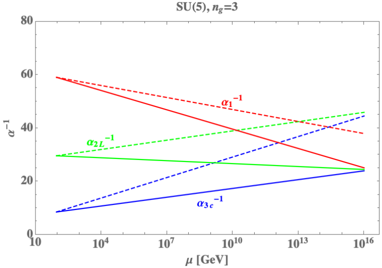
References

- Early works:
  a “Towards a Grand Unified Theory of Flavor”, Nucl.Phys.B 156 (1979) 126, Howard Georgi.
  b “Doubly Lopsided Mass Matrices from Unitary Unification”, Phys. Rev. D 78 (2008) 075001, 0804.1356, Stephen Barr.
- Recent papers:
  α “The global $B-L$ symmetry in the flavor-unified $\mathrm{SU}(N)$ theories”, JHEP 04 (2024) 046, 2307.07921, **NC**, Ying-nan Mao, Zhaolong Teng.
  β “The Standard Model quark/lepton masses and the Cabibbo-Kobayashi-Maskawa mixing in an $\mathrm{SU}(8)$ theory”, 2402.10471, **NC**, Ying-nan Mao, Zhaolong Teng.
  γ “The gauge coupling unification in an $\mathrm{SU}(8)$ theory”, *in preparation*, **NC**, Zhanpeng Hou, Ying-nan Mao, Zhaolong Teng.

Historical Reviews: *minimal* GUTs and the Flavor Puzzle

Historical reviews

- GUTs were proposed in terms of the minimal gauge group of $\mathrm{SU}(5)$ with $3 \times \left[\overline{\mathbf{5_F}} \oplus \mathbf{10_F} \right]$ by [‘74, Georgi-Glashow] (GG), and $\mathrm{SO}(10)$ with $3 \times \mathbf{16_F}$ by [‘75, Fritzsch-Minkowski]. The main ingredients: (i) gauge symmetries $\mathcal{G}_{\rm SM} \equiv \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_W \otimes \mathrm{U}(1)_Y \subset \mathcal{G}_{\rm GUT}$, AF of the QCD [‘73, Gross, Wilczek, Politzer], (ii) the two-generational chiral fermions (with charm quark theorized in ‘70 by Glashow-Iliopoulos-Maiani, and discovered in late ‘74).
- The supersymmetric (susy) extension to the $\mathrm{SU}(5)$ can unify three SM gauge couplings at $\mu \sim 10^{16}\,\mathrm{GeV}$ [‘81, Dimopoulos-Georgi], with $\mathcal{O}(1)$ TeV sparticles.



Historical reviews

- The chiral fermions in $\mathrm{SU}(5)$ are decomposed as $\overline{\mathbf{5}_{\mathbf{F}}} = \underbrace{(\overline{\mathbf{3}}\,, \mathbf{1}\,, +\frac{1}{3})_{\mathbf{F}}}_{d_R{}^c} \oplus \underbrace{(\mathbf{1}\,, \overline{\mathbf{2}}\,, -\frac{1}{2})_{\mathbf{F}}}_{\ell_L}$ and $\mathbf{10}_{\mathbf{F}} = \underbrace{(\mathbf{3}\,, \mathbf{2}\,, +\frac{1}{6})_{\mathbf{F}}}_{q_L} \oplus \underbrace{(\overline{\mathbf{3}}\,, \mathbf{1}\,, -\frac{2}{3})_{\mathbf{F}}}_{u_R{}^c} \oplus \underbrace{(\mathbf{1}\,, \mathbf{1}\,, +1)_{\mathbf{F}}}_{e_R{}^c}$.
- The susy GG $\mathrm{SU}(5)$ model contains Higgs fields of $\mathbf{24}_{\mathbf{H}} \oplus \mathbf{5}_{\mathbf{H}} \oplus \overline{\mathbf{5}_{\mathbf{H}}}$, with the GUT symmetry breaking of $\mathrm{SU}(5) \xrightarrow{\langle \mathbf{24}_{\mathbf{H}} \rangle} \mathcal{G}_{\mathrm{SM}}$.
- The Yukawa couplings come from the superpotential of

$$W_Y = Y_D \overline{\mathbf{5}_{\mathbf{F}}} \mathbf{10}_{\mathbf{F}} \overline{\mathbf{5}_{\mathbf{H}}} + Y_U \mathbf{10}_{\mathbf{F}} \mathbf{10}_{\mathbf{F}} \mathbf{5}_{\mathbf{H}} \,. \quad (1)$$

At the EW scale, the Higgs spectrum include two doublets of $\Phi_u \equiv (\mathbf{1}\,, \mathbf{2}\,, +\frac{1}{2})_{\mathbf{H}} \subset \mathbf{5}_{\mathbf{H}}$ and $\Phi_d \equiv (\mathbf{1}\,, \overline{\mathbf{2}}\,, -\frac{1}{2})_{\mathbf{H}} \subset \overline{\mathbf{5}_{\mathbf{H}}}$.

Historical reviews

- Besides of the well-acknowledged challenges within the *minimal* GUTs, there are two longstanding problems within the SM that have never been solved with/without the susy extension, which are: (i) the SM flavor puzzle, (ii) the PQ quality problem of the QCD axion.
- The formulation of the QM solved several fundamental puzzles in the late $19^{\rm th}$ century: (i) blackbody radiation [1900, Max Planck], (ii) photoelectric effects [1905, Albert Einstein], (iii) hydrogen spectrum [1913, Niels Bohr], by hypothesizing the quantized energies/angular momenta of particles.
- This talk: the SM flavor puzzle can be addressed by extending the *minimal* $\mathrm{SU}(5)/\mathrm{SO}(10)$ GUTs to the $\mathrm{SU}(8)$ GUT (minimally), with the $[N\,,k]_{\mathbf{F}}$ $(k\geq 3)$ irreps (to avoid the exotic fermions).

The flavor puzzle: origin

- The SM flavor puzzle: (i) inter-generational mass hierarchies, (ii) intra-generational mass hierarchies with non-universal splitting patterns, and (iii) the CKM mixing pattern of the quarks and the PMNS mixing pattern of the neutrinos.
- Why/how $n_g = 3$? Both the SM and the *minimal* GUTs admit the simple repetitive structure in terms of their chiral irreducible anomaly-free fermion sets (IRAFFSs).

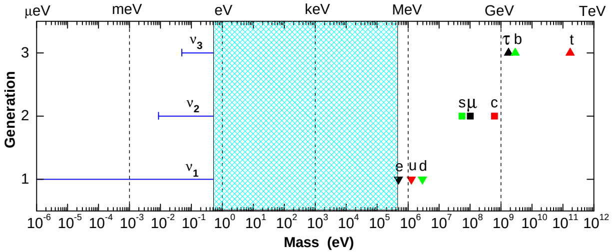


Figure: The SM fermion mass spectrum, 1909.09610, Z.Z. Xing.

The flavor puzzle: Yukawa couplings

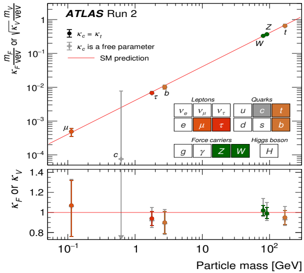


Figure: The LHC measurements of the SM Higgs boson, 2207.00092.

- The flavor puzzle: to look for the origin of the hierarchical Yukawa couplings of the *single* SM Higgs boson $y_f = \sqrt{2} m_f / v_{\rm EW}$ for all SM quarks/leptons.
- Symmetry dictates interactions ['80, Chen-Ning Yang].

The flavor puzzle

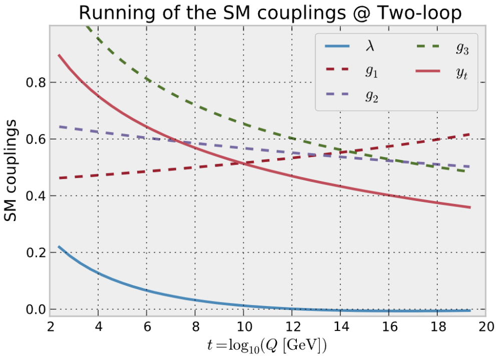


Figure: The RGEs of SM couplings, by PyR@TE.

- The RGEs cannot generate large mass hierarchies ['78, Froggatt, Nielsen].

Flavors and Global Symmetries in the $\mathrm{SU}(8)$ Theory

The origin of generations

- The main conjecture: three generations do not repeat but are non-trivially embedded in the UV theories such as the GUT [‘79, Georgi, ‘80, Nanopolous].
- This was first considered by [‘79, Georgi] based on a unified group of $\mathrm{SU}(N)$, with the anti-symmetric chiral fermions of

$$\{f_L\}_{\mathrm{SU}(N)} = \sum_k n_k \, [N\,, k]_{\mathbf{F}} \,, \; n_k \in \mathbb{Z} \,. \tag{2}$$

No exotic fermions in the spectrum with the $[N\,, k]_{\mathbf{F}}$.

- The anomaly-free condition can be expressed in terms of a Diophantine equation

$$\sum_k n_k \, \mathrm{Anom}([N\,, k]_{\mathbf{F}}) = 0 \,, \tag{3}$$

$$\mathrm{Anom}([N\,, k]_{\mathbf{F}}) = \frac{(N-2k)\,(N-3)!}{(N-k-1)!\,(k-1)!} \,. \tag{4}$$

Georgi's counting of the SM generations

- To decompose the $\mathrm{SU}(N)$ irreps under the $\mathrm{SU}(5)$, e.g., $\mathbf{N_F} = (N-5)\times \mathbf{1_F} \oplus \mathbf{5_F}$. Decompositions of other irreps can be obtained by tensor products, [‘79, Georgi].
- All fermion irreps in Eq. (2) can be decomposed into the $\mathrm{SU}(5)$ irreps of $(\mathbf{1_F}\,, \mathbf{5_F}\,, \mathbf{10_F}\,, \overline{\mathbf{10_F}}\,, \overline{\mathbf{5_F}})$, and we denote their multiplicities as $(\nu_{\mathbf{1_F}}\,, \nu_{\mathbf{5_F}}\,, \nu_{\mathbf{10_F}}\,, \nu_{\overline{\mathbf{10_F}}}\,, \nu_{\overline{\mathbf{5_F}}})$.
- Their multiplicities should satisfy $\nu_{\mathbf{5_F}} + \nu_{\mathbf{10_F}} = \nu_{\overline{\mathbf{5_F}}} + \nu_{\overline{\mathbf{10_F}}}$ from the anomaly-free condition.
- The total SM fermion generations are determined by

$$n_g = \nu_{\overline{\mathbf{5_F}}} - \nu_{\mathbf{5_F}} = \nu_{\mathbf{10_F}} - \nu_{\overline{\mathbf{10_F}}}\,. \quad (5)$$

Georgi's counting of the SM generations in GUTs

- The net $\mathbf{10_F}$'s from a particular $\mathrm{SU}(N)$ irrep [2209.11446]

$$\nu_{\mathbf{10_F}}\,[N\,,k]_{\mathbf{F}} - \nu_{\overline{\mathbf{10_F}}}\,[N\,,k]_{\mathbf{F}} = \frac{(N-2k)(N-5)!}{(k-2)!\,(N-k-2)!}\,. \tag{6}$$

- The usual rank-2 GG models can only give $\nu_{\mathbf{10_F}}\,[N\,,2]_{\mathbf{F}} - \nu_{\overline{\mathbf{10_F}}}\,[N\,,2]_{\mathbf{F}} = 1$. This means one can only repeat the set of anomaly-free fermion irreps to form multiple generations in rank-2 GG models.
- Alternatively, to embed multiple generations non-trivially in the GUTs, one must consider at least the rank-3 GG models. The leading candidate group must be $\mathrm{SU}(7)$, ['79, Frampton], since the $[6\,,3]_{\mathbf{F}}$ irrep of $\mathrm{SU}(6)$ is self-conjugate.
- Note that the non-minimal GUTs are likely to loose the asymptotic freedom (AF), since $T([N\,,2]) \sim N$ and $T([N\,,3]) \sim N^2$. For the $n_g = 3$ case, we find that the GUTs with $\mathcal{G} \geq \mathrm{SU}(9)$ loose the AF.

Georgi's counting of the SM generations in GUTs

- $\mathrm{SO}(2N)$ cannot embed multiple generations non-trivially, e.g., spinor irrep of $\mathbf{64_F}$ in the $\mathrm{SO}(14)$ is decomposed into $\mathrm{SO}(10)$ as

$$\mathbf{64_F} \to \ldots \to 2 \times \left[\mathbf{16_F} \oplus \overline{\mathbf{16_F}} \right] \, , \qquad (7)$$

  $\mathbf{16_F} = \mathbf{1_F} \oplus \overline{\mathbf{5_F}} \oplus \mathbf{10_F}$, and $\overline{\mathbf{16_F}} = \mathbf{1_F} \oplus \mathbf{5_F} \oplus \overline{\mathbf{10_F}}$. One can only obtain $n_g = 2 - 2 = 0$ at the EW scale.
- The realistic symmetry breaking patterns of the $\mathrm{SU}(N)$ usually do not follow the $\mathrm{SU}(N) \to \ldots \to \mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ one, which is dangerous in terms of the proton decay. n_g is independent of the symmetry breaking patterns.
- The zeroth stage would better be achieved by the $\mathrm{SU}(N)$ adjoint Higgs field [‘74, L.F.Li] as $\mathrm{SU}(N) \to \mathrm{SU}(k_1)_S \otimes \mathrm{SU}(k_2)_W \otimes \mathrm{U}(1)_X \, , k_1 = [\frac{N}{2}]$, since we wish to set the proton decay scale as high as possible. Currently, $\tau_p \gtrsim 10^{34}\,\mathrm{yr}$ [‘20, SuperK].

Georgi's counting of the SM generations in GUTs

- Georgi's "third law" of GUT [‘79]: no repetition of a particular irrep of $\mathrm{SU}(N)$, i.e., $n_k = 0$ or $n_k = 1$ in Eq. (2). The minimal theory is

$$\{f_L\}_{\mathrm{SU}(11)} = [11\,,4]_{\mathbf{F}} \oplus [11\,,8]_{\mathbf{F}} \oplus [11\,,9]_{\mathbf{F}} \oplus [11\,,10]_{\mathbf{F}}\,. \tag{8}$$

  with $\dim_{\mathbf{F}} = 561$.

- My understanding: to prevent the simple repetitions of one generational anomaly-free fermions, such as $3 \times \left[\overline{\mathbf{5}_{\mathbf{F}}} \oplus \mathbf{10}_{\mathbf{F}}\right]$ in an $\mathrm{SU}(5)$ chiral theory. The fermions of $\left[\overline{\mathbf{5}_{\mathbf{F}}} \oplus \mathbf{10}_{\mathbf{F}}\right]$ is a chiral irreducible anomaly-free fermion set (IRAFFS) of the $\mathrm{SU}(5)$.
- My "third law" [2307.07921]: *only distinctive chiral IRAFFSs without simple repetitions and can lead to $n_g = 3$ at the EW scale are allowed in an $\mathrm{SU}(N)$ theory.*

The $\mathrm{SU}(8)$ theory

- The $\mathrm{SU}(8)$ theory with rank-2 and rank-3 chiral IRAFFSs of

$$\{f_L\}^{n_g=3}_{\mathrm{SU}(8)} = \left[\overline{\mathbf{8_F}}^{\omega} \oplus \mathbf{28_F}\right] \bigoplus \left[\overline{\mathbf{8_F}}^{\dot\omega} \oplus \mathbf{56_F}\right] , \ \dim_{\mathbf{F}} = 156 ,$$
$$\omega = (3 \,, \mathrm{IV} \,, \mathrm{V} \,, \mathrm{VI}) \,, \ \dot\omega = (\dot{1} \,, \dot{2} \,, \dot{\mathrm{VII}} \,, \dot{\mathrm{IIX}} \,, \dot{\mathrm{IX}}) \,. \quad (9)$$

  Six $(\mathbf{5_F} \,, \overline{\mathbf{5_F}})$ pairs, one $(\mathbf{10_F} \,, \overline{\mathbf{10_F}})$ pair from the $\mathbf{56_F}$, and $3 \times \left[\overline{\mathbf{5_F}} \oplus \mathbf{10_F}\right]_{\mathrm{SM}}$.

- The Higgs fields and the Yukawa couplings:

$$-\mathcal{L}_Y = Y_{\mathcal{B}} \overline{\mathbf{8_F}}^{\omega} \mathbf{28_F} \overline{\mathbf{8_H}}_{,\omega} + Y_{\mathcal{T}} \mathbf{28_F} \mathbf{28_F} \mathbf{70_H}$$
$$+ Y_{\mathcal{D}} \overline{\mathbf{8_F}}^{\dot\omega} \mathbf{56_F} \overline{\mathbf{28_H}}_{,\dot\omega} + \frac{c_4}{M_{\mathrm{pl}}} \mathbf{56_F} \mathbf{56_F} \overline{\mathbf{28_H}}^{\dagger}_{,\dot\omega} \mathbf{63_H} + H.c. \,. \quad (10)$$

  NB: $\mathbf{56_F} \mathbf{56_F} \mathbf{28_H} = 0$ ['08, S. Barr], and one has to use $d = 5$ operator suppressed by $1/M_{\mathrm{pl}}$, with $M_{\mathrm{pl}} = (8\pi G_N)^{1/2} = 2.4 \times 10^{18}\,\mathrm{GeV}$.

- *Gravity breaks global symmetries.*

Global symmetries in the $\mathrm{SU}(8)$ theory

- The global symmetries of the $\mathrm{SU}(8)$ theory:

$$\widetilde{\mathcal{G}}_{\rm flavor}\left[\mathrm{SU}(8)\right] = \left[\widetilde{\mathrm{SU}}(4)_{\omega} \otimes \widetilde{\mathrm{U}}(1)_{T_2} \otimes \widetilde{\mathrm{U}}(1)_{\mathrm{PQ}_2}\right]$$
$$\bigotimes \left[\widetilde{\mathrm{SU}}(5)_{\dot\omega} \otimes \widetilde{\mathrm{U}}(1)_{T_3} \otimes \widetilde{\mathrm{U}}(1)_{\mathrm{PQ}_3}\right] ,$$
$$\left[\mathrm{SU}(8)\right]^2 \cdot \widetilde{\mathrm{U}}(1)_{T_{2\,,3}} = 0 \,, \quad \left[\mathrm{SU}(8)\right]^2 \cdot \widetilde{\mathrm{U}}(1)_{\mathrm{PQ}_{2\,,3}} \neq 0 \,. \qquad (11)$$

Fermions	$\overline{\mathbf{8_F}}^{\Omega=\omega\,,\dot\omega}$	$\mathbf{28_F}$	$\mathbf{56_F}$	
$\widetilde{\mathrm{U}}(1)_T$	$-3t$	$+2t$	$+t$	
$\widetilde{\mathrm{U}}(1)_{\mathrm{PQ}}$	p	q_2	q_3	
Higgs	$\overline{\mathbf{8}}_{\mathbf{H}\,,\omega}$	$\overline{\mathbf{28}}_{\mathbf{H}\,,\dot\omega}$	$\mathbf{70_H}$	$\mathbf{63_H}$
$\widetilde{\mathrm{U}}(1)_T$	$+t$	$+2t$	$-4t$	0
$\widetilde{\mathrm{U}}(1)_{\mathrm{PQ}}$	$-(p+q_2)$	$-(p+q_3)$	$-2q_2$	0

Table: The $\widetilde{\mathrm{U}}(1)_T$ and the $\widetilde{\mathrm{U}}(1)_{\mathrm{PQ}}$ charges, $p : q_2 \neq -3 : +2$ and $p : q_3 \neq -3 : +1$.

Global symmetries in the $\mathrm{SU}(8)$ theory

- The global $\widetilde{\mathrm{U}}(1)_T$ symmetries at different stages

$$
\begin{aligned}
\mathrm{SU}(8) \to \mathcal{G}_{441_{X_0}} &: \mathcal{T}' = \mathcal{T} - 4t\mathcal{X}_0 \,, \\
\mathcal{G}_{441_{X_0}} \to \mathcal{G}_{341_{X_1}} &: \mathcal{T}'' = \mathcal{T}' + 8t\mathcal{X}_1 \,, \\
\mathcal{G}_{341_{X_1}} \to \mathcal{G}_{331_{X_2}} &: \mathcal{T}''' = \mathcal{T}'' \,, \quad \mathcal{G}_{331_{X_2}} \to \mathcal{G}_{\mathrm{SM}} \;:\; \mathcal{B} - \mathcal{L} = \mathcal{T}''' \,. \qquad (12)
\end{aligned}
$$

Higgs	$\mathcal{G}_{441} \to \mathcal{G}_{341}$	$\mathcal{G}_{341} \to \mathcal{G}_{331}$	$\mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$	$\mathcal{G}_{\mathrm{SM}} \to \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\mathrm{EM}}$
$\overline{\mathbf{8_H}}_{,\omega}$	✓	✓	✓	✓
$\overline{\mathbf{28_H}}_{,\dot\omega}$	✗	✓	✓	✓
$\mathbf{70_H}$	✗	✗	✗	✓

Table: The Higgs fields and their symmetry-breaking directions in the $\mathrm{SU}(8)$ theory. The ✓and ✗represent possible and impossible symmetry breaking directions for a given Higgs field.

Global symmetries in the $\mathrm{SU}(8)$ theory

- Consistent global $\widetilde{\mathrm{U}}(1)_{B-L}$ relations of $(\mathcal{B}-\mathcal{L})(q_L)=\frac{4}{3}t$, $(\mathcal{B}-\mathcal{L})(\ell_L)=-4t$, and etc.
- The global $\widetilde{\mathrm{U}}(1)_{B-L}$

$$\mathbf{70_H} \supset \underline{(\mathbf{4}\,,\overline{\mathbf{4}}\,,+\frac{1}{2})_{\mathbf{H}}} \oplus (\overline{\mathbf{4}}\,,\mathbf{4}\,,-\frac{1}{2})_{\mathbf{H}} \supset \boxed{\underbrace{(\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime}}_{\mathcal{B}-\mathcal{L}=0}} \oplus \underbrace{(\mathbf{1}\,,\mathbf{2}\,,-\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime}}_{\mathcal{B}-\mathcal{L}=-8t}\,. \quad (13)$$

We conjecture that the $(\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime}\subset\mathbf{70_H}$ is the only SM Higgs doublet.

- We also find 23 out of 27 left-handed sterile neutrinos $\check{\mathcal{N}}_L^{\Omega}$ remain massless through the 't Hooft anomaly matching of the global $\widetilde{\mathrm{U}}(1)_T$ to $\widetilde{\mathrm{U}}(1)_{B-L}$.

The $\mathrm{SU}(8)$ fermions

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\overline{\mathbf{8_F}}^{\Omega}$	$(\overline{\mathbf{4}}, \mathbf{1}, +\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\Omega}$	$(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\Omega}$	$(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\Omega} ~:~ \mathcal{D}_R^{\Omega\,c}$
		$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\Omega}$	$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\Omega}$	$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\Omega} ~:~ \check{\mathcal{N}}_L^{\Omega}$
	$(\mathbf{1}, \overline{\mathbf{4}}, -\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\mathbf{1}, \overline{\mathbf{4}}, -\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\Omega}$	$(\mathbf{1}, \overline{\mathbf{2}}, -\frac{1}{2})_{\mathbf{F}}^{\Omega} ~:~ \mathcal{L}_L^{\Omega} = (\mathcal{E}_L^{\Omega}, -\mathcal{N}_L^{\Omega})^T$
				$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\Omega'} ~:~ \check{\mathcal{N}}_L^{\Omega'}$
			$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\Omega''}$	$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\Omega''} ~:~ \check{\mathcal{N}}_L^{\Omega''}$

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{28_F}$	$(\mathbf{6}, \mathbf{1}, -\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})_{\mathbf{F}} ~:~ \mathfrak{D}_L$
		$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}$	$\underline{(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}} ~:~ {t_R}^c}$
	$(\mathbf{1}, \mathbf{6}, +\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{6}, +\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{3}, +\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})_{\mathbf{F}} ~:~ ({\mathfrak{e}_R}^c, {\mathfrak{n}_R}^c)^T$
				$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}} ~:~ \check{\mathfrak{n}}_R^c$
			$(\mathbf{1}, \overline{\mathbf{3}}, +\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{1}, \overline{\mathbf{2}}, +\frac{1}{2})_{\mathbf{F}}' ~:~ ({\mathfrak{n}_R'}^c, -{\mathfrak{e}_R'}^c)^T$
				$\underline{(\mathbf{1}, \mathbf{1}, +1)_{\mathbf{F}} ~:~ {\tau_R}^c}$
	$(\mathbf{4}, \mathbf{4}, 0)_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{4}, -\frac{1}{12})_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{3}, 0)_{\mathbf{F}}$	$\underline{(\mathbf{3}, \mathbf{2}, +\frac{1}{6})_{\mathbf{F}} ~:~ (t_L, b_L)^T}$
				$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})_{\mathbf{F}}' ~:~ \mathfrak{D}_L'$
			$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})_{\mathbf{F}}''$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})_{\mathbf{F}}'' ~:~ \mathfrak{D}_L''$
		$(\mathbf{1}, \mathbf{4}, +\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{3}, +\frac{1}{3})_{\mathbf{F}}''$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})_{\mathbf{F}}'' ~:~ ({\mathfrak{e}_R''}^c, {\mathfrak{n}_R''}^c)^T$
				$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}' ~:~ \check{\mathfrak{n}}_R'^c$
			$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}''$	$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}'' ~:~ \check{\mathfrak{n}}_R''^c$

The SU(8) fermions

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{56_F}$	$(\mathbf{1},\overline{\mathbf{4}},+\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{4}},+\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}^{\prime}$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}^{\prime\prime\prime}\ :\ (\mathfrak{n}_R^{\prime\prime\prime c},-\mathfrak{e}_R^{\prime\prime\prime c})^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}^{\prime}\ :\ {\mu_R}^c$
			$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}^{\prime\prime}$	$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}^{\prime\prime}\ :\ {\mathfrak{E}_R}^c$
	$(\overline{\mathbf{4}},\mathbf{1},-\frac{3}{4})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}^{\prime}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}^{\prime}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}^{\prime}\ :\ {u_R}^c$
		$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}\ :\ \mathfrak{E}_L$
	$(\mathbf{4},\mathbf{6},+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{6},+\frac{1}{6})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}^{\prime}$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}^{\prime}\ :\ (c_L,s_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}^{\prime\prime\prime}\ :\ \mathfrak{D}_L^{\prime\prime\prime}$
			$(\mathbf{3},\overline{\mathbf{3}},+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\overline{\mathbf{2}},+\frac{1}{6})_{\mathbf{F}}^{\prime\prime}\ :\ (\mathfrak{d}_L,-\mathfrak{u}_L)^T$
				$(\mathbf{3},\mathbf{1},+\frac{2}{3})_{\mathbf{F}}\ :\ \mathfrak{U}_L$
		$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}^{\prime}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}^{\prime}$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}^{\prime\prime\prime\prime}\ :\ (\mathfrak{e}_R^{\prime\prime\prime\prime c},\mathfrak{n}_R^{\prime\prime\prime\prime c})^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\prime\prime\prime}\ :\ \check{\mathfrak{n}}_R^{\prime\prime\prime c}$
			$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}^{\prime\prime}$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}^{\prime\prime\prime\prime\prime}\ :\ (\mathfrak{n}_R^{\prime\prime\prime\prime\prime c},-\mathfrak{e}_R^{\prime\prime\prime\prime\prime c})^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}^{\prime\prime\prime}\ :\ {e_R}^c$
	$(\mathbf{6},\mathbf{4},-\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{4},-\frac{1}{12})_{\mathbf{F}}^{\prime}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}^{\prime\prime}$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}^{\prime\prime\prime}\ :\ (u_L,d_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}^{\prime\prime\prime\prime}\ :\ \mathfrak{D}_L^{\prime\prime\prime\prime}$
			$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}^{\prime\prime\prime\prime\prime}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}^{\prime\prime\prime\prime\prime}\ :\ \mathfrak{D}_L^{\prime\prime\prime\prime\prime}$
		$(\overline{\mathbf{3}},\mathbf{4},-\frac{5}{12})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{3},-\frac{1}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{2},-\frac{1}{6})_{\mathbf{F}}\ :\ ({\mathfrak{d}_R}^c,{\mathfrak{u}_R}^c)^T$
				$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}^{\prime\prime}\ :\ {\mathfrak{U}_R}^c$
			$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}^{\prime\prime\prime}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}^{\prime\prime\prime}\ :\ {c_R}^c$

Symmetry breaking pattern in the $\mathrm{SU}(8)$ theory

- The symmetry breaking pattern ['74, L.F.Li] of $\mathrm{SU}(8) \to \mathcal{G}_{441} \to \mathcal{G}_{341} \to \mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}} \to \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\mathrm{EM}}$.
- The intermediate symmetry breaking stages and massive vectorlike fermions:

  0 : $\mathrm{SU}(8) \xrightarrow{\mathbf{63_H}} \mathcal{G}_{441}$, all fermions remain massless.

  1 : $\mathcal{G}_{441} \xrightarrow{\overline{\mathbf{8_H}}_{,\mathrm{IV}}} \mathcal{G}_{341}$, a pair of $(\mathbf{5_F}, \overline{\mathbf{5_F}})$.

  2 : $\mathcal{G}_{341} \xrightarrow{\overline{\mathbf{8_H}}_{,\mathrm{V}}, \overline{\mathbf{28_H}}_{,\dot{1},\dot{\mathrm{VII}}}} \mathcal{G}_{331}$, two pairs of $(\mathbf{5_F}, \overline{\mathbf{5_F}})$ and a pair of $(\mathbf{10_F}, \overline{\mathbf{10_F}})$.

  3 : $\mathcal{G}_{331} \xrightarrow{\overline{\mathbf{8_H}}_{,3,\mathrm{VI}}, \overline{\mathbf{28_H}}_{,\dot{2},\mathrm{I}\dot{\mathrm{X}},\mathrm{I}\dot{\mathrm{X}}}} \mathcal{G}_{\mathrm{SM}}$, three pairs of $(\mathbf{5_F}, \overline{\mathbf{5_F}})$.
- Dimensionless parameters

$$
\zeta_1 \equiv \frac{W_{\overline{\mathbf{4}},\mathrm{IV}}}{M_{\mathrm{pl}}}, \quad \zeta_2 \equiv \frac{w_{\overline{\mathbf{4}},\mathrm{V}}}{M_{\mathrm{pl}}}, \quad \dot{\zeta}_2 \equiv \frac{w_{\overline{\mathbf{4}},\dot{1},\dot{\mathrm{VII}}}}{M_{\mathrm{pl}}},
$$
$$
\zeta_3 \equiv \frac{V_{\overline{\mathbf{3}},3,\mathrm{VI}}}{M_{\mathrm{pl}}}, \quad \dot{\zeta}_3' \equiv \frac{V'_{\overline{\mathbf{3}},\dot{2},\mathrm{I}\dot{\mathrm{X}}}}{M_{\mathrm{pl}}}, \quad \dot{\zeta}_3 \equiv \frac{V_{\overline{\mathbf{3}},\mathrm{I}\dot{\mathrm{X}}}}{M_{\mathrm{pl}}},
$$
$$
\zeta_1 \gg \zeta_2 \sim \dot{\zeta}_2 \gg \zeta_3 \sim \dot{\zeta}_3' \sim \dot{\zeta}_3 . \tag{14}
$$

Vectorlike fermions in the $\mathrm{SU}(8)$ theory

stages	$Q_e = -\frac{1}{3}$	$Q_e = +\frac{2}{3}$	$Q_e = -1$	$Q_e = 0$
v_{441} $\{\Omega\}$	$\mathfrak{D}$ $\rm IV$	-	$(\mathfrak{e}'' \,, \mathfrak{n}'')$ $\rm IV$	$\{\check{\mathfrak{n}}' \,, \check{\mathfrak{n}}''\}$ $\{{\rm IV}' \,, {\rm IV}''\}$
v_{341} $\{\Omega\}$	$\mathfrak{d} \,, \{\mathfrak{D}'' \,, \mathfrak{D}'''''\}$ $\{{\rm V} \,, \dot{\rm V}{\rm II}\}$	$\mathfrak{u} \,, \mathfrak{U}$	$\mathfrak{E} \,, (\mathfrak{e} \,, \mathfrak{n}) \,, (\mathfrak{e}'''' \,, \mathfrak{n}'''')$ $\{{\rm V} \,, \dot{\rm V}{\rm II}\}$	$\{\check{\mathfrak{n}} \,, \check{\mathfrak{n}}'''\}$ $\{{\rm V}' \,, \dot{\rm V}{\rm II}'\}$
v_{331} $\{\Omega\}$	$\{\mathfrak{D}' \,, \mathfrak{D}''' \,, \mathfrak{D}''''\}$ $\{{\rm VI} \,, {\rm II}\dot{\rm X} \,, {\rm I}\dot{\rm X}\}$	-	$(\mathfrak{e}' \,, \mathfrak{n}') \,, (\mathfrak{e}''' \,, \mathfrak{n}''') \,, (\mathfrak{e}''''' \,, \mathfrak{n}''''')$ $\{{\rm VI} \,, {\rm II}\dot{\rm X} \,, {\rm I}\dot{\rm X}\}$	-

Table: The vectorlike fermions at different intermediate symmetry breaking scales in the $\mathrm{SU}(8)$ theory.

The SM Quark/Lepton Masses and the CKM mixing in the $\mathrm{SU}(8)$ Theory

SM fermion masses in the $\mathrm{SU}(8)$ theory

- The natural top quark mass from the tree level

$$
\begin{aligned}
& Y_{\mathcal{T}} \mathbf{28_F} \mathbf{28_F} \mathbf{70_H} \supset Y_{\mathcal{T}} (\mathbf{6}\,,\mathbf{1}\,,-\frac{1}{2})_{\mathbf{F}} \otimes (\mathbf{4}\,,\mathbf{4}\,,0)_{\mathbf{F}} \otimes (\mathbf{4}\,,\overline{\mathbf{4}}\,,+\frac{1}{2})_{\mathbf{H}} \\
\supset & \ \ldots \supset Y_{\mathcal{T}} (\overline{\mathbf{3}}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}} \otimes (\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{6})_{\mathbf{F}} \otimes (\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime} \\
\Rightarrow & \ \frac{1}{\sqrt{2}} Y_{\mathcal{T}} t_L {t_R}^c v_{\mathrm{EW}} \,.
\end{aligned}
\tag{15}
$$

- With the identification of $(\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{6})_{\mathbf{F}} \equiv (t_L\,,b_L)^T$ and $(\overline{\mathbf{3}}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}} \equiv {t_R}^c$ within the $\mathbf{28_F}$, it is straightforward to infer that $(\mathbf{1}\,,\mathbf{1}\,,+1)_{\mathbf{F}} \equiv {\tau_R}^c$ also lives in the $\mathbf{28_F}$. This explains why do the third-generational SM $\mathbf{10_F}$ reside in the $\mathbf{28_F}$, while the first- and second-generational SM $\mathbf{10_F}$'s must reside in the $\mathbf{56_F}$.

SM fermion masses in the SU(8) theory

- To generate other lighter SM fermion masses: the gravitational effects through $d = 5$ operators, which break the global symmetries in Eq. (11) explicitly.
- The direct Yukawa couplings of $\mathcal{O}_{\mathcal{F}}^{d=5}$:

$$\begin{aligned}
&\mathcal{O}_{\mathcal{F}}^{(3\,,2)} \equiv \overline{\mathbf{8_F}}^{\dot\omega} \mathbf{56_F} \cdot \overline{\mathbf{28_H}}^{\dagger}_{,\dot\kappa} \cdot \mathbf{70_H}^{\dagger} \\
\Rightarrow &\left[\dot\zeta_3 (s_L \mathcal{D}_R^{\dot\omega\,c} - \mathcal{E}_L^{\dot\omega} \mu_R{}^c) + \dot\zeta_3^\prime (d_L \mathcal{D}_R^{\dot\omega\,c} - \mathcal{E}_L^{\dot\omega} e_R{}^c) \right] v_{\rm EW} \,, \\
&\mathcal{O}_{\mathcal{F}}^{(4\,,1)} \equiv \mathbf{56_F} \mathbf{56_F} \cdot \overline{\mathbf{28_H}}_{\,,\dot\omega} \cdot \mathbf{70_H} \Rightarrow \dot\zeta_2 (c_L u_R{}^c + \cancel{u_L c_R{}^c}) v_{\rm EW} \,, \\
&\mathcal{O}_{\mathcal{F}}^{(5\,,1)} \equiv \mathbf{28_F} \mathbf{56_F} \cdot \overline{\mathbf{8_H}}_{\,,\omega} \cdot \mathbf{70_H} \\
\Rightarrow &\left[\zeta_1 (u_L t_R{}^c + t_L u_R{}^c) + \zeta_2 (c_L t_R{}^c + t_L c_R{}^c) \right] v_{\rm EW} \,.
\end{aligned} \tag{16}$$

- All $(u\,, c\,, t)$ obtain hierarchical masses, while only the $(s\,, \mu)$ become massive.

SM fermion masses in the SU(8) theory

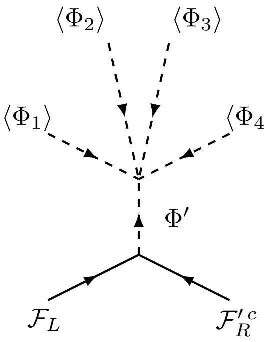


Figure: The indirect Yukawa couplings.

These are achievable through the EWSB components in renormalizable Yukawa couplings of $\overline{\mathbf{8_F}}^{\omega}\mathbf{28_F}\overline{\mathbf{8_H}}_{,\omega}$ and $\overline{\mathbf{8_F}}^{\dot\omega}\mathbf{56_F}\overline{\mathbf{28_H}}_{,\dot\omega}$.

SM fermion masses in the $\mathrm{SU}(8)$ theory

- There are two indirect Yukawa couplings with the irreducible Higgs mixing operators of $\mathcal{O}_{\mathcal{H}}^{d=5}$:

$$\begin{aligned} \mathcal{O}_{\mathscr{A}}^{d=5} \equiv\ & \epsilon_{\omega_1\omega_2\omega_3\omega_4} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_1} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_2} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_3} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_4} \mathbf{70_H}^{\dagger} \,, \\ & \mathcal{PQ} = 2(2p + 3q_2) \neq 0 \,, \end{aligned} \tag{17a}$$

$$\begin{aligned} \mathcal{O}_{\mathscr{B}}^{d=5} \supset\ & (\overline{\mathbf{28_H}}^{\dagger}_{,\dot{1}} \overline{\mathbf{28_H}}_{,\dot{\mathrm{VII}}}) \cdot \overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{I}\dot{\mathrm{I}}\mathrm{X}} \overline{\mathbf{28_H}}^{\dagger}_{,\dot{1}} \mathbf{70_H}^{\dagger} \,, \\ & (\overline{\mathbf{28_H}}^{\dagger}_{,\dot{1}} \overline{\mathbf{28_H}}_{,\dot{\mathrm{VII}}}) \cdot \overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{I}\dot{\mathrm{X}}} \overline{\mathbf{28_H}}^{\dagger}_{,\dot{2}} \mathbf{70_H}^{\dagger} \,, \\ & \mathcal{PQ} = 2(p + q_2 + q_3) \,. \end{aligned} \tag{17b}$$

- Each operator of $\mathcal{O}_{\mathcal{H}}^{d=5}$
  1. *breaks the global symmetries explicitly*;
  2. *can not be further partitioned into subset of renormalizable operators, among which any of them can be allowed by both the gauge and the global symmetries.*

SM fermion masses in the $\mathrm{SU}(8)$ theory

- The $(u\,,c\,,t)$ masses

$$\mathcal{M}_u = \frac{v_{\rm EW}}{\sqrt{2}} \begin{pmatrix} 0 & 0 & c_5\zeta_1/\sqrt{2} \\ c_4\dot{\zeta}_2/\sqrt{2} & 0 & c_5\zeta_2/\sqrt{2} \\ c_5\zeta_1/\sqrt{2} & c_5\zeta_2/\sqrt{2} & Y_{\mathcal{T}} \end{pmatrix} \,. \tag{18}$$

- The $(d\,,s\,,b)$ masses

$$\mathcal{M}_d \approx \frac{v_{\rm EW}}{4} \begin{pmatrix} (2c_3 + Y_{\mathcal{D}} d_{\mathscr{B}})\dot{\zeta}_3' & (2c_3 + Y_{\mathcal{D}} d_{\mathscr{B}} \Delta_{\dot{2}})\dot{\zeta}_3' & 0 \\ (2c_3 + Y_{\mathcal{D}} d_{\mathscr{B}} \Delta_{\dot{1}}')\dot{\zeta}_3 & (2c_3 + Y_{\mathcal{D}} d_{\mathscr{B}} \zeta_{23}^{-2})\dot{\zeta}_3 & 0 \\ 0 & 0 & Y_{\mathcal{B}} d_{\mathscr{A}} \zeta_{23}^{-1}\zeta_1 \end{pmatrix} . \tag{19}$$

The charged lepton masses are $\mathcal{M}_\ell = (\mathcal{M}_d)^T$.

- Natural fermion mass textures by following the gravity-induced $d = 5$ operators.

SM fermion masses in the SU(8) theory

- The $(u\,,c\,,t)$ masses

$$m_u \approx c_4 \frac{\zeta_2^2}{\sqrt{2}\zeta_1} v_{\rm EW}\,, \quad m_c \approx c_5^2 \frac{\zeta_1^2}{\sqrt{2} Y_{\mathcal{T}}} v_{\rm EW}\,, \quad m_t \approx \frac{Y_{\mathcal{T}}}{\sqrt{2}} v_{\rm EW}\,. \tag{20}$$

- The $(d\,,s\,,b)$ and $(e\,,\mu\,,\tau)$ masses

$$m_d = m_e \approx \frac{c_3 \dot{\zeta}_3}{2} |\tan\lambda| v_{\rm EW}\,, \quad m_s = m_\mu \approx \frac{1}{4}(2c_3 + Y_{\mathcal{D}} d_{\mathscr{B}} \zeta_{23}^{-2}) \dot{\zeta}_3 v_{\rm EW}\,,$$
$$m_b = m_\tau \approx Y_{\mathcal{B}} \frac{d_{\mathscr{A}} \zeta_1 \zeta_2}{4\zeta_3} v_{\rm EW}\,. \tag{21}$$

- The CKM mixing:

$$\hat{V}_{\rm CKM}\Big|_{\rm SU(8)} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & \frac{c_5}{Y_{\mathcal{T}}}\zeta_2 \\ -\lambda & 1-\lambda^2/2 & -\frac{c_5}{Y_{\mathcal{T}}}\zeta_1 \\ -\frac{c_5}{Y_{\mathcal{T}}}(\lambda\zeta_1+\zeta_2) & -\frac{c_5}{Y_{\mathcal{T}}}\zeta_1 & 1 \end{pmatrix}\,. \tag{22}$$

SM fermion masses in the $\mathrm{SU}(8)$ theory: benchmark

ζ_1	ζ_2	ζ_3	$Y_{\mathcal{D}}$	$Y_{\mathcal{B}}$	$Y_{\mathcal{T}}$
6.0×10^{-2}	2.0×10^{-3}	2.0×10^{-5}	0.5	0.5	0.8
c_3	c_4	c_5	$d_{\mathscr{A}}$	$d_{\mathscr{B}}$	λ
1.0	0.2	1.0	0.01	0.01	0.22
m_u	m_c	m_t	$m_d = m_e$	$m_s = m_\mu$	$m_b = m_\tau$
1.6×10^{-3}	0.6	139.2	0.5×10^{-3}	6.4×10^{-2}	1.5
$\lvert V_{ud}\rvert$	$\lvert V_{us}\rvert$	$\lvert V_{ub}\rvert$			
0.98	0.22	3.0×10^{-3}			
$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$	$\lvert V_{cb}\rvert$			
0.22	0.98	7.5×10^{-2}			
$\lvert V_{td}\rvert$	$\lvert V_{ts}\rvert$	$\lvert V_{tb}\rvert$			
0.019	7.5×10^{-2}	1			

Table: The parameters of the $\mathrm{SU}(8)$ benchmark point and the predicted SM quark/lepton masses (in unit of GeV) as well as the CKM mixings.

Summary

Summary

- We propose an $\mathrm{SU}(8)$ theory to address the SM flavor puzzle. Different generations transform differently in the UV-complete theory and their repetitive structure only emerge in the IR, which lead to the flavor non-universality of SM quarks/leptons.
- Our construction relaxes Georgi's "third law" in 1979, and we avoid the repetitions of one IRAFFS. The global symmetries based on the chiral IRAFFSs are vital to: (i) determine one single SM Higgs doublet through the non-anomalous $\widetilde{\mathrm{U}}(1)_{B-L}$ symmetry, (ii) count the massless left-handed sterile neutrinos precisely through the 't Hooft anomaly matching, (iii) organize the $d = 5$ operators for the SM fermion mass (mixing) terms.
- The symmetry-breaking pattern of the $\mathrm{SU}(8)$ theory is described, and all light SM fermion masses besides of the top quark are due to the inevitable gravitational effects that break the emergent global symmetries explicitly.

Summary and Outlook

- Crucial assumptions: (i) the VEV assignments of three intermediate symmetry-breaking scales in Eq. (14), (ii) the SM flavor IDs in the $\mathbf{28_F}$ and $\mathbf{56_F}$, and (iii) the $d = 5$ operators of direct and indirect Yukawa couplings containing the SM Higgs doublet.
- Main results: all SM quark/lepton masses, as well as the CKM mixing pattern can be quantitatively recovered with $\mathcal{O}(0.1) - \mathcal{O}(1)$ direct Yukawa couplings and $\mathcal{O}(0.01)$ Higgs mixing coefficients.
- All SM quark/leptons are flavor non-universal under the extended strong/weak symmetries, while the SM neutrinos $\nu_L \in \overline{\mathbf{8_F}}^\Omega$ are flavor universal.
- The degenerate $m_{d^i} = m_{\ell^i}$ will be further probed based on the RGEs of $\frac{d m_f(\mu)}{d \log \mu} \equiv \gamma_{m_f} m_f(\mu)$, $\gamma_{m_f}(\alpha^\Upsilon) = \frac{\alpha^\Upsilon}{4\pi} \gamma_0(\mathcal{R}_f^\Upsilon)$.
- The gauge coupling unification? The alternative symmetry breaking patterns? SUSY extensions? Proton lifetime?