SMEFT vs HEFT in the case of type-II seesaw

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Overview

- SMEFT v.s. HEFT
- Type-II seesaw
- Matching onto SMEFT
- Matching onto HEFT
- Numerical evaluation

SMEFT v.s. HEFT

Standard Model Effective Field Theory (SMEFT)

$$\mathcal{L}_{\rm SMEFT}^{(\rm LO+NLO)} \supset (D_\mu H)^\dagger (D^\mu H) + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

Symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$

Field contents Written in a basis *before* the EW symmetry breaking

Power coiunting $1/M$ expansion

$H,\ W_\mu^a,\ B_\mu,\ \cdots$

Higgs Effective Field Theory (HEFT)

$$\mathcal{L}_{\rm HEFT}^{(\rm LO)} \supset m_W^2 W_\mu^+ W^{-\mu}\left[1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right]$$

Symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ $\ast$ $SU(2)_L \times U(1)_Y$ is non-linearly realized

Field contents Written in a basis *after* the EW symmetry breaking

Power coiunting Loop expansion

$h,\ W_\mu^\pm,\ Z_\mu,\ A_\mu, \cdots$

SM + singlet scalar

$$\mathcal{L} = (D^\mu H)^\dagger (D_\mu H) + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$$

Buchalla et al. 1608.03564

$$H \in \mathbf{2}_{1/2}\,, \quad S \in \mathbf{1}_0 \qquad V(H,S) = -\frac{\mu_1^2}{2}H^\dagger H - \frac{\mu_2^2}{2}S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{4}S^4 + \frac{\lambda_3}{2}H^\dagger H S^2$$

SMEFT

E

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\mathcal{L}^{(\mathrm{LO+NLO})}_{\mathrm{SMEFT}} \supset (D_\mu H)^\dagger (D^\mu H) + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

HEFT

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\mathcal{L}^{(\mathrm{LO})}_{\mathrm{HEFT}} \supset m_W^2 W_\mu^+ W^{-\mu}\left[1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right]$$

SM + singlet scalar

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SMEFT

E

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

Integrate out $\tilde{s}$

$$\mathcal{L}_{\rm NLO} = -\frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_s^2}(H^\dagger H)\Box(H^\dagger H)$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$(D_\mu H)^\dagger (D^\mu H) - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_s^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\mathcal{L}_{\rm SMEFT}^{(\rm LO+NLO)} \supset (D_\mu H)^\dagger (D^\mu H) + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

HEFT

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\mathcal{L}_{\rm HEFT}^{(\rm LO)} \supset m_W^2 W_\mu^+ W^{-\mu}\left[1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right]$$

SM + singlet scalar

$$\mathcal{L} = (D^\mu H)^\dagger (D_\mu H) + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$$

Buchalla et al. 1608.03564

$$H \in \mathbf{2}_{1/2}\,, \quad S \in \mathbf{1}_0 \qquad V(H,S) = -\frac{\mu_1^2}{2}H^\dagger H - \frac{\mu_2^2}{2}S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{4}S^4 + \frac{\lambda_3}{2}H^\dagger H S^2$$

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$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$(D_\mu H)^\dagger (D^\mu H) - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_s^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\mathcal{L}_{\text{SMEFT}}^{(\text{LO+NLO})} \supset (D_\mu H)^\dagger (D^\mu H) + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

HEFT

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

$$m_W^2 W_\mu^+ W^{-\mu}\left[2c_\chi\left(\frac{h}{v}\right) + \left(c_\chi^4 - s_\chi^3 c_\chi \frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3)\right]$$

$$\mathcal{L}_{\text{HEFT}}^{(\text{LO})} \supset m_W^2 W_\mu^+ W^{-\mu}\left[1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right]$$

SM + singlet scalar

$$\mathcal{L} = (D^\mu H)^\dagger (D_\mu H) + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$$

Buchalla et al. 1608.03564

$H \in \mathbf{2}_{1/2}$, $S \in \mathbf{1}_0$

$$V(H,S) = -\frac{\mu_1^2}{2} H^\dagger H - \frac{\mu_2^2}{2} S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{4} S^4 + \frac{\lambda_3}{2} H^\dagger H S^2$$

SMEFT

E

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

Integrate out $\tilde{s}$

$$\mathcal{L}_{\rm NLO} = -\frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_s^2}(H^\dagger H)\Box(H^\dagger H)$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$(D_\mu H)^\dagger (D^\mu H) - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_s^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\supset\ m_W^2 W_\mu^+ W^{-\mu}\left[(2-\chi^2)\left(\frac{\tilde{h}}{v}\right) + (1-2\chi^2)\left(\frac{\tilde{h}}{v}\right)^2 + \mathcal{O}(h^3)\right]$$

$$\chi^2 \doteq \frac{v^2}{2}\frac{\lambda_3^2}{\lambda_2 M_s^2}$$

HEFT

M

$$S = \frac{v_s + \tilde{s}}{\sqrt{2}}$$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

$$m_W^2 W_\mu^+ W^{-\mu}\left[2c_\chi\left(\frac{h}{v}\right) + \left(c_\chi^4 - s_\chi^3 c_\chi \frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3)\right]$$

$$c_\chi \simeq 1 - \frac{\chi^2}{2}\ ,\quad s_\chi \simeq 0$$

Recent development

HEFT must be used   T. Cohen, N. Craig, X. Lu, D. Sutherland, JHEP 03 (2021) 237, 2008.08597

- when there is a new particle with a mass relatively close to the EW scale

Or
- when the new particle is a new source of the EW symmetry-breaking

Calculations in specific models

- Matching Singlet extension model onto SMEFT & HEFT

  G. Buchalla, O. Cata, A. Celis, C. Krause Nucl.Phys.B 917 (2017) 209-233, 1608.03564

- Matching Triplet ($Y=0$) extension model onto SMEFT & HEFT

  T. Cohen, N. Craig, X. Lu, D. Sutherland, JHEP 03 (2021) 237, 2008.08597

- Matching 2HDM onto SMEFT & HEFT

  S. Dawson, D. Fontes, C. Quezala -Calonge, J. J. SanzCill, Phys.Rev.D. (2021) 237, 2008.08597

Type-II seesaw

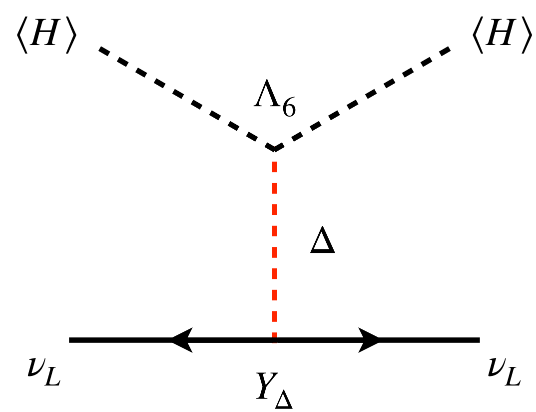
- In addition to $SU(2)_L$ doublet scalar $H \in (\mathbf{2}, 1/2)$, triplet scalar $\Delta \in (\mathbf{3}, 1)$ is introduced.

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

- Neutrino mass is generated by the $U(1)_L$ violating interactions

Lepton number violating operator

$$\mathscr{L}_{\not{L}} = \Lambda_6 H^T \Delta^\dagger H - Y_\Delta^{ij} \bar{L}_i^C \Delta L_j + \mathrm{H.c.}$$



- Triplet VEV induces large deviations in ρ parameter from unity (custodial sym. breaking)

➡ Triplet VEV must be tiny: $v_\Delta \ll v_H$

Matching onto SMEFT

This is already done by IHEP group X. Li, D. Zhang, S. Zhou, JHEP04(2022)038, 2201.05082

Symbol	Operator	dimensionless WC
dim 5		
$O^{(5)}_{pr}$	$\bar{L}^p \tilde{H} \tilde{H}^T L^{rC}$	$-\frac{\Lambda_6}{2M_\Delta} Y^{*pr}_\Delta$
dim 6		
O_H	$(H^\dagger H)^3$	$\frac{1}{2}(8\lambda - \lambda_4 + \lambda_5)\frac{\Lambda_6^2}{M_\Delta^2} - \frac{\Lambda_6^4}{M_\Delta^4}$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\frac{\Lambda_6^2}{2M_\Delta^2}$
O_{HD}	$(H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)$	$\frac{\Lambda_6^2}{M_\Delta^2}$
O^{pr}_{eH}	$(H^\dagger H)(\bar{L}^p H e^r_R)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y^{pr}_l$
O^{pr}_{uH}	$(H^\dagger H)(\bar{Q}^p \tilde{H} u^r_R)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y^{pr}_{\rm u}$
O^{pr}_{dH}	$(H^\dagger H)(\bar{Q}^p H d^r_R)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y^{pr}_{\rm d}$
$O^{prst}_{\ell\ell}$	$(\bar{L}^p \gamma_\mu L^r)(\bar{L}^s \gamma^\mu L^t)$	$\frac{1}{4} Y^{*ps}_\Delta Y^{rt}_\Delta$

We rewrite the results of the matching onto SMEFT in terms of the broken phase fields.

$$\mathscr{L}^{\rm smeft}_{\rm dim6} \supset (D_\mu H)^\dagger D^\mu H + \frac{C_{H\Box}}{M_\Delta^2}(H^\dagger H)\Box(H^\dagger H) + \frac{C_{HD}}{M_\Delta^2}(H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)$$

$$\xrightarrow{H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}} \frac{1}{2}\left[1 - 2c_{H,kin}\left(1 + \frac{h}{\tilde{v}_T}\right)^2\right]\partial_\mu h \partial^\mu h$$

To make h canonically normalized, we must perform

$$\tilde{h} = \int_0^h {\rm d}s \sqrt{1 - 2c_{\Phi,kin}\left(1 + \frac{s}{\tilde{v}_T}\right)^2}$$

R. Alonso, E. E. Jenkins, A. V. Manohar, M. Trott, 10.1007/JHEP04(2014)159, 1312.2014

Matching onto HEFT

In the broken phase, there are too many particles to be integrated out, which makes the HEFT matching complicated.

	Before SSB	After SSB
Light field	$H \in (\mathbf{2}, 1)$	h
Heavy field	$\Delta \in (\mathbf{3}, 1)$	$H_0,\ A_0,\ H^{\pm},\ H^{\pm\pm}$

① Utilize restrictions on the ρ parameter.

$$\rho = \frac{M_W^2}{M_z^2 \cos^2\theta_W} = 1.00038 \pm 0.00020$$

$$\Rightarrow \quad \epsilon \lesssim 10^{-2} \qquad \epsilon := \frac{v_\Delta}{\sqrt{v_H^2 + 2v_\Delta^2}}$$

We perform double expansion w.r.t. ϵ and $1/M_\Delta$

Matching onto HEFT

There are too many particles to be integrated out … $\{H_0,\ A_0,\ H^{\pm},\ H^{\pm\pm}\}$

② We can utilize remnant of $U(1)_L$ sym.

$U(1)_L$ violating terms

$$\mathscr{L}_{\rm typeII} = \mathscr{L}_{\rm SM} - \lambda HH\Delta\Delta - (\Delta\Delta)^2 - \Lambda_6 H\Delta H - Y_\Delta L\Delta L$$

$\mathscr{L}_{\rm typeII}$ is invariant under hypothetical Z_2

$$\Lambda_6 \to -\Lambda_6 \qquad Y_\Delta \to -Y_\Delta \qquad \Delta \to -\Delta$$

In the broken phase, this Z_2 becomes

$$\epsilon \to -\epsilon \qquad Y_\Delta \to -Y_\Delta \qquad \Phi_{\rm heavy} \to -\Phi_{\rm heavy}$$

$\Phi_{\rm heavy} \in \{H_0, A_0, H^{\pm}, H^{\pm\pm}\}$

$$\epsilon := \frac{v_\Delta}{\sqrt{v_H^2 + 2v_\Delta^2}}$$

This suggests the solution of E.o.M. for $\Phi_{\rm heavy}$ must take the form of

$$\Phi_{\rm heavy}[\phi_{\rm SM}] = \epsilon\left(\frac{\phi_{\rm SM}^2}{M_\Delta} + \frac{\phi_{\rm SM}^3}{M_\Delta^2} + \cdots\right) + Y_\Delta\left(\frac{\bar{L}L}{M_\Delta^2} + \cdots\right)$$

4-point $H_0^4,\ A_0^4,\ (H^-H^+)^2, \cdots$ generate $\left(\Phi_{\rm heavy}[\phi_{\rm SM}]\right)^4 = \mathcal{O}(\epsilon^3)\ \mathcal{O}(1/M_\Delta^3)$, which can be neglected.

We get the list pf the operator coefficients for the SMEFT and HEFT

Ex.) dim 5

dim 5		
Operator	WCs in SMEFT (Ref. [35])	WCs in HEFT (this work)
$\Delta L = 0$		
h^5	Eq. (5.6)	Eq. (5.7)
$h^3 W^-_\mu W^{+\mu}$	$\frac{2g^2}{3\tilde{v}_T}\tilde{\epsilon}^2\left[1+r_\Delta^2(\lambda_4-\lambda_5)\right]$	Eq. (5.8)
$hW^-_\mu W^{+\mu} W^-_\nu W^{+\nu}$	0	$-2\epsilon^2 g^4(\kappa-1)\frac{v_T\left(2M_\Delta^2\kappa+v_T^2(2\kappa-1)(\lambda_4-\lambda_5)\right)}{\left(2M_\Delta^2+v_T^2(\lambda_4-\lambda_5)\right)^2}$
$hW^-_\mu W^+_\nu W^{-\mu} W^{+\nu}$	0	$2\epsilon^2 g^4\frac{v_T\left(4M_\Delta^2\kappa+v_T^2(2\kappa-1)(\lambda_4+\lambda_5)\right)}{\left(2M_\Delta^2+v_T^2(\lambda_4+\lambda_5)\right)^2}$
$h^3 Z_\mu Z^\mu$	$\frac{4g^2}{3c_W^2\tilde{v}_T}\tilde{\epsilon}^2\left[1+r_\Delta^2(\lambda_4-\lambda_5)\right]$	Eq. (5.9)
$hZ_\mu Z^\mu Z_\nu Z^\nu$	0	$-\epsilon^2\frac{g^4}{c_W^4}(\kappa-2)\frac{v_T\left(3M_\Delta^2\kappa+v_T^2(2\kappa-1)(\lambda_4-\lambda_5)\right)}{\left(2M_\Delta^2+v_T^2(\lambda_4-\lambda_5)\right)^2}$
$hW^-_\mu W^{+\mu} Z_\nu Z^\nu$	0	$-\epsilon^2 g^2\frac{g^2}{c_W^2}\frac{v_T\left(2M_\Delta^2\kappa(4\kappa-5)+v_T^2(6\kappa^2-11\kappa+4)(\lambda_4-\lambda_5)\right)}{\left(2M_\Delta^2+v_T^2(\lambda_4-\lambda_5)\right)^2}$
$hW^-_\mu W^+_\nu Z^\mu Z^\nu$	0	$-2\epsilon^2 g^2\frac{g^2}{c_W^2}\frac{M_\Delta^2\left((4\kappa-2)c_W^2-8\kappa+2\right)+v_T^2(c_W^2-2)(2\kappa-1)\lambda_4}{\left(2M_\Delta^2+v_T^2\lambda_4\right)^2}$
$ihW^-_\mu W^+_\nu Z^{\mu\nu}$	0	$\epsilon^2 g^2\frac{g}{c_W}\frac{2\kappa-1}{2M_\Delta^2+v_T^2\lambda_4}$
$i(h\overleftrightarrow{D}_\mu W^-_\nu)W^{+\mu}Z^\nu$	0	$-\epsilon^2 g^2\frac{g}{c_W}\frac{2\kappa-1}{2M_\Delta^2+v_T^2\lambda_4}$
$h^2\bar{\psi}_L\psi_R$	$\sqrt{2}\tilde{\epsilon}^2 Y_\psi^{pr}\frac{1}{\tilde{v}_T}\left(1+(\lambda_4-\lambda_5)r_\Delta^2\right)$	$\sqrt{2}\epsilon^2 Y_\psi^{pr}\kappa\frac{M_\Delta^2(4-6\kappa)+v_T^2\kappa(\lambda_4-\lambda_5)}{v_T\left(2M_\Delta^2+v_T^2(\lambda_4-\lambda_5)\right)}$
$W^-_\mu W^{+\mu}\bar{\psi}_L\psi_R$	0	$\epsilon^2 g^2 Y_\psi^{pr}\frac{2\sqrt{2}v_T^2\kappa(\kappa-1)}{v_T\left(2M_\Delta^2+v_T^2(\lambda_4-\lambda_5)\right)}$
$Z_\mu Z^\mu\bar{\psi}_L\psi_R$	0	$\epsilon^2\frac{g^2}{c_W^2}Y_\psi^{pr}\frac{\sqrt{2}v_T^2\kappa(\kappa-2)}{v_T\left(2M_\Delta^2+v_T^2(\lambda_4-\lambda_5)\right)}$

What we found : There are operators that are present in HEFT but not in SMEFT, especially those involving gauge fields

List of operators present in HEFT but absent in SMEFT

dim 4	dim 5 ($\Delta L = 0$)	dim 5 ($\Delta L = 2$)	dim 6
$Z_\mu Z^\mu Z_\nu Z^\nu$	$hW^-_\mu W^{+\mu} W^-_\nu W^{+\nu}$	$W^-_\mu W^{+\mu}(\nu_L^T C \nu_L)$	$(\partial_\mu h)(\partial_\nu h) W^{-\mu} W^{+\nu}$
	$hW^-_\mu W^+_\nu W^{-\mu} W^{+\nu}$	$W^+_\mu W^{+\mu}(e_L^T C e_L)$	$hh(D_\mu W^{-\mu})(D_\nu W^{+\nu})$
	$hZ_\mu Z^\mu Z_\nu Z^\nu$	$W^+_\mu Z^\mu(\nu_L^T C e_L)$	$hh(D_\mu D_\nu W^{-\mu}) W^{+\nu}$
	$hW^-_\mu W^{+\mu} Z_\nu Z^\nu$	$Z_\mu Z^\mu(\nu_L^T C \nu_L)$	⋮
	$hW^-_\mu W^+_\nu Z^\mu Z^\nu$		⋮
	$ihW^-_\mu W^+_\nu Z^{\mu\nu}$		
	$i(h\overleftrightarrow{D}_\mu W^-_\nu) W^{+\mu} Z^\nu$		
	$W^-_\mu W^{+\mu}(\bar{\psi}_L \psi_R)$		
	$Z_\mu Z^\mu(\bar{\psi}_L \psi_R)$		

SMEFT operators

$\bar{L}\tilde{H}\tilde{H}^T L^C$

$(H^\dagger H)^3$

$(H^\dagger H)\Box(H^\dagger H)$

$(H^\dagger D_\mu H)(H^\dagger D^\mu H)$

$(H^\dagger H)(\bar{L} H e_R)$

$(H^\dagger H)(\bar{Q}\tilde{H} u_R)$

$(H^\dagger H)(\bar{Q} H d_R)$

$(H^\dagger H)(\bar{Q} H d_R)$

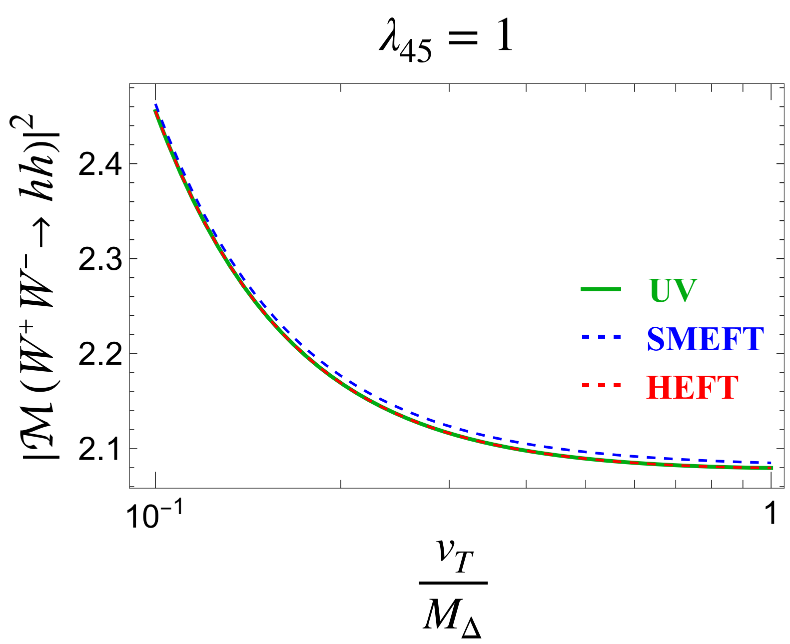
$(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$

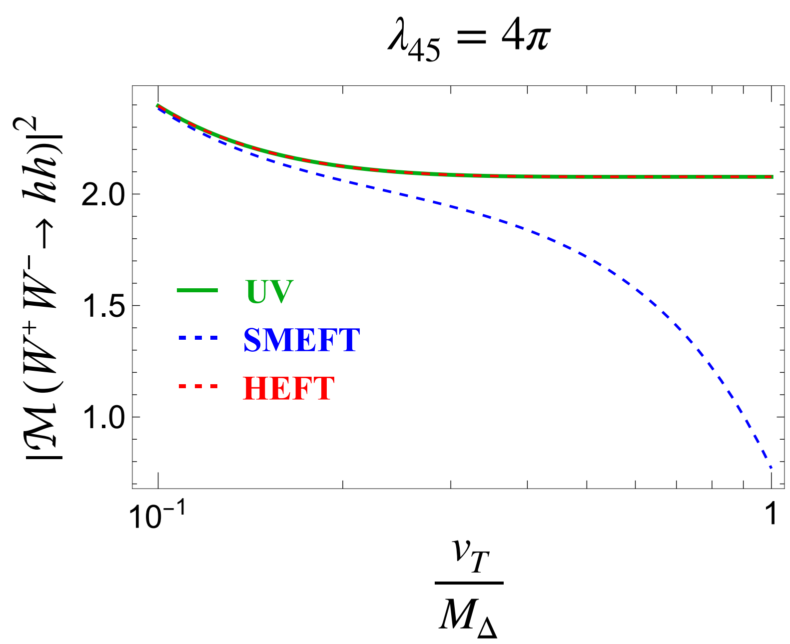
Numerical evaluation

We evaluate Higgs pair production through vector boson fusion: $W^+W^- \to hh$

$$\lambda_1 = 1, \quad \epsilon = v_\Delta / v_T = 10^{-2}, \quad \sqrt{s} = 250\text{GeV}, \quad \theta = \pi/4, \quad \lambda_{45} := \lambda_4 - \lambda_5$$

v_T : true vacuum $\quad$ θ : scattering angle $\quad$ M_Δ : scalar triplet mass





HEFT reproduces UV theory more accurately than SMEFT in the regime that v_T / M_Δ is close to unity.

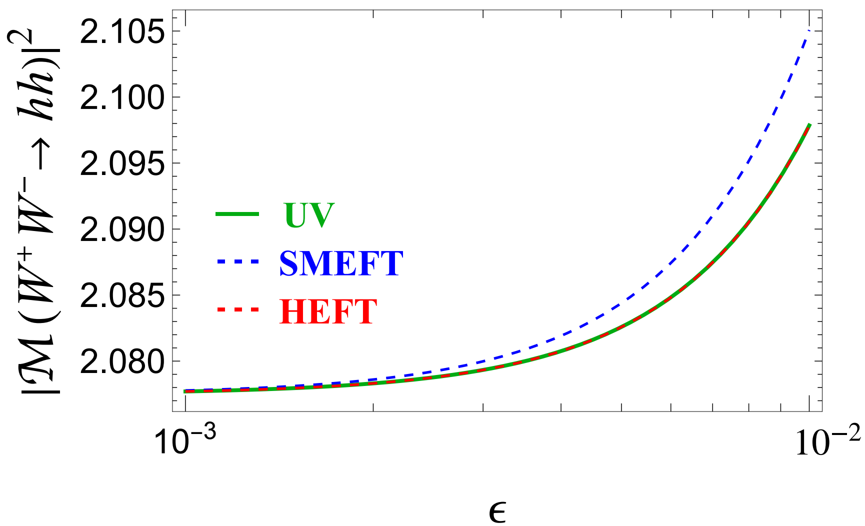
Numerical evaluation

We evaluate Higgs pair production through vector boson fusion: $W^+W^- \to hh$

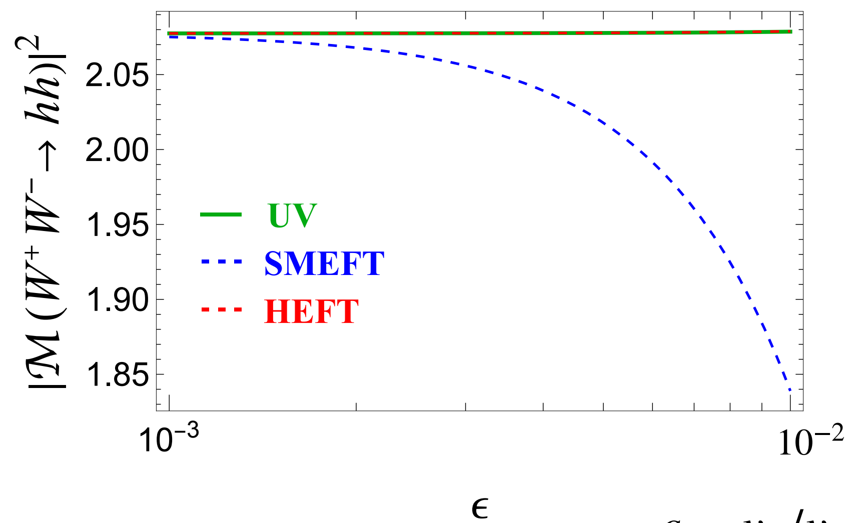
$$\lambda_1 = 1, \qquad \frac{v_T}{M_\Delta} = 0.4, \qquad \sqrt{s} = 250\text{GeV}, \qquad \theta = \pi/4, \qquad \lambda_{45} := \lambda_4 - \lambda_5$$

v_T : true vacuum $\quad \theta$: scattering angle $\quad M_\Delta$: scalar triplet mass

$\lambda_{45} = 1$



$\lambda_{45} = 4\pi$



$\epsilon = v_\Delta / v_T$

HEFT reproduces UV theory more accurately than SMEFT in the regime that ϵ is large. However, the differences between them are slight.

Summary

- We match type-II seesaw model onto SMEFT and HEFT and examine which EFT more accurately reproduces UV theory.

- We rewrite previous studies matching Type-II seesaw onto SMEFT in a basis after the EW symmetry breaking.

- We perform matching the Type-II seesaw onto the HEFT.

- We performed numerical calculations and found that HEFT reproduces UV theory more accurately than SMEFT in the regime that v_T/M_Δ is close to unity.

Back Up

Matching onto HEFT

There are too many particles to be integrated out …

	SMEFT	HEFT
Light	$H \in (\mathbf{2},1)$	h
Heavy	$\Delta \in (\mathbf{3},1)$	$H_0,\ A_0,\ H^{\pm},\ H^{\pm\pm}$

We can utilize remnant of $U(1)_L$ sym.

$U(1)_L$ violating terms

$$\mathscr{L}_{\text{typeII}} = \mathscr{L}_{\text{SM}} - \lambda HH\Delta\Delta - (\Delta\Delta)^2 - \Lambda_6 H\Delta H - Y_\Delta L\Delta L$$

$\mathscr{L}_{\text{typeII}}$ is invariant under hypothetical Z_2

$$\Lambda_6 \to -\Lambda_6 \qquad Y_\Delta \to -Y_\Delta \qquad \Delta \to -\Delta$$

In the broken phase, this Z_2 becomes

$$\epsilon \to -\epsilon \qquad Y_\Delta \to -Y_\Delta \qquad \Phi_{\text{heavy}} \to -\Phi_{\text{heavy}}$$

$\Phi_{\text{heavy}} \in \{H_0, A_0, H^{\pm}, H^{\pm\pm}\}$

$$\epsilon := \frac{v_\Delta}{\sqrt{v_H^2 + 2v_\Delta^2}}$$

This suggests the solution of E.o.M. must take the form of

$$\Phi_{\text{heavy}}[\phi_{\text{SM}}] = \epsilon\left(\frac{\phi_{\text{SM}}^2}{M_\Delta} + \frac{\phi_{\text{SM}}^3}{M_\Delta^2} + \cdots\right) + Y_\Delta\left(\frac{\bar{L}L}{M_\Delta^2} + \cdots\right)$$

Higgs sector & Yukawa sector in HEFT

Higgs sector in SM

$$\mathcal{L}^{\rm SM}_{\rm Higgs} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$
$$= \frac{1}{2}{\rm Tr}\left[(D_\mu \hat{\Phi})^\dagger D^\mu \hat{\Phi}\right] + \frac{\mu^2}{2}{\rm Tr}\left[\hat{\Phi}^\dagger\hat{\Phi}\right] - \frac{\lambda}{4}{\rm Tr}\left[\hat{\Phi}^\dagger\hat{\Phi}\right]^2$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　　但し $\tilde{H} := i\sigma_2 H^*$

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} - i g W^a_\mu \frac{\tau^a}{2}\hat{\Phi} + i g' \hat{\Phi} B_\mu \frac{\tau^3}{2}$$

Higgs sector in HEFT

h : singlet under non-linearly realized SU(2)xU(1)
(125GeV Higgs)

$w^\pm, z$: transform nontrivially under non-linearly realized SU(2)xU(1)
(would-be Goldstone)

➡ We must separate h from $w^\pm, z$ because they transform differently

Polar decomposition : $\hat{\Phi} = (v + h) U$　　　$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$

$$\mathcal{L}^{\rm SM}_{\rm Higgs} = \frac{v^2}{4}\left(1 + \frac{h}{v}\right)^2 {\rm Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V_{\rm SM}(h)$$

Higgs sector & Yukawa sector in HEFT

Higgs sector in SM

$$\mathcal{L}^{\rm SM}_{\rm Higgs} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$

$$= \frac{1}{2}{\rm Tr}\left[(D_\mu \hat{\Phi})^\dagger D^\mu \hat{\Phi}\right] + \frac{\mu^2}{2}{\rm Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right] - \frac{\lambda}{4}{\rm Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right]^2$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　　但し $\tilde{H} := i\sigma_2 H^*$

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} - igW^a_\mu \frac{\tau^a}{2}\hat{\Phi} + ig'\hat{\Phi} B_\mu \frac{\tau^3}{2}$$

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$$\mathcal{L}_{\rm Higgs}^{\rm HEFT} = \frac{v^2}{4}\ F(h)\ {\rm Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

Higgs sector & Yukawa sector in HEFT

SM Yukawa

$$\mathcal{L}^{\rm SM}_{\rm Yukawa} = -y_u\, \bar{q}_L \tilde{H} u_R - y_d\, \bar{q}_L H d_R - y_e\, \bar{l}_L \tilde{H} e_R + \text{h.c.}$$

$$= -y_u\, \bar{q}_L \hat{\Phi} P_+ q_R - y_d\, \bar{q}_L \hat{\Phi} P_- q_R - y_e\, \bar{l}_L \hat{\Phi} P_- l_R + \text{h.c.}$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　但し $\tilde{H} := i\sigma_2 H^*$

$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$, $l_R = \begin{pmatrix} 0 \\ e_R \end{pmatrix}$: right-handed fermions　　$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$: projection operator

HEFT Yukawa

Substituting $\hat{\Phi} = (v+h)U$ into $\mathcal{L}^{\rm SM}_{\rm Yukawa}$, we get

$$\mathcal{L}^{\rm SM}_{\rm Yukawa} = -v\, y_u \left(1 + \frac{h}{v}\right) \bar{q}_L U P_+ q_R - v\, y_d \left(1 + \frac{h}{v}\right) \bar{q}_L U P_- q_R - v\, y_e \left(1 + \frac{h}{v}\right) U P_- l_R + \text{h.c.}$$

Replacement

$$\mathcal{L}^{\rm HEFT}_{\rm Yukawa} = -v\, Y_u(h)\, \bar{q}_L U P_+ q_R - v\, Y_d(h)\, \bar{q}_L U P_- q_R - v\, Y_e(h)\, U P_- l_R + \text{h.c.}$$

Buchalla et al., 1307.5017

HEFT (LO)

$$\mathcal{L}_{\mathrm{HEFT,\,LO}} = \mathcal{L}_0 + \mathcal{L}_{Uh}$$

$$\mathcal{L}_{Uh} = \frac{v^2}{4} F(h) \mathrm{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

Higgs sector

$$- v\left[Y_u(h)\,\bar{q}_L\, U P_+ q_R + Y_d(h)\,\bar{q}_L\, U P_- q_R + Y_e(h)\,\bar{l}_L\, U P_- l_R + \mathrm{h.c.}\right]$$

Yukawa項

$$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$$

$$SU(2)_L \times U(1)_Y \quad : \quad \hat{\Phi} \to \mathfrak{g}_L \hat{\Phi} \mathfrak{g}_Y^\dagger \quad \longleftrightarrow \quad h \to h\,, \quad U \to \mathfrak{g}_L\, U\, \mathfrak{g}_Y^\dagger$$

$$\mathcal{L}_0 = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ \bar{q}_L i \not{D} q_L + \bar{l}_L i \not{D} l_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{e}_R i \not{D} e_R$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in \mathbf{2}_{1/6} \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \in \mathbf{2}_{-1/2}$$

Identical to those in SM

$$u_R \in \mathbf{1}_{2/3} \qquad d_R \in \mathbf{1}_{-1/3} \qquad e_R \in \mathbf{1}_{-1}$$

SMEFT v.s. HEFT

Representative effective field theories (EFT) with SM particles

Standard Model Effective Field Theory (SMEFT)

$$\mathcal{L}_{\rm SMEFT}^{(\rm LO+NLO)} \supset (D_\mu H)^\dagger (D^\mu H) + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

⇨ Deviation from the SM appears from NLO

Higgs Effective Field Theory (HEFT)

$$\mathcal{L}_{\rm HEFT}^{(\rm LO)} \supset m_W^2 W_\mu^+ W^{-\mu} \left[1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)} \left(\frac{h}{v}\right)^2 + \cdots\right]$$

⇨ Deviation from the SM appears from LO

SMEFT v.s. HEFT

Two representative EFTs that treat SM particles as dynamical particles:

- Standard Model Effective Field Theory (SMEFT)
- Higgs Effective Field Theory (HEFT)

SMEFT	HEFT
$SU(2)_L \times U(1)_Y$ is linearly realized	$SU(2)_L \times U(1)_Y$ is non-linearly realized
Written in terms of <u>symmetric</u> phase fields	Written in terms of <u>broken</u> phase fields
$1/M$ expansion	Loop expansion

How to choose between SMEFT and HEFT?