Detecting the Fifth-Force via Superconducting Josephson Junctions

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YC, Jie Sheng and T. T. Yanagida,
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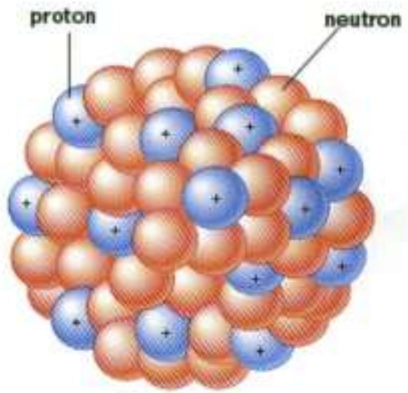
Fundamental Forces

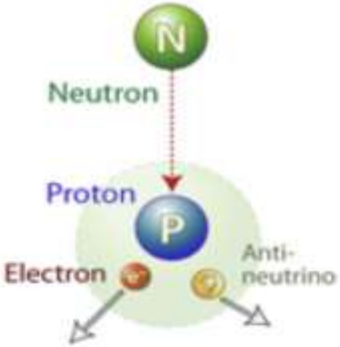
What we know: four fundamental forces

Gravity

Electromagnetism



Strong



Weak

Is there a fifth fundamental force?

- **Theory**
- **Experiment**

Fifth force: Well-motivated Theory

An Example: **B-L Extension of the SM:** $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

1. SM: (B and L) anomalous, (B-L) non-anomalous.
2. Gauge anomalies canceled.

$U(1)_{B-L}$ Gauge Boson mediate the B-L force.

- Force range: $1/m_{A'}$
- Yukawa potential:

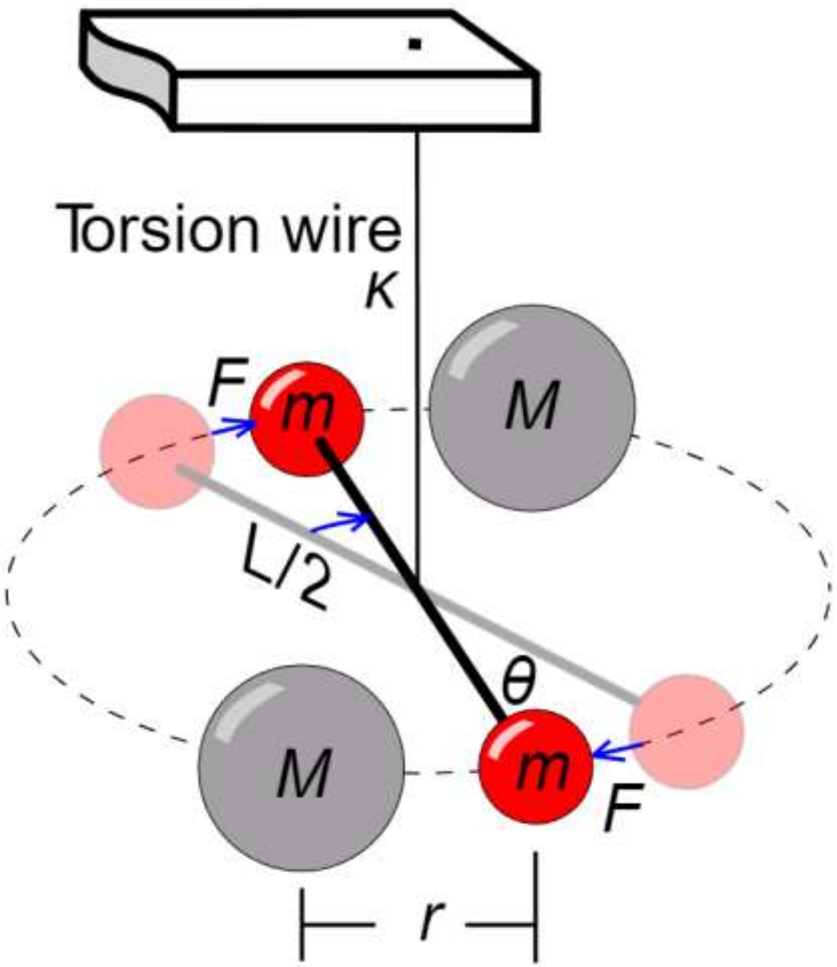
$$V_{B-L}(r) = g_{B-L}^2 \frac{Q_{B-L} e^{-m_{A'} r}}{r}.$$

Charge table	l	ν_R	e_R	q	u_R	d_R
$SU(1)_L \times U(1)_Y$	(2,-1/2)	(1,0)	(1,-1)	(2,1/6)	(1,2/3)	(1,1/3)
$U(1)_{B-L}$	-1	-1	-1	1/3	1/3	1/3

e^-(electron) : -1
p^+(proton) : +1
n (neutron) : +1
Atom : +N(neutron)

How to Probe? Classical way

Torsion Balance



- Test of EP: $\frac{\mathrm{GMm}}{\mathrm{r}^2} = \mathrm{F} = \mathrm{ma} \rightarrow \mathrm{a} = \frac{\mathrm{GM}}{\mathrm{r}^2}$
  All objects experience the same acceleration in the same gravitational field.

- Change the materials of the test body (red ball)
- Measure the Eotvos parameter:

$$\eta_{1,2} = \frac{a_1 - a_2}{(a_1 + a_2)/2} = \frac{(m_g/m_i)_1 - (m_g/m_i)_2}{[(m_g/m_i)_1 + (m_g/m_i)_2]/2}$$

For B-L case:

$$V_i(r) \equiv V_G^i(r) + V_{B-L}^i(r) = -G\frac{m_i M}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$V_1(r) \neq V_2(r) \Rightarrow a_1 \neq a_2 \qquad \eta \sim 10^{-13\sim15}$$

Constraint on B-L

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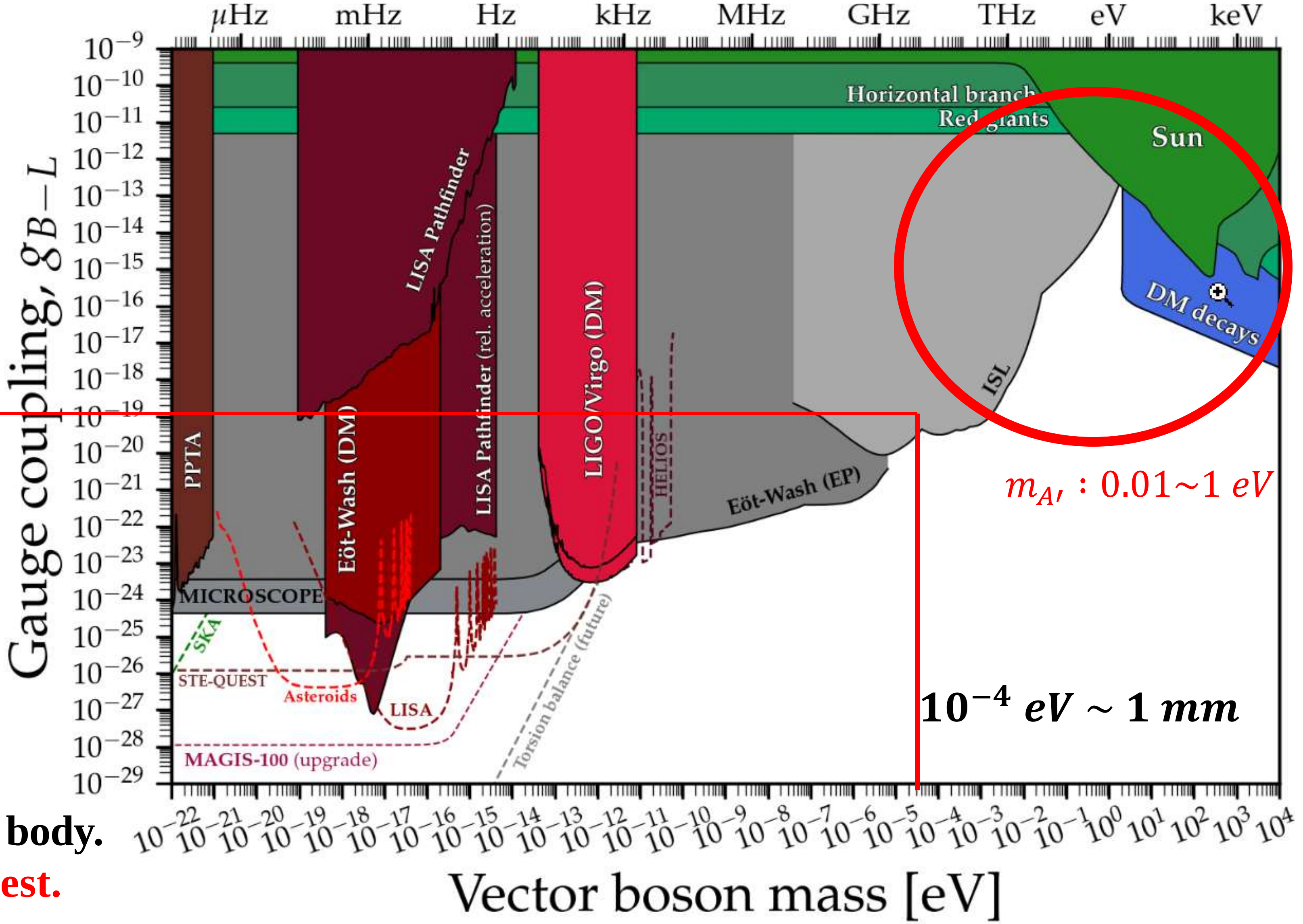
$$V_{B-L}(r) = g_{B-L}^2 \frac{Q_{B-L} e^{-m_{A'} r}}{r}.$$

Comparison with gravity:

$$g_{B-L}^2 \sim \frac{m_N^2}{M_P^2} \rightarrow$$

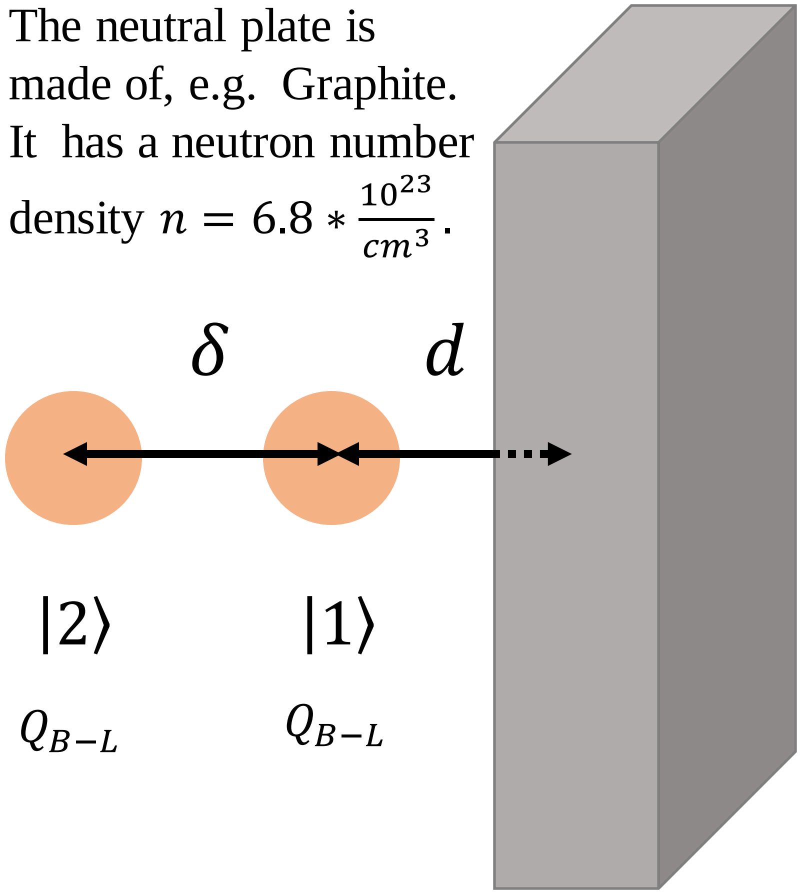
$$g_{B-L} \sim 10^{-19}$$



- **Sensitivity: limited by the size of test body.**
- **Classical test. How about Quantum test.**

Phase Evolution

The neutral plate is made of, e.g. Graphite. It has a neutron number density $n = 6.8 * \frac{10^{23}}{cm^3}$.



- Time dependent Schrodinger equation:

$$i\partial_t |\psi\rangle = \hat{H} |\psi\rangle$$

- The state's phase can evolve with time.

(H: time independent, position dependent)

$$|1\rangle \rightarrow e^{iH(\hat{x})t}|1\rangle = e^{iV(x_1)t}|1\rangle, \qquad \phi_1(t) = \mathrm{V}(\mathrm{x}_1)\mathrm{t}$$

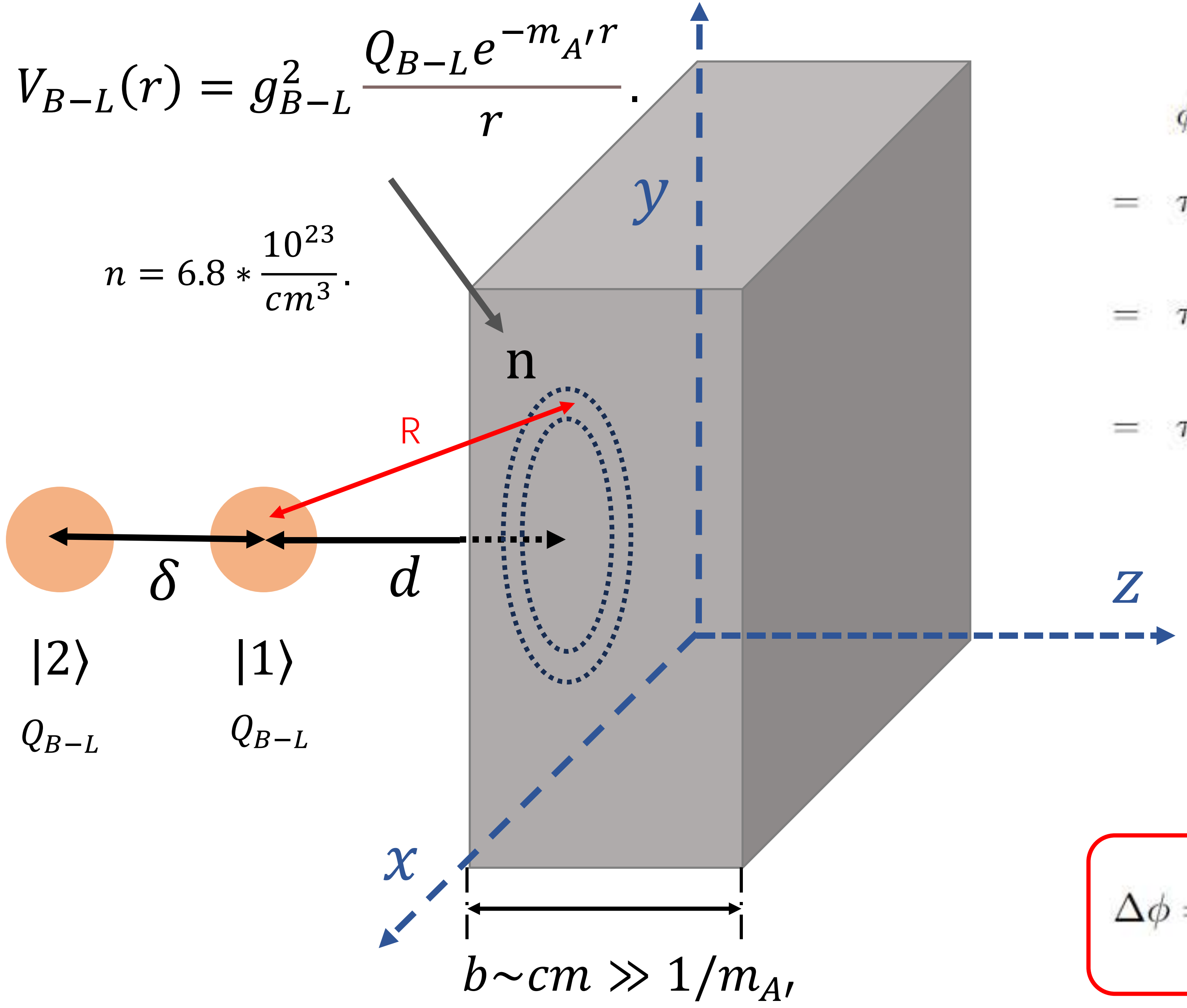
$$|2\rangle \rightarrow e^{iH(\hat{x})t}|2\rangle = e^{iV(x_2)t}|2\rangle, \qquad \phi_2(t) = \mathrm{V}(\mathrm{x}_2)\mathrm{t}$$

- Phase difference between 1 and 2:

$$\Delta\phi = \phi_1 - \phi_2 = [V(x_1) - V(x_2)] * t$$

$$V_{B-L}(r) = g_{B-L}^2 \frac{Q_{B-L} e^{-m_{A'} r}}{r}.$$

$$n = 6.8 * \frac{10^{23}}{cm^3}.$$



B-L (feeton) Potential

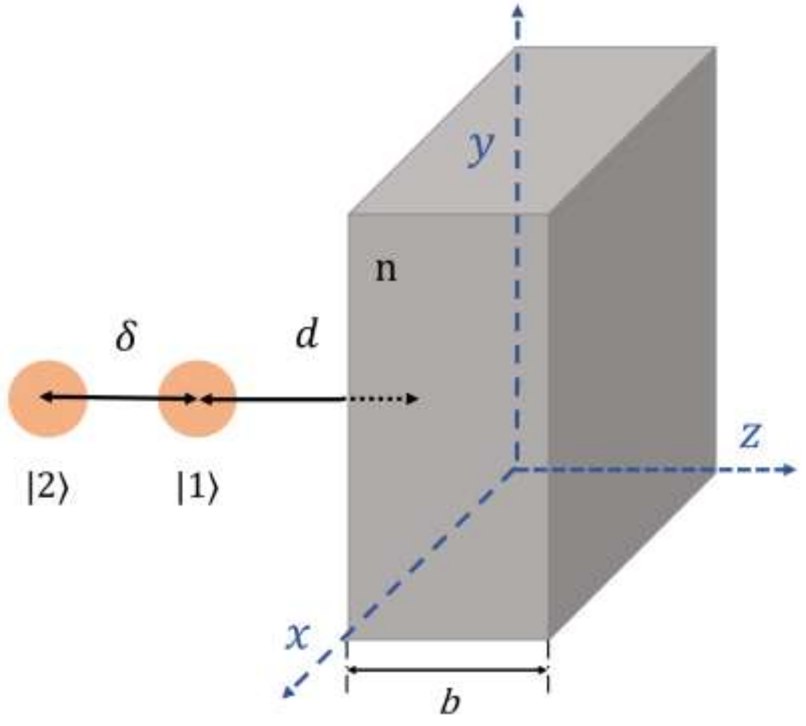
$$\begin{aligned}
&\phi_1(\tau, d) \\
&= \tau g_{B-L}^2 Q_{B-L} n \int\int\int \frac{e^{-m_{A'} R(x,y,z)}}{R(x,y,z)} dx dy dz \\
&= \tau g_{B-L}^2 Q_{B-L} n \int_0^b \int_0^\infty \frac{e^{-m_{A'}\sqrt{(d+z)^2+r^2}} 2\pi r}{\sqrt{(d+z)^2+r^2}} dr dz \\
&= \tau g_{B-L}^2 Q_{B-L} n \frac{2\pi e^{-m_{A'} d}\left(1 - e^{-m_{A'} b}\right)}{m_{A'}^2}.
\end{aligned}$$

Similarly, we can have the phase for state 2,

$$\phi_2 = \phi(\tau, d + \delta).$$

And, $\Delta\phi = \phi_1 - \phi_2$.

$$\Delta\phi = \tau g_{B-L}^2 Q_{B-L} n \frac{2\pi\left(e^{-m_{A'} d} - e^{-m_{A'}(d+\delta)}\right)}{m_{A'}^2}$$



e.g, $m_{A'} \sim 0.01\ eV$, $\frac{1}{m_{A'}} \sim 10^{-3}$cm.

1. Thickness of plate large enough as $m_{A'}\, b \gg 1$. Take $b = 1cm$.

2. Short-distance limit, $m_{A'}\, d \sim 0$, such as $d = 1\mu m = 10^{-4} cm$.

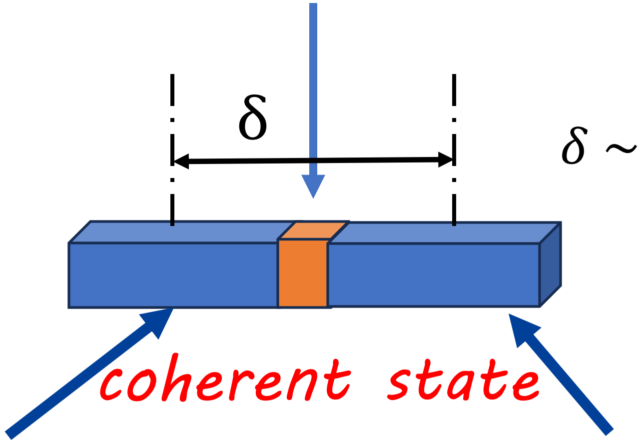
$$\Delta\phi = \tau g_{B-L}^2 Q_{B-L} n \frac{2\pi\left(e^{-m_{A'}d} - e^{-m_{A'}(d+\delta)}\right)}{m_{A'}^2} \quad \Rightarrow \quad \Delta\phi_f \simeq \frac{2\pi\tau g_{B-L}^2 Q_{B-L} n\delta}{m_{A'}}.$$

$$\Delta\phi_f = \boxed{3\times 10^{-3}} \times \left(\frac{g_{B-L}}{10^{-16}}\right)^2 \left(\frac{\tau}{1\,\text{min}}\right) \left(\frac{10^{-2}\,\text{eV}}{m_{A'}}\right) \left(\frac{\delta}{100\,\text{nm}}\right)$$

Q: How to detect a small phase difference induced by B-L gauge potential?

Josephson Effects

Insulator $\epsilon \sim 1\ nm$



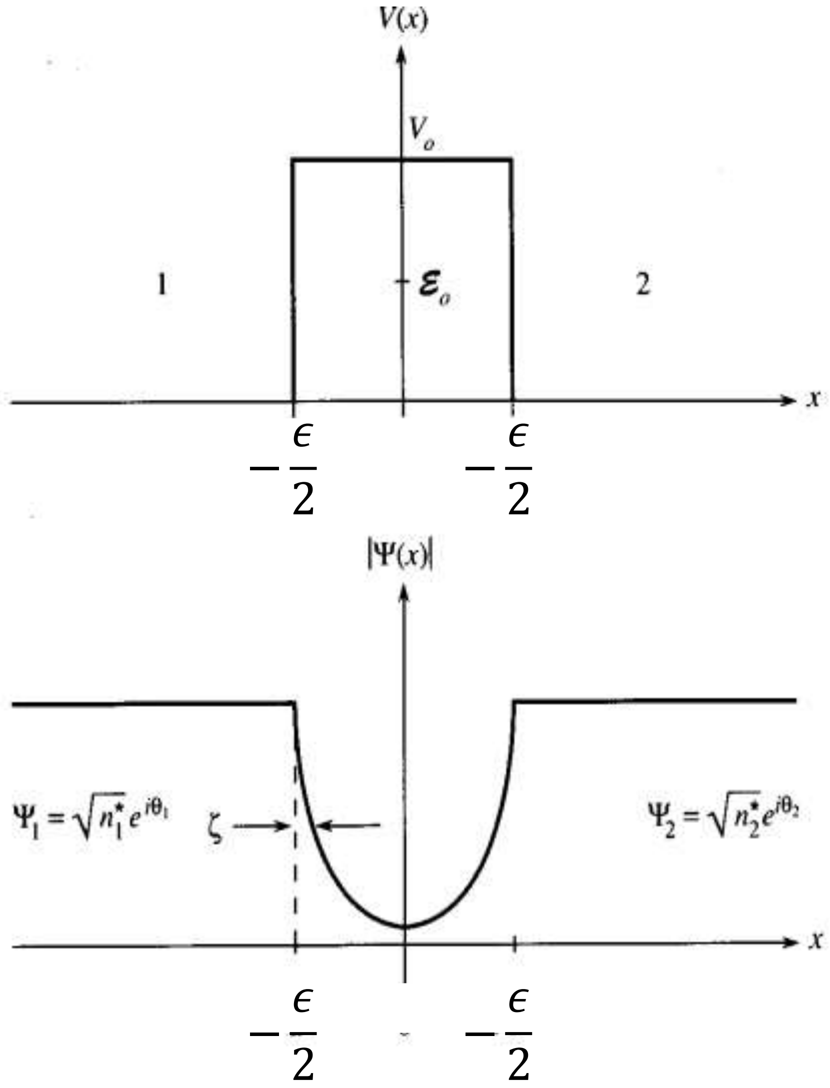
$\delta \sim a \sim 100$ nm

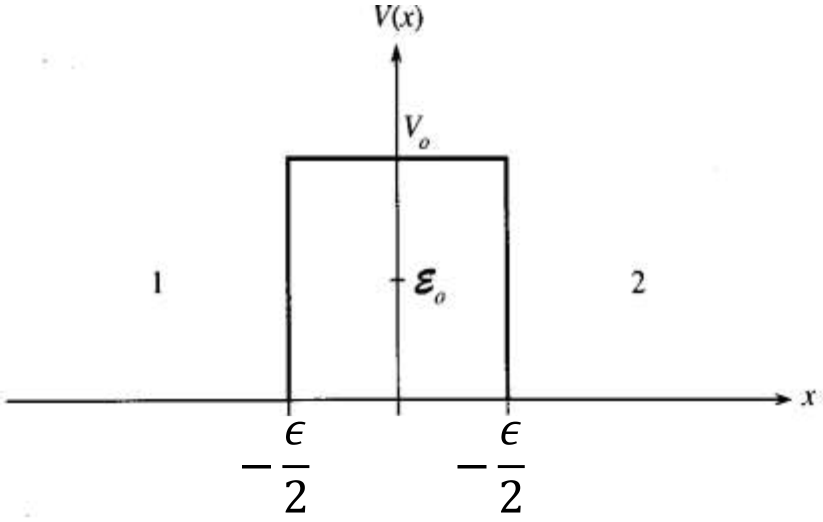
Superconductor 2

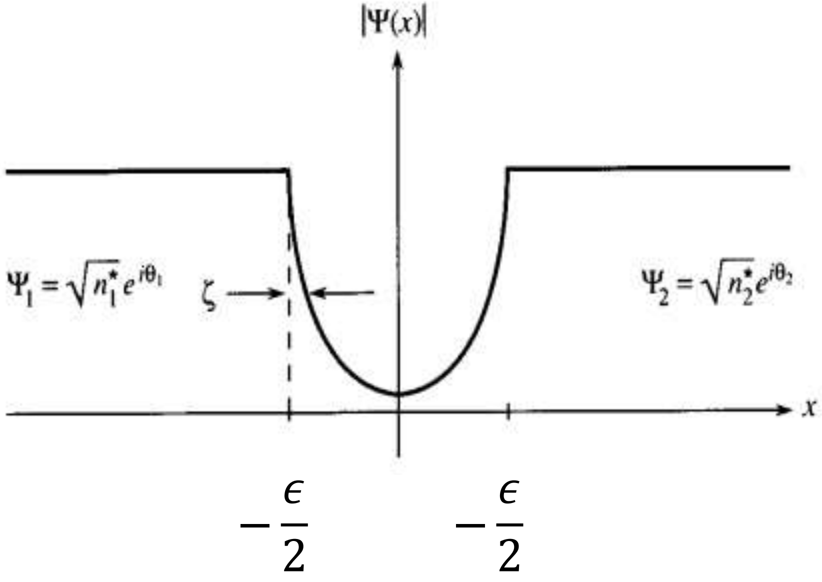
$$\Psi_2 = \sqrt{n_2} e^{i\phi_2}$$

Superconductor 1

$$\Psi_1 = \sqrt{n_1} e^{i\phi_1}$$







The Schrodinger equation within the insulator region is:

$$-\frac{\hbar^2}{2(2m_e)}\nabla^2\Psi(\mathbf{r}) = (E - V)\,\Psi(\mathbf{r})$$

The general solution can be parametrized as,

$$\Psi(x) = C_1 \cosh x/\xi + C_2 \sinh x/\xi,$$

with $\xi = \sqrt{1/4m_e(V - E)}.$

Furthermore, by considering the requirements of continuity, we match the boundary conditions and get,

$$C_1 = \frac{\sqrt{n_1}e^{i\phi_1} + \sqrt{n_2}e^{i\phi_2}}{2\cosh(\epsilon/2\xi)}, C_2 = \frac{\sqrt{n_1}e^{i\phi_1} - \sqrt{n_2}e^{i\phi_2}}{2\sinh(\epsilon/2\xi)}.$$

Ref: *Applied Superconductivity* by Rudolf Gross & Achim Marx

The current density is defined as,

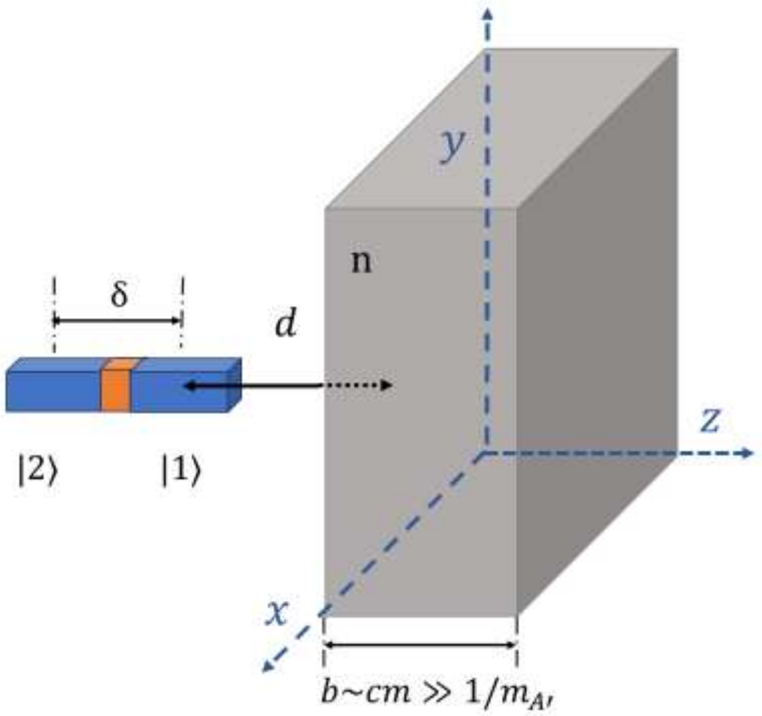
$$J = -(2e/m_e)\mathrm{Re}(\Psi^* i\nabla\Psi)$$

The current is induced by phase difference

$$J = \frac{e\sqrt{n_1 n_2}}{m_e \epsilon}\sin\Delta\phi\,, \quad \Delta\phi \equiv (\phi_1 - \phi_2).$$

Ref: *Applied Superconductivity* by Rudolf Gross & Achim Marx

We relate the phase difference with a physical phenomenon!



1. Source is still the graphite cubic.

2. State 1 and 2 becomes the single cooper-pair state.

$$\Delta\phi_f \simeq \frac{2\pi\tau g_{B-L}^2 Q_{B-L} n\delta}{m_{A'}}.$$

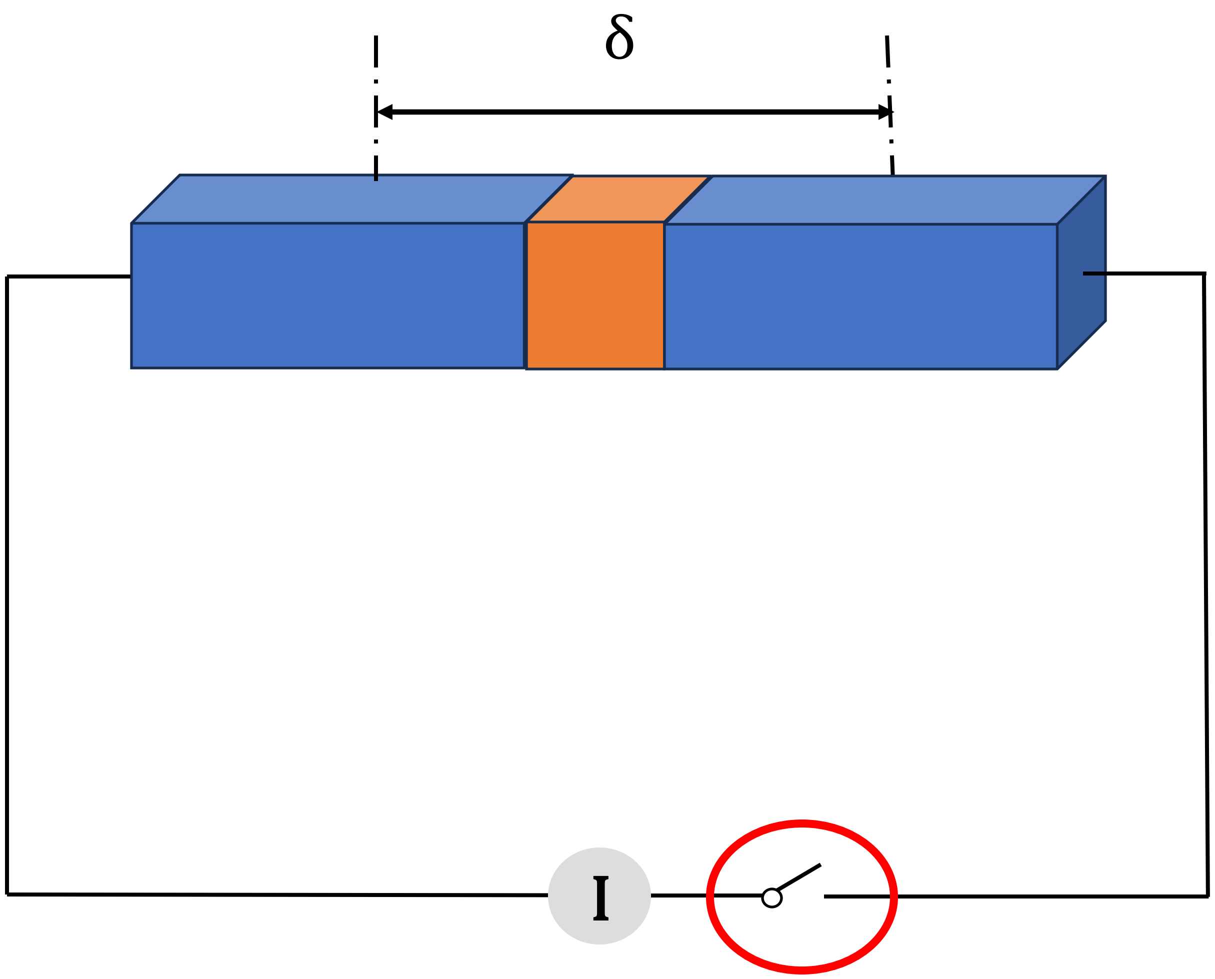
$$\Delta\phi_f = 3\times 10^{-3} \times \left(\frac{g_{B-L}}{10^{-16}}\right)^2 \left(\frac{\tau}{1\,\mathrm{min}}\right)\left(\frac{10^{-2}\,\mathrm{eV}}{m_{A'}}\right)\left(\frac{\delta}{100\,\mathrm{nm}}\right)$$

Experimental Proposal

Step1.

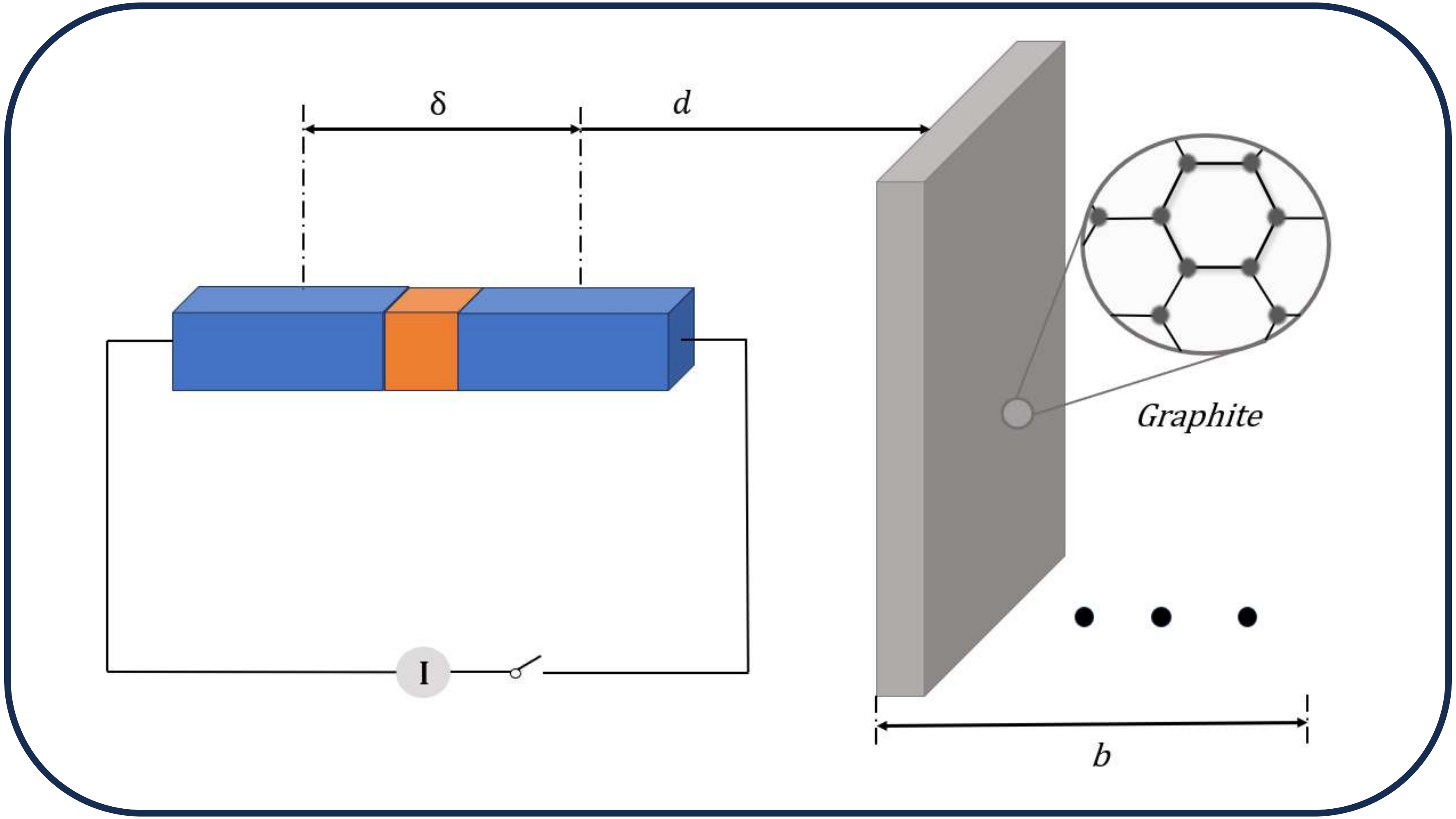
A Josephson junction (JJ) may have some random initial phase.

Close the loop and consume the current by resistance to make $\Delta\phi_{ini} = 0$.

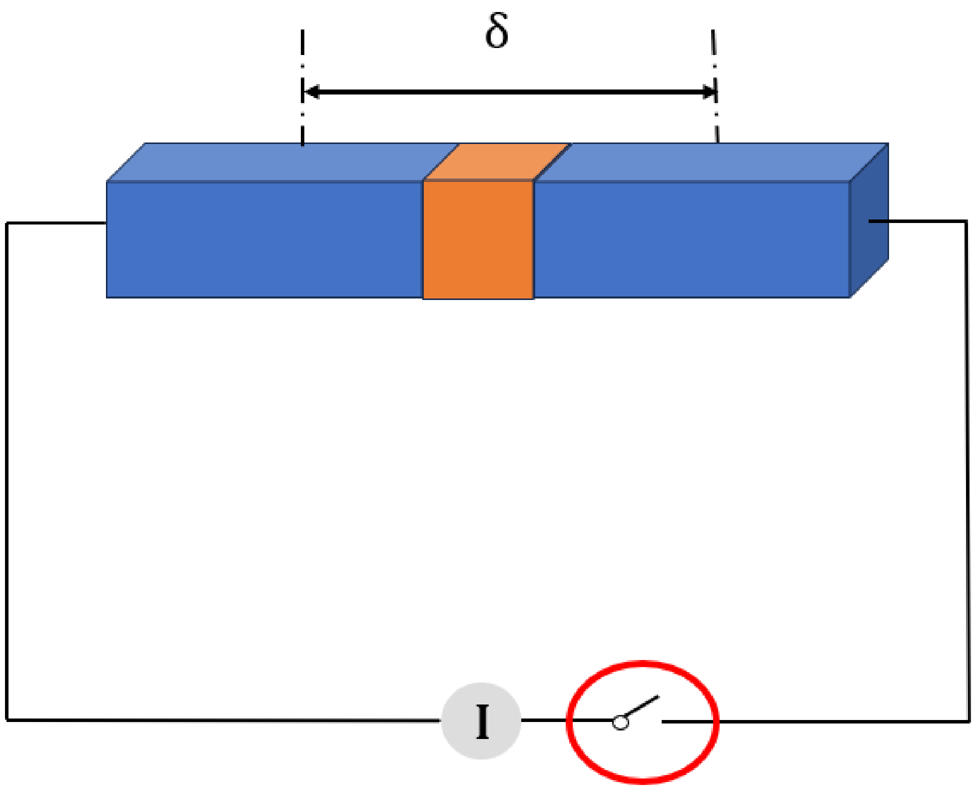


Step2.

Disconnect the loop and move the plate nearby. The phase starts to evolve.



We should shield the background EM field (for illustration, by a conductor) as much as possible to ensure E = B = 0



Step3.
At the time moment τ, we close the circuit again, measure the current, and compare it with the predicted values from our theoretical calcualtion.

$$\Delta\phi_f \simeq \frac{2\pi\tau g_{B-L}^2 Q_{B-L} n\delta}{m_{A'}}.$$

$$\Delta\phi_f = 3\times10^{-3}\times\left(\frac{g_{B-L}}{10^{-16}}\right)^2\left(\frac{\tau}{1\,\mathrm{min}}\right)\left(\frac{10^{-2}\,\mathrm{eV}}{m_{A'}}\right)\left(\frac{\delta}{100\,\mathrm{nm}}\right)$$

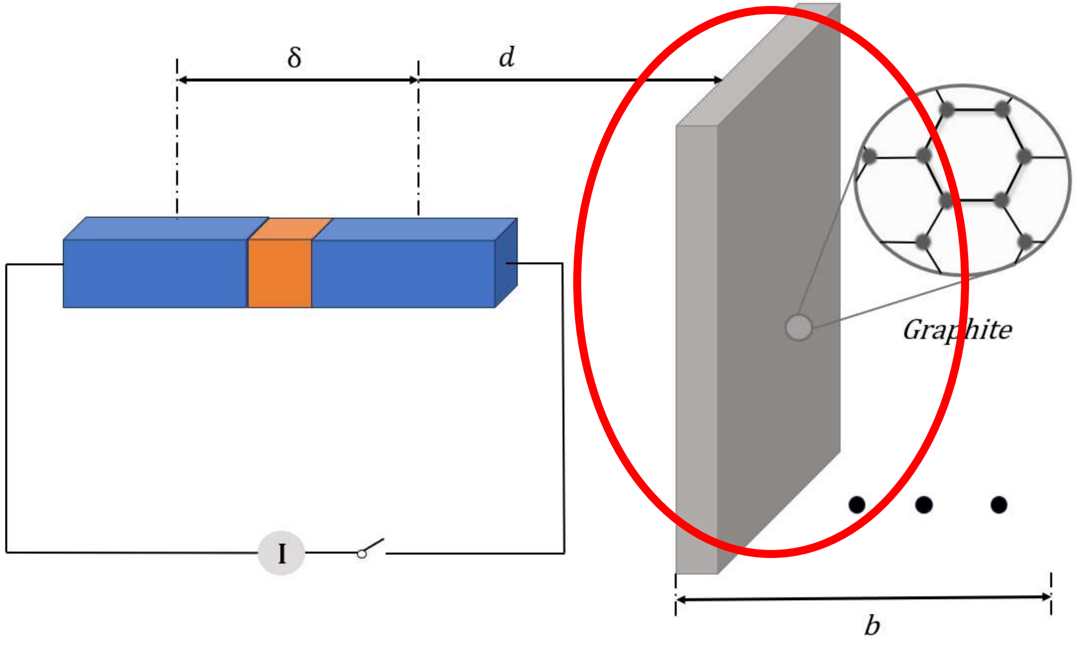
$Q_{B-L} = 2$ for a Cooper-pair

$$I_f = I_c \sin\Delta\phi_f\,, \quad \text{where} \quad I_c = \frac{e n_1 a^2}{m_e \epsilon}.$$

$$I_f = 6\times10^{-3}\,\mathrm{A}\times\left(\frac{g_{B-L}}{10^{-16}}\right)^2\left(\frac{\tau}{1\,\mathrm{min}}\right)\left(\frac{10^{-2}\,\mathrm{eV}}{m_{A'}}\right)\left(\frac{\delta}{100\,\mathrm{nm}}\right).$$

Possible Backgrounds

(1)· Gravity



The plate also contributes a gravity potential. Again, we can estimate that,

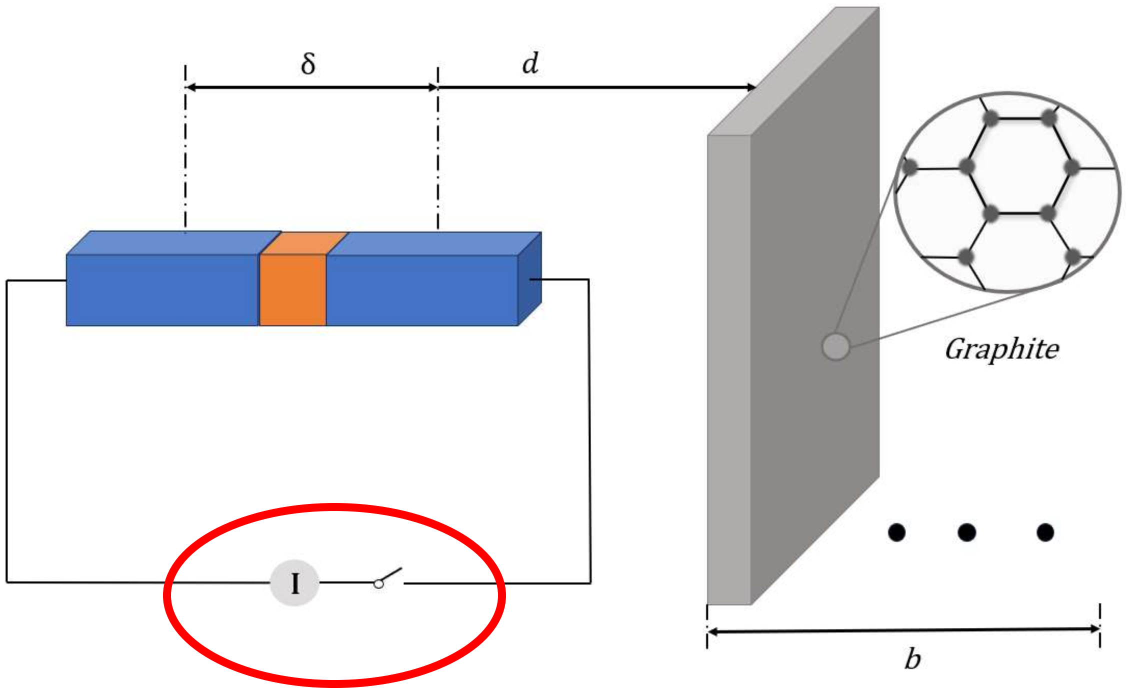
Comparison with gravity:

$$g_{B-L}^2 \sim \frac{m_N m_e}{M_P^2} \quad ----\rightarrow \quad g_{B-L} \sim 10^{-20}$$

$$\phi_g(d) = \tau \times \int_0^b \oiint_S \frac{2Gm_e\rho_C}{\sqrt{(d+z)^2+x^2+y^2}} dSdz,$$

$$\Delta\phi_g = \phi_g(d) - \phi_g(d+\delta) \simeq 10^{-11}$$

(2). Thermal Noise

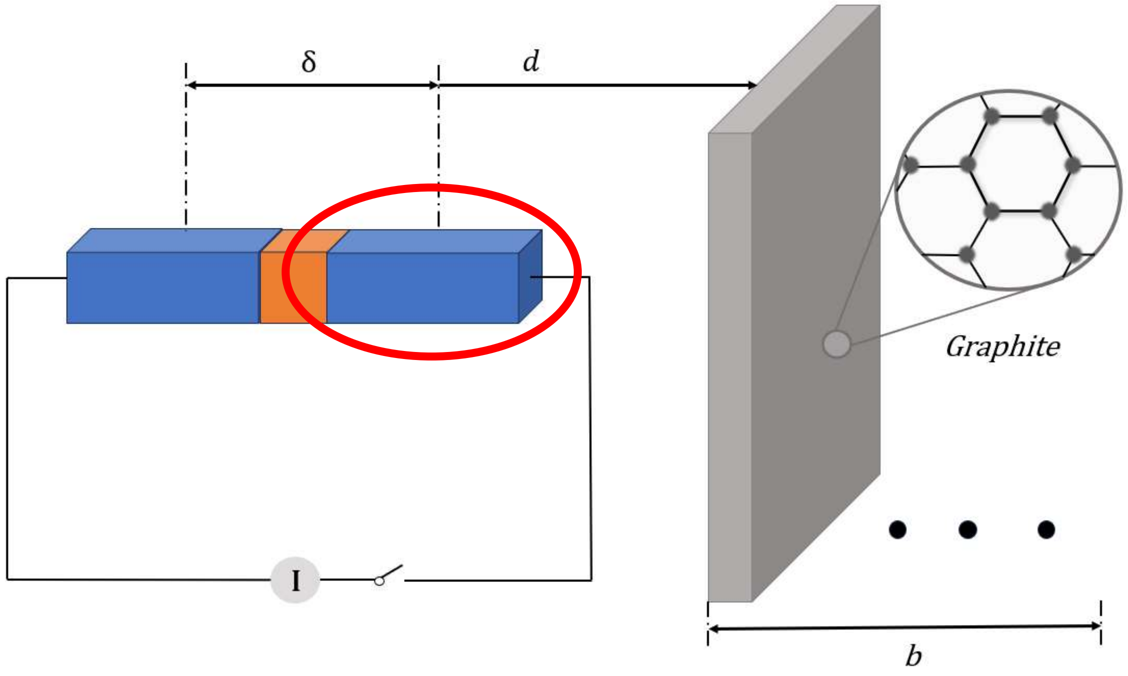


As long as the temperature is non-zero, the electrons in circuit have some thermal motions and induces some background current.

$$I_T \approx ekT/\hbar \approx 10^{-7}(T/1K)A$$

The lower the temperature, the better. Currently, in the laboratory, we can achieve temperatures as low as 1 millikelvin (mK).

(3)· Quantum Fluctuation

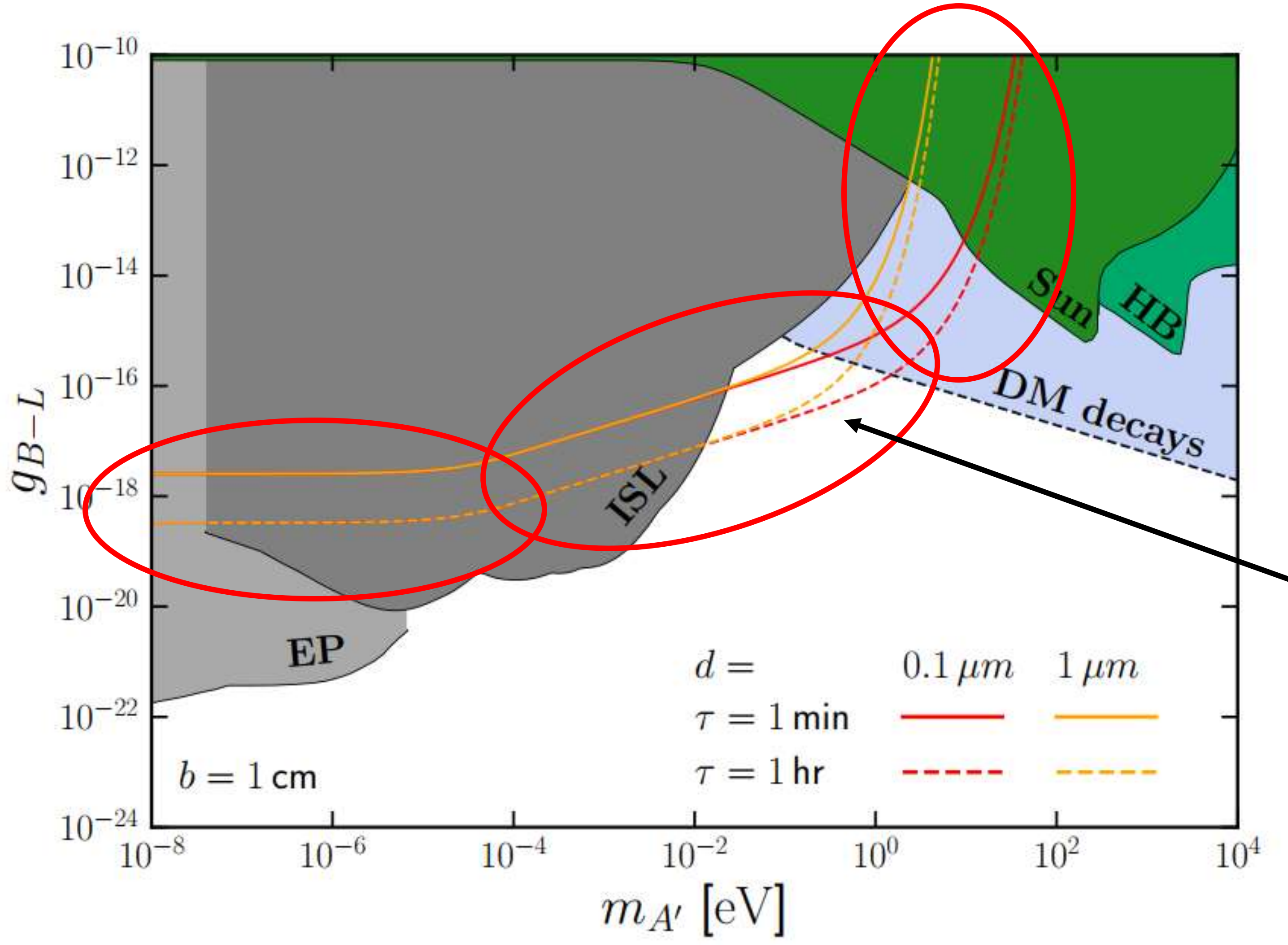


For a coherent state containing N particles, its phase has a quantum fluctuation due to the number-phase uncertainty relationship,

$$\Delta N \cdot \Delta \phi > \hbar$$

Taking the setups above, the total number of Cooper-pairs is around $N \sim 10^7$. The fluctuation of number is then $\Delta N \sim \sqrt{N} \sim 10^3$. As a result, the sensitivity of phase is $\Delta\phi > 10^{-3}$.

Projected Constraint $\Delta\phi > 10^{-3}$



$$\Delta\phi_f \simeq \frac{2\pi\tau g_{B-L}^2 Q_{B-L} n\delta}{m_{A'}}.$$

Detecting Feeton DM

Conclusion and Discussions

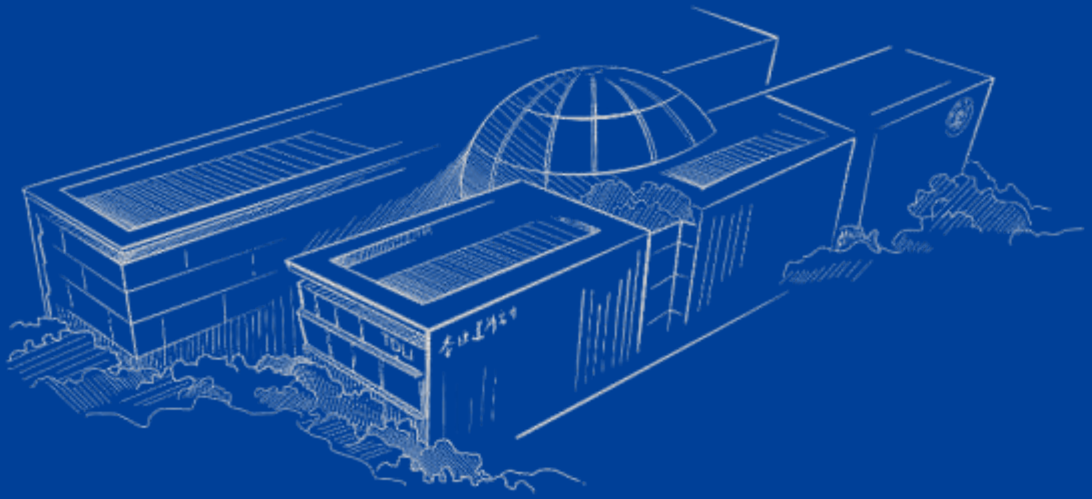
1. Interplay between quantum mechanics and new physics.

2. The quantum mechanical tests of fifth force at scale of millimeters.

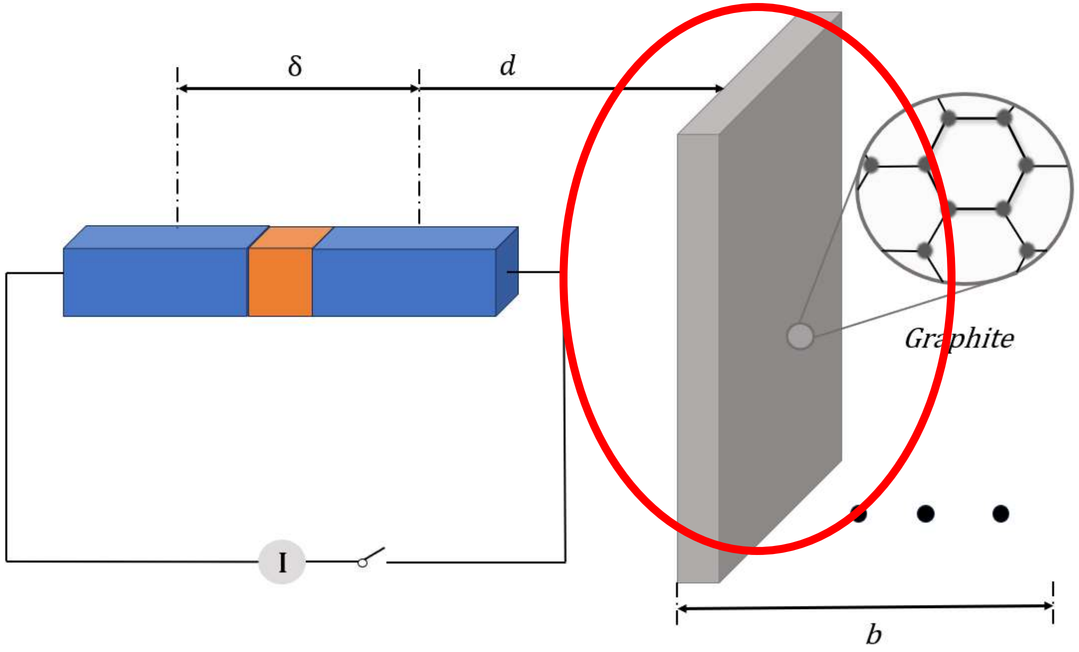
3. Strongest constraint on Yukawa fifth force in the mass range (0.01, 10)eV. Strongest constraint for Feeton DM candidate in that mass range.

4. A new application of the Josephson junction.

Back Up



1. Review the Gravity Background



Comparison with gravity:

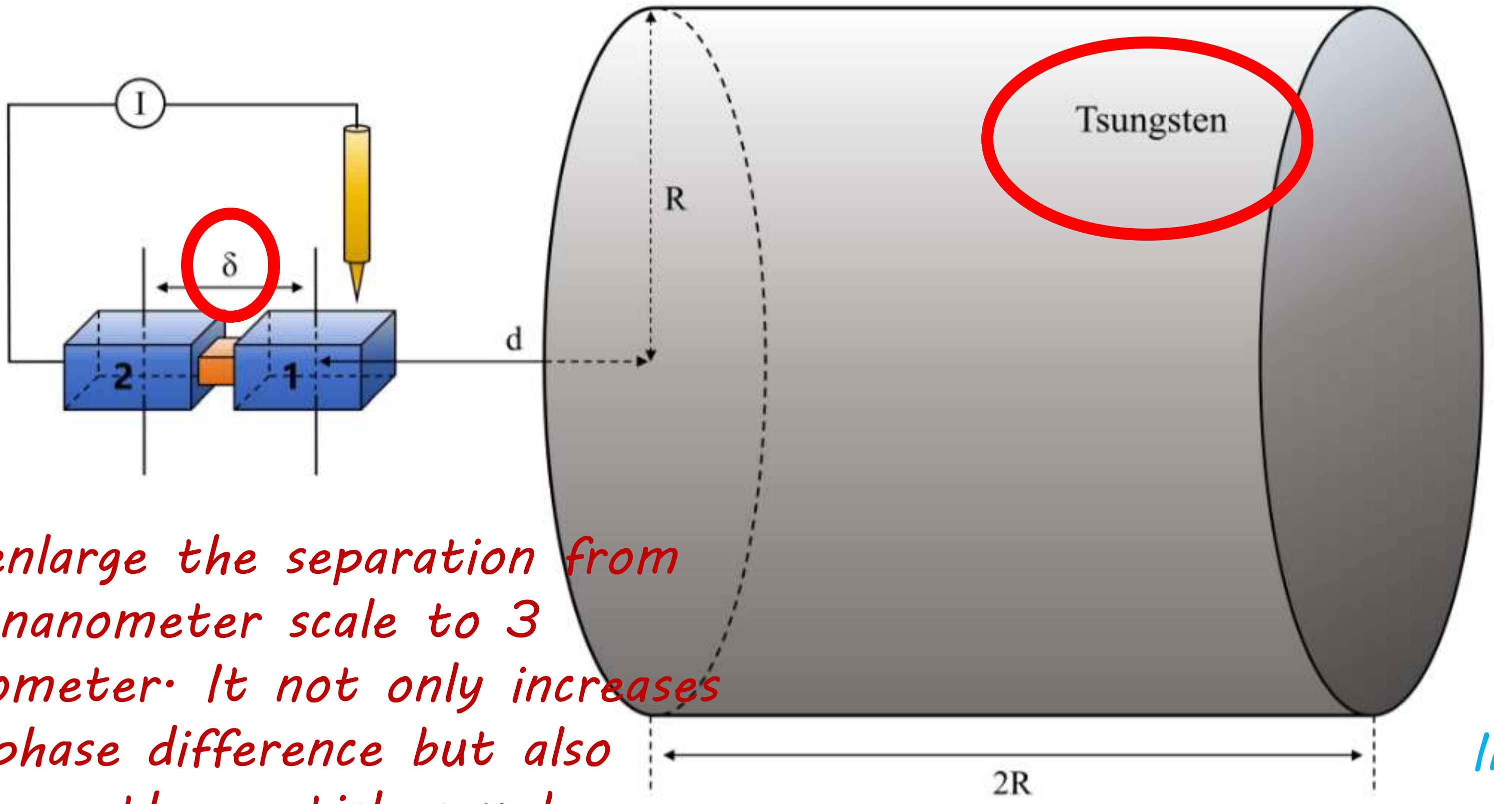
$$g_{B-L}^2 \sim \frac{m_N m_e}{M_P^2} \quad - - - - \rightarrow \quad g_{B-L} \sim 10^{-20}$$

The disadvantages of gravity in detection compared to the fifth force.

(1). The gravity coupling is suppressed by mass of electron. (The test of lepton gravity is also a long standing challenge!)

(2). Gravity does not decay much along the separation δ.

2. Improvements in the experimental setup.

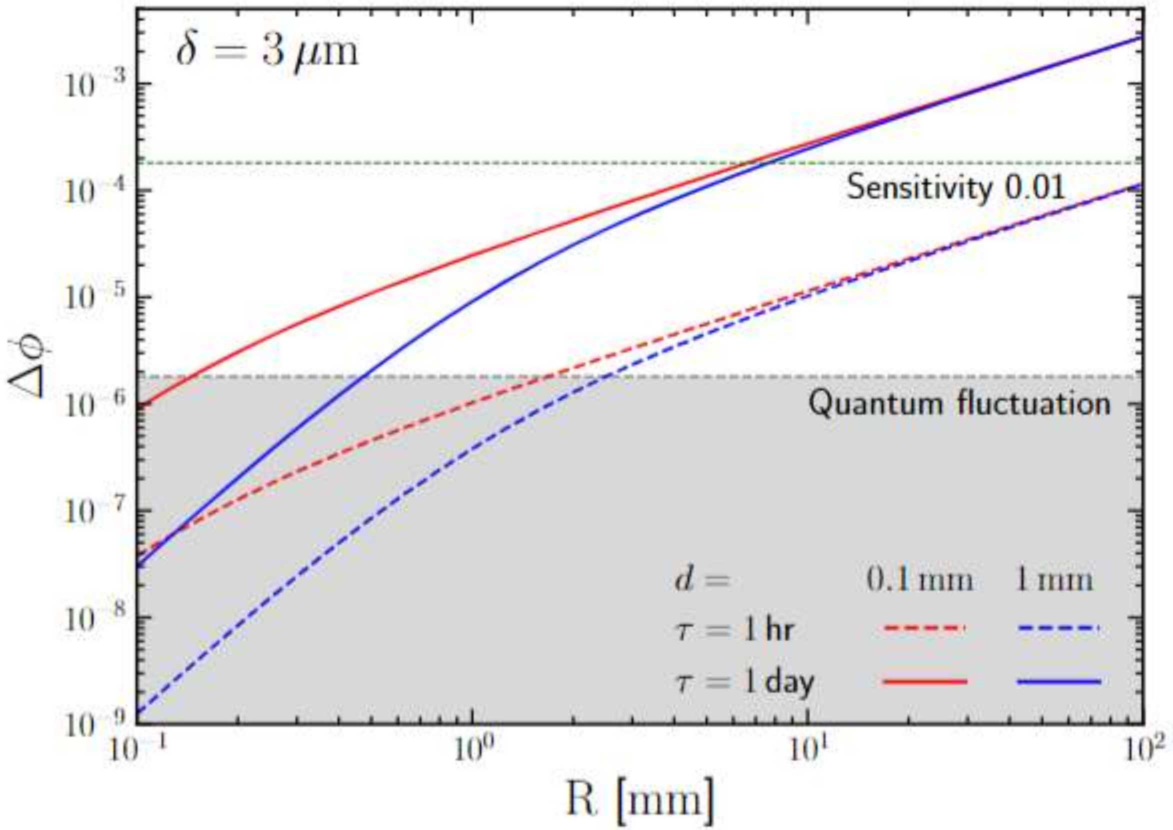


The density of tsungsten is $\rho = \frac{20\mathrm{g}}{\mathrm{cm}^3}$. But it does not make too much change.

We enlarge the separation from 100 nanometer scale to 3 micrometer. It not only increases the phase difference but also increases the particle number, thereby reducing the fluctuation.

Increase the running time.

3. Quantum Test of mm Scale Gravity.



It is the first time that the quantum mechanical measurement of gravity can reach such a small scale.

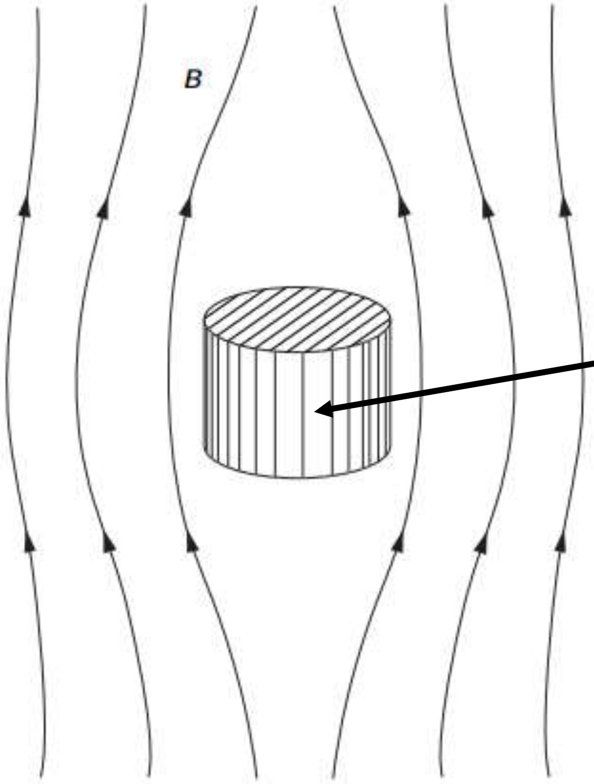
1. Through electron-lattice-electron (phonon mediated) interaction, the electrons have a small net attractive force between each other around fermion surface.

2. Electrons pair to form Cooper-pairs, which is an effective boson. They attend to form a coherent state because of bose-enhancement. The state is described by a classical wave:

$$\Psi_1 = \sqrt{n_1} e^{i\phi_1}$$

3. To excite a coherent ground state, we need a huge amount of energy. A normal scattering can not provide such an energy. Thus it will no longer scatter.

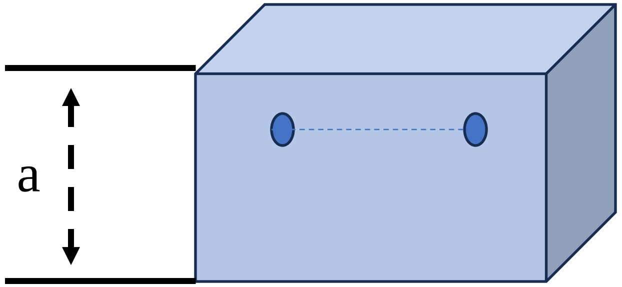
Ref: Feynman's lecture on superconductivity



Based on Lenz's law, superconductors can easily generate a sufficiently large opposing current.

$$E = B = 0$$

Superconductor naturally provides a electromagnetic free environment which is helpful for a tiny-force detection.



$$\Psi_1 = \sqrt{n_1} e^{i\phi_1}$$

The London penetration length (λ) refers to the depth at which a magnetic field can penetrate into a superconductor. The size of the superconductor should be larger than this length. In our case, we assume that the penetration length, $\lambda = 50$nm, length of the superconductor is $a = 100$ nm. Additionally, the length, width, and height of the superconductor are approximately of the same order.

$$n_1 = \frac{m_e}{e^2 \lambda_L^2} = 1.22 \times 10^{22}\ \mathrm{cm}^{-3}$$