# *QCD Matter at Finite $T, \mu_B$ and $B$*

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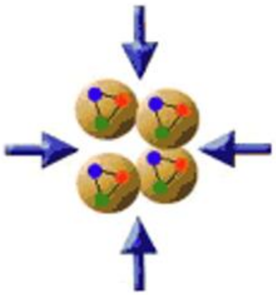
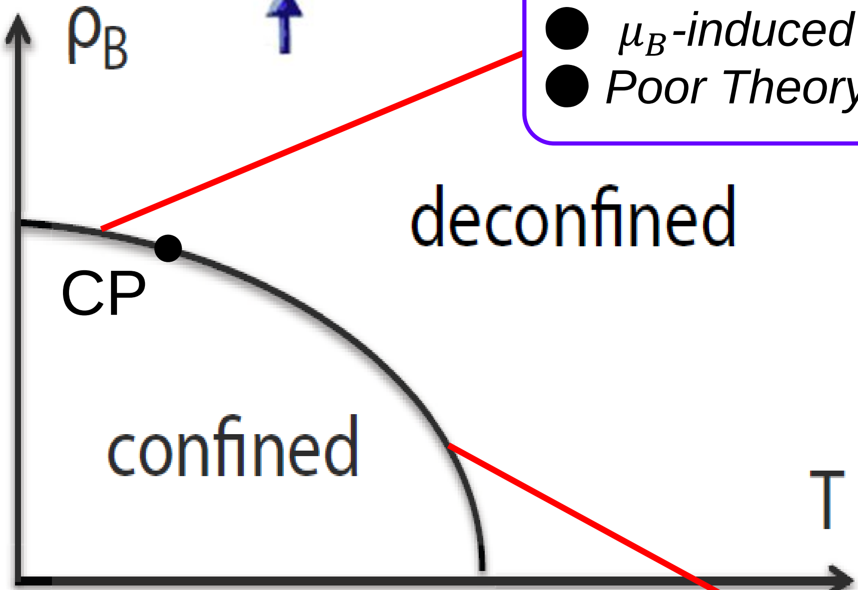
# *QCD Phase Transitions at Finite $T$ and $\mu_B$*



*Matter compression*

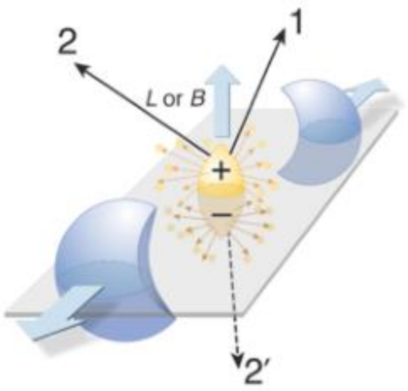
- *$\mu_B$-induced QUANTUM Phase Transitions*
- *Poor Theory*




*Vacuum excitation*

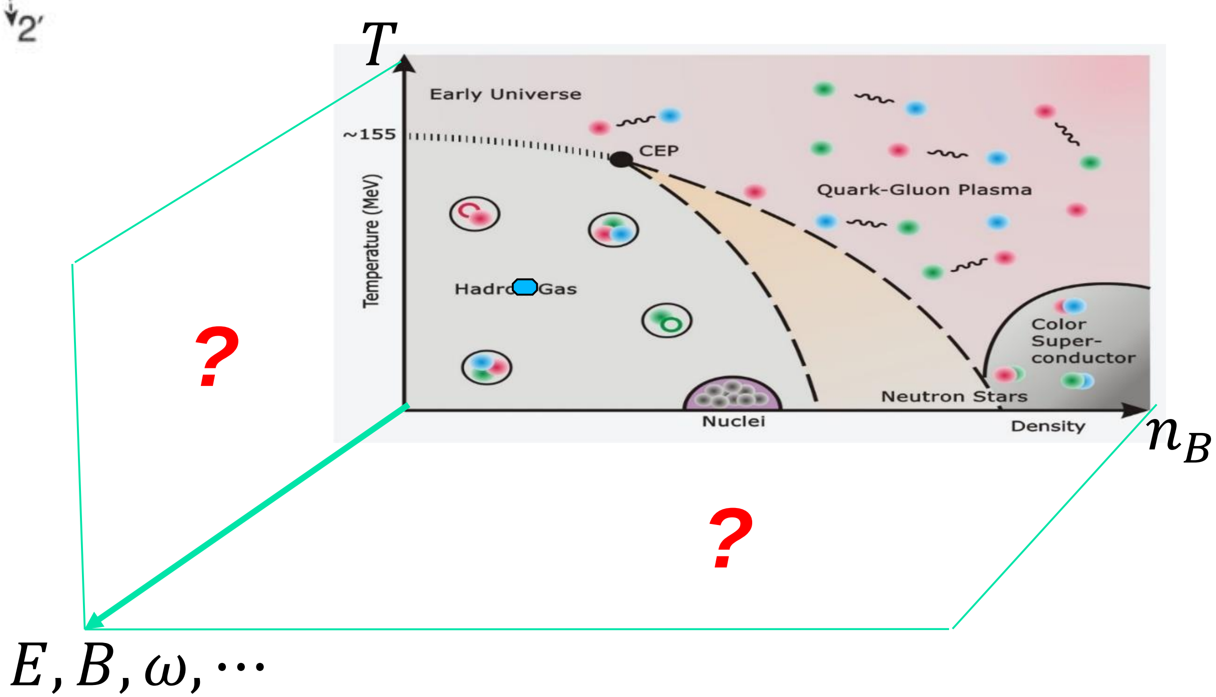
- *T-induced CLASSICAL Phase Transitions*
- *Lattice QCD, (resumed) pQCD, FRG, DSE, Models*

# Including E, B and ω




- $|eB| \sim 5m_\pi^2$ *at RHIC and* $70m_\pi^2$ *at LHC, the strongest magnetic field in nature!*

- $\omega \sim 10^{21}/s$ *at RHIC, the strongest rotation in nature!*



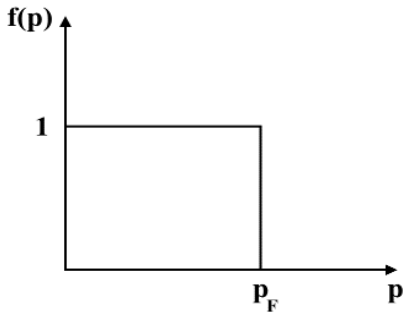
# Competition between $T, \mu_B$ and $B$

*1) Competition between B and T:*

*$\vec{B}$ breaks down the translation invariance, but T restores the invariance.*

*→ Does the cancellation reduce the B effect in QGP ?*

*2) Competition between B and $\mu_B$:*




*Will the jump at the Fermi surface enhance the B effect ?*

*We need detailed calculation in QCD.*

# Feynman Rules in Magnetic Field

A.Kostenko and C.Thompson, Astrophys J. 869, 44(2018), 875, 23(2019).

External lines:

$$[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$$

$$\psi^\sigma_\mp(x,p) = \begin{cases} e^{-ip\cdot x}u_\sigma(\boldsymbol{x},p) \\ e^{ip\cdot x}v_\sigma(\boldsymbol{x},p) \end{cases}$$

$$u_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_zp_n\phi_{n-1} \\ (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \\ -ip_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -p_z(\epsilon_n+m)\phi_n \end{bmatrix}, \qquad v_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -p_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -ip_z(\epsilon_n+m)\phi_n \\ -p_zp_n\phi_{n-1} \\ i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \end{bmatrix},$$

$$u_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ -ip_zp_n\phi_n \\ p_z(\epsilon_n+m)\phi_{n-1} \\ ip_n(\epsilon+\epsilon_n)\phi_n \end{bmatrix} \qquad v_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z(\epsilon_n+m)\phi_{n-1} \\ -p_n(\epsilon+\epsilon_n)\phi_n \\ -i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ p_zp_n\phi_n \end{bmatrix}$$

Quark propagator:

$$G(x'-x) = -i\left(\frac{L}{2\pi\lambda}\right)^2 \int dp_z da \sum_{\sigma,n}\left[\theta(t'-t)u_\sigma(\boldsymbol{x}',p)\bar{u}_\sigma(\boldsymbol{x},p)e^{-ip\cdot(x'-x)} - \theta(t-t')v_\sigma(\boldsymbol{x}',p)\bar{v}_\sigma(\boldsymbol{x},p)e^{ip\cdot(x'-x)}\right]$$

$$G(p) = -\int_0^\infty \frac{dv}{|qB|}\left\{[m+(\gamma\cdot p)_\parallel][1 - isgn(q)\gamma_1\gamma_2\tanh(v)] - \frac{(\gamma\cdot p)_\perp}{cosh^2(v)}\right\} e^{-\frac{v}{|qB|}\left[m^2 - p_\parallel^2 + \frac{\tanh(v)}{v}p_\perp^2\right]}$$

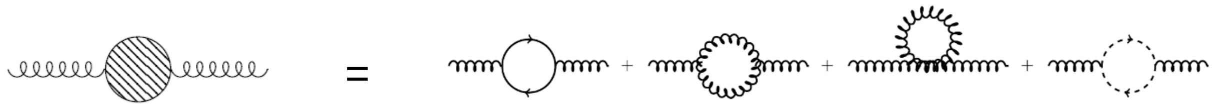
Schwinger propagator, 1951

- no more translation invariance.
- the two Schwinger phases for $q$ and $\bar{q}$ are cancelled to each other in loop calculation.

# QCD Thermodynamics under $B$

G.Huang, J.Zhao, PZ, PRD107, 114035(2023)

*Gluon self-energy in magnetic field:*



*for quark loop*

$$\Pi^{||}_{\mu\mu}(T,B) = g^2 T|qB| \sum_{n p_z n_1} \frac{(2-\delta_{n_1 0})\,(\delta^{||}_{\mu\mu} + g^{||}_{\mu\mu})(-\omega_n^2 + p_z^2)}{(m^2 + \omega_n^2 + p_z^2 + 2n_1|qB|)^2}$$

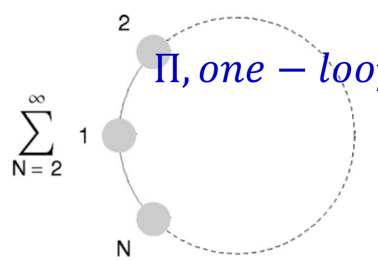
$$\Pi^{\perp}_{\mu\mu}(T,B) = 0$$

*Matsubara frequencies* $p_0 = i\omega_n = i(2n+1)\pi T$
*quark longitudinal momentum* $p_z$
*transverse Landau energy* $\varepsilon_k = 2n_1|qB|$

*for gluon and ghost loops*

$$\overline{\Pi}_{\mu\nu}(T,B) = \overline{\Pi}_{\mu\nu}(T,0)$$

*To include non-perturbative effect, we take the summation of ring diagrams*
*→ gluon propagator and thermodvnamic potential:*

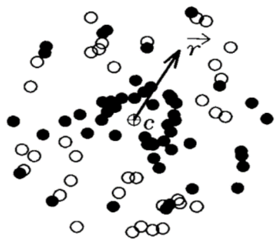



$\Pi, one - loop\ gluon\ energy$

# Color Screening in Hot QCD Matter

G.Huang, J.Zhao, PZ, PRD107, 114035(2023)

*Debye screening of a pair of charged particles $q\bar{q}$:*




$$\frac{1}{r} \to \frac{1}{r} e^{-m_D r} = \frac{1}{r} e^{-r/r_D}$$

*screening mass $m_D$*

*screening length $r_D$*

*Pole of the gluon propagator → screening mass:*

$$m_D^2(T,B) = m_Q^2(T,B) + m_G^2(T),$$

$$m_Q^2(T,B) = -\Pi_{00}^{||}(T,B),$$

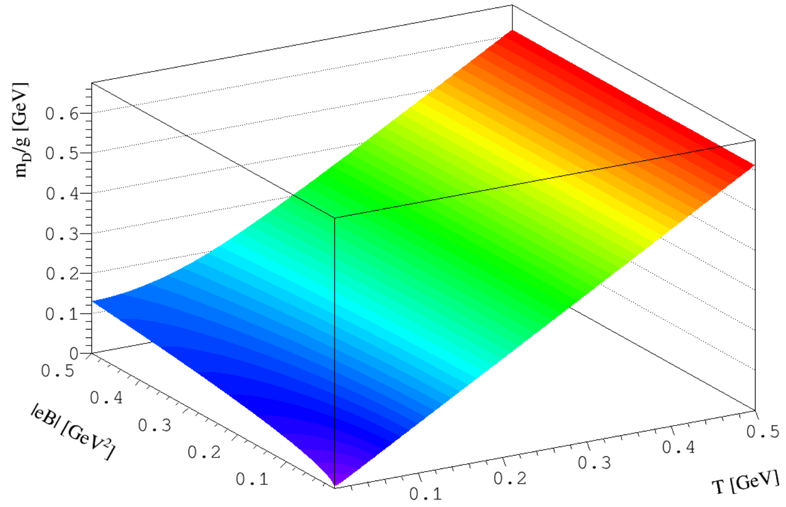
$$m_G^2(T) = -\overline{\Pi}_{00}^{||}(T).$$

$$m_Q^2(T,B) = -g^2 T|qB| \sum_{n p_z n_1} \left[ (2-\delta_{n_1,0}) \frac{m^2 - \omega_n^2 + p_z^2 + 2n_1|qB|}{(m^2 + \omega_n^2 + p_z^2 + 2n_1|qB|)^2} \right]$$

$$m_G^2(T) = \frac{N_c}{3} g^2 T^2$$

## $m_D(T, B)$

*G.Huang, J.Zhao, PZ, PRD107, 114035(2023)*




*The B effect is gradually cancelled by thermal motion.*

***Conclusion :***
***Even if the magnetic field in QGP is strong enough, its effect on QCD matter is almost cancelled by the fireball temperature.***

# Color Screening in Dense QCD Matter

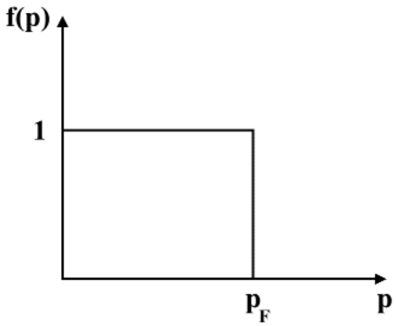
G.Huang, J.Zhao, PZ, PRD108, L091503(2023)

*In the frame of ring diagram summation*

$$m_D^2(\mu_f, B) = \frac{g^2}{(2\pi)^2} \sum_{n=0} (2-\delta_{n0}) \frac{\mu_f |qB|}{\sqrt{\mu_f^2 - 2n|qB|}}$$

*For chiral quarks, the distribution*

$$f(p) = \theta(\mu_f - p)$$




*the Fermi surface is determined by*

$$p_z^2 + 2n|qB| = \mu_f^2$$

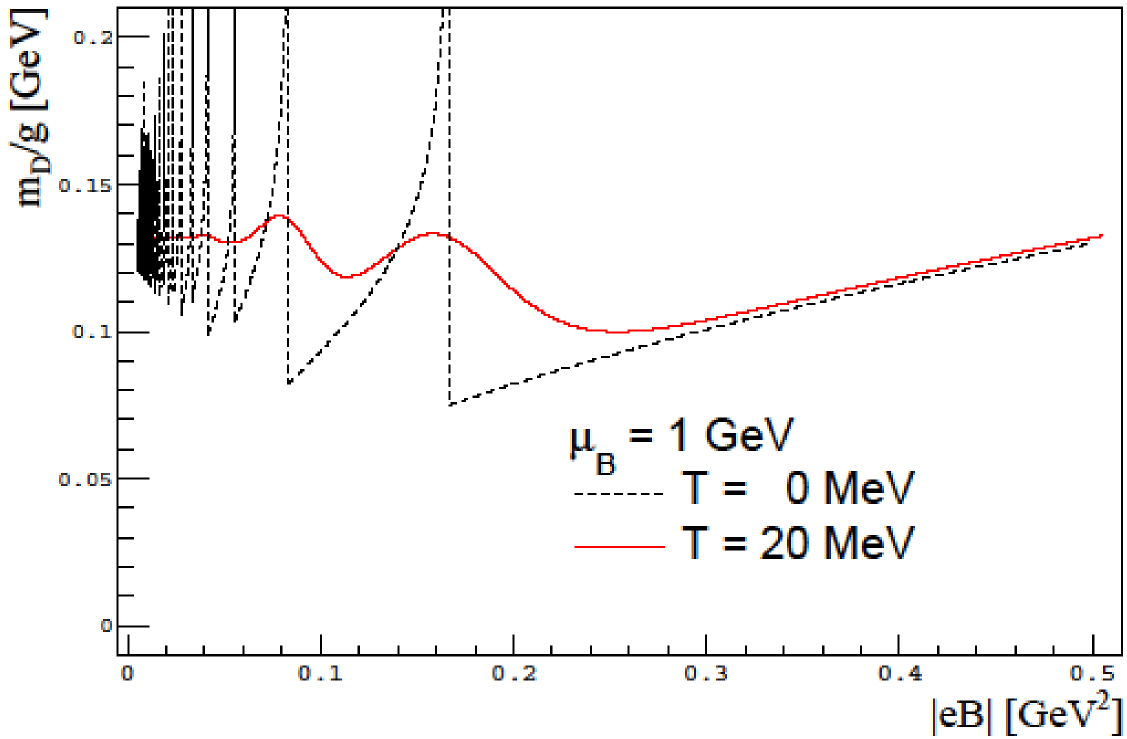
*when $\mu_f$ and Landau levels are matched*

$$\mu_f^2 = 2n|qB|$$

*the infrared divergence at the Fermi surface induces a complete screening $m_D \to \infty$, called resonant screening.*

# Resonant Screening

G.Huang, J.Zhao, PZ, PRD108, L091503(2023)




***The color interaction in dense and magnetized QCD is completely screened, when the Fermi surface matches the Landau levels*** $\mu_f^2 - 2n|qB| = 0$.

*Possible oscillation induced by rotation and magnetic field (at zero density but finite temperature)? Question raised by Defu and Xuguang yesterday.*

# Summary

***Conclusion 1:***
***Even if the magnetic field in QGP is strong enough, its effect on hot QCD matter is almost cancelled by the thermal motion.***

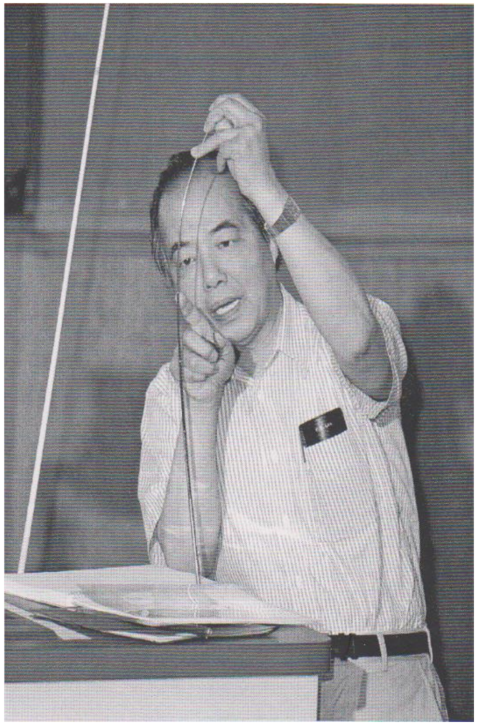
***Question:***
***What is the magnetic effect in HIC at RHIC and LHC? Should we pay more attention to the initial stage, like heavy flavor production (see arXiv: 2401.17559 by Shile Chen et al.)?***

***Conclusion 2:***
***The color interaction in dense and magnetized QCD can be completely screened, when the Fermi surface matches the Landau levels*** $\mu_f^2 - 2n|qB| = 0.$

***Question:***
***What is the magnetic effect in compact stars and HIC at BES, FAIR, NICA and HIAF?***

# 要想认识最小的，需要知道最大的

李政道, 1996



*Large things are made of small*
*And even smaller.*
*To know the smallest*
*We need also the largest*

*All lie in vacuum*
*Everywhen and everywhere.*
*How can the micro*
*Be separate from the macro?*

*Let vacuum be a condensate*
*Violating harmony*
*We can then penetrate*
*Through asymmetry into symmetry*

大事物由小事物组成
甚至是更小的。
要想认识最小的
我们也需要知道最大的。
一切都取决于真空
无论何时何地。
微观的事物怎能
与宏观相分离？
真空其实是一种凝聚
破坏了和谐。
如此我们方可洞穿
不对称中的对称。

【杨振伟翻译】