GW signatures of dynamical tides:

heaviest neutron stars vs lightest black holes

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Outline

- Mass upper limit of neutron stars M_TOV

  2.6 Msun: lightest BH or heaviest NS ?
- Normal modes of neutron stars
- Signature of mode excitation in BNS GWs
- Interface mode as an example
- Summary

M_TOV

- M_TOV maximum mass of non-rotating neutron stars

  M_TOV ~ (2.2--2.4) Msun

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- M_max maximum mass of fastest-rotating neutron stars

  M_max = 1.2 M_TOV ~2.64–2.88 Msun

M_TOV

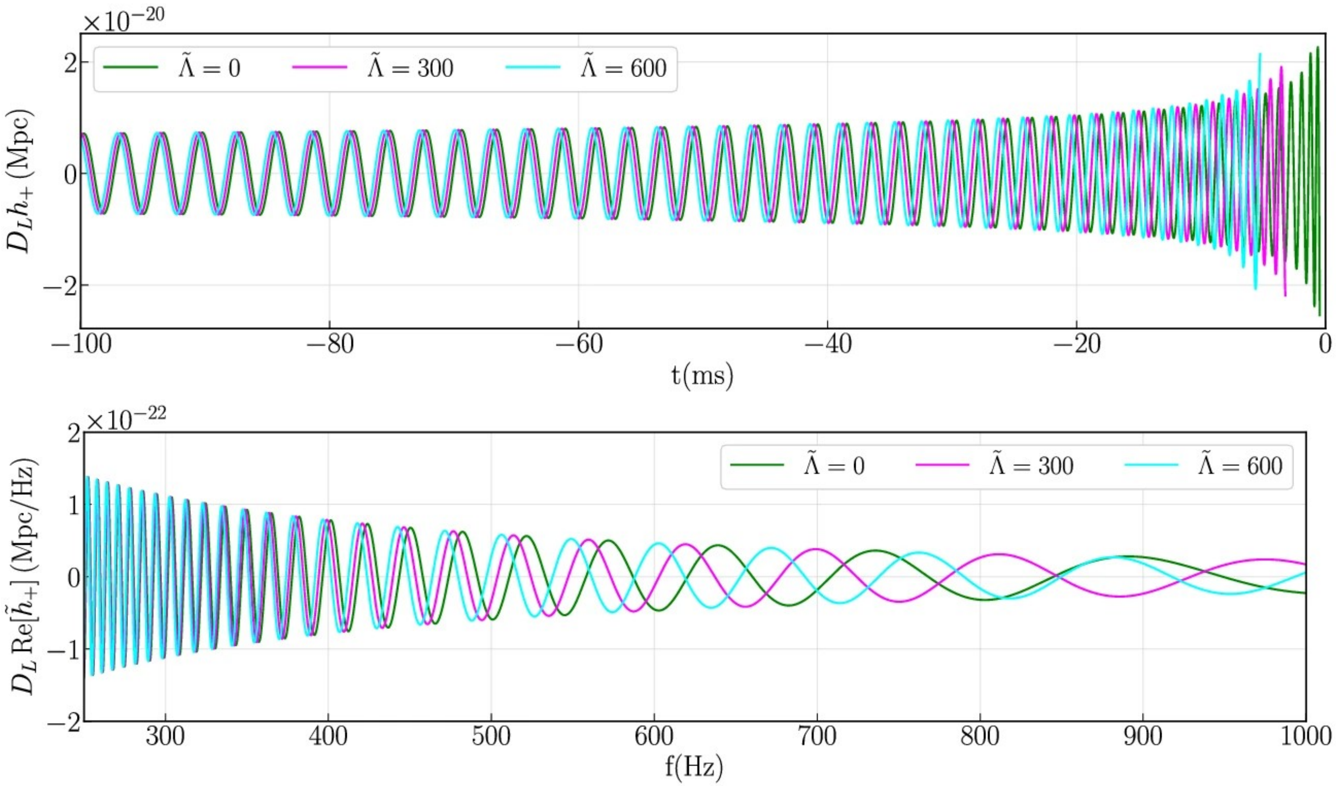
M_max = 1.2 M_TOV ~ 1.2 * (2.2--2.4) Msun = 2.64–2.88 Msun

- GW190814 23 Msun + 2.6 Msun

  BH + lightest BH/heaviest NS ?

Tidal deformability   EM signals   Dynamical tides

NSs vs BHs: tidal deformability



NS tidal deformability

dimensionless tidal deformability (Chatziioannou 2020)

$$\Lambda_{\rm NS} := \lambda / M_{\rm NS}^5 \propto (R_{\rm NS}/M_{\rm NS})^5 \sim M_{\rm NS}^{-6}$$

GW170817: $\Lambda_{1.3 M_\odot} \sim 200 \pm 400$ (LVC 2017)

$$\Lambda_{2.6 M_\odot} \sim 200 \times (2)^{-6} \sim \mathcal{O}(1)$$

GW only: tidal deformability

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Unmeaureable small diff in the tidal deformability: O(1) vs 0

GW + EM observations

- 1.3 Msun + 2.6 Msun

  GW + GRB/kilonova → Cannot distinguish NS + BH/NS

- 2.6 Msun + 23 Msun

  GW only → Cannot distinguish BH/NS + BH

- 2.6 Msun + 2.6 Msun

  GW + GRB/kilonova → BNSs

  GW only → BBHs **Can but RARE^2**

Normal modes

stellar response to a tidal field

$$\vec{\xi}(\mathbf{r},t) = \sum_{\alpha} a_{\alpha}(t)\vec{\xi}(\mathbf{r})$$

$$\vec{\xi}_{\alpha}(\mathbf{r}) = \vec{\xi}_{nlm}(\mathbf{r}) = (U_{nl}(r)\hat{r} + rV_{nl}(r)\nabla)\, Y_{lm}(\theta,\phi)$$

$$\left[\mathcal{L}(r) - \omega_0^2\right]\vec{\xi}_{\alpha} = 0$$

McDermott 85,88, Lai94

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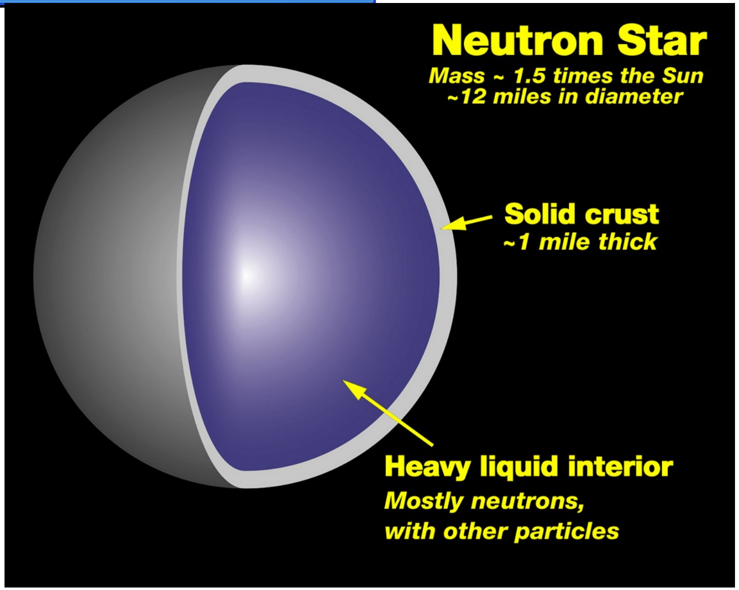

pressure → f-, p-modes
gravity → g-modes
crust shear → s-modes

Mode excitation in a tidal field

$$M + M', \quad D(t)$$

$$U(\mathbf{r}) = -\frac{GM'}{|\mathbf{r} - \mathbf{D}|} = -GM' \sum_{lm} W_{lm} \frac{r^l}{D^{l+1}} e^{-im\Phi(t)} Y^*_{lm}(\theta, \phi)$$

$$\frac{d^2}{dt^2} a_\alpha + \omega_\alpha^2 a_\alpha = \frac{GM' W_{lm} Q_{nl}}{D(t)^{l+1}} e^{-im\Phi(t)}$$

Lai1994

$$Q_{nl} = \int d^3x \, \rho \vec{\xi}^*_{nlm} \cdot \nabla [r^l Y_{lm}(\theta, \phi)]$$

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f-mode: $f_0 \simeq 2kHz,\ Q \simeq 0.5$

p-mode: $f_0 > 2kHz,\ Q \simeq 0.5$

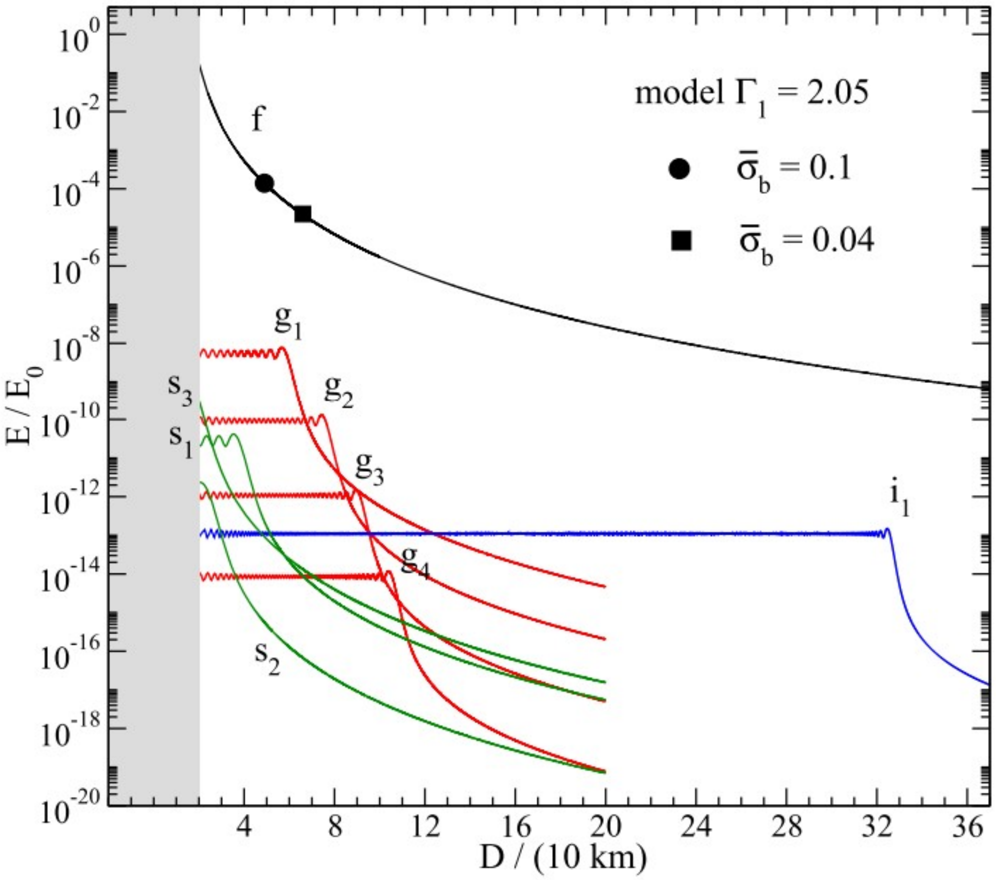
g-mode: $f_0 \simeq 100Hz,\ Q \simeq 10^{-4}$

Larger g-mode Q if quark core Miao

Mode excitation in a tidal field

Passamonti+21

$$E/E_0 = E/(GM^2/R) = 2|a_\alpha|^2$$



Signature of mode excitation

Mode resonance at frequency $f_0 \Rightarrow$ energy exchange with orbit motion δE

$\Rightarrow$ dephasing in GWs $\delta\phi$

$$|\delta\phi| = 2\pi f_0 \frac{\delta E}{\dot{E}_{\rm GW}}$$

$$|\delta\phi| \simeq 5 \times 10^{-3} \left(\frac{f_0}{100 Hz}\right)^{-2} \left(\frac{Q}{3 \times 10^{-4}}\right)^2 M_{1.4}^{-4} R_{10}^2$$

Lai1994

Signature of mode excitation in GWs

$$h(f) = A(f)e^{i\psi(f)}$$

$$\psi(f;\phi_c,t_c) = 2\pi f t_c - \phi_c + \frac{3}{4}(8\pi\mathcal{M}f)^{-5/3}$$

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Yu2017

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Yu2017

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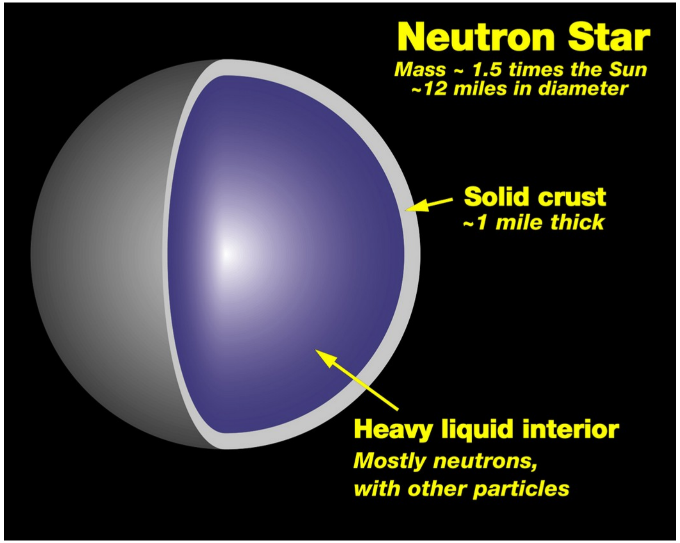
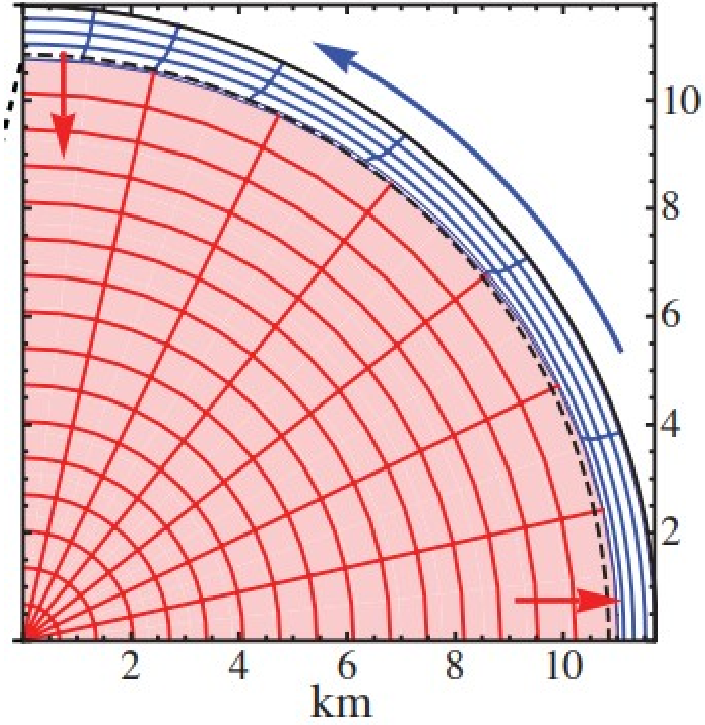
Note: mode resonance must happen in the detector sensitivity band to be detected.

Interface mode as an example



Credit:blackradius.com



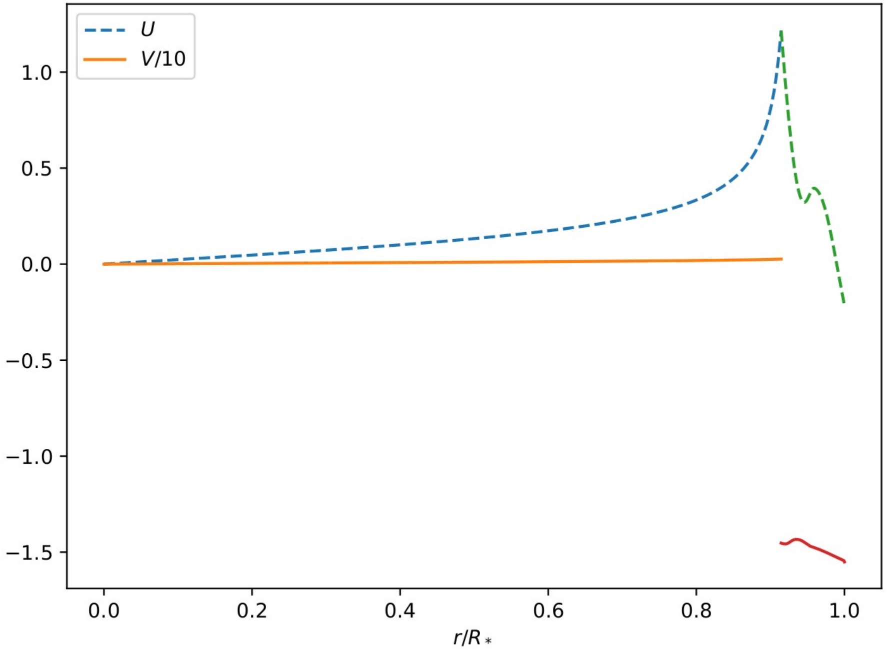
Credit:Tsang2012

Interface mode

$$\vec{\xi}(\vec{r}, t) = \sum_{m=0,\pm 2} a_m(t)\vec{\xi}_m(\vec{r})$$

$$\vec{\xi}_m = U(r)Y_{2m}(\theta, \phi)\hat{r} + V(r)\nabla Y_{2m}(\theta, \phi)$$

> 99% mode energy is stored in the solid crust



Interface mode

i-mode frequency f_0 depends on eos $(p(\rho), \mu(\rho))$ at the crust base.

$$|\delta\phi| \simeq 5 \times 10^{-3} \left(\frac{f_0}{100Hz} \right)^{-2} \left(\frac{Q}{3 \times 10^{-4}} \right)^{2} M_{1.4}^{-4} R_{10}^{2}$$

$f_0 \simeq (30, 200)Hz, \ Q \sim 10^{-2} \Rightarrow \delta\phi \sim (1, 50)$

detectable by LIGO (sensitivity $O(1)$) ?

Nope!

i-mode cannot be fully excited because NS crust melts before $f_{\rm gw}$ goes up to f_0.

Crust melting driven by i-mode

BNS $1.3M_\odot + 1.3M_\odot$ and $R_\star = 11.7$ km, $f_{0,ini} = 190$ Hz.

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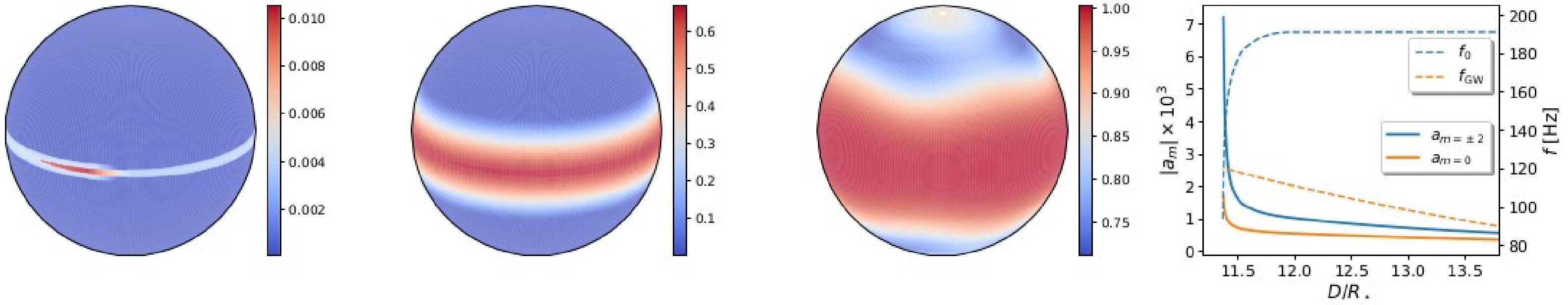


Left three: $e_i/e_{\rm melt}$ at $D/R_* = 12.0/11.6/11.4$.

Right: $a_m(t), f_0(t), f_{\rm gw}(t)$

Dynamical signature on GWs

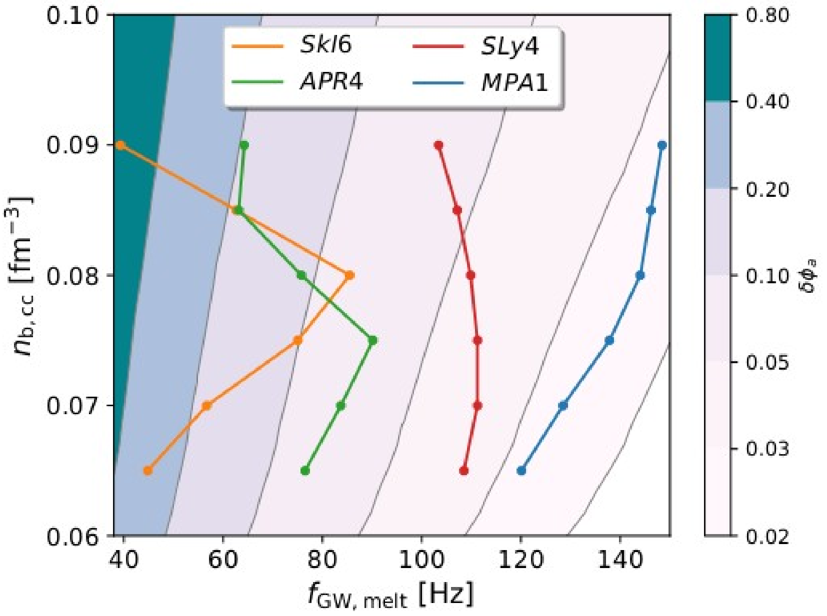
BNS $(M_\star + qM_\star)$:

$$\delta\phi = \frac{2\omega_{\rm orb} E_{\rm melt}}{P_{\rm gw}} = \frac{0.1}{q^2}\left(\frac{1+q}{2}\right)^{2/3} E_{47} M_{1.3}^{-10/3} f_{70}^{-7/3}$$

$\omega_{\rm orb} = \pi f_{\rm gw,melt}$, $E_{47} = E_{\rm melt}/10^{47}$ ergs, $f_{70} = f_{\rm gw}/70$ Hz

Note: melting energy instead of mode energy,
Frequency at melt instead of at resonance.

Phase shifts examples



BNSs (1.3 Msun+1.3 Msun)

I-mode search in GW170817

$$h(f) = A(f)e^{i\Psi(f)}, \delta\Psi(f) = \delta\phi_a(1 - f/f_a)\Theta(f - f_a)$$

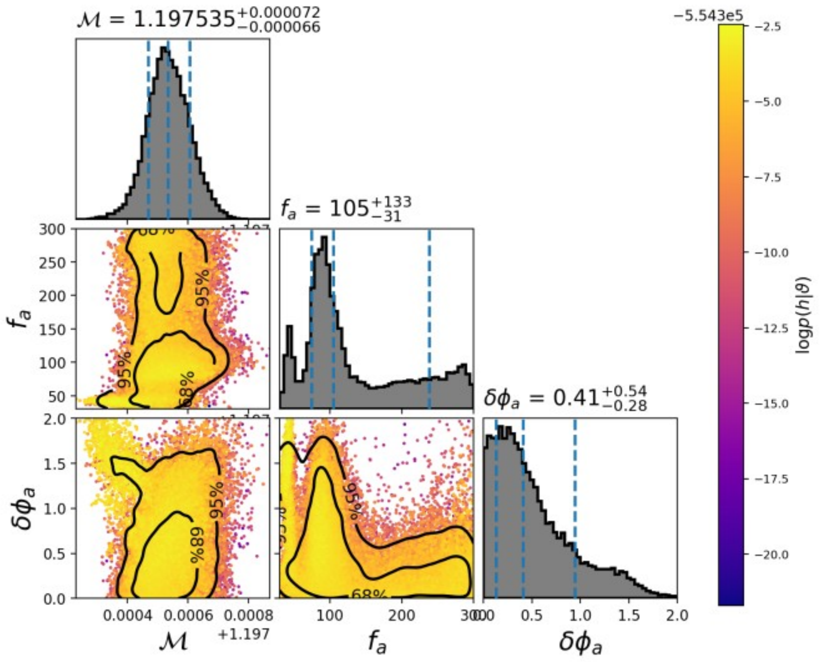


Figure: GW170817 search. No evidence of i-mode found: $\delta\phi_a < 1$ (@2σ)

Forecast search in GW170817 like events with LIGO A+

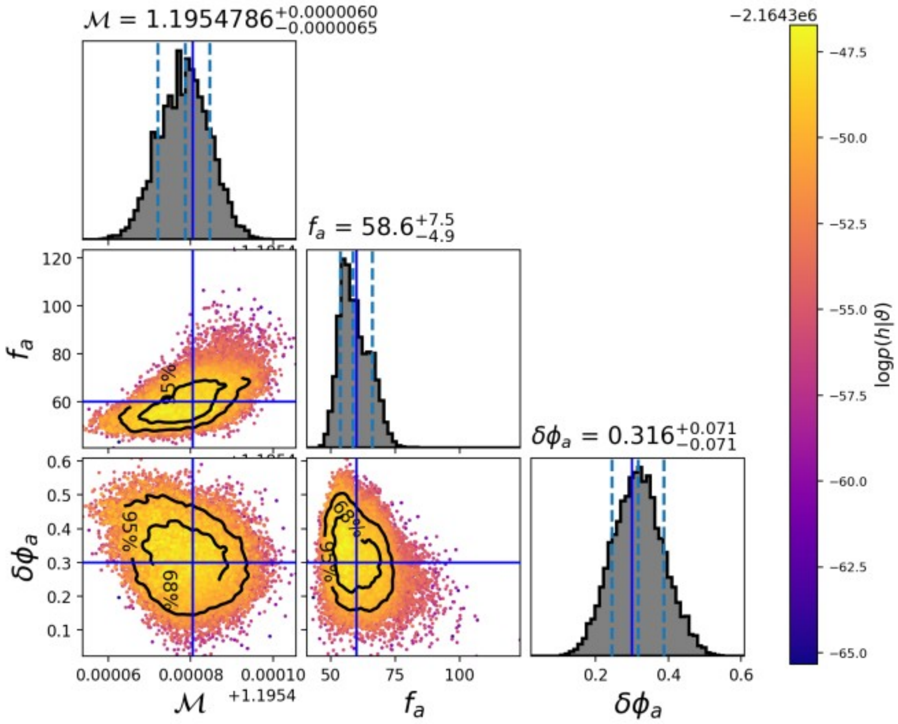


Figure: a GW170817 like event with i-mode injection $f_a = 60$ Hz, $\delta\phi_a = 0.3$

Summary

2.6 Msun → Lightest BH or most massive NS ?

Tidal deformability	no
GW+EM observations	(very likely) no
Normal modes	(very likely) yes