Gravitational waves from inflaton decay

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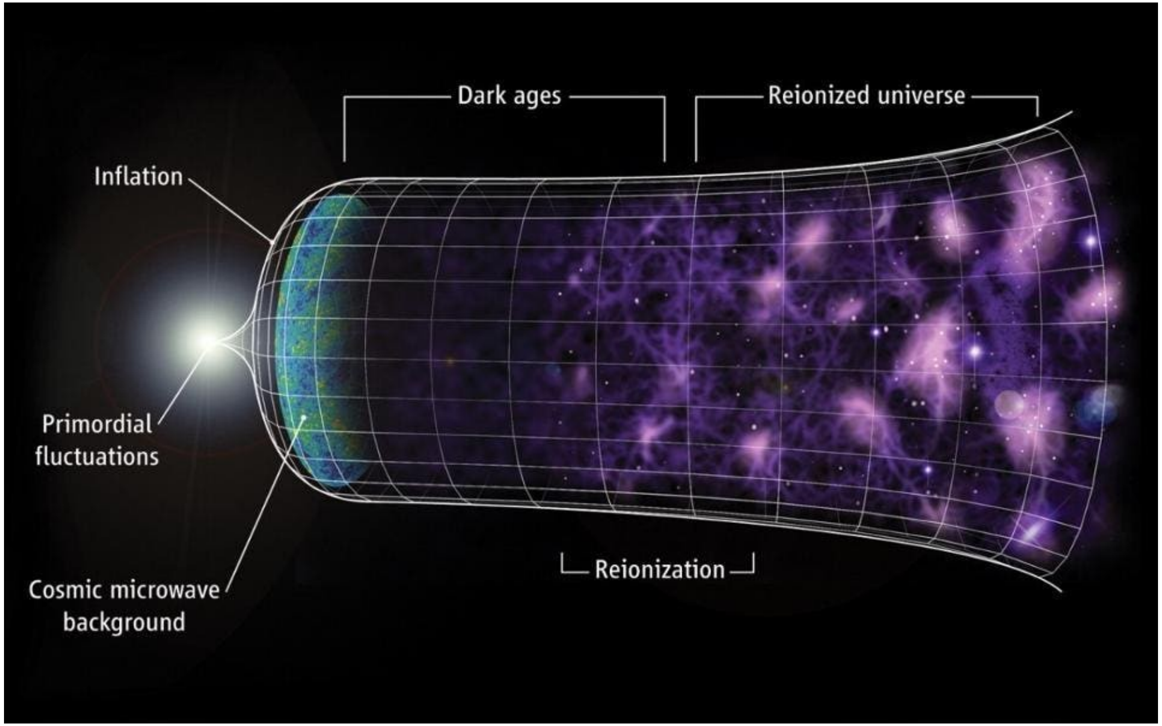
Based on A.Koshelev, A. Starobinsky, AT, Phys.Lett.B 838 (2023) 137686
AT, arXiv:2312.16691 (accepted to PLB)

Early Universe inflation: Why do we need it?



- Initial conditions for Hot Big Bang
- The best explanation for homogeneity and isotropy of the present Universe
- Natural mechanism of generation of 'seeds' for CMB anisotropies and structures in the late Universe

$$a(t) = \text{const} \cdot e^{H_{vac}\, t}$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j$$

Realization of inflation and reheating

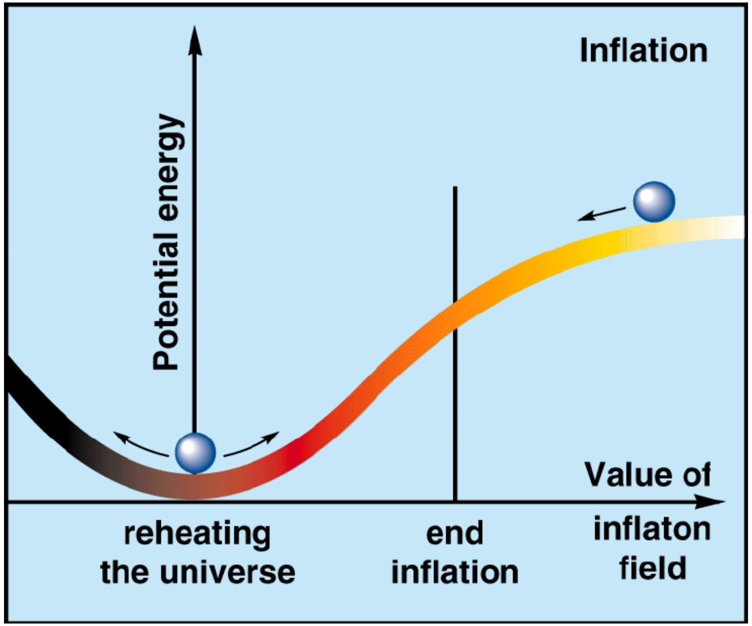
$$p = -\rho. \qquad a(t) = \text{const} \cdot \mathrm{e}^{H_{vac}\, t}$$

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

Slowly rolling scalar field is a solution!



Oscillations after inflation decay to the SM particles $\Longrightarrow$ reheating of the Universe

Reheating temperature is unknown: from 1 GeV to 10^{16} GeV

Why inflation is so attractive?

Nothing depends on initial conditions — attractor dynamics

⇓

Solution to the problems of Hot Big Bang related to initial conditions

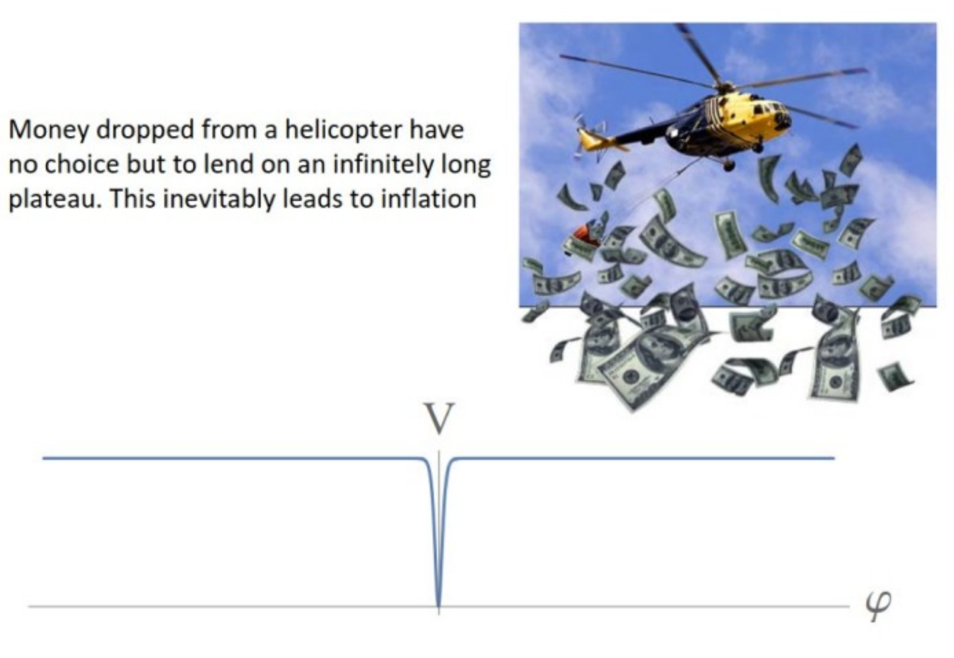


Figure 4: Inflation in economy and in the universe.

A. Linde, arXiv: 1710.04278

Origin of CMB perturbations

Scalar field in de-Sitter space

$$S_\varphi = \frac{1}{2}\int d^4x a^2(\eta)[(\partial_\eta\varphi)^2 - (\partial_i\varphi)^2]. \qquad \varphi = \frac{\chi}{a(\eta)}.$$

$$\chi'' - \frac{a''}{a}\chi + k^2\chi = 0.$$

Bunch-Davies vacuum:

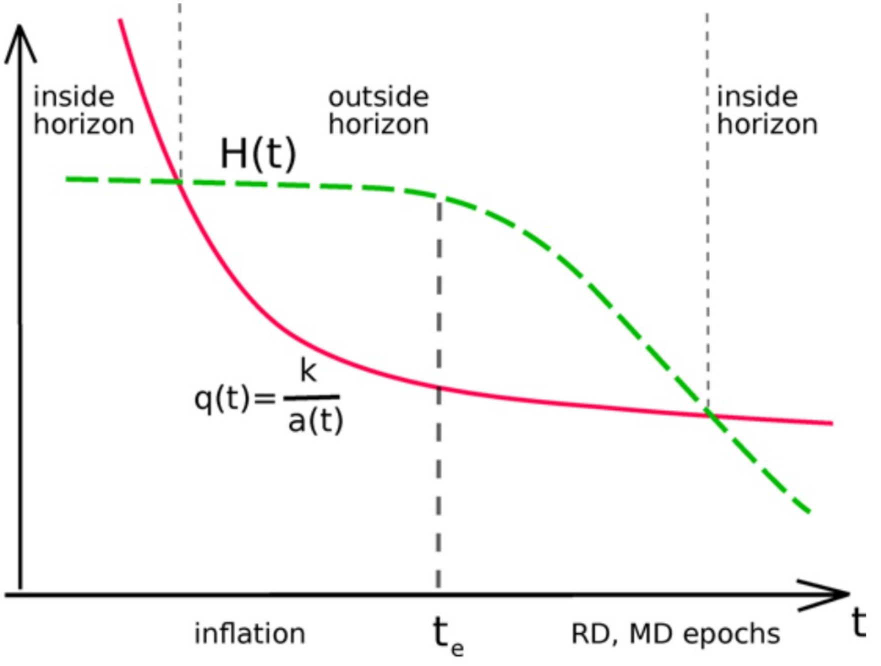
$$\chi(\mathbf{x},\eta) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}(\mathrm{e}^{ik\eta - i\mathbf{kx}}A^\dagger_{\mathbf{k}} + \mathrm{e}^{-ik\eta + i\mathbf{kx}}A_{\mathbf{k}}), \quad \eta \to -\infty,$$

$$\chi(\mathbf{x},\eta)_{\{k\}} = \int_{\{k\}} \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}(\mathrm{e}^{-i\mathbf{kx}}\chi_k^{(+)}(\eta)A^\dagger_{\mathbf{k}} + \mathrm{e}^{i\mathbf{kx}}\chi_k^{(-)}(\eta)A_{\mathbf{k}}),$$

$$\chi_k^{(+)} = \mathrm{e}^{ik\eta}\left(1 + \frac{i}{k\eta}\right) \qquad \chi_k^{(-)} = [\chi_k^{(+)}]^*$$

$$|\eta| \ll k^{-1},$$

$$\chi(\mathbf{x},\eta)_{\{k\}} = \int_{\{k\}} \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\left(-\frac{1}{k\eta}\right)(\mathrm{e}^{-i\mathbf{kx}+i\alpha_k}A^\dagger_{\mathbf{k}} + \mathrm{e}^{i\mathbf{kx}-i\alpha_k}A_{\mathbf{k}}).$$

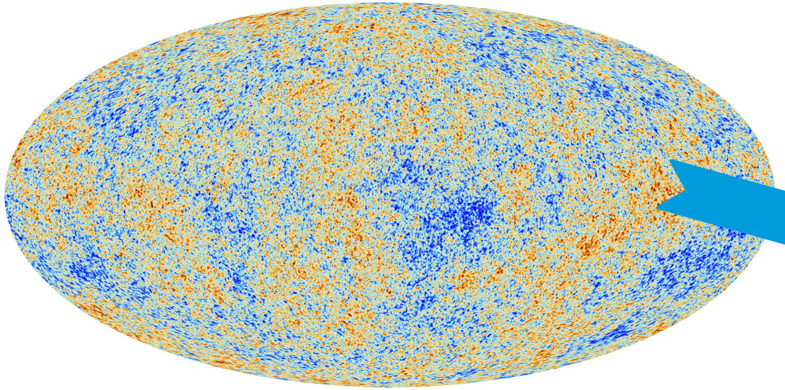


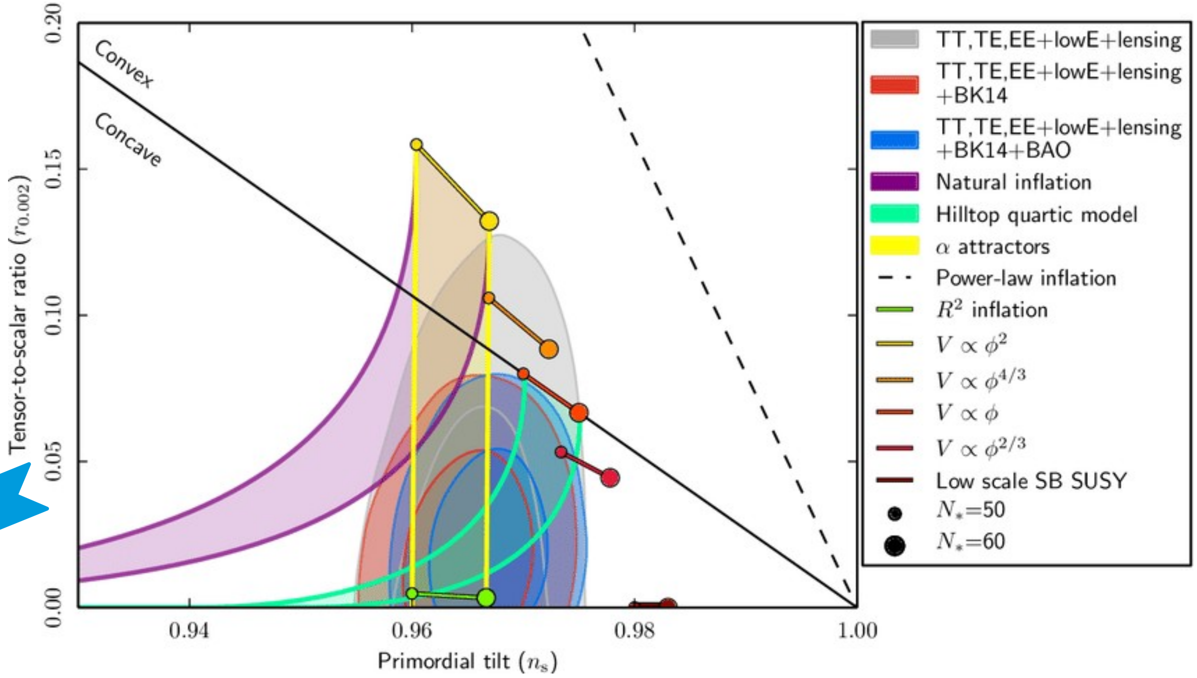
$$\varphi(\mathbf{x},\eta)_{\{k\}} = \int_{\{k\}} \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\frac{H}{k}(\mathrm{e}^{-i\mathbf{kx}+i\alpha_k}A^\dagger_{\mathbf{k}} + \mathrm{e}^{i\mathbf{kx}-i\alpha_k}A_{\mathbf{k}})$$

$$\langle\varphi(\mathbf{x})^2\rangle_{\{k\}} = \int_{\{k\}} \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3} = \int_{\{k\}} \frac{dk}{k}\frac{H^2}{(2\pi)^2}$$

Planck Constraints on the potential

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H^2}{2\pi\dot{\phi}_c}\right)^2_{\eta_k}$$



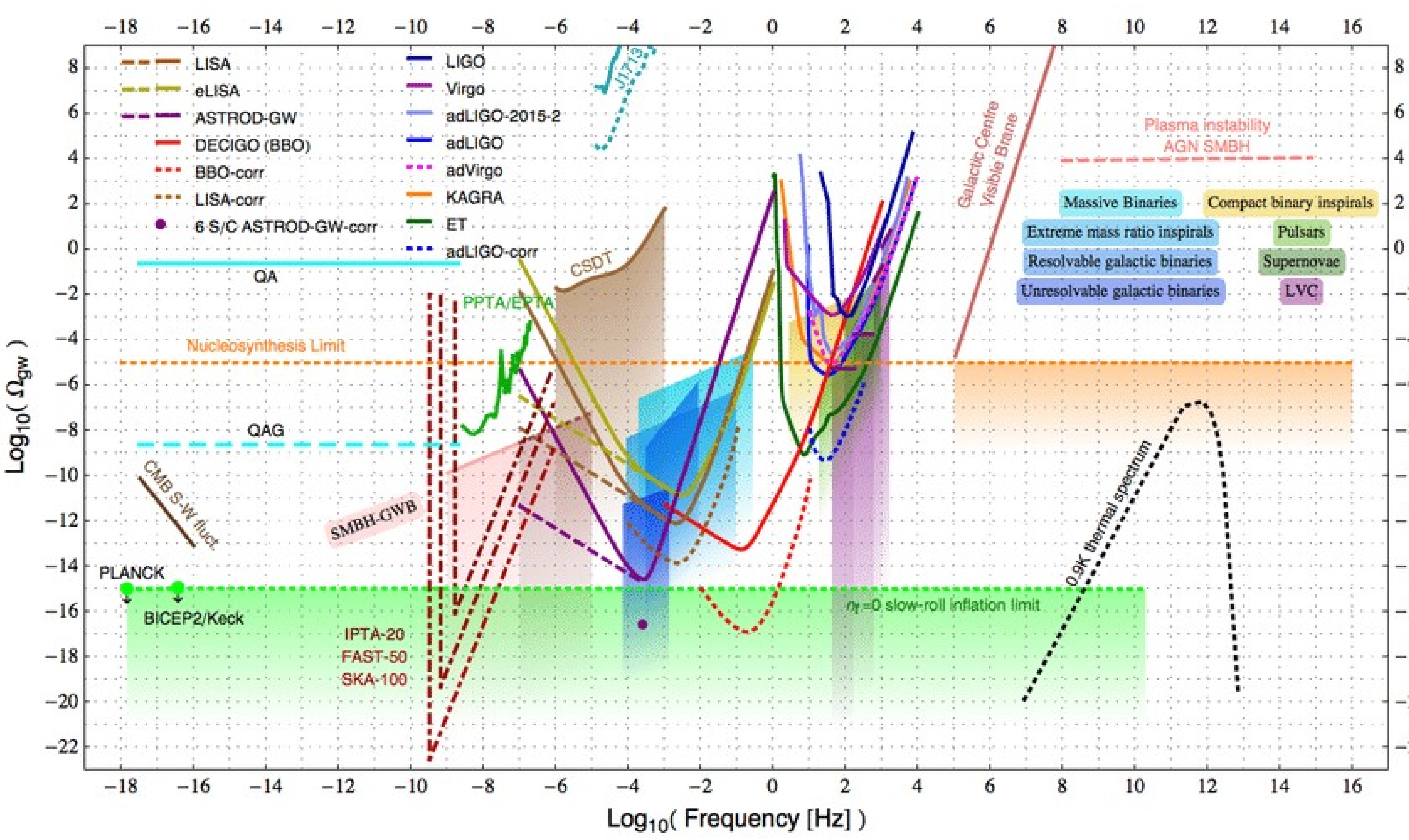


Serious QFT challenge – there is no renormalizable model left!

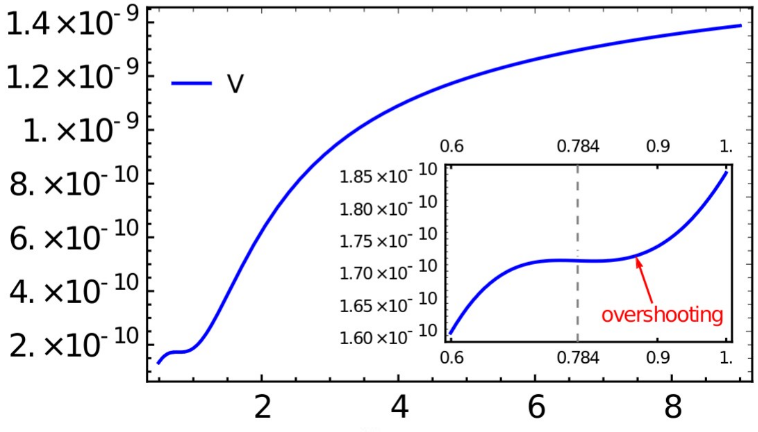
$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}}\left(\frac{k}{k_*}\right)^{n_s-1}$$

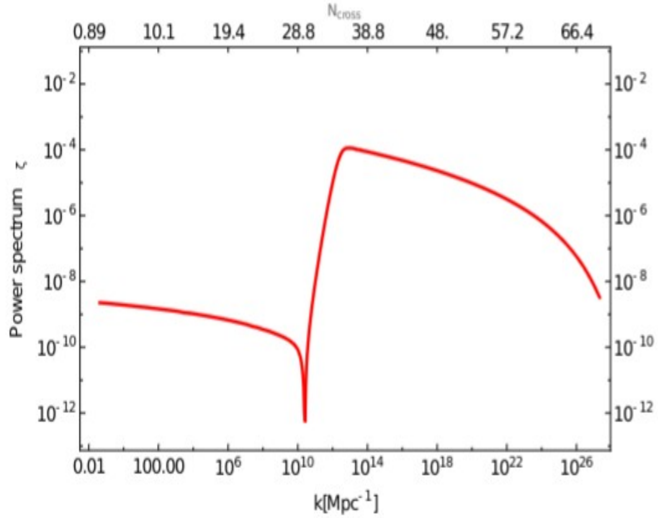
$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = \frac{1}{\pi}\frac{M_{Pl}^2 V'^2}{V} = 16\epsilon$$

$$n_s(k) - 1 = \frac{M_{Pl}^2}{4\pi}\left(\frac{V''}{V} - \frac{3}{2}\left(\frac{V'}{V}\right)^2\right)$$

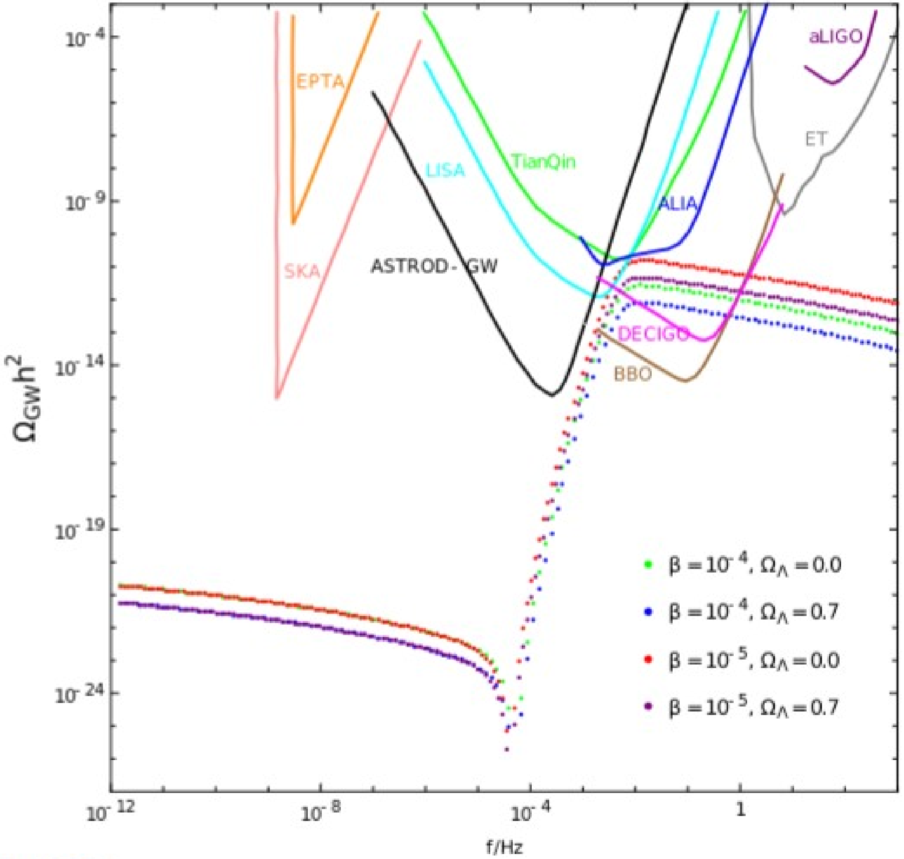

LISA
eLISA
ASTROD-GW
DECIGO (BBO)
BBO-corr
LISA-corr
6 S/C ASTROD-GW-corr
LIGO
Virgo
adLIGO-2015-2
adLIGO
adVirgo
KAGRA
ET
adLIGO-corr
QA
QAG
CSDT
PPTA/EPTA
Nucleosynthesis Limit
CMB S-W fluct
SMBH-GWB
PLANCK
BICEP2/Keck
IPTA-20
FAST-50
SKA-100
Galactic Centre
Visible Brane
Plasma instability
AGN SMBH
Massive Binaries
Compact binary inspirals
Extreme mass ratio inspirals
Pulsars
Resolvable galactic binaries
Supernovae
Unresolvable galactic binaries
LVC
$n_t = 0$ slow-roll inflation limit
0.9K thermal spectrum
$\mathrm{Log}_{10}(\Omega_{gw})$
Log_{10}(Frequency [Hz])

Gravitational waves from inflation





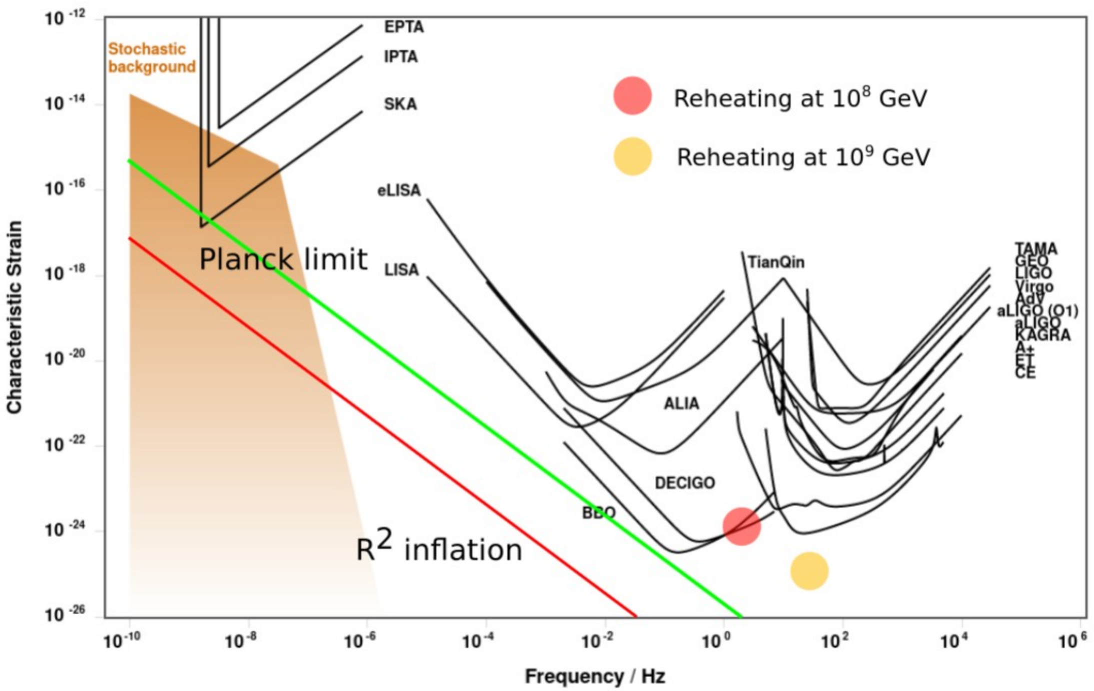
Overshooting the local minimum or slowing down of the inflaton rolling lead to the peak in scalar power spectrum and induced GW



M.Drees, Y.Xu, 1905.13581
Ezquiaga, Garcia-Bellido et al, 1705.04861

Gravitational waves from reheating

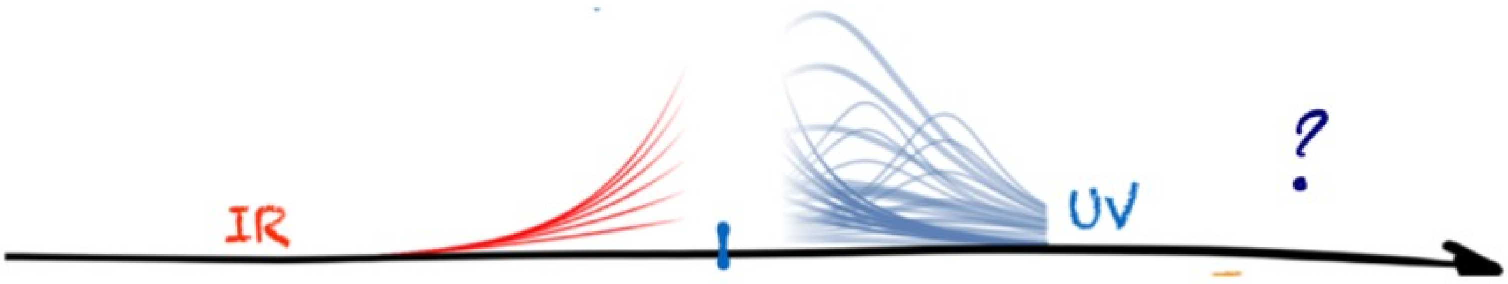
- Perturbative decay of inflaton to SM particles
- More complicated dynamics: parametric resonance, tachyonic instability… - require numerical simulations
- Inflaton clumping, structure formation – lead to inhomogeneous reheating → more GWs



K. Jedamzik, M. Lemoine and J. Martin, arXiv:1002.3278

D. Gorbunov, AT, arXiv:1212.4466

How to deal with non-renormalizable theories?



- We write all couplings in the Lagrangian which are compatible with the symmetries of low energy theory
- The Wilson coefficients are arbitrary and should be got from experiment
- This approach is working for energies below cutoff scale (minimal suppression scale of higher derivative operators)

EFT of inflaton and gravity

Expansion around the flat space:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

$$S_{NR} = \int d^4x\sqrt{-g}\left(\frac{\phi}{\Lambda_1}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + \frac{\phi}{\Lambda_2}R_{\mu\nu}R^{\mu\nu} + \frac{\phi}{\Lambda_3}R^2 + \frac{1}{\Lambda_4^2}G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi\right)$$

$$S_{int}^{SM} = \int d^4x\sqrt{-g}\left(-|D_\mu H|^2 + \mu\phi H^\dagger H + \frac{1}{\Lambda_5^2}G_{\mu\nu}D^\mu H^\dagger D^\nu H\right)$$

Leading contribution to graviton production after inflation?

EFT of inflaton and gravity

Expansion around the flat space:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

$$S_{NR} = \int d^4x\sqrt{-g}\left(\frac{\phi}{\Lambda_1}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + \frac{\phi}{\Lambda_2}R_{\mu\nu}R^{\mu\nu} + \frac{\phi}{\Lambda_3}R^2 + \frac{1}{\Lambda_4^2}G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi\right)$$

$$S_{int}^{SM} = \int d^4x\sqrt{-g}\left(-|D_\mu H|^2 + \mu\phi H^\dagger H + \frac{1}{\Lambda_5^2}G_{\mu\nu}D^\mu H^\dagger D^\nu H\right)$$

Decay to gravitons (the $\frac{\phi}{\Lambda_1}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$ term)

$$\Gamma = \frac{m^7}{32\pi M_p^4\Lambda_1^2}$$

reheating (the $\mu\phi H^\dagger H$ term)

bremsstrahlung (the $-|D_\mu H|^2$ and $\frac{1}{\Lambda_5^2}G_{\mu\nu}D^\mu H^\dagger D^\nu H$ terms)

Other operators are suppressed by higher powers of Λs

Results are valid for ANY UV completion for quantum gravity

Example: non-local UV completion to gravity

$$S = -\frac{M_P^2}{2}\int\sqrt{-g}d^4x\left(R - \frac{R^2}{6M^2}\right)$$

A. A. Starobinsky, Phys. Lett. B **91** (1980)

What could be the UV completion?

$+\beta W_{\mu\nu\lambda\rho}W^{\mu\nu\lambda\rho}$ – renormalizable Stelle gravity $\Rightarrow$ *ghost*

K. S. Stelle, Phys. Rev. D **16** (1977), 953-969

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{\lambda}{2}\left(RF(\Box)R + WF_W(\Box)W\right)\right)$$

E. T. Tomboulis, [arXiv:hep-th/9702146 [hep-th]].

Ghost-free if

$$F(\Box) = \frac{M_P^2}{6M^2\Box}\left[(\Box - M^2)e^{\sigma(\Box)} + M^2\right]$$

$$F_W(\Box) = M_P^2\frac{e^{\sigma(\Box)} - 1}{2\Box}$$

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

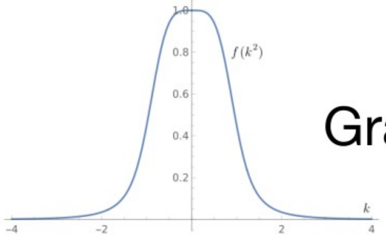
How does it work?

Examlple:

$$L = \frac{1}{2}\phi f(\Box)\phi - V(\phi)\,, \quad f(\Box) = (\Box - m^2)\,e^{\sigma(\Box)}$$

$\sigma(\Box)$ is an entire function, for example $\sigma(\Box) = \Box/\Lambda^2$

UV-finite theory for ANY $V(\phi)$!



Graviton propagator

Graviton production in non-local model

We prove that graviton production is determined only by the term $W\Box W$, irregardless of the presence of higher derivatives

$$L = A\, W_{\alpha\beta\gamma\delta}\,\Box W^{\alpha\beta\gamma\delta} + B\, R\, W_{\alpha\beta\gamma\delta} W^{\alpha\beta\gamma\delta}$$

$$\Gamma = \frac{6}{\pi}\frac{M^{11}}{M_P^6}(A+2B)^2 = \frac{3}{2\pi}\alpha_1^2\frac{M^3}{M_P^2}\left(\frac{M}{\Lambda}\right)^8, \ \alpha_1 \sim 1,$$

Λ – scale of non-locality

Izu.jpg

Connecting to the observables:

$$\Delta N_{eff} = 2.85\frac{\rho_{GW}}{\rho_{SM}} = 2.85\frac{\Gamma_{GW}}{\Gamma_H} = 821\alpha_1^2\frac{M^8}{\Lambda^8}$$

Planck: $\Delta N_{eff} < 0.2$

Bound on the non-locality scale: $\Lambda \gtrsim 3M$

Graviton production in non-local model

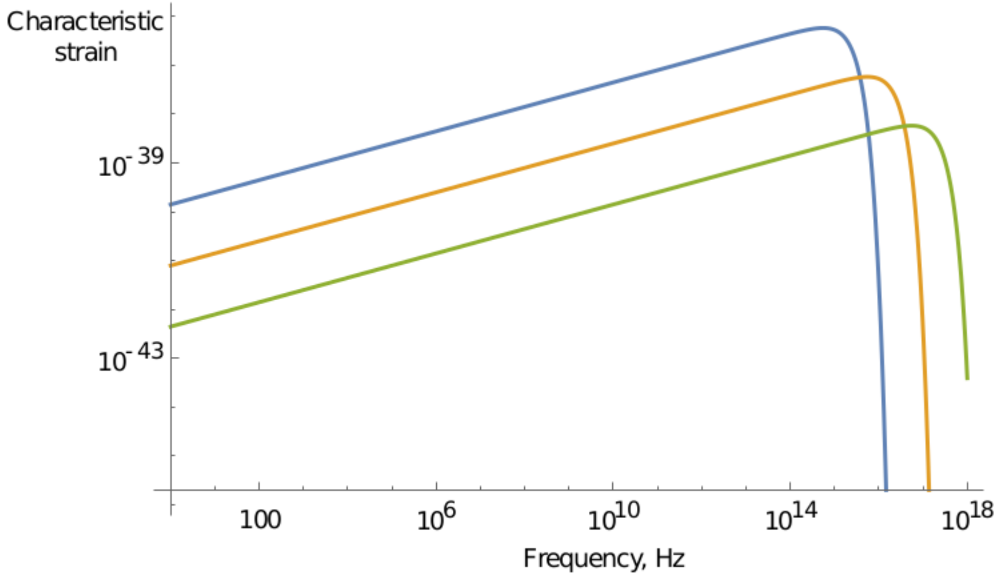


Figure 1: The blue curve shows the gravitational wave signal for $T_{reh} = 10^{10}$ GeV. The orange curve is for $T_{reh} = 10^9$ GeV and the green curve is plotted for $T_{reh} = 10^8$ GeV. In all cases we assume $\Gamma_{GW}/\Gamma_{SM} = 10^{-3}$. The low-frequency slopes of the plots correspond to the universal $h_c \propto f^{1/4}$ behaviour following from (21). The lowest characteristic strain available for future gravitational wave detectors is 10^{-24} for the frequencies $1 - 10^6$ Hz. One can see that the predicted signal is well below that level even for more intensive reheating and GW production.

Inflaton decay to gravitons: selected results

- Planck-suppressed operators **do matter** for low T_{reh} !

$$T_{reh} \lesssim 0.15\, g_{reh}^{1/4} \frac{m^{7/2}}{M_P^{3/2}\Lambda_1} \left(\frac{\Delta N_{eff}}{0.2}\right)^{-1/2}$$

Overproduction of dark radiation

$$m = 10^{13}\ \text{GeV} \quad T_{reh}^{min} = 1\ \text{GeV}$$

$$m = 10^{16}\ \text{GeV} \quad T_{reh}^{min} = 10^{10}\ \text{GeV}$$

$$\Delta N_{eff} = 2.85 \frac{\rho_{GW}}{\rho_{SM}} = 2.85 \frac{\Gamma_{GW}}{\Gamma_H}$$

$$\Delta N_{eff} \lesssim 0.2$$

$$T_{reh} = 0.3\, g_{reh}^{1/4} \sqrt{\Gamma_{SM} M_P}.$$

$$\Gamma_{GW} = \frac{m^7}{64\pi M_p^4 \Lambda_1^2}$$

$$\Gamma_{SM} = \frac{\mu^2}{8\pi M}.$$

$$S_{NR} = \int d^4x \sqrt{-g} \left(\frac{\phi}{\Lambda_1} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}\right)$$

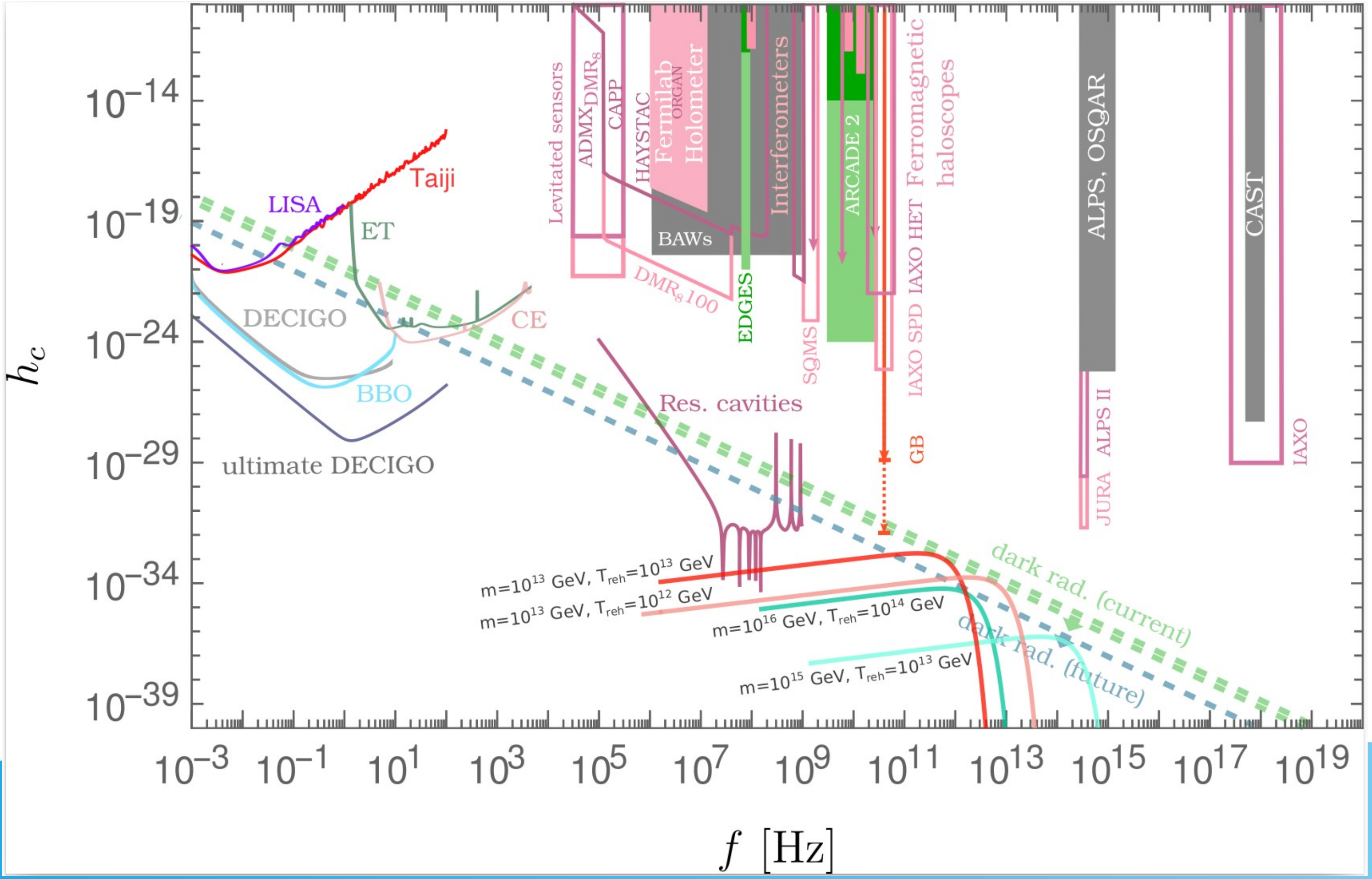
Gravitational waves from inflaton decay

$$\frac{d\Omega_{GW}}{d\log E} = \frac{16E^4}{M^4}\frac{\rho_{reh}}{\rho_0}\frac{\Gamma_{GW}}{H_{reh}}\frac{1}{\gamma(E)}e^{-\gamma(E)}$$

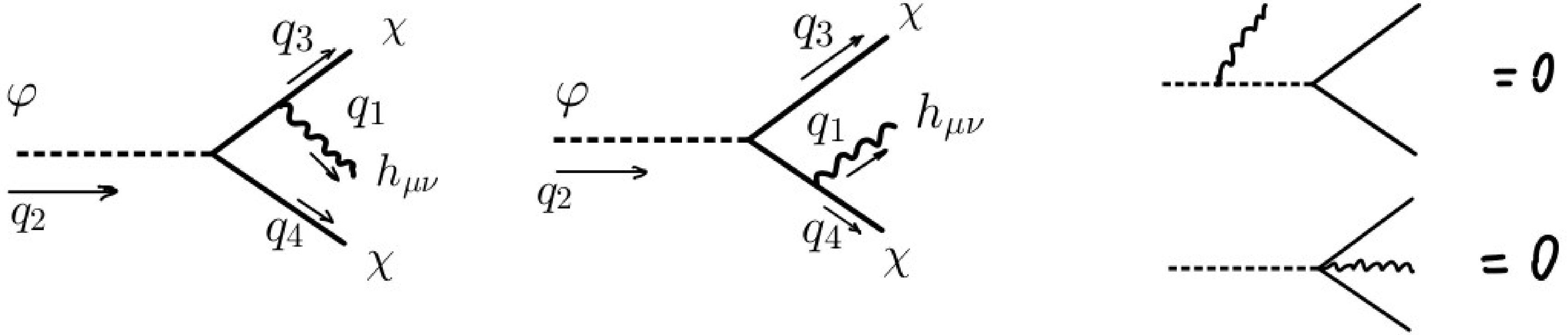
$$\gamma(E) = \left(\left(\frac{g_{reh}}{g_0}\right)^{1/3}\frac{T_{reh}}{T_0}\frac{2E}{M}\right)^{3/2}$$

A. Koshelev, A. Starobinsky, AT, PLB, arXiv:2211.02070

$$h_c(f) = \sqrt{\frac{3H_0^2}{\pi f^2}\frac{d\Omega_{GW}}{df}}$$



Graviton bremsstrahlung during reheating



Not sensitive to inflaton-graviton coupling

$$G(k)=\frac{\partial\Gamma}{\partial k}=A\frac{(m-2k)^2}{m\,k},\quad A=\frac{1}{64\pi^3}\frac{\mu^2}{3M_p^2}$$

$$G(k)=\frac{\partial\Gamma}{\partial k}=A\frac{(m-2k)^2}{m\,k}+B_{UV}(k)\qquad A=\frac{1}{64\pi^3}\frac{\mu^2}{2M_p^2}\left(\frac{m^2}{\Lambda_5^2}+1\right)^2$$

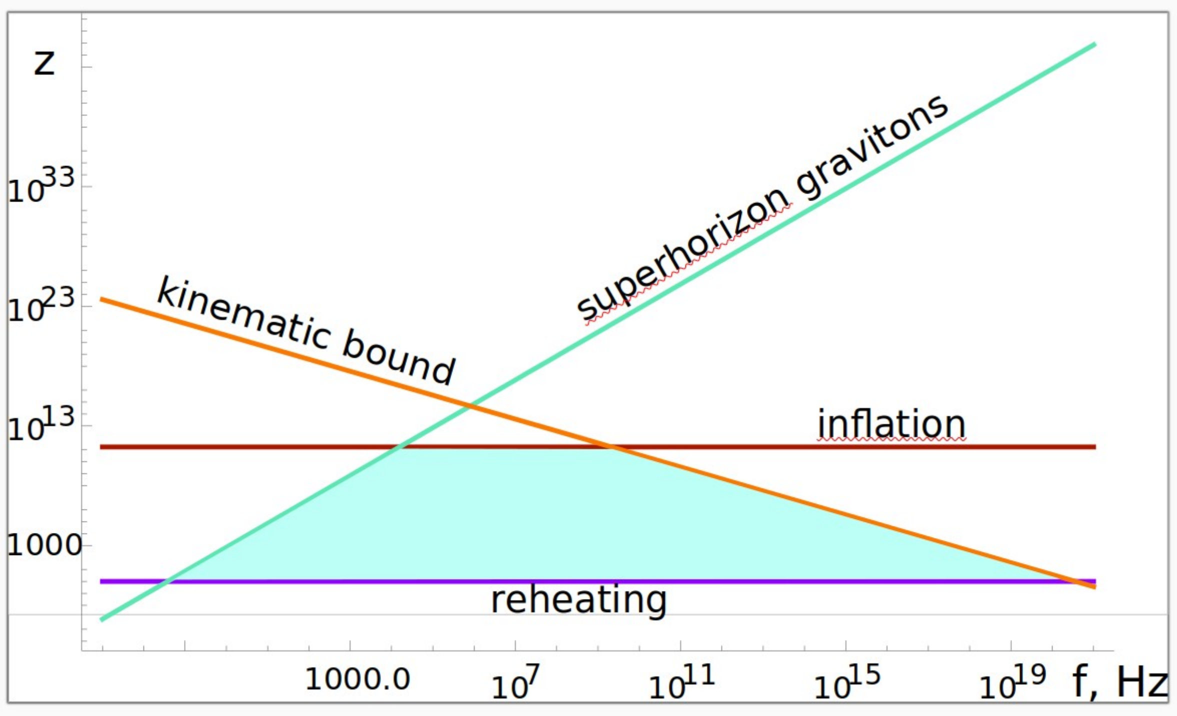
$$B_{UV}(k)=\frac{1}{64\pi^3}\frac{\mu^2}{2M_p^2}\frac{2(m-2k)^2}{15\Lambda_5^2}\left(\frac{m\,(7k-10m)}{\Lambda_5^2}-10\right)$$

$$\frac{d\rho_{GW}}{dk}=\int\frac{kdN}{a_0^3}=\int dt\frac{kn_\phi(t)a(t)^3}{a_0^3}G(k\frac{a_0}{a(t)})$$

$$n_\phi=\frac{\rho_{reh}}{M}\left(\frac{a_{reh}}{a}\right)^3e^{-\Gamma_{tot}t}$$

Limits on GW frequencies

$$\frac{d\Omega_{GW}}{d\log k} = \frac{k^2}{M\, H_{reh}} \frac{a_{reh}^2}{a_0^2} \frac{\rho_{reh}}{\rho_0} \int_{z_{min}}^{z_{max}} dz\, G\!\left(kz \frac{a_0}{a_{reh}}\right) z^{-3/2} e^{-2z^{-3/2}/3}$$

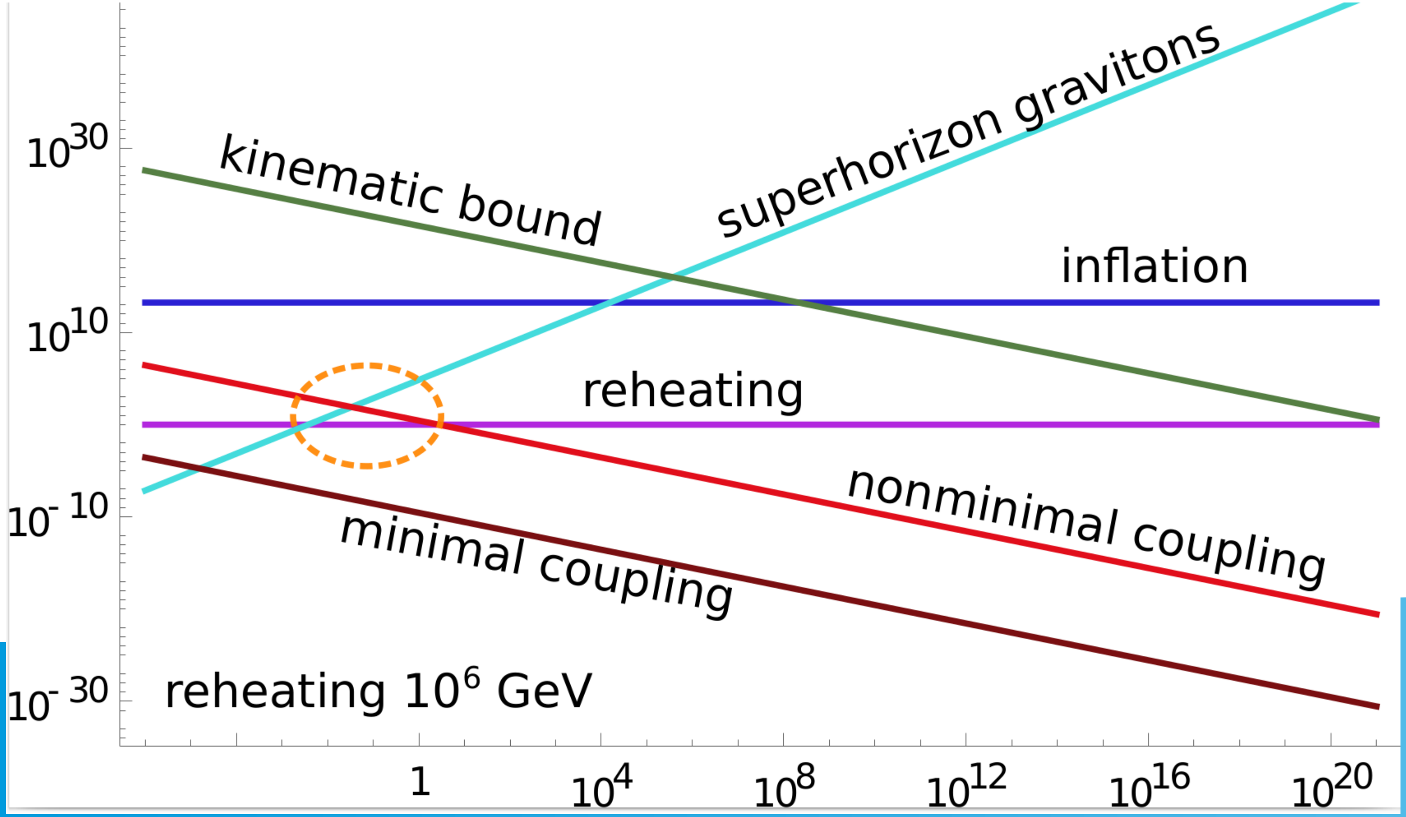


- Kinematic bound – comoving momentum is less than m/2
- Causality requirement - no superhorizon gravitons!
- Gravitons were emitted between inflation and reheating

More limitations: IR singularity

$$G(k) = \frac{\partial \Gamma}{\partial k} = A\frac{(m-2k)^2}{m\,k}$$

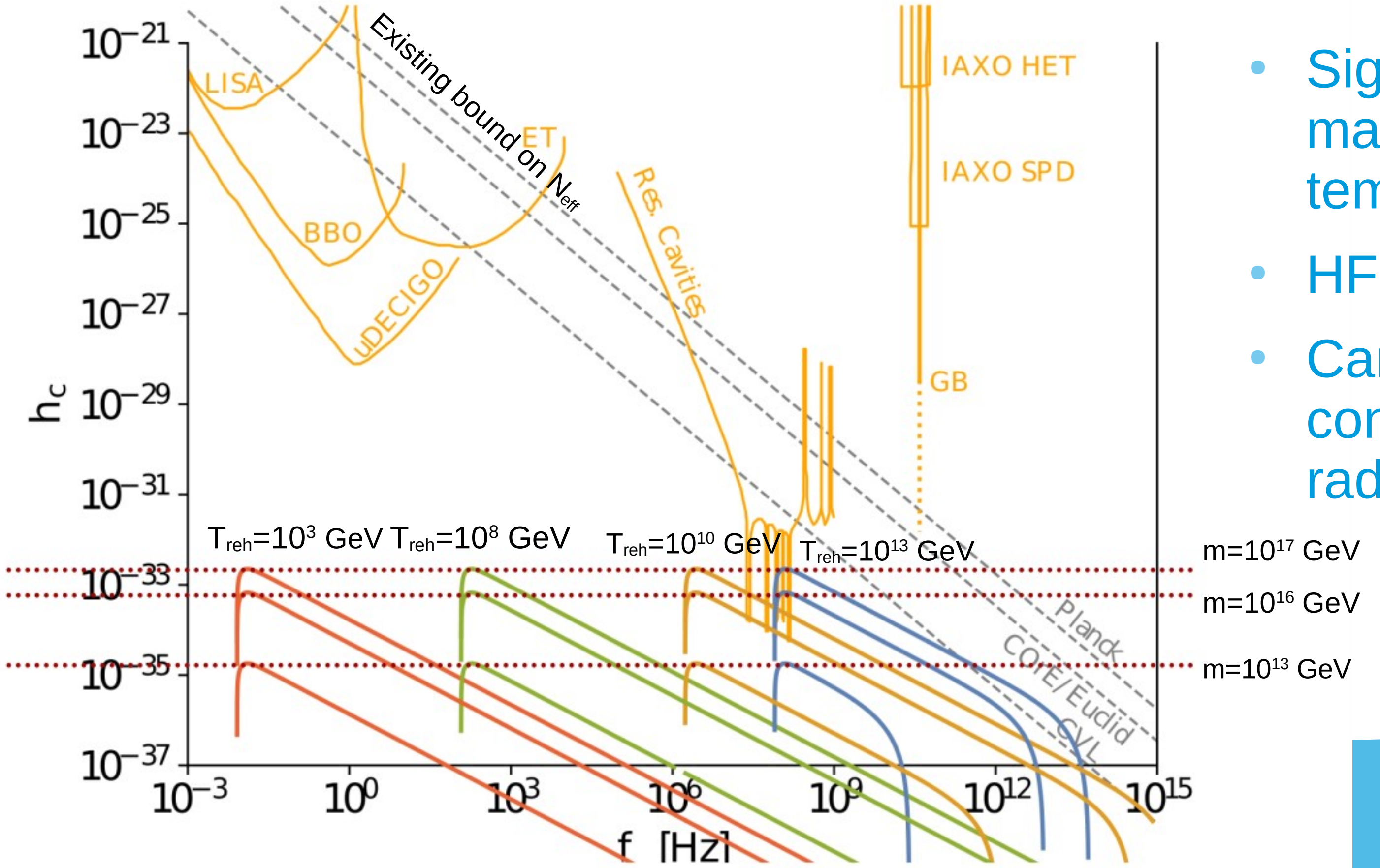
The computation is incorrect when $G(k) \sim 1$



- Resummation of the result including soft graviton emission may be required

- The GW spectrum cannot be pushed to high values in a consistent weakly coupled EFT

Gravitational waves from bremsstrahlung: $\Lambda_5 = M_P$



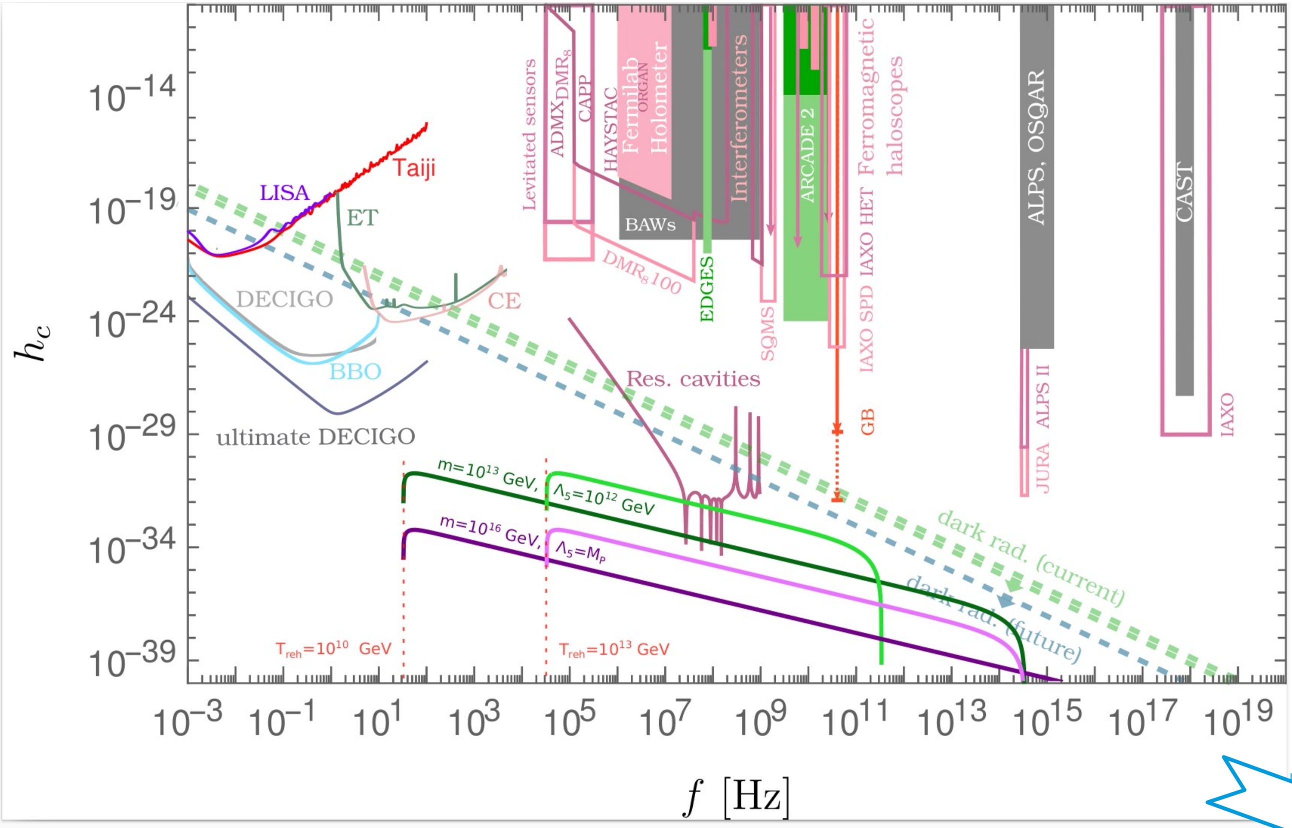
Detection prospects from Barman, Bernal, Xu, Zapata, 2301.11325

- Signals for different inflaton masses and reheating temperatures
- HF GW domain
- Can be also probed as contribution to the dark radiation

$$h_c(f) = \sqrt{\frac{3H_0^2}{\pi f^2}\frac{d\Omega_{GW}}{df}}$$

Results coincide with 2301.11325, except the IR cutoff

What if the quantum gravity scale is lower?



- GW signals for inflaton mass $m=10^{13}$ GeV
- The shape does not change, the amplitude is becoming higher
- The unitarity breaking scale is $\Lambda_{UV}=(\Lambda_5\ M_P)^{1/2} > m$
- From $\Lambda_{UV}=10^{15}$ GeV – tension with N_{eff} bound

Reheating-dependend bounds on quantum gravity scale!

Conclusions

- High frequency gravitational waves can be sensitive to the quantum gravity effects
- Perturbative decay of inflation to gravitons can be non-negligible for low reheating temperatures $\rightarrow$ high frequency GWs
- Graviton bremsstrahlung during reheating can provide a sizable HF GW signal $\rightarrow$ constraints on EFT
- Reheating-dependent constraints on quantum gravity scale from gravitational waves !

Thank you!