



Distinguishing Thermal Histories of Dark Matter from Structure Formation

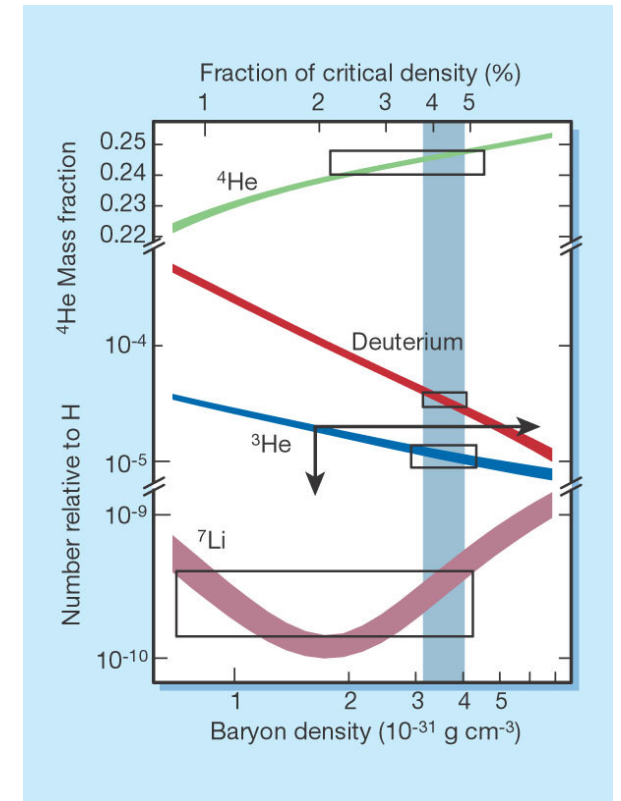
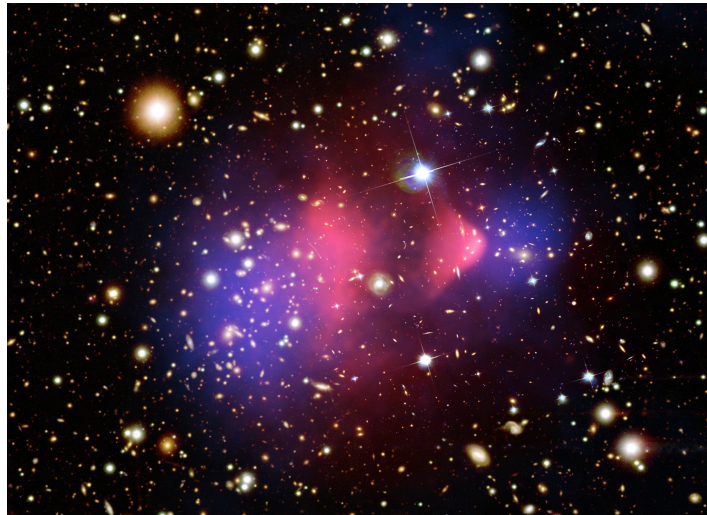
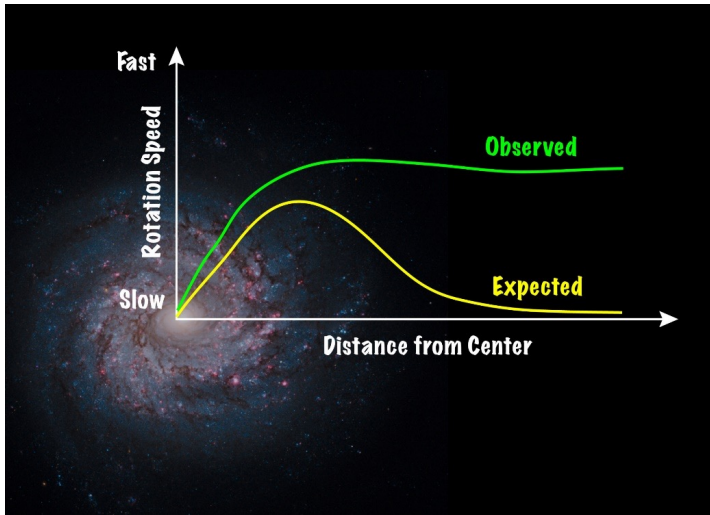
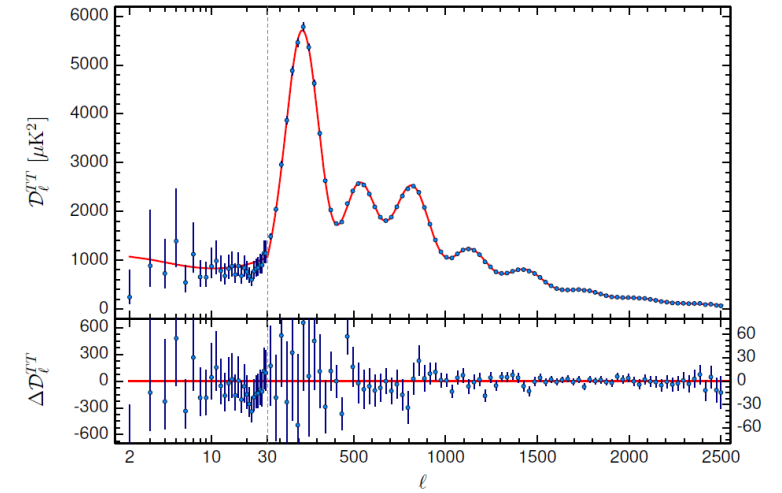
JCAP 01 (2024) 023

Fei Huang

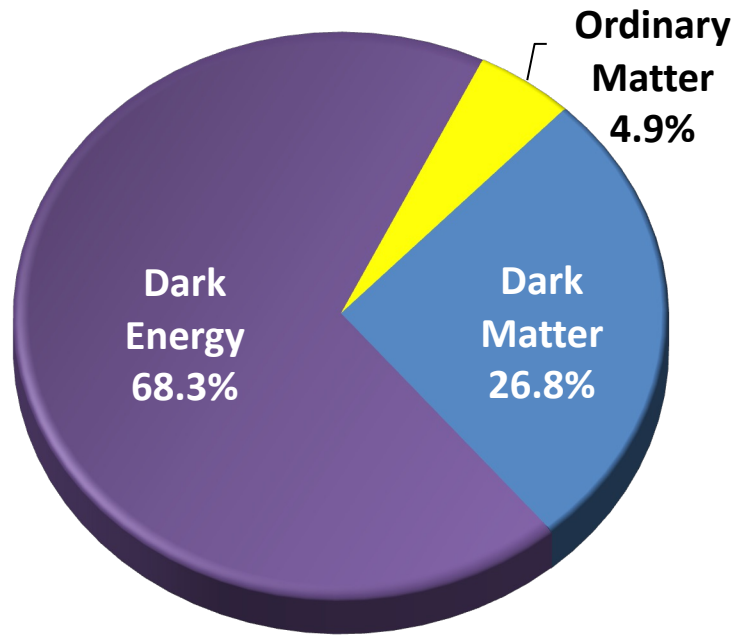
with Yuan-Zhen Li and Jiang-Hao Yu

May 13, 2024

The existence of dark matter is supported by observations across many different scales



What have we learnt?



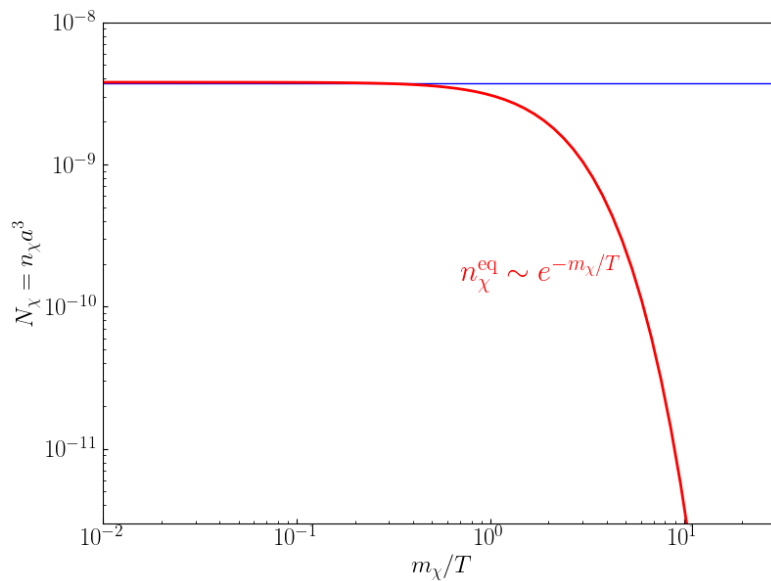
We still don't know:

- **Abundance**: $\sim 26\%$ of the universe
- **Cold**: Non-relativistic, massive
- **Dark**: Negligible nongravitational interaction with Standard Model fields
- **Nonbaryonic**: Baryonic matter is simply not enough
- **BSM**: Not Standard Model particle

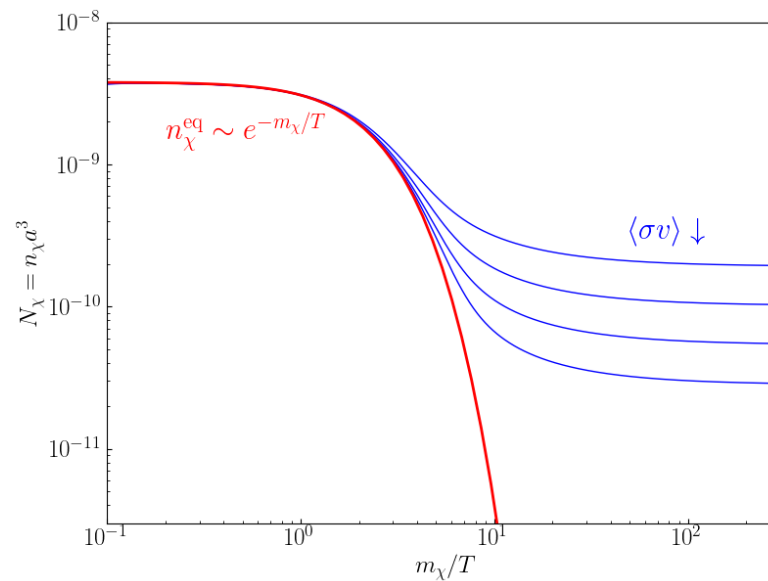
- **Particle properties**: mass, spin, fundamental or composite, single-component or multi-component, etc.
- **Production mechanism**: freeze-out, freeze-in, decays, misalignment?

DM Production from a Thermal Bath

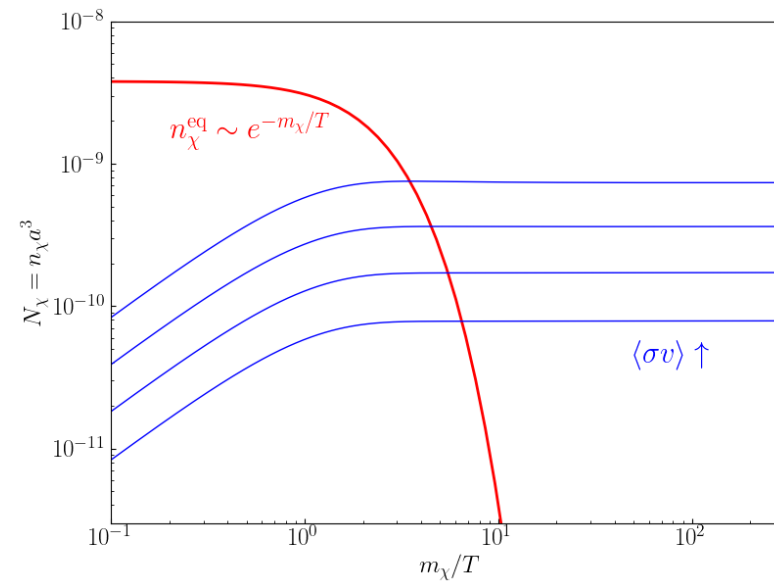
Relativistic freeze-out “Warm Dark Matter”



Non-Relativistic Freeze-Out



Freeze-In



Phase-Space Distribution

For any mechanism, there is a DM distribution function

$$f(p, t)$$

After DM production is complete, the evolution of $f(p, t)$ is only subject to cosmological redshift:

$$p \sim a^{-1}$$

For **freeze-out**, DM was in thermal equilibrium.

At decoupling:

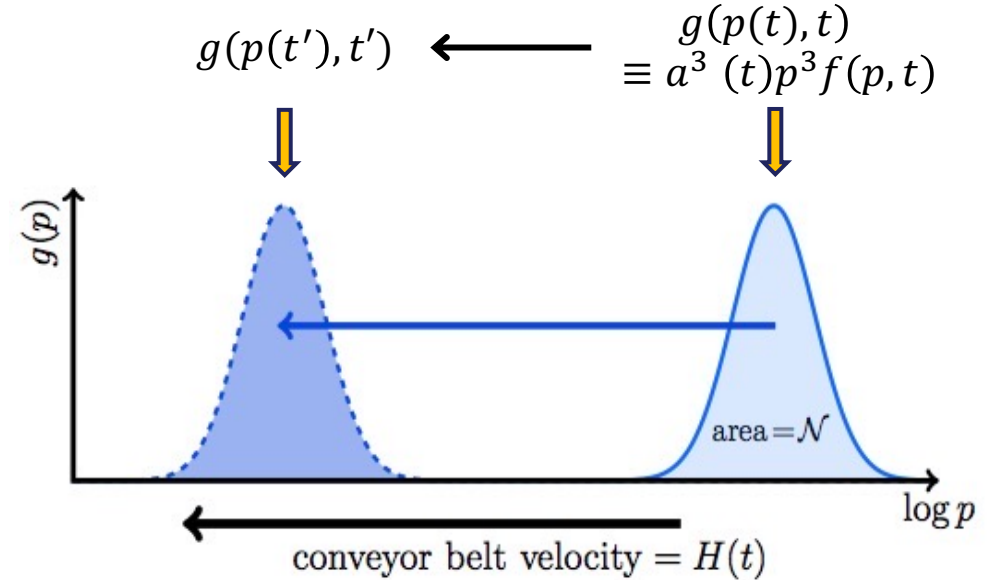
$$f_{\chi}(p, t_{dec}) = \left[\exp \left(\frac{E - \mu_{dec}}{T_{\chi}(t_{dec})} \right) \pm 1 \right]^{-1}$$

Relativistic freeze-out: $T_{\chi}(t_{dec}) \gg m_{\chi}$, $E \sim p$

At later times

$$f_{\chi}(p, t) \approx \left[\exp \left(-\frac{p - \mu}{T_{\chi}} \right) \pm 1 \right]^{-1}$$

$$T_{\chi}(t) = T_{\chi}(t_{dec}) \frac{a_{dec}}{a(t)}, \quad \mu(t) = \mu_{dec} \frac{a_{dec}}{a(t)}$$



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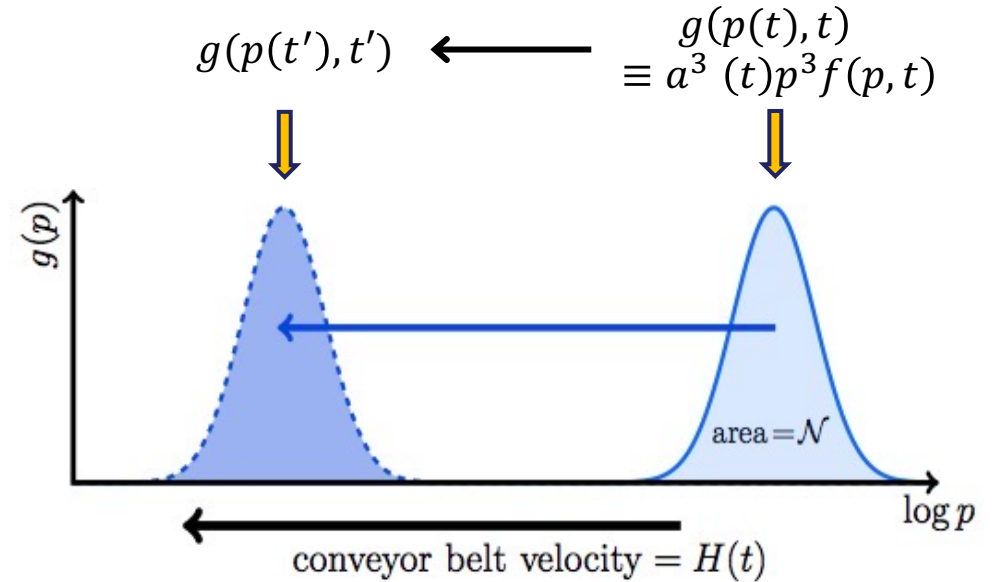
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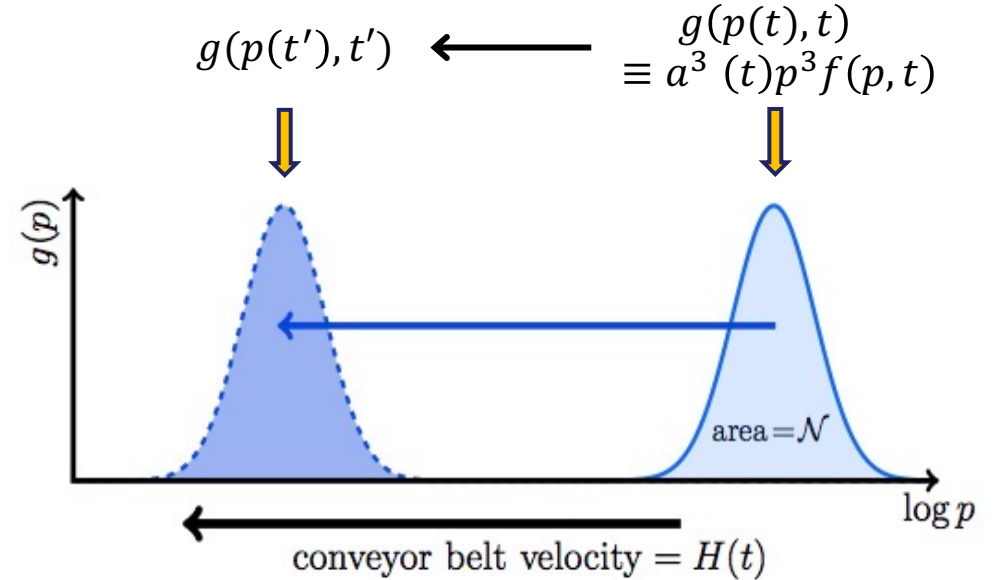
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Non-Relativistic freeze-out: $E_{dec} \sim m_{\chi} + \frac{p_{dec}^2}{2m_{\chi}}$

At later times

$$f_{\chi}(p, t) \approx \left[\exp \left(- \frac{m_{\chi} + p^2/(2m_{\chi}) - \mu}{T_{\chi}} \right) \pm 1 \right]^{-1}$$

$$T_{\chi}(t) = T_{\chi}(t_{dec}) \frac{a_{dec}^2}{a^2(t)}, \quad \mu(t) = m_{\chi} - (m_{\chi} - \mu_{dec}) \frac{a_{dec}^2}{a^2(t)}$$



Phase-Space Distribution

For any mechanism, there is a DM distribution function

$$f(p, t)$$

After DM production is complete, the evolution of $f(p, t)$ is only subject to cosmological redshift:

$$p \sim a^{-1}$$

For **freeze-in**, DM was never in equilibrium

$$f_\chi(p, t_{dec}) = C \frac{\exp(-p/M)}{\sqrt{p/M}}$$

F. D'Eramo & A. Lenoci
JCAP 10 (2021) 045

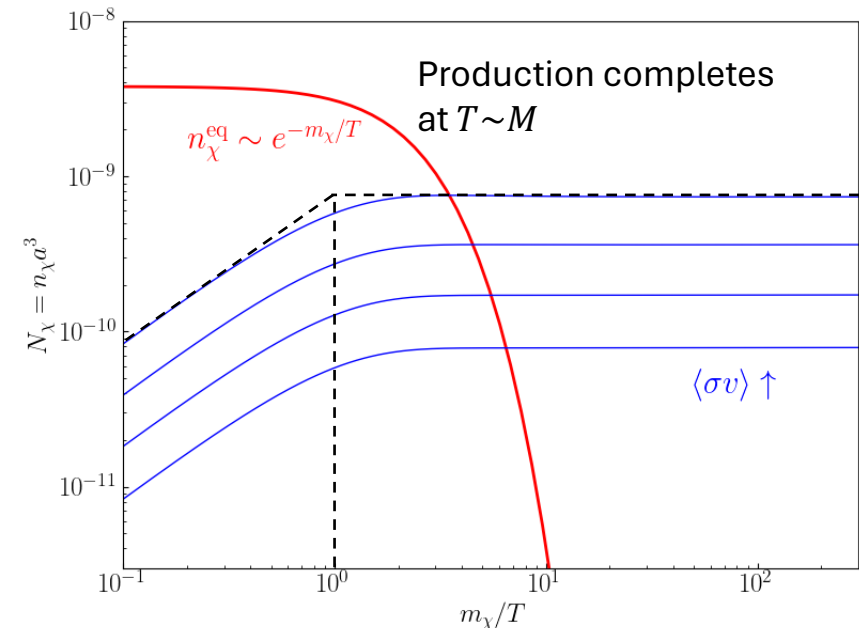
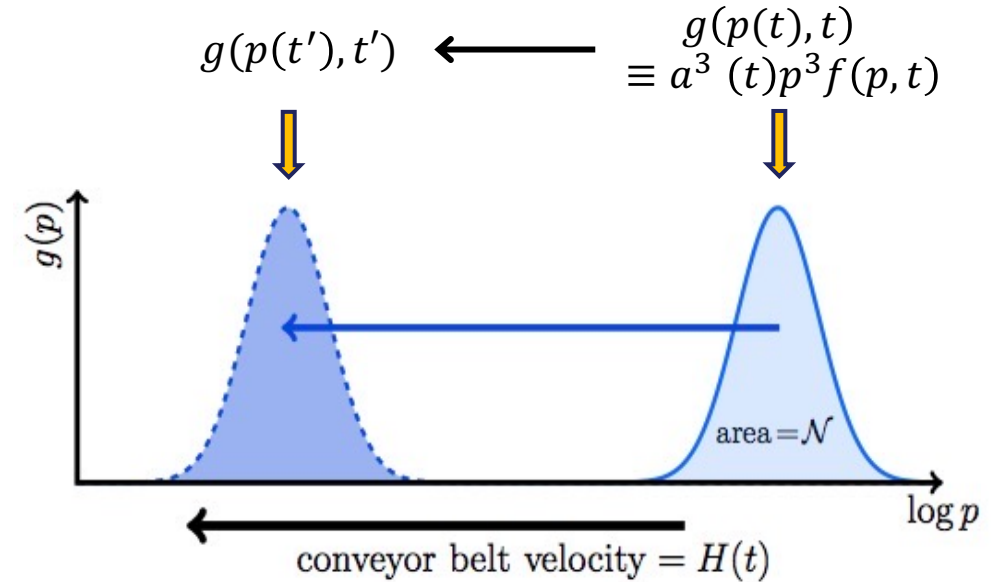
suitable for $2 \rightarrow 2$ processes, e.g.,

$$\psi + \psi \rightarrow \chi + \chi$$

Define an **effective** temperature:

$$T_\chi(t) = M \frac{a_{dec}}{a(t)}$$

$$f_\chi(p, t) = C \frac{\exp(-p/T_\chi)}{\sqrt{p/T_\chi}}$$



A little exercise with WDM

$$f_{\chi}(p) \approx \exp(-p/T_{\chi})$$

*relativistic form, even
after DM becomes non-rel*

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Energy density :

$$\rho_{\chi} \sim \int d^3p \, E \, f_{\chi}(p) \sim m_{\chi} T_{\chi}^3$$

Average velocity:

$$\langle v \rangle \sim \frac{1}{m_{\chi}} \int d^3p \, |\vec{p}| \, f_{\chi}(p) \sim \frac{T_{\chi}}{m_{\chi}}$$

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$$\langle v \rangle_0 \approx 1.1 \times 10^{-7} \times \left(\frac{2}{g_\chi} \right)^{1/3} \left(\frac{\Omega_\chi}{0.25} \right)^{1/3} \left(\frac{1 \text{ keV}}{m_\chi} \right)^{4/3}$$

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*1/a
+
entropy conservation
 $a^3 T^3 g_{*,s}(T) = \text{const.}$*

$$\begin{aligned} \langle v \rangle_0 &\approx 1.1 \times 10^{-7} \times \left(\frac{2}{g_\chi} \right)^{1/3} \left(\frac{\Omega_\chi}{0.25} \right)^{1/3} \left(\frac{1 \text{ keV}}{m_\chi} \right)^{4/3} \\ &\approx 6.5 \times 10^{-7} \times \frac{1 \text{ keV}}{m_\chi} \times \left(\frac{5}{g_{*,s}(T_{\text{dec}})} \right)^{1/3} \frac{T_\chi(t_{\text{dec}})}{T_{\text{dec}}} \end{aligned}$$

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*physical quantities
at production time*

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*physical quantities
at production time*

e.g., Lyman- α constraint

*DM velocity is constrained
by structure formation!*

$$\begin{aligned} m_{\text{WDM}} &\geq 3.5 \text{ keV} \\ &\geq 5.3 \text{ keV} \end{aligned}$$

V. Iršič et al.
Phys. Rev. D 96 (2017) 023522

Constraints on decoupling temperatures

$$\begin{aligned} m_{\text{WDM}} &\geq 3.5 \text{ keV} \\ &\geq 5.3 \text{ keV} \end{aligned}$$



$$\begin{aligned} \langle v \rangle_0 &\lesssim 2.1 \times 10^{-8} \\ &\lesssim 1.2 \times 10^{-8} \end{aligned}$$

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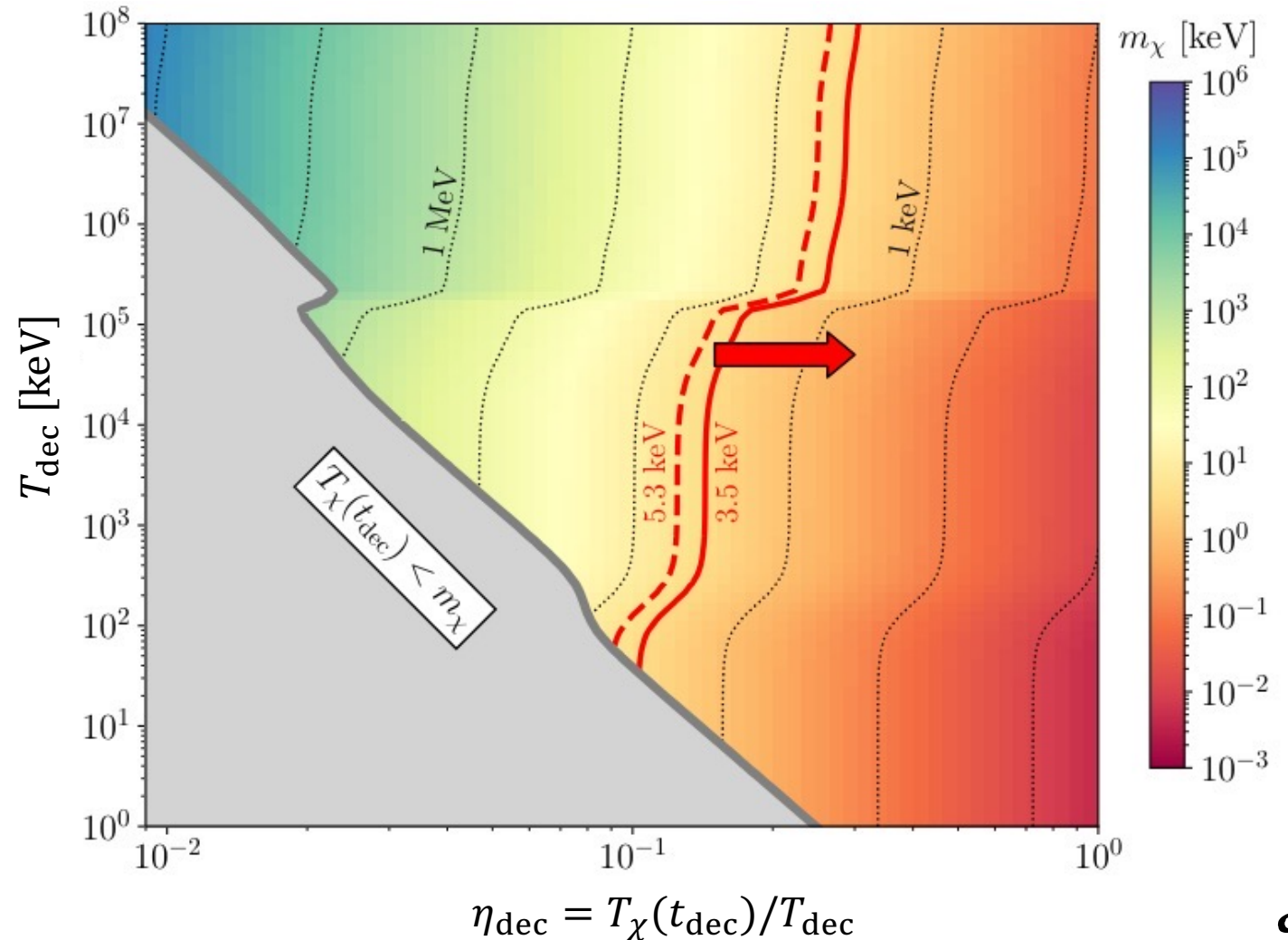


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- ❖ Mass contours = velocity contours
- ❖ $\eta_{\text{dec}} < 1 \rightarrow$ WDM cannot freeze out from SM thermal bath
- ❖ Freeze-out of WDM occurs in a colder DS thermal bath

Relativistic Freeze-Out

$$m_\chi \approx 1.9 \times 10^{-3} \text{ keV} \times \frac{\Omega_\chi}{0.25} \frac{g_{*,s}(T_{\text{dec}})}{g_\chi} \eta_{\text{dec}}^{-3}$$



Constraints on decoupling temperatures

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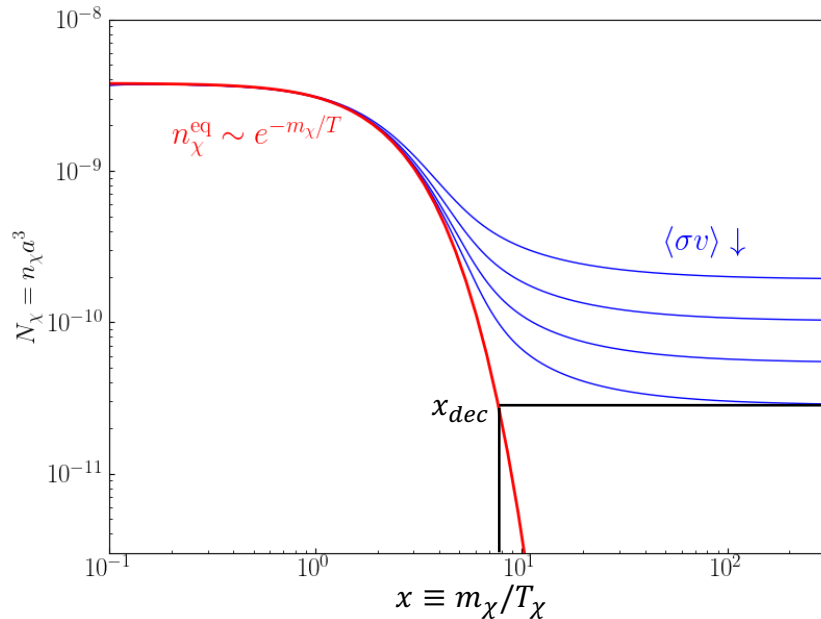
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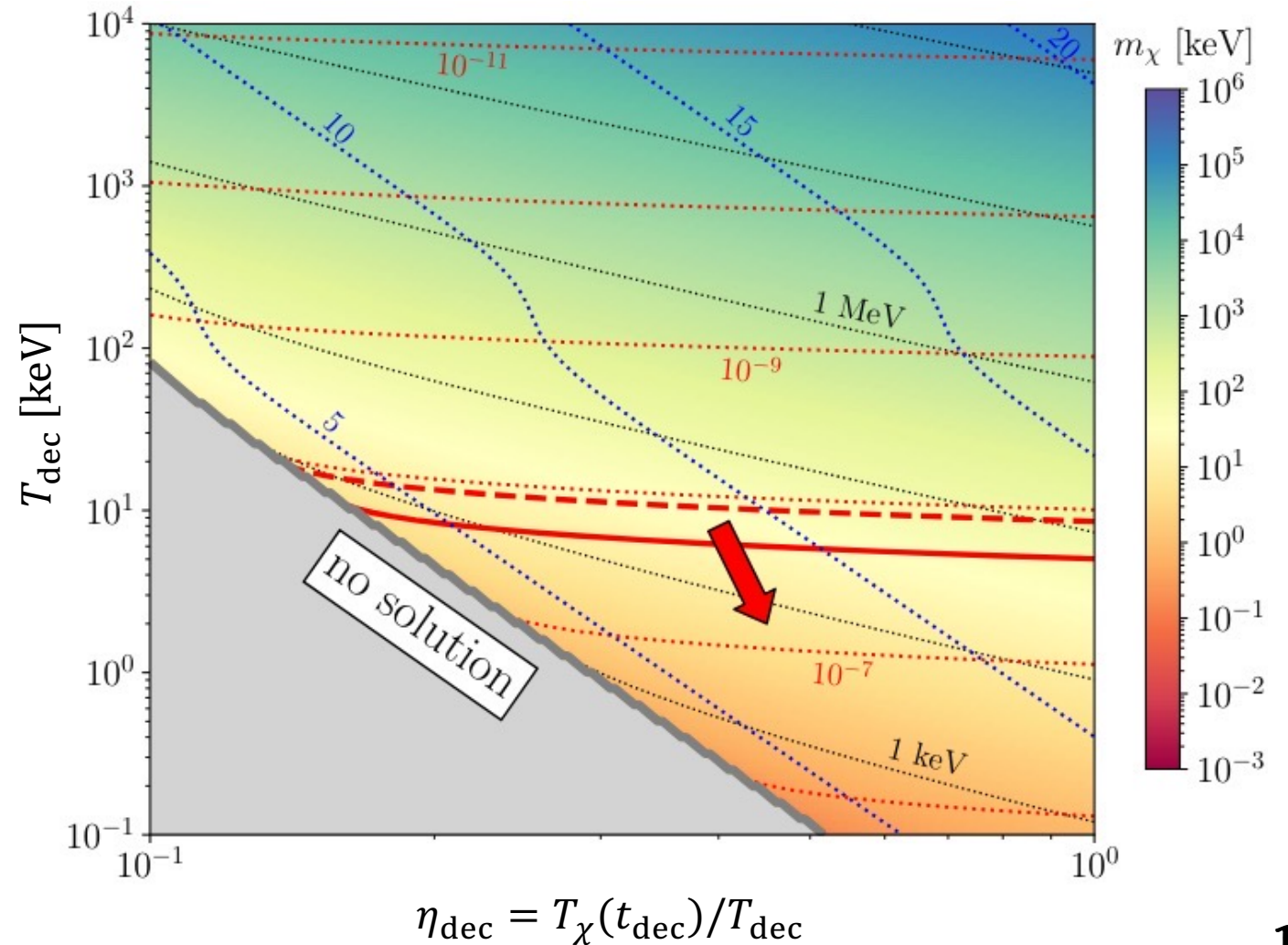
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- ❖ Additional dependence on x_{dec}
- ❖ $\eta_{\text{dec}} = 1$ allowed



Non-Relativistic Freeze-Out

$$m_\chi \approx 2.1 \times \text{keV} \times \frac{\Omega_\chi}{0.25} \frac{g_{*,s}(T_{\text{dec}})}{g_\chi} \eta_{\text{dec}}^{-3} \left(\frac{10}{x_{\text{dec}}} \right)^{3/2} \frac{e^{x_{\text{dec}}}}{e^{10}}$$



Constraints on decoupling temperatures

$$m_{\text{WDM}} \geq 3.5 \text{ keV}$$

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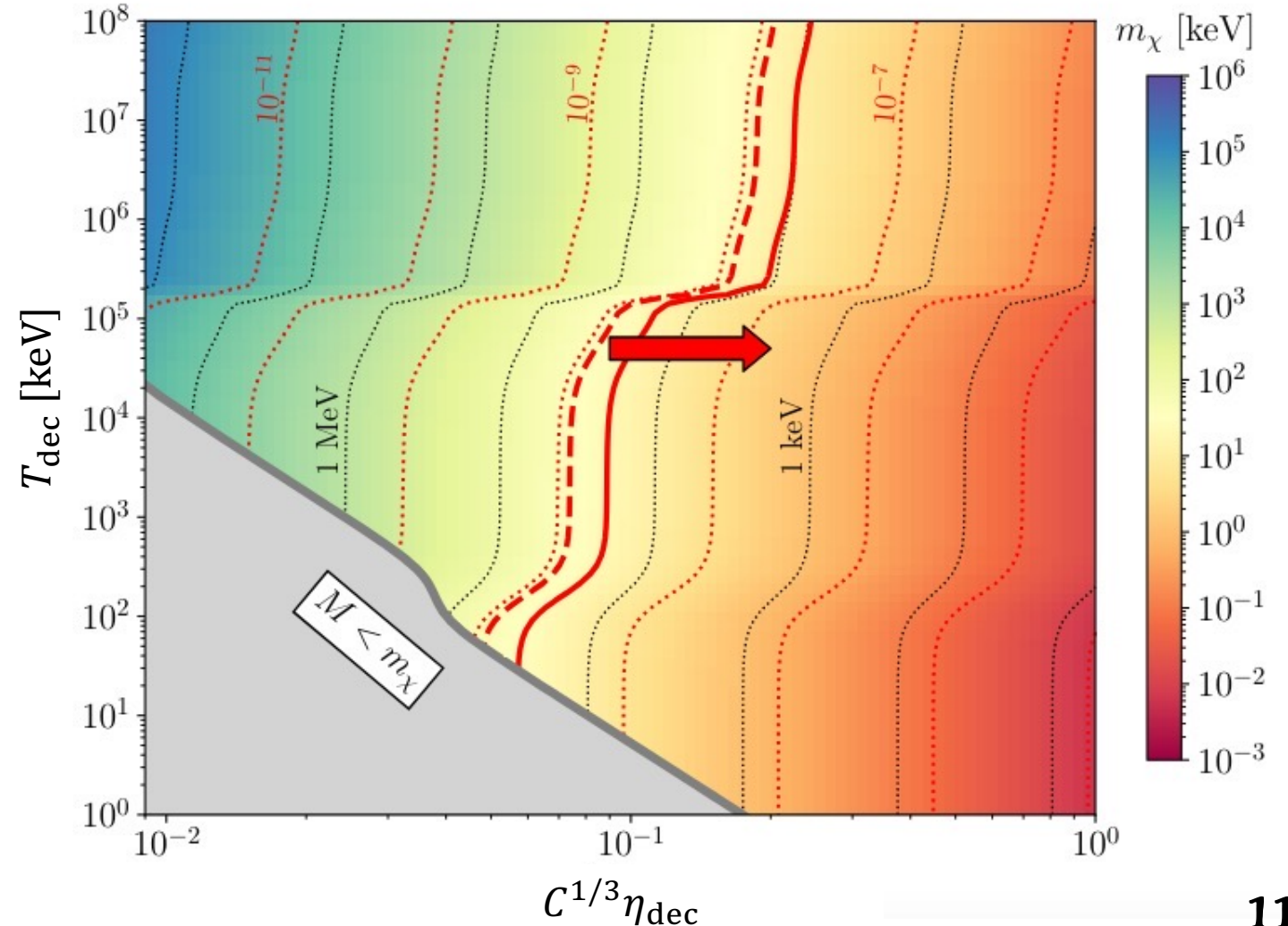
$$\langle v \rangle_0 \lesssim 2.1 \times 10^{-8}$$

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- ❖ Additional degeneracy from C
 - $f_\chi(p, t) = C \frac{\exp(-p/T_\chi)}{\sqrt{p/T_\chi}}$
 - Mass and velocity contours identical only if C fixed
- ❖ $\eta_{\text{dec}} = 1$ also allowed

Freeze-In

$$m_\chi \approx 2.7 \times 10^{-3} \text{ keV} \times \frac{\Omega_\chi}{0.25} \frac{g_{*,s}(T_{\text{dec}})}{g_\chi} C^{-1} \eta_{\text{dec}}^{-3}$$



An optimistic possibility

What if

- deviation from CDM is observed in future observations
- consistent with free-streaming effects from non-negligible primordial velocity of DM

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- A lower bound on DM velocities

*The allowed region
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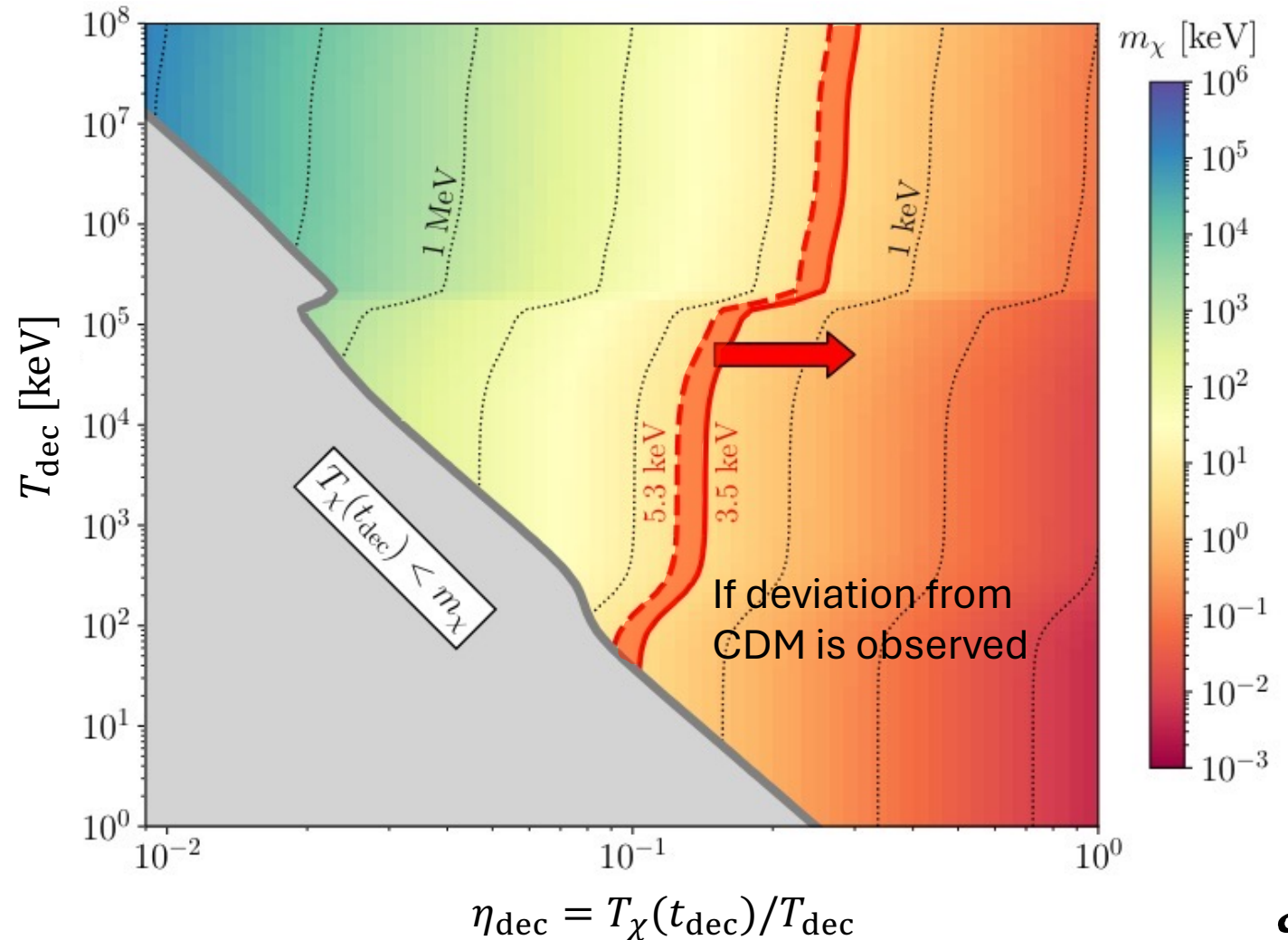


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- Different implications for different production mechanisms

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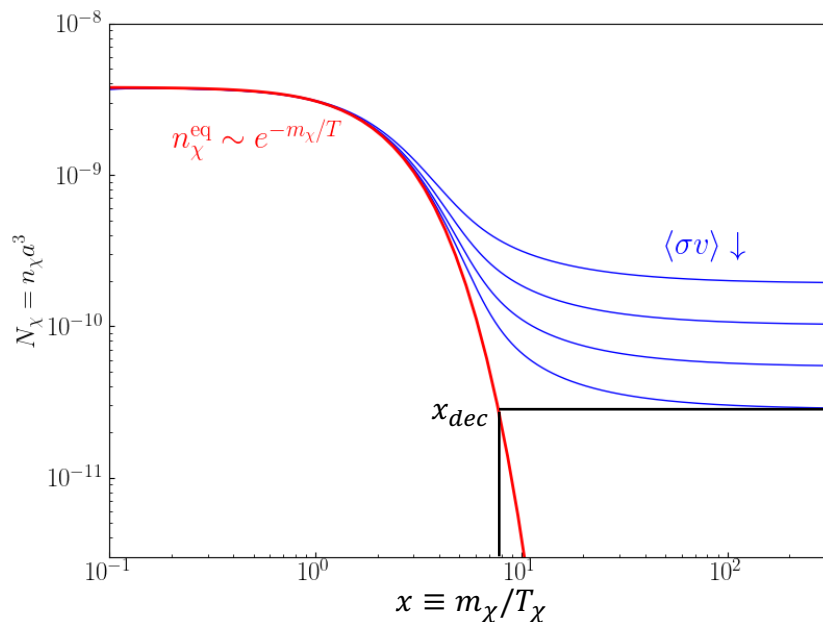
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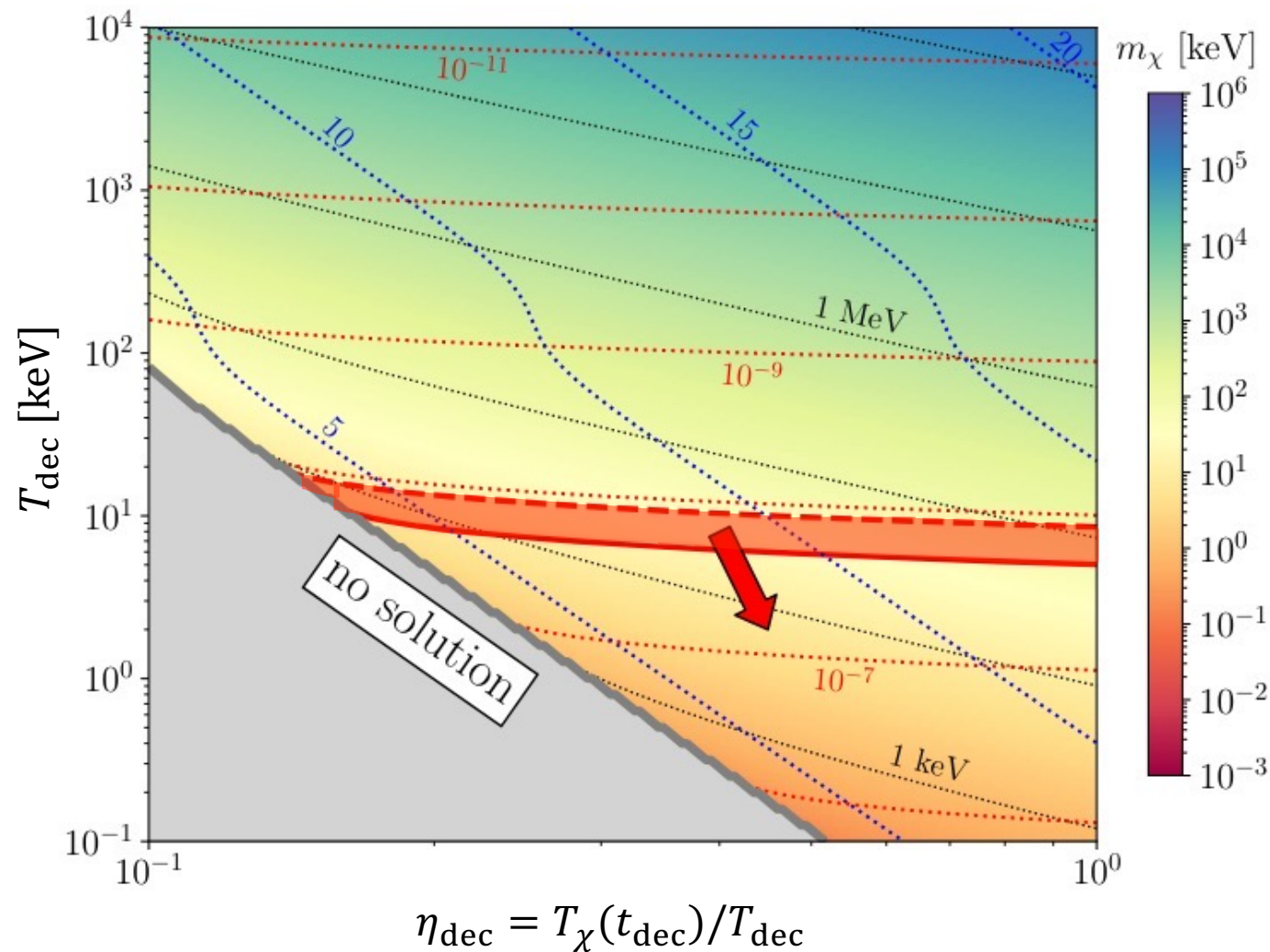
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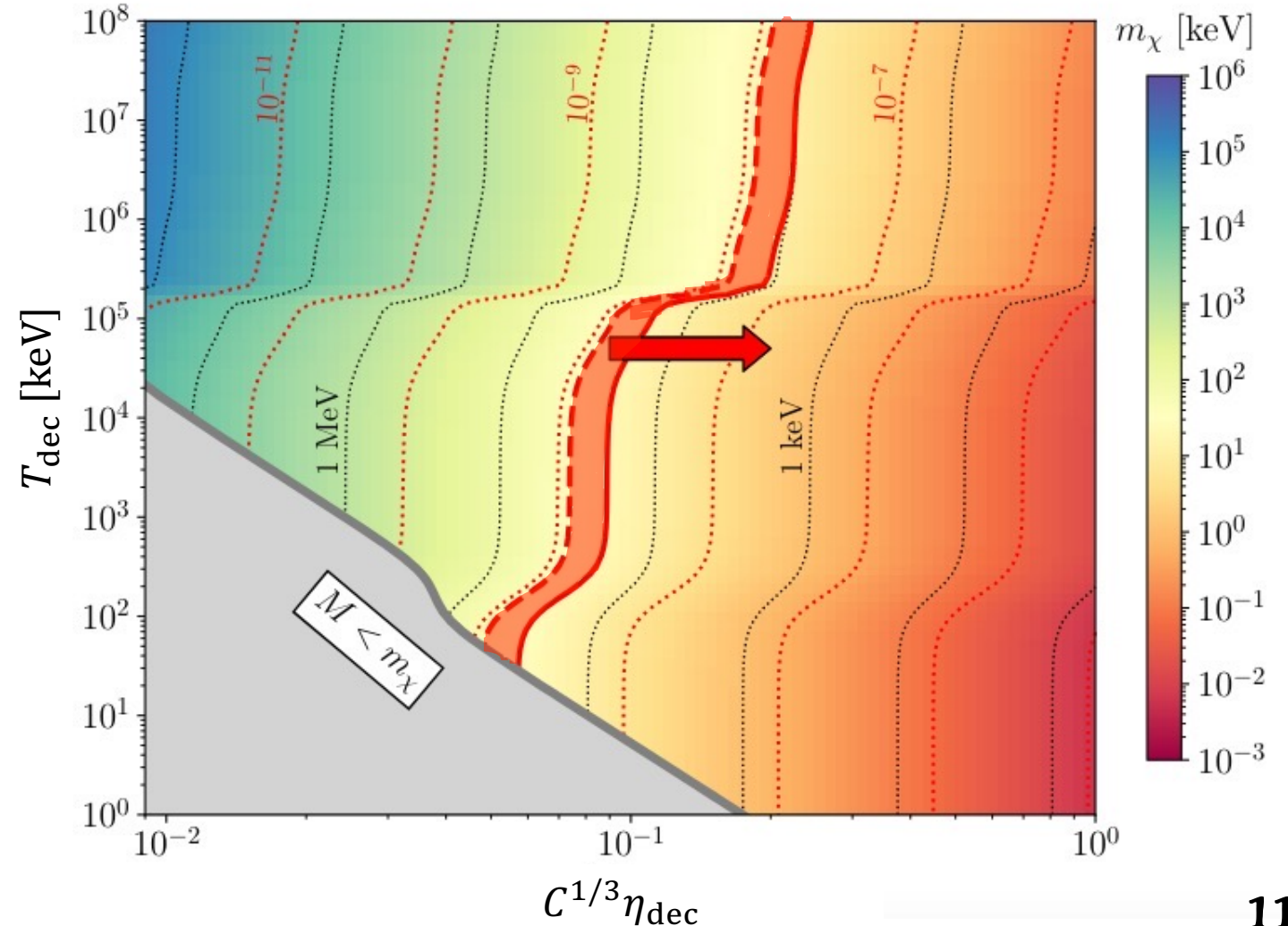
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- deviation from CDM is observed in future observations
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What can we learn?

*Straightforward
expectations*

- A lower bound on DM velocities
- Different implications for different production mechanisms

*The allowed region
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Can we do more?

An optimistic possibility

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- deviation from CDM is observed in future observations
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What can we learn?

Straightforward expectations

- A lower bound on DM velocities
- Different implications for different production mechanisms

The allowed region becomes a band

Can we do more?

- Differences between different production mechanisms?
 - Different production mechanisms can give rise to the same average velocity today
 - Need to go beyond the average velocity and look at the full distribution function!

Beyond the average velocity

Solve Boltzmann equations numerically
to obtain the distribution function

$$\left(\frac{\partial}{\partial t} - Hp \frac{\partial}{\partial p} \right) f_{\chi}(p, t) = \mathcal{C}[f]$$

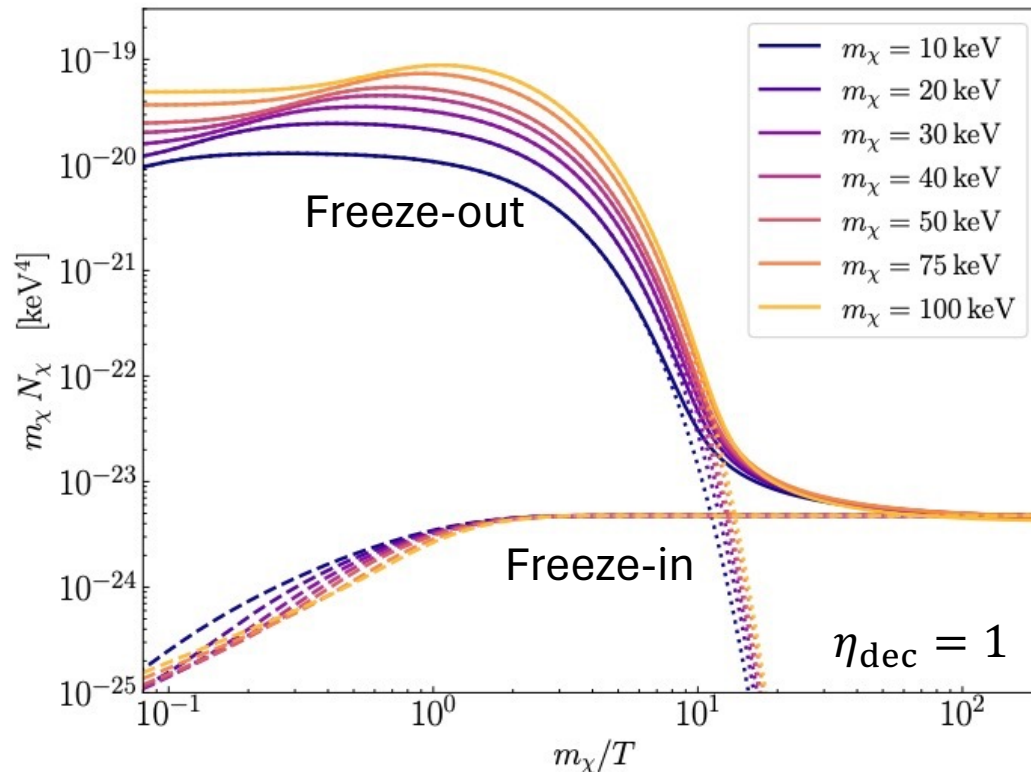
Assuming $\psi + \psi \leftrightarrow \chi + \chi, m_{\psi} \ll m_{\chi}$

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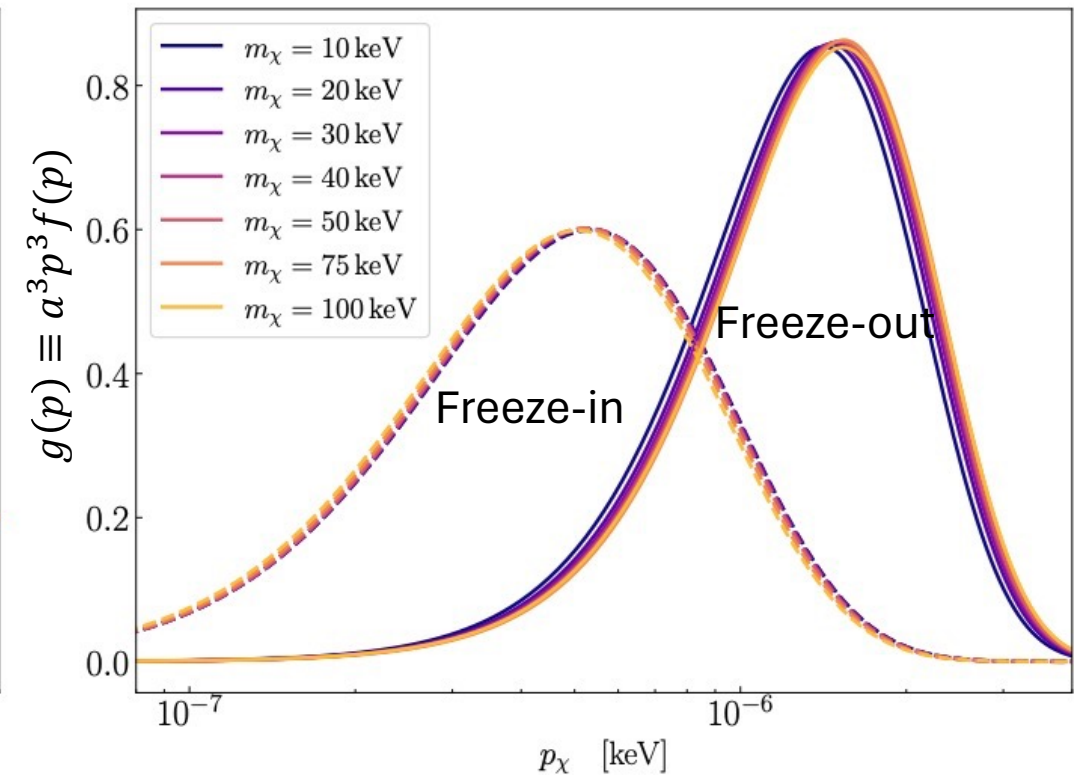
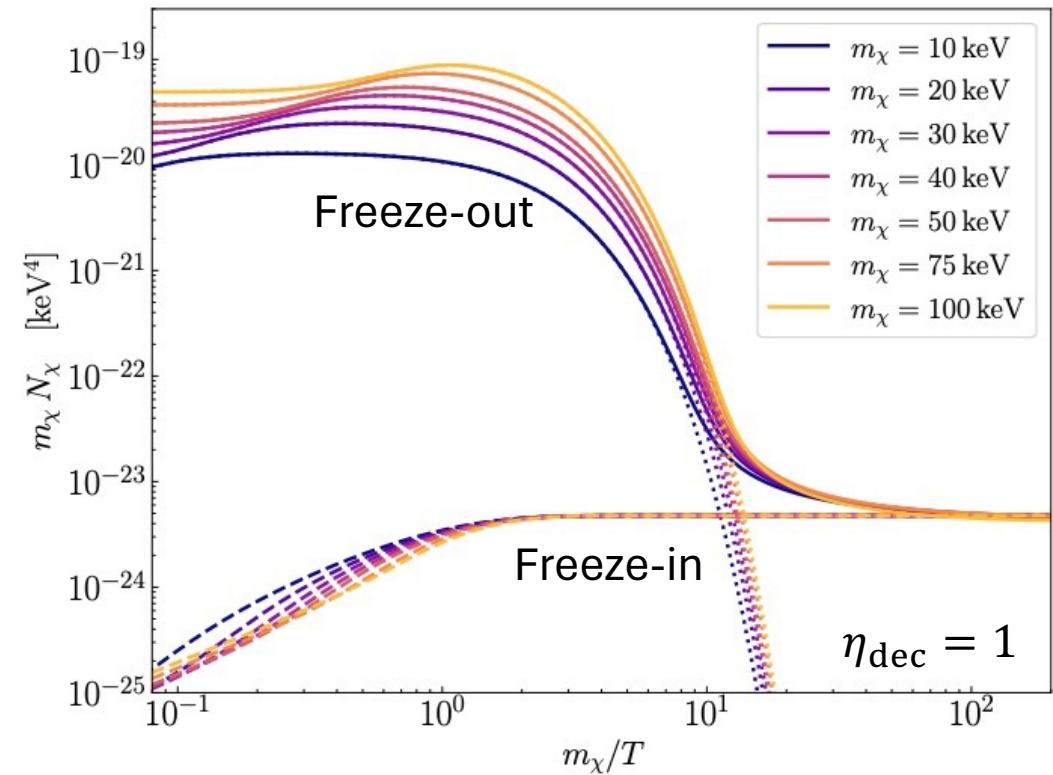


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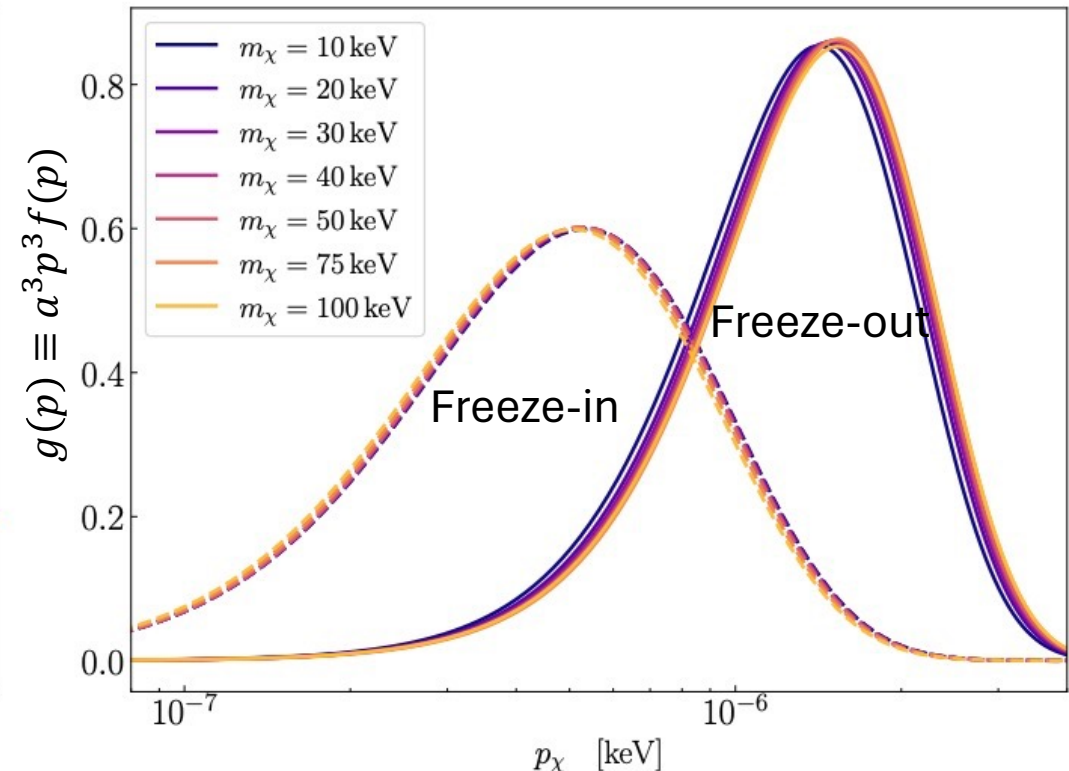
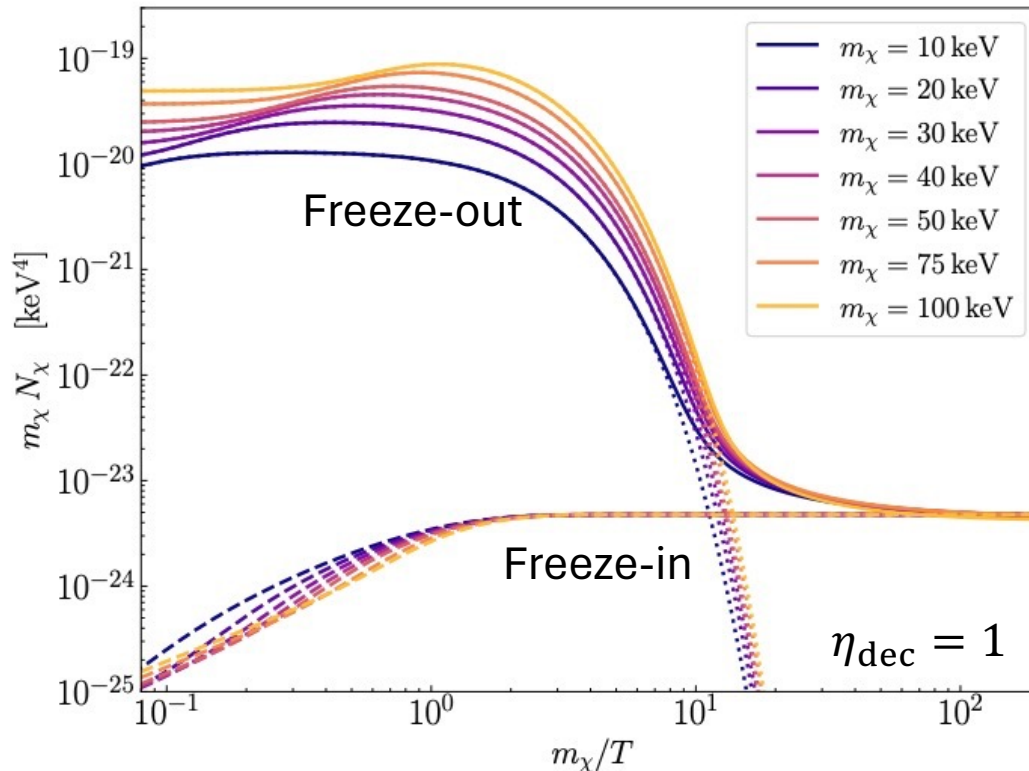
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Assuming $\psi + \psi \leftrightarrow \chi + \chi, m_\psi \ll m_\chi$

- For the same production mechanism \rightarrow **similar** distributions
 - Larger mass \rightarrow Smaller overall velocity
- Distributions from freeze-in and freeze-out are **distinct**
 - same mass \rightarrow freeze-in distribution is **colder**
 - Even if same average velocity \rightarrow **different shape**



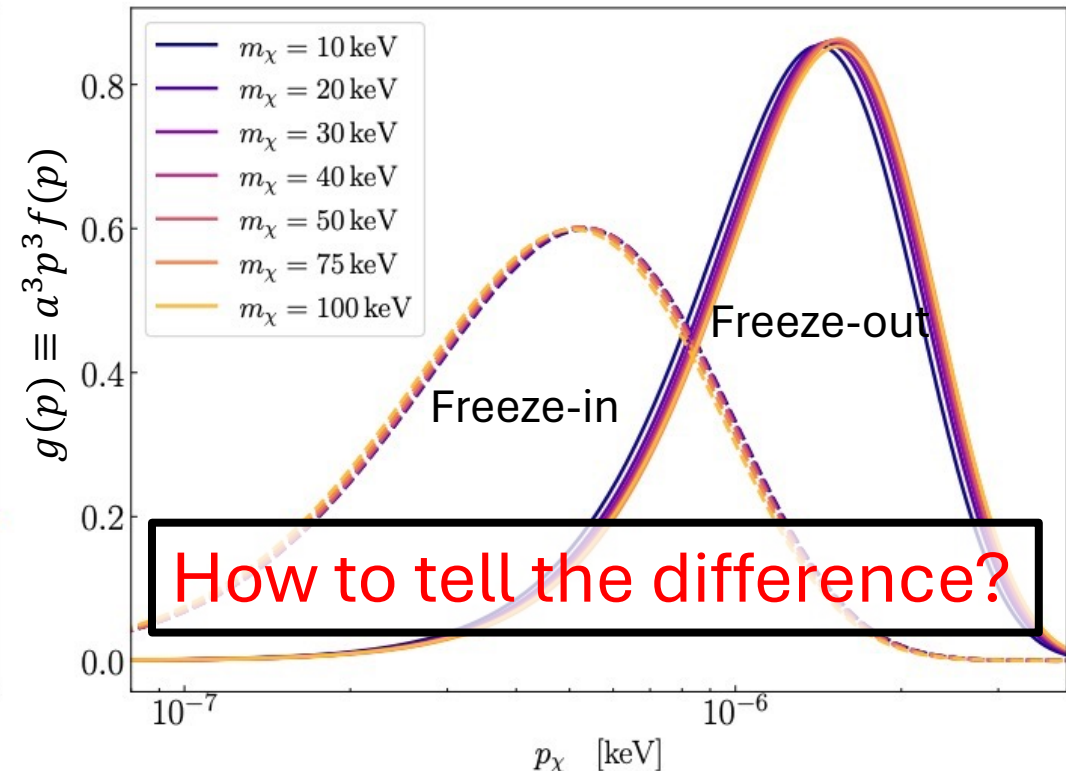
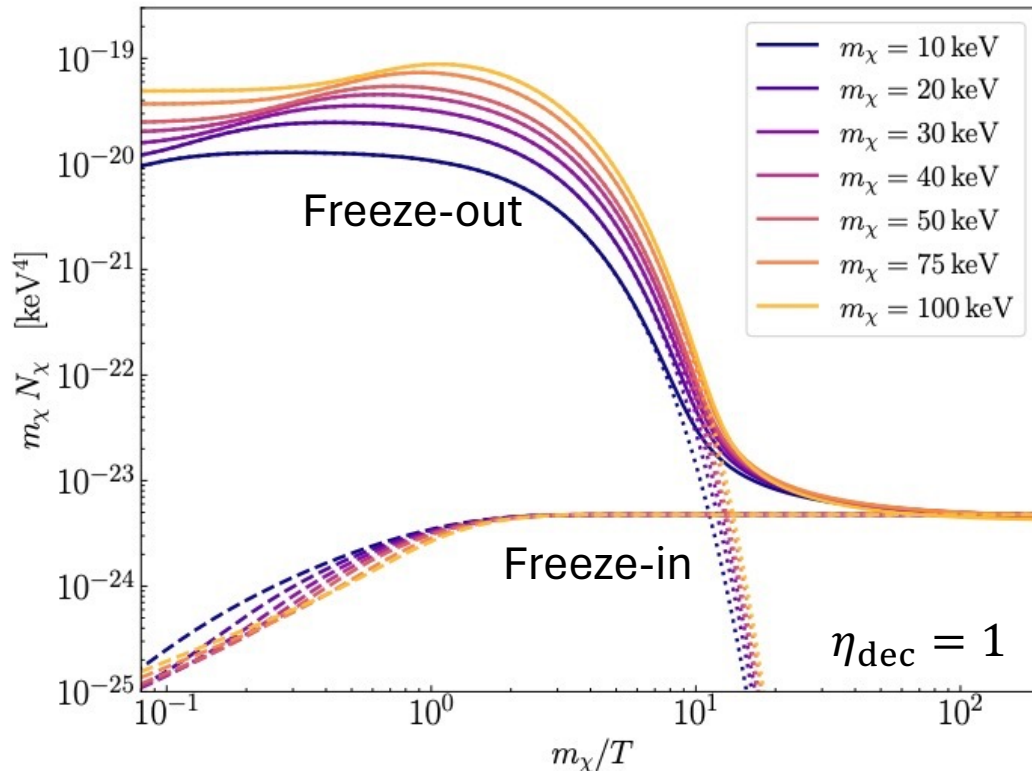
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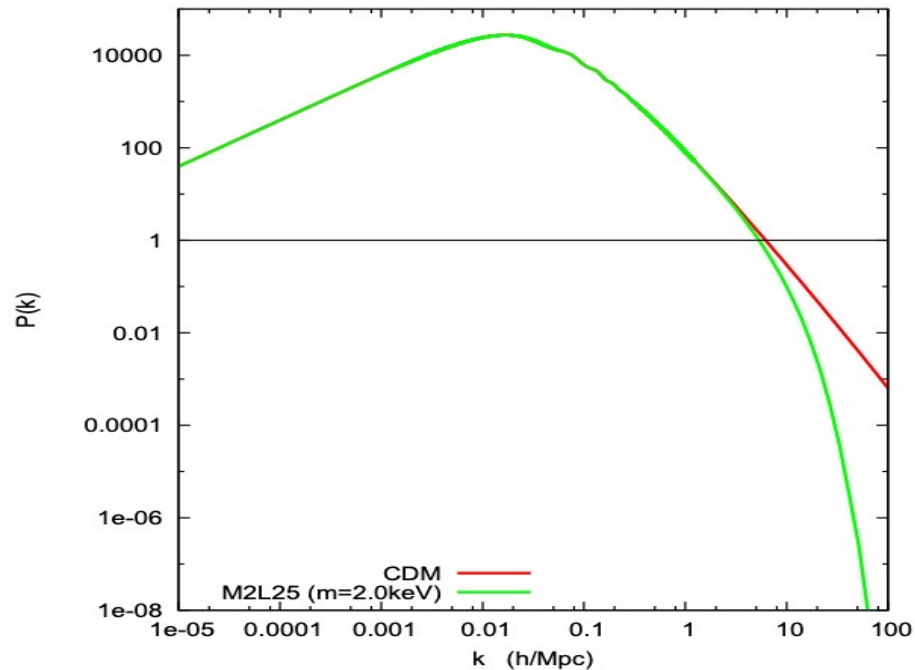
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Imprints on cosmic structure

- Non-negligible velocities suppresses structure formation, reflected in the ***matter power spectrum***

$$P(k)$$



Imprints on cosmic structure

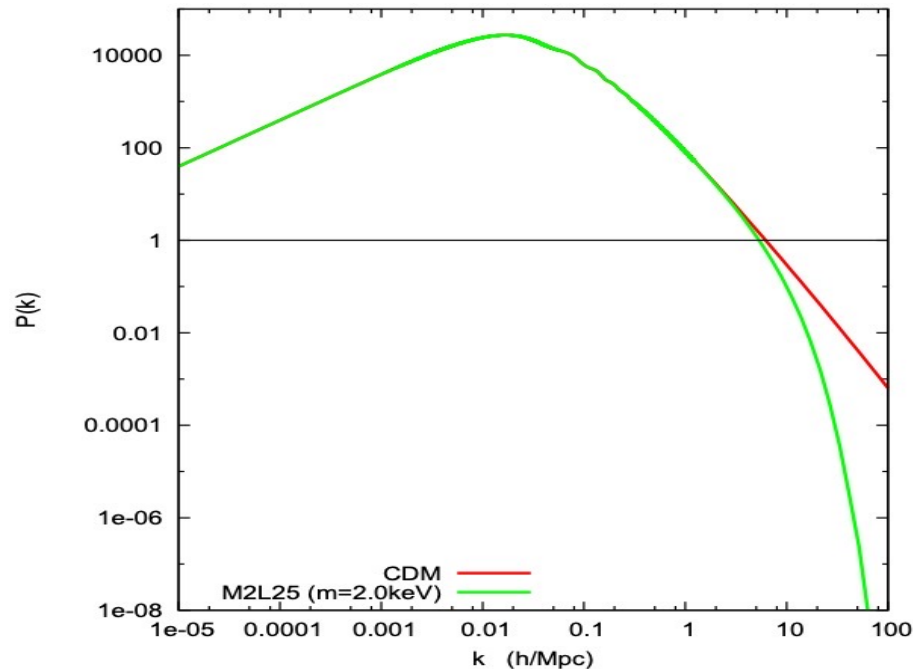
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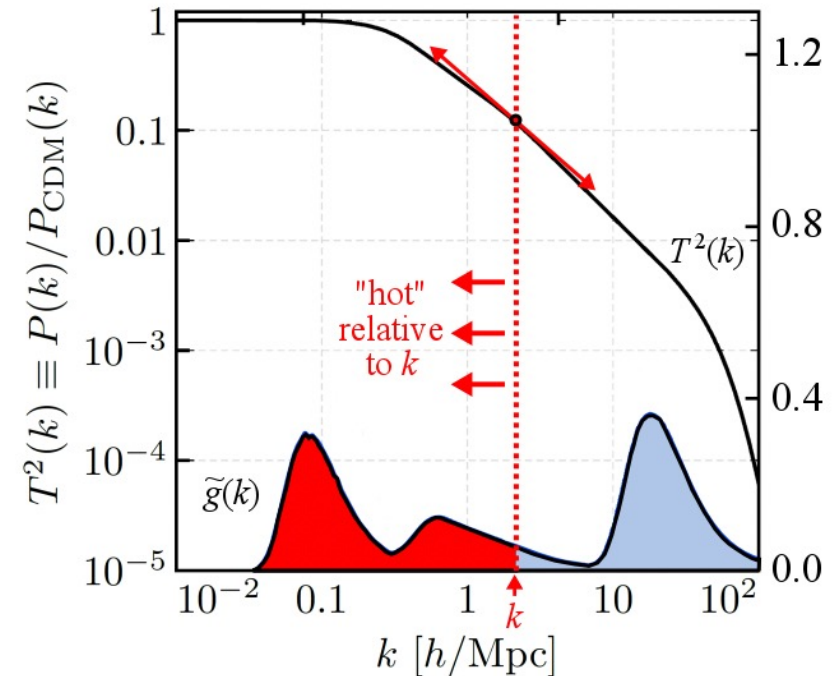
- Often represented by the **squared transfer function**

$$T^2(k) \equiv P(k)/P_{\text{CDM}}(k)$$

- The shape of $T^2(k)$ **contains the information** of the distribution function



Example taken from [J. Lesgourgues and T. Tram](#)
[JCAP 09 \(2011\) 032](#)



[K. Dienes, FH, J. Kost, S. Su, B. Thomas](#)
[Phys.Rev.D 101 \(2020\) 12](#)

Imprints on cosmic structure

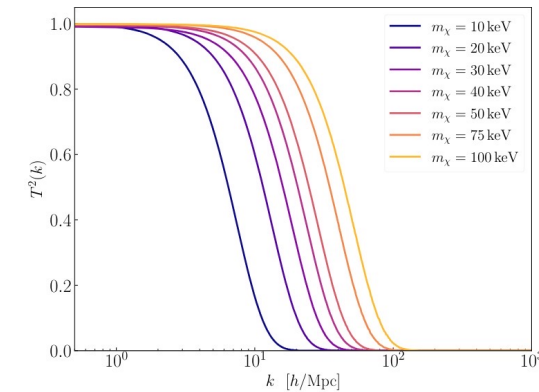
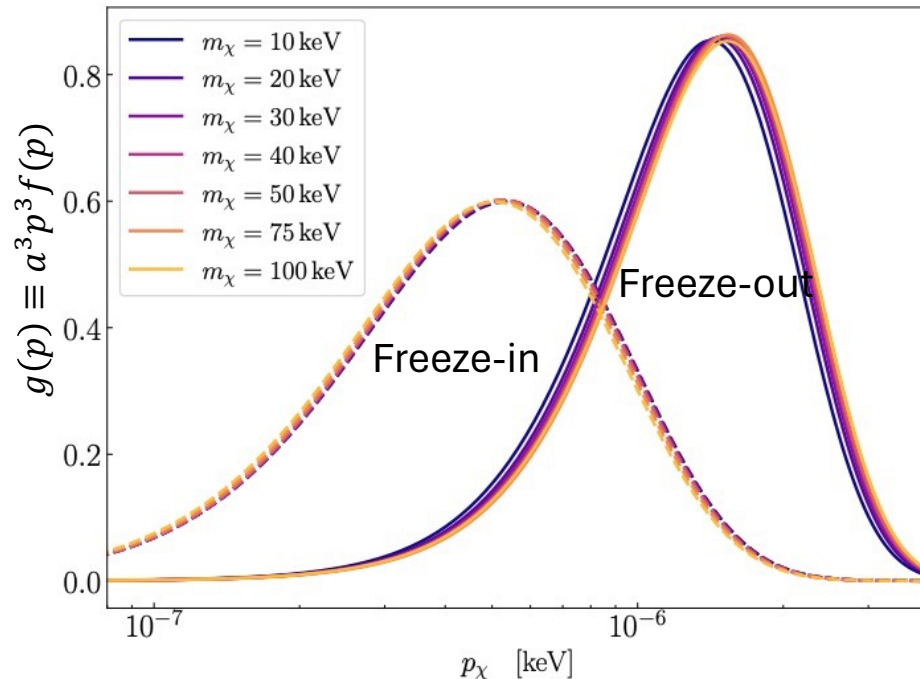
- Non-negligible velocities suppresses structure formation, reflected in the **matter power spectrum**

$$P(k)$$

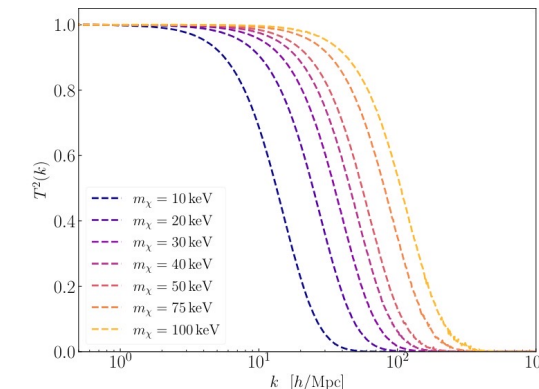
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Freeze-out



Freeze-in

Imprints on cosmic structure

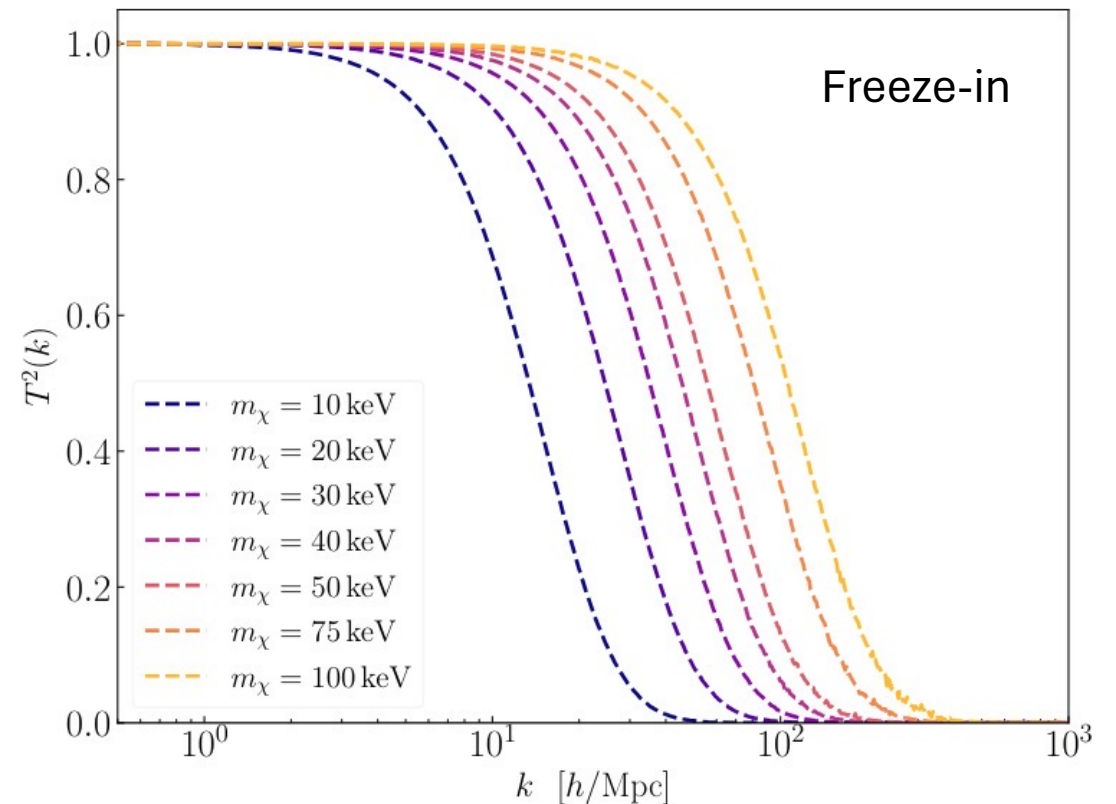
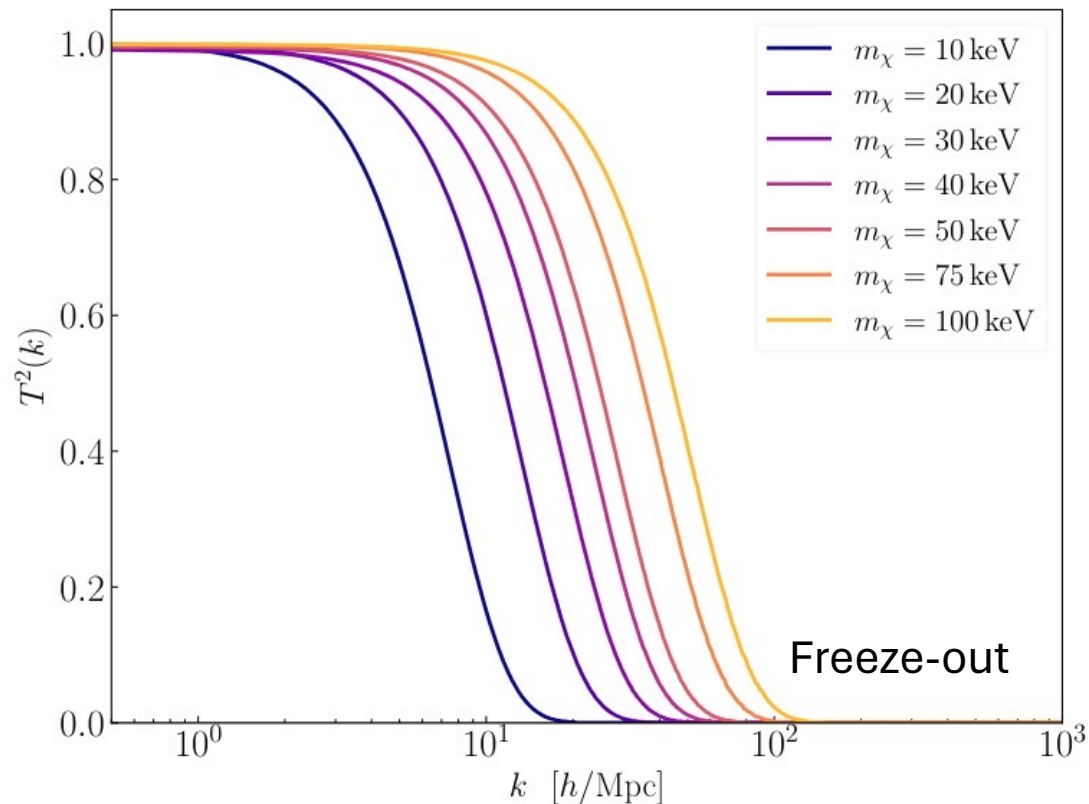
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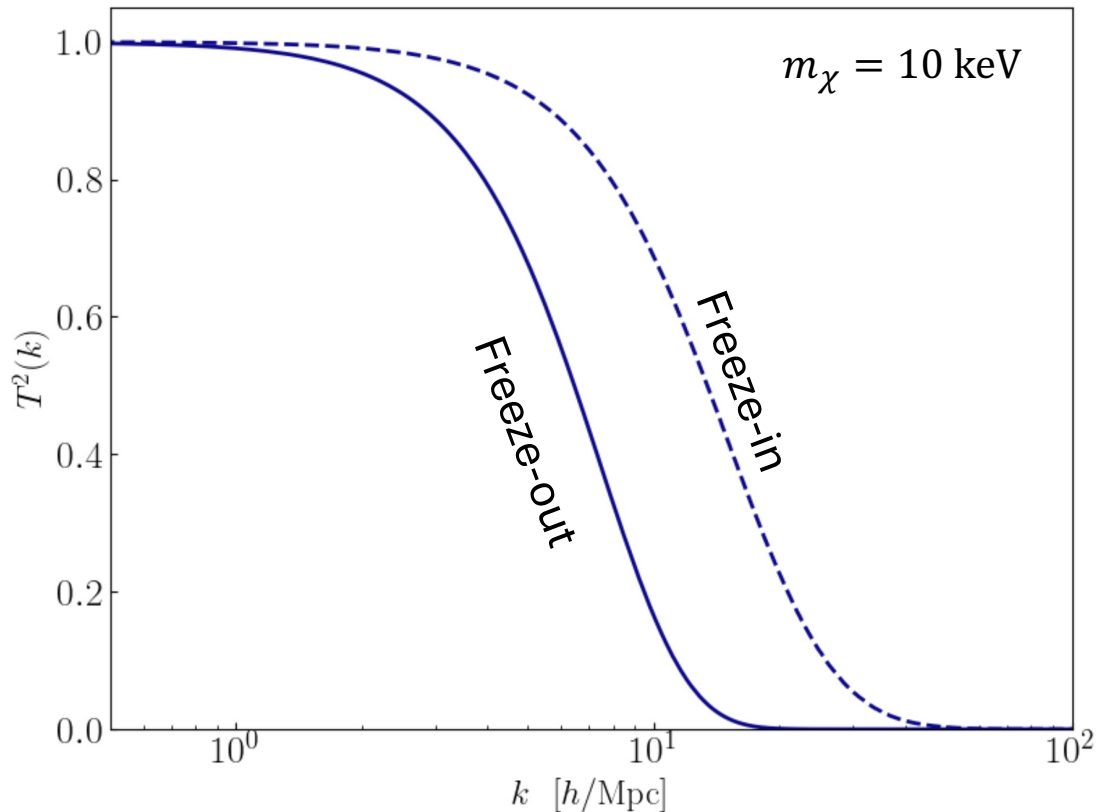
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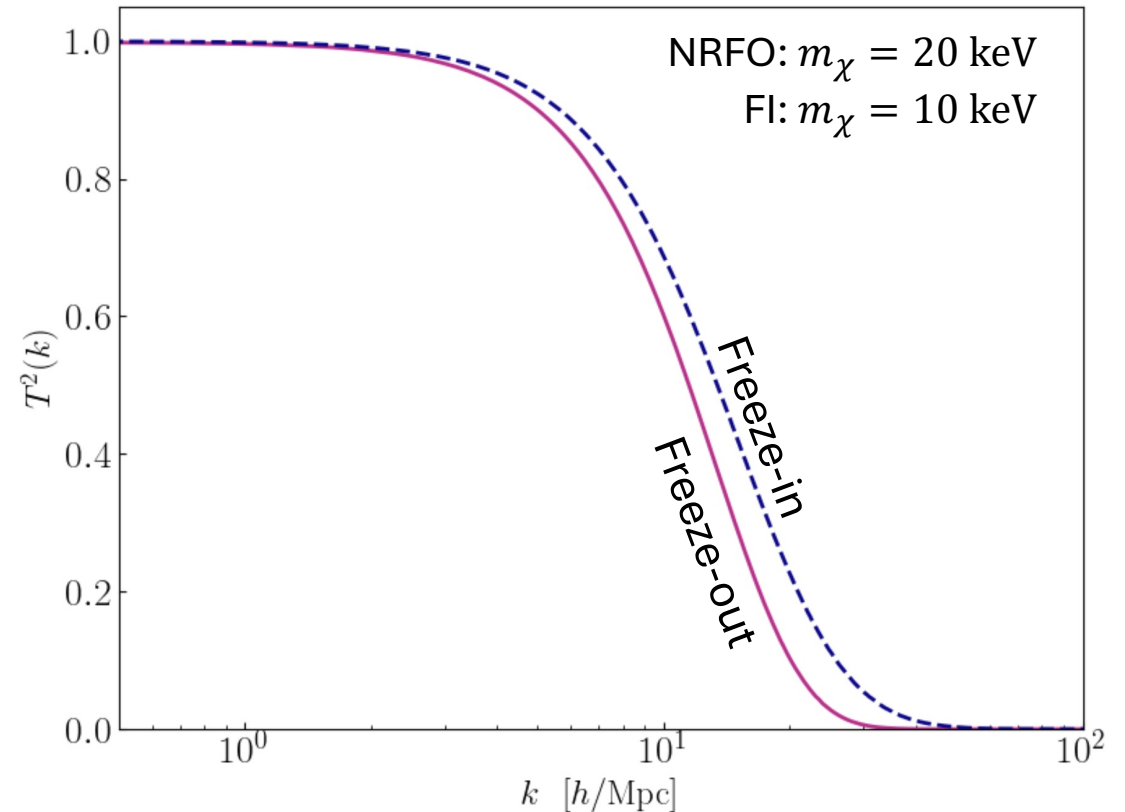
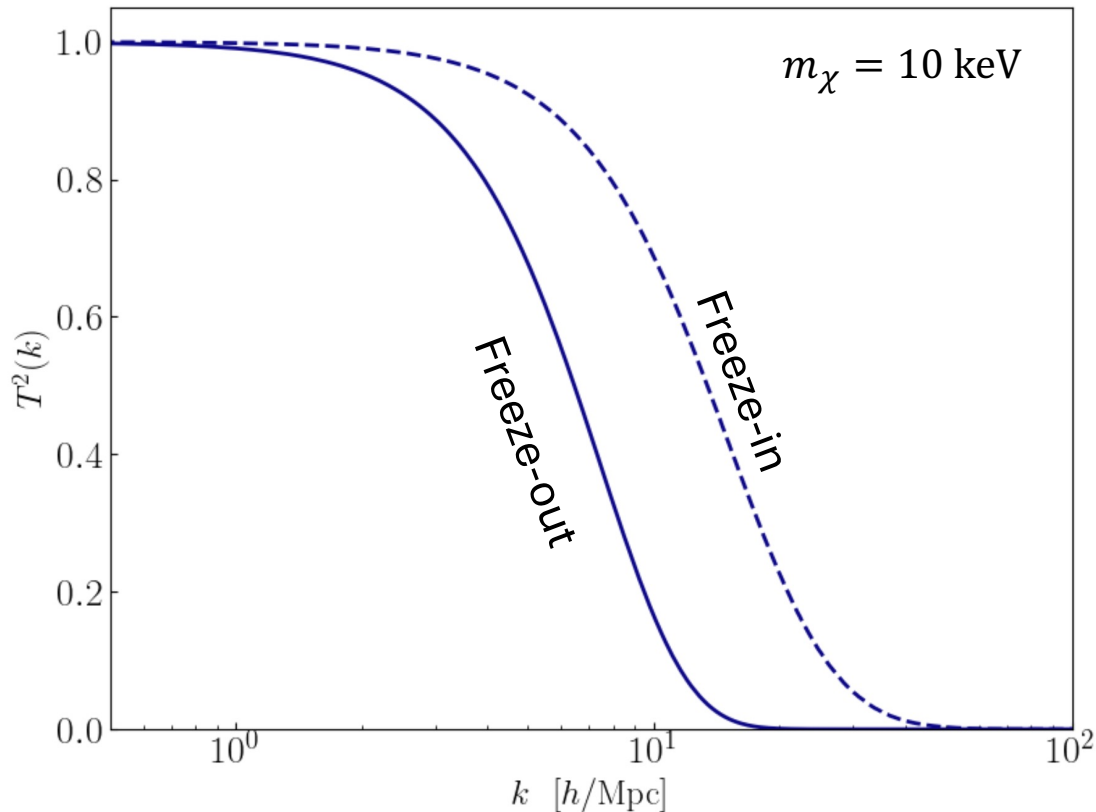
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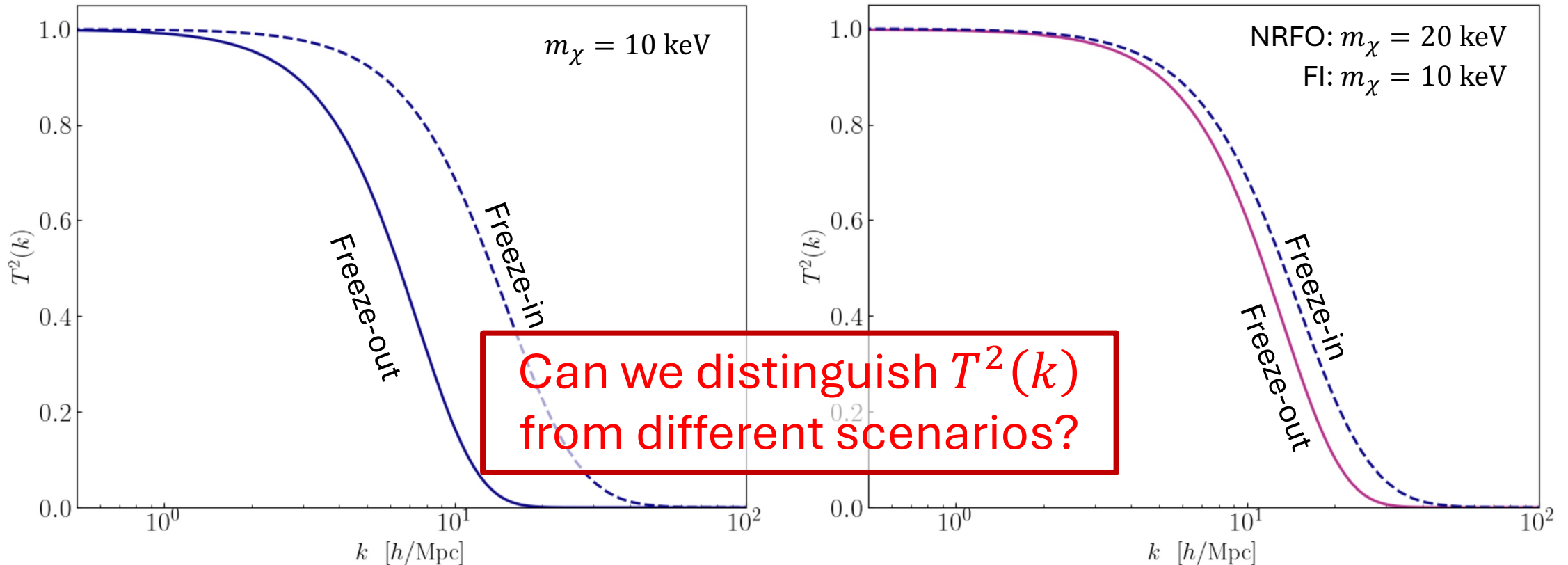
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Distinguishing different scenarios

- If future measurements on $P(k)$ finds deviation from CDM, use WDM as a baseline model

$$P_{\text{WDM}}(k)[1 + \sigma_P^+(k)] > P(k) > P_{\text{WDM}}(k)[1 - \sigma_P^-(k)]$$

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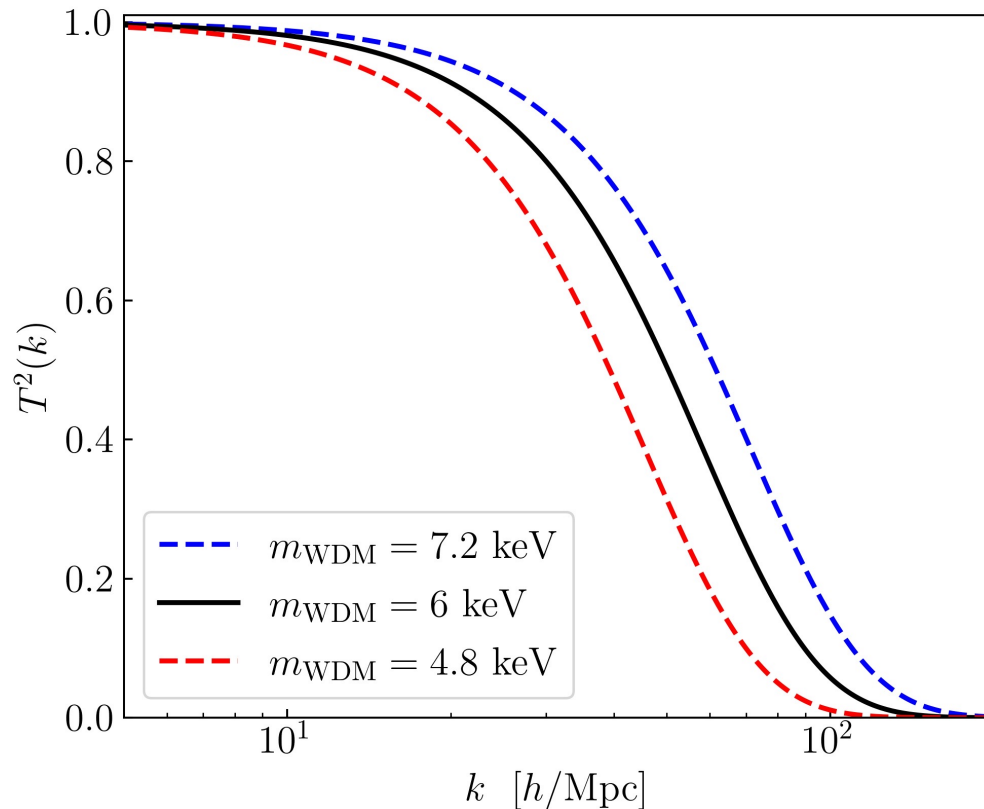
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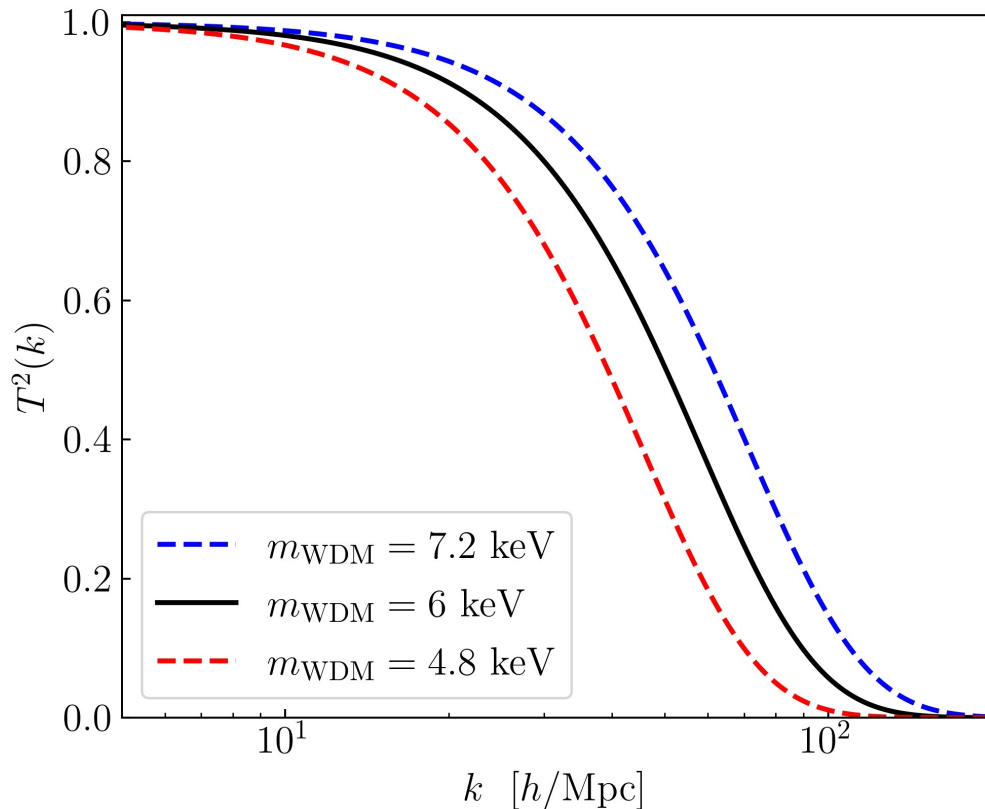
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δA analysis: [R. Murgia et al. JCAP 11 \(2017\) 046](#)

$$P_{1D}(k) \equiv \frac{1}{2\pi} \int_k^\infty dk' k' P(k')$$

$$A \equiv \int_{k_{\min}}^{k_{\max}} dk \frac{P_{1D}(k)}{P_{1D}^{\text{CDM}}(k)} \quad \begin{array}{l} k_{\max} = 20 \, h/\text{Mpc} \\ k_{\min} = 0.5 \, h/\text{Mpc} \end{array}$$

$$\delta A \equiv 1 - A/A_{\text{CDM}}$$

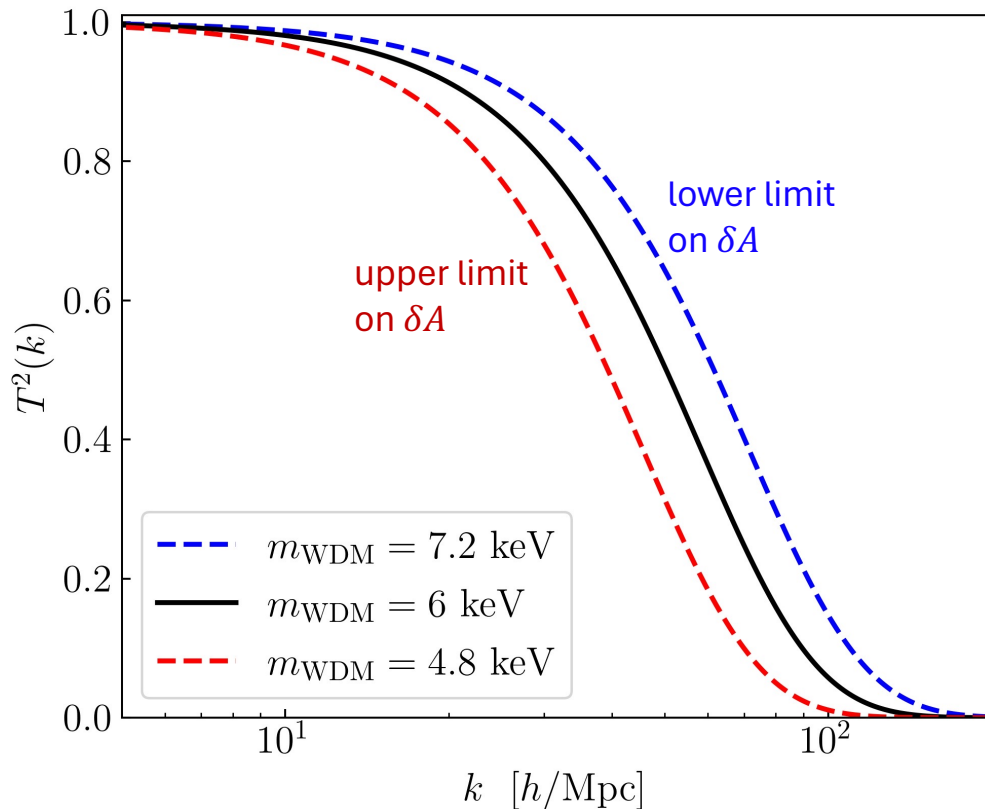
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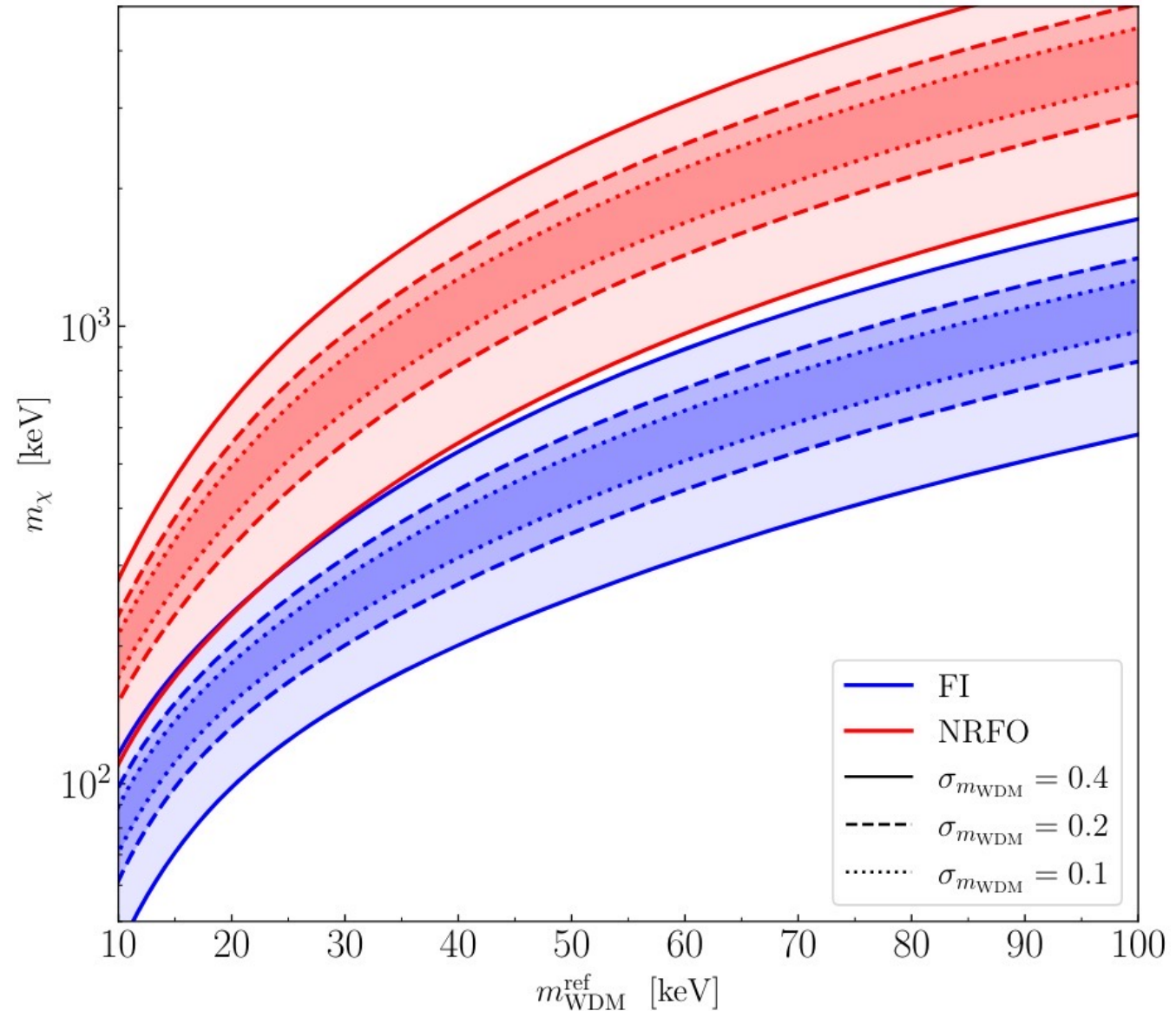
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Distinguishing different scenarios

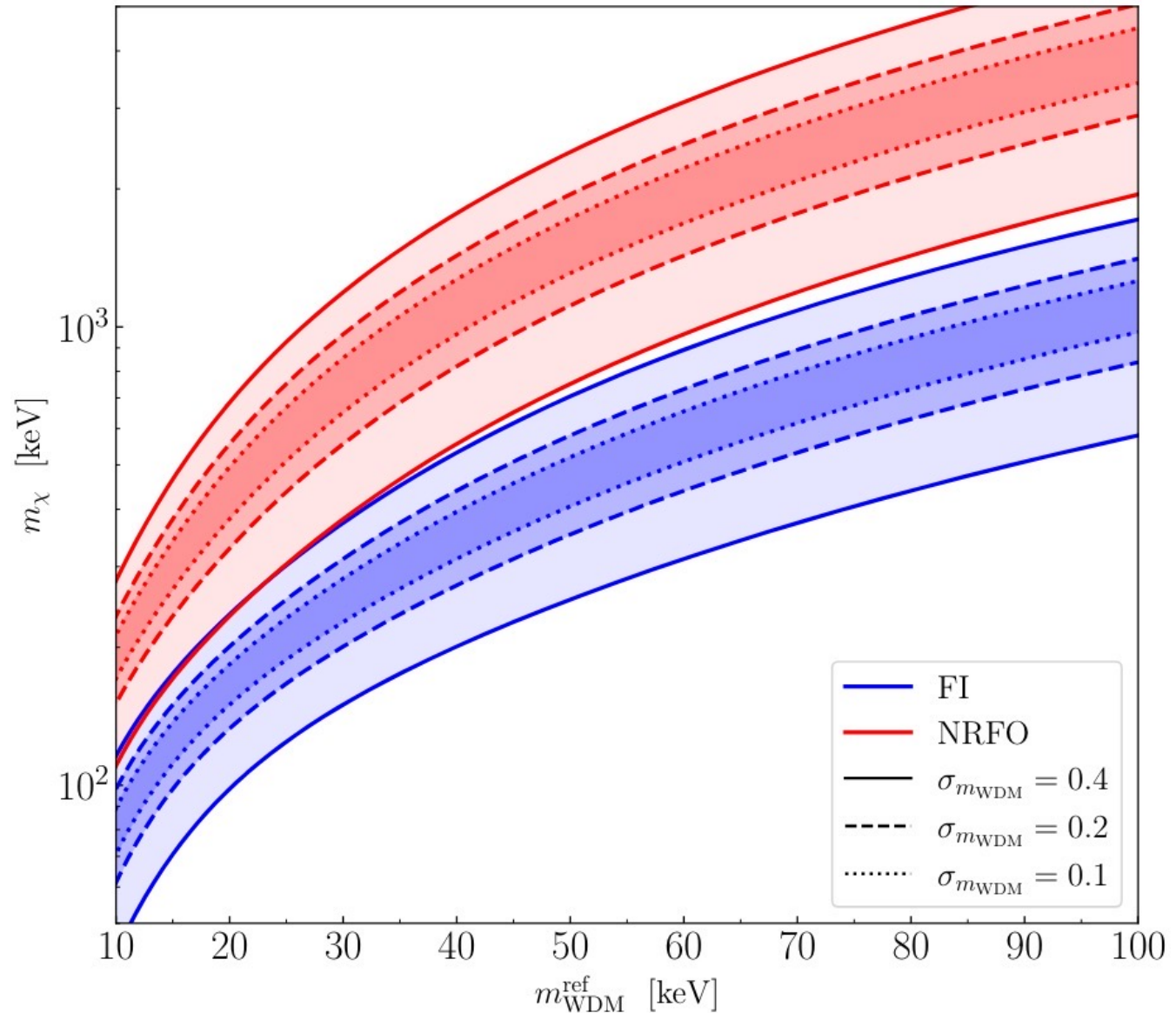
- Different allowed mass ranges for FI and NRFO
- Allowed regions are completely separated given sufficiently small uncertainties



Distinguishing different scenarios

- Different allowed mass ranges for FI and NRFO
- Allowed regions are completely separated given sufficiently small uncertainties
- < 1 keV uncertainty from future global 21 cm measurement forecasted by J. Hibbard et al.

[Astrophys.J. 929 \(2022\) 2, 151](#)



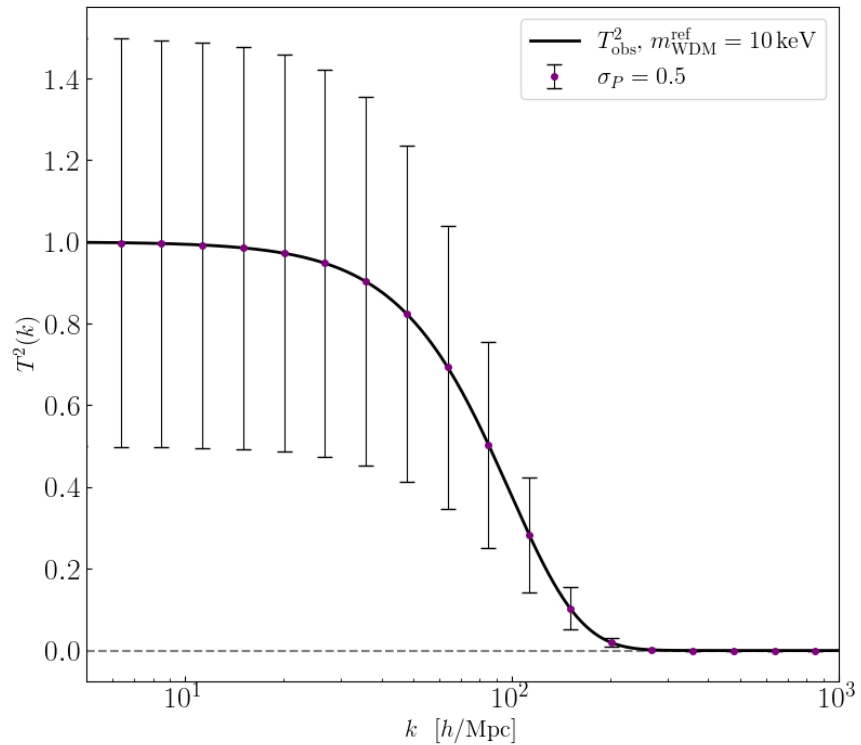
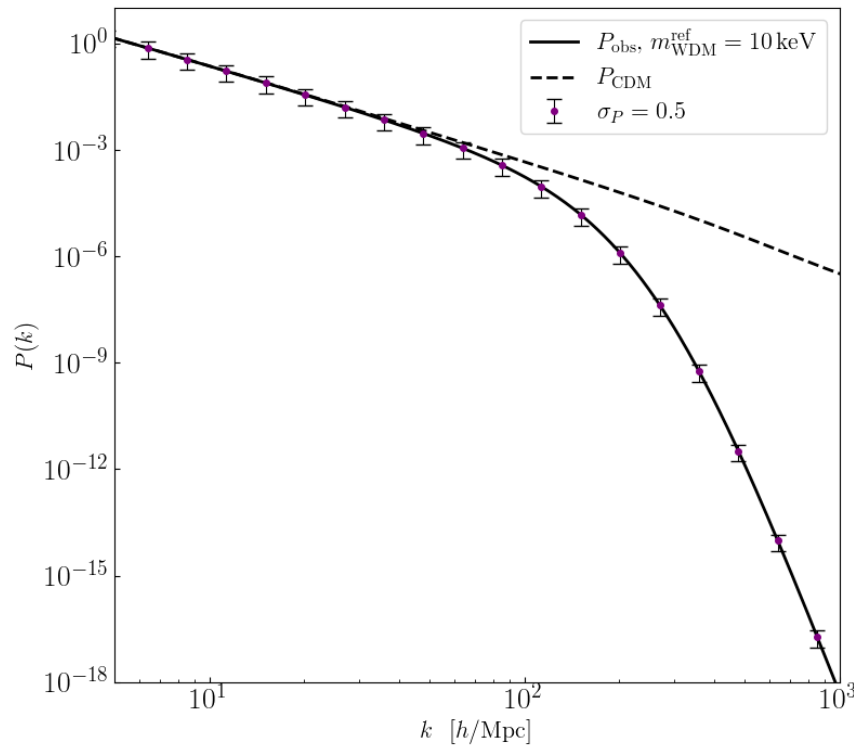
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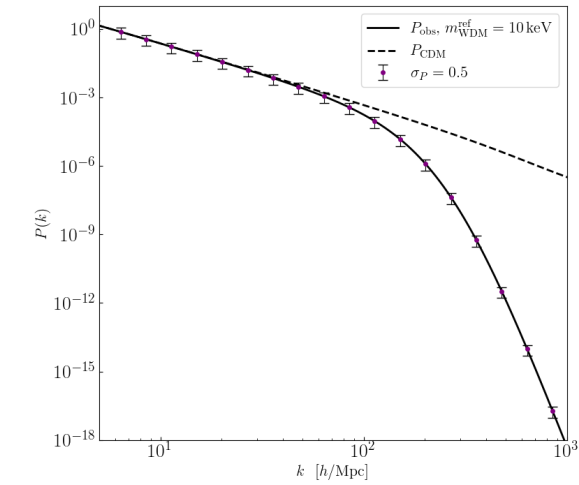
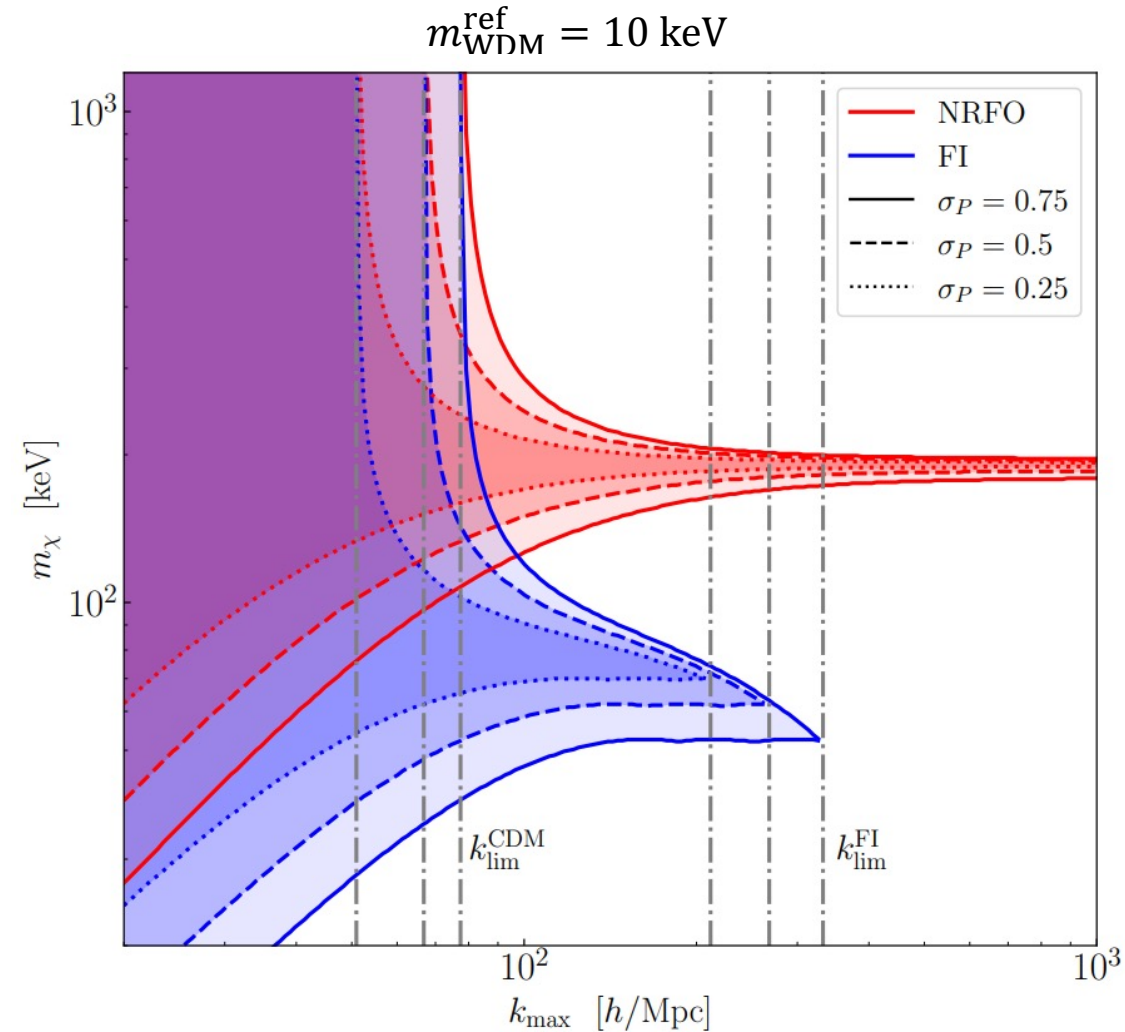
- Explore different possibilities: e.g., **constant symmetric relative errors** on $P(k)$

$$\sigma_P^\pm(k) = \sigma_P$$



Distinguishing different scenarios

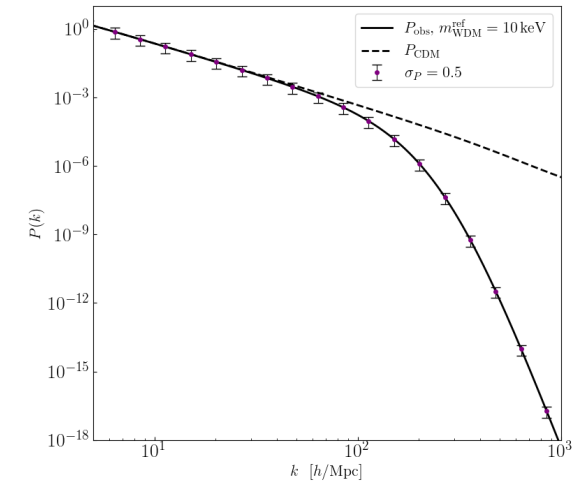
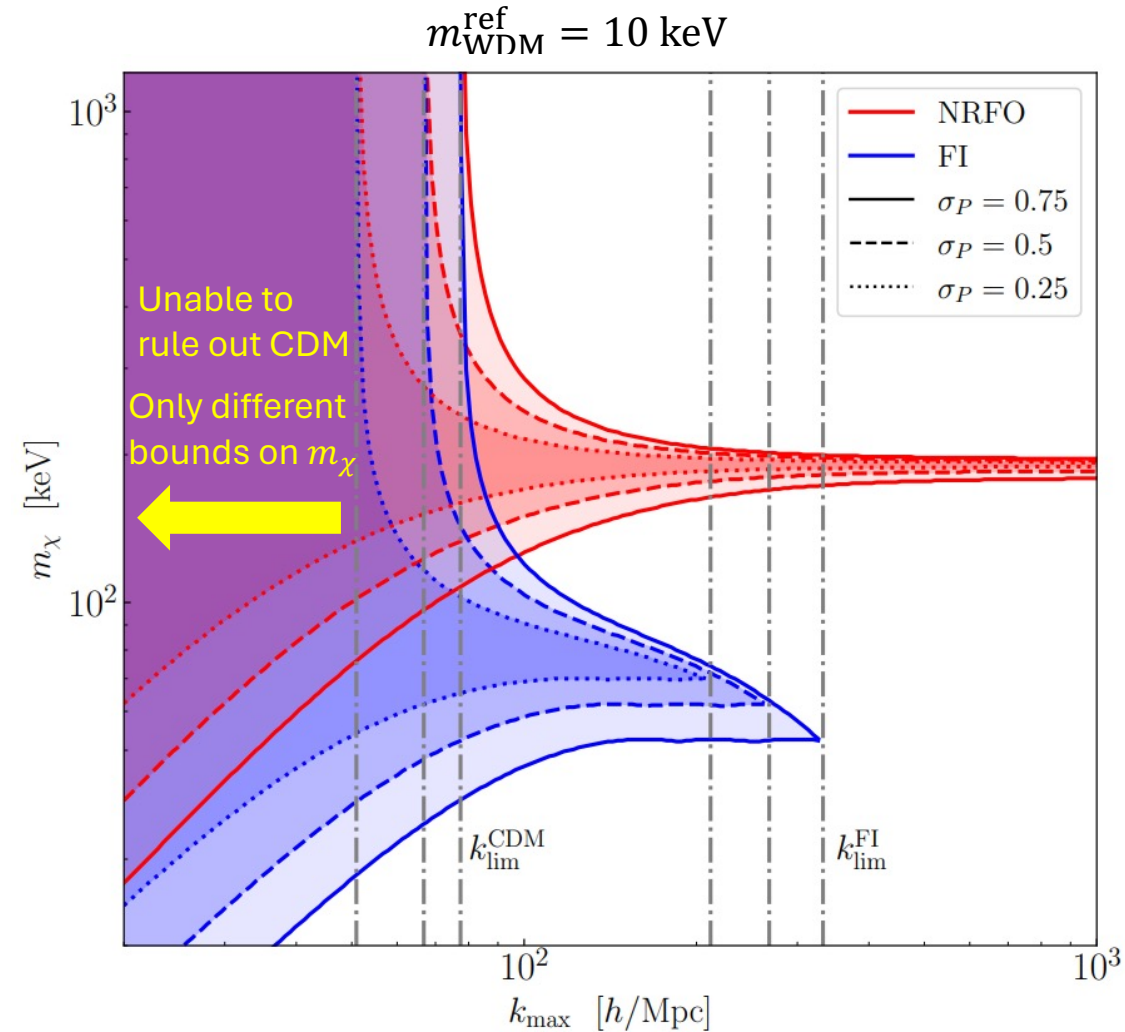
- Assuming $N = 20$ data points evenly distributed on log-scale from $k_{\min} = 1 \text{ h/Mpc}$ to k_{\max}



constant symmetric
relative errors on $P(k)$

Distinguishing different scenarios

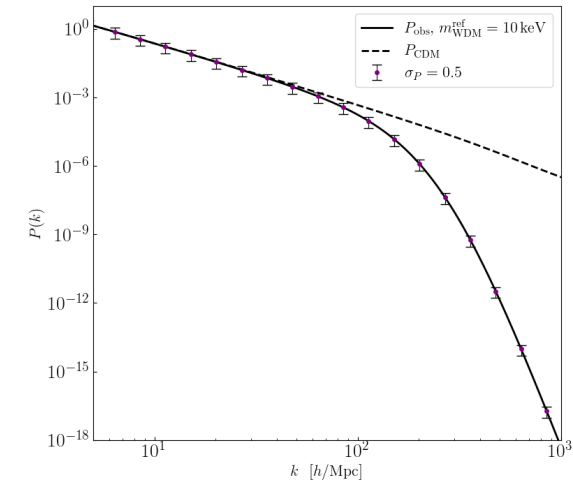
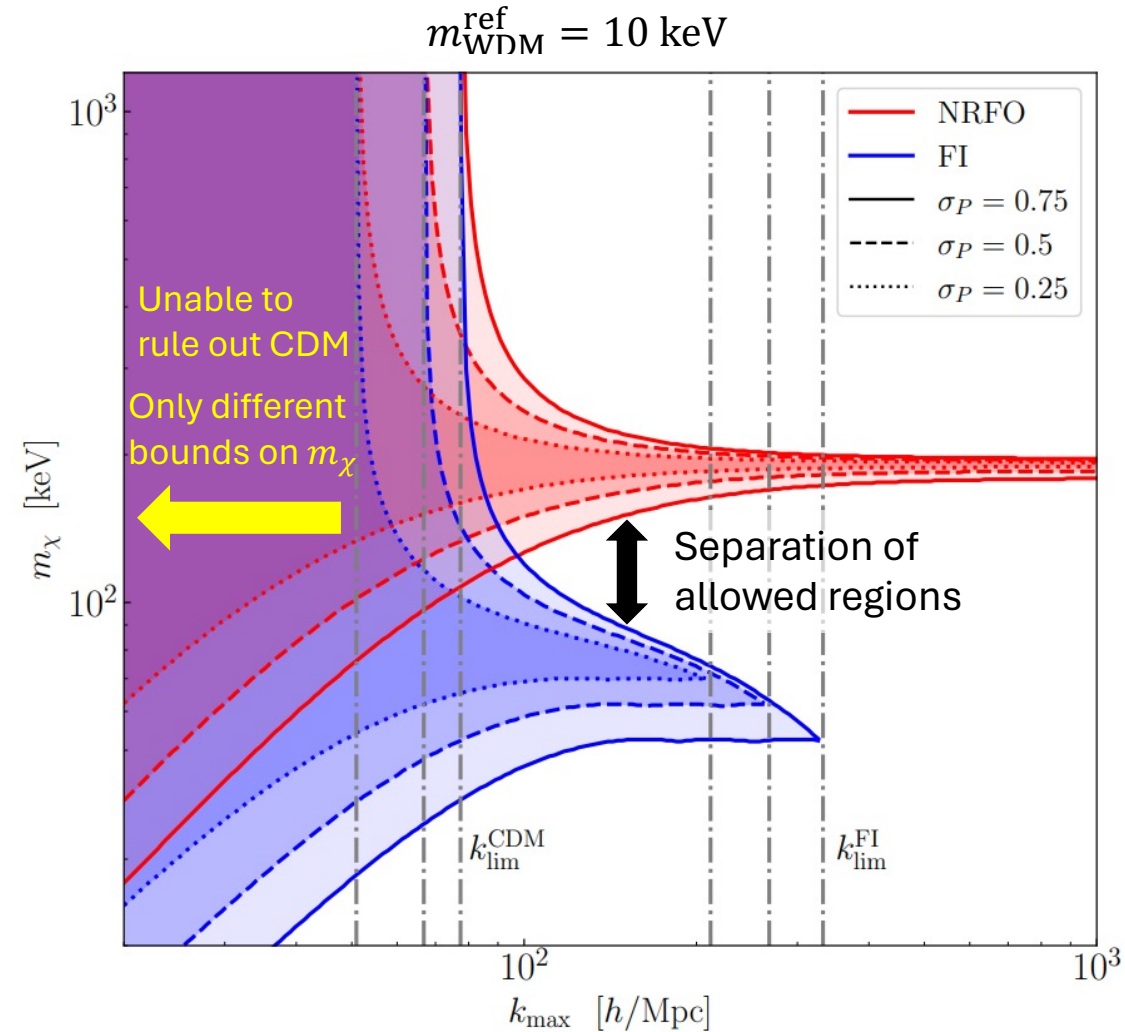
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constant symmetric
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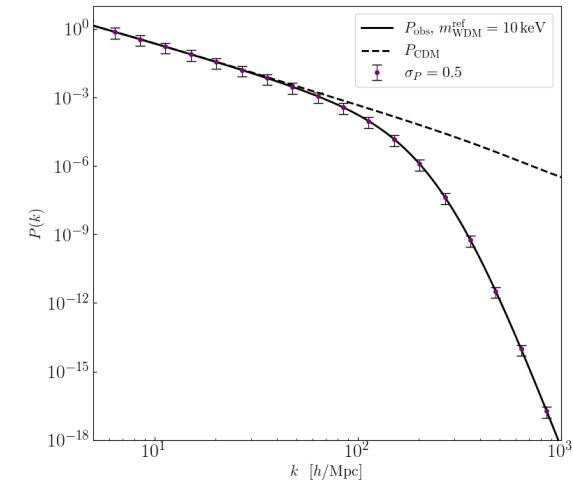
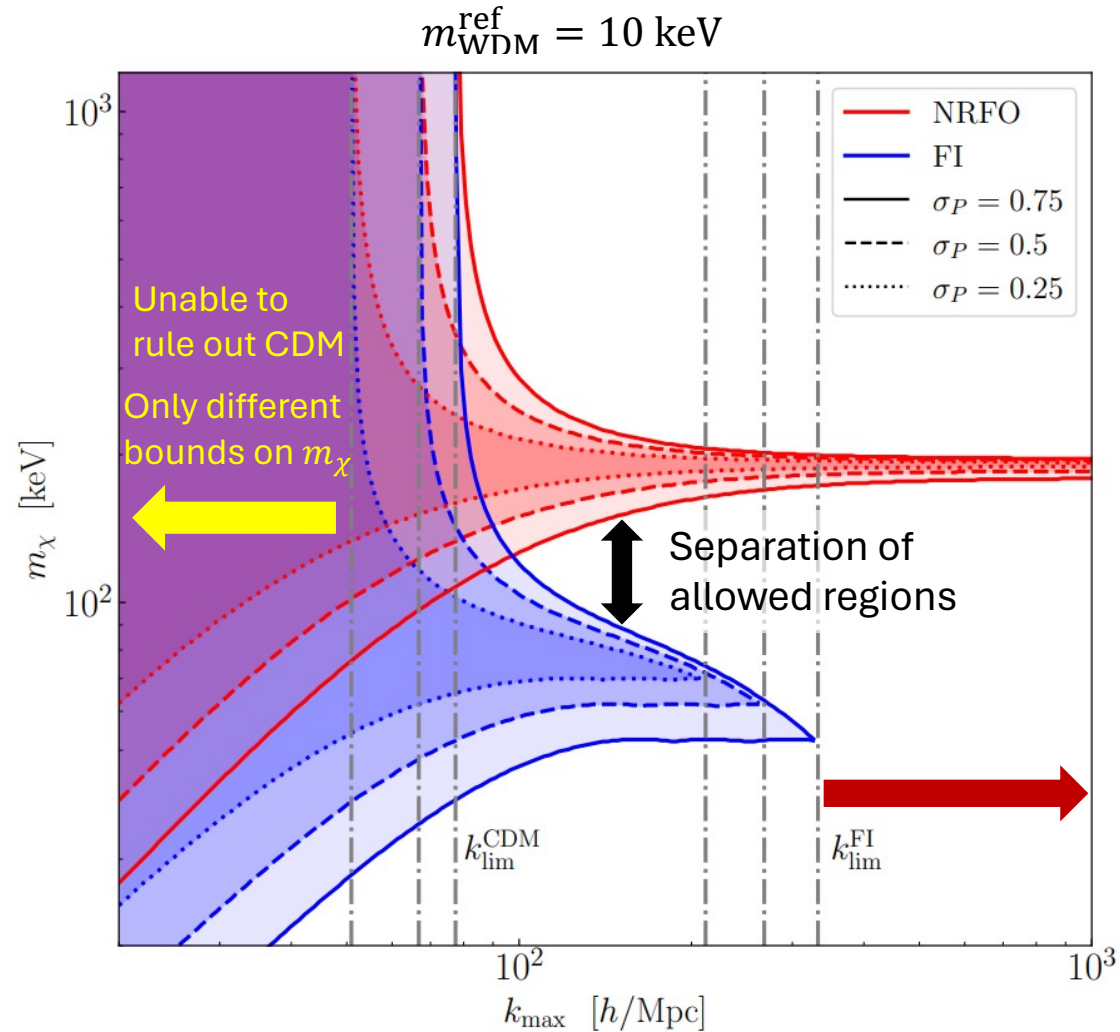
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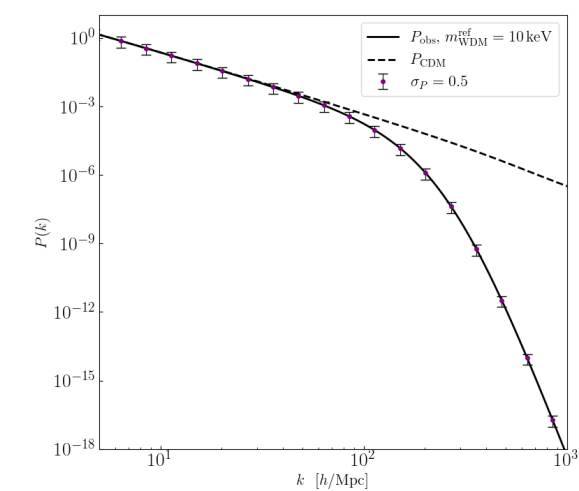
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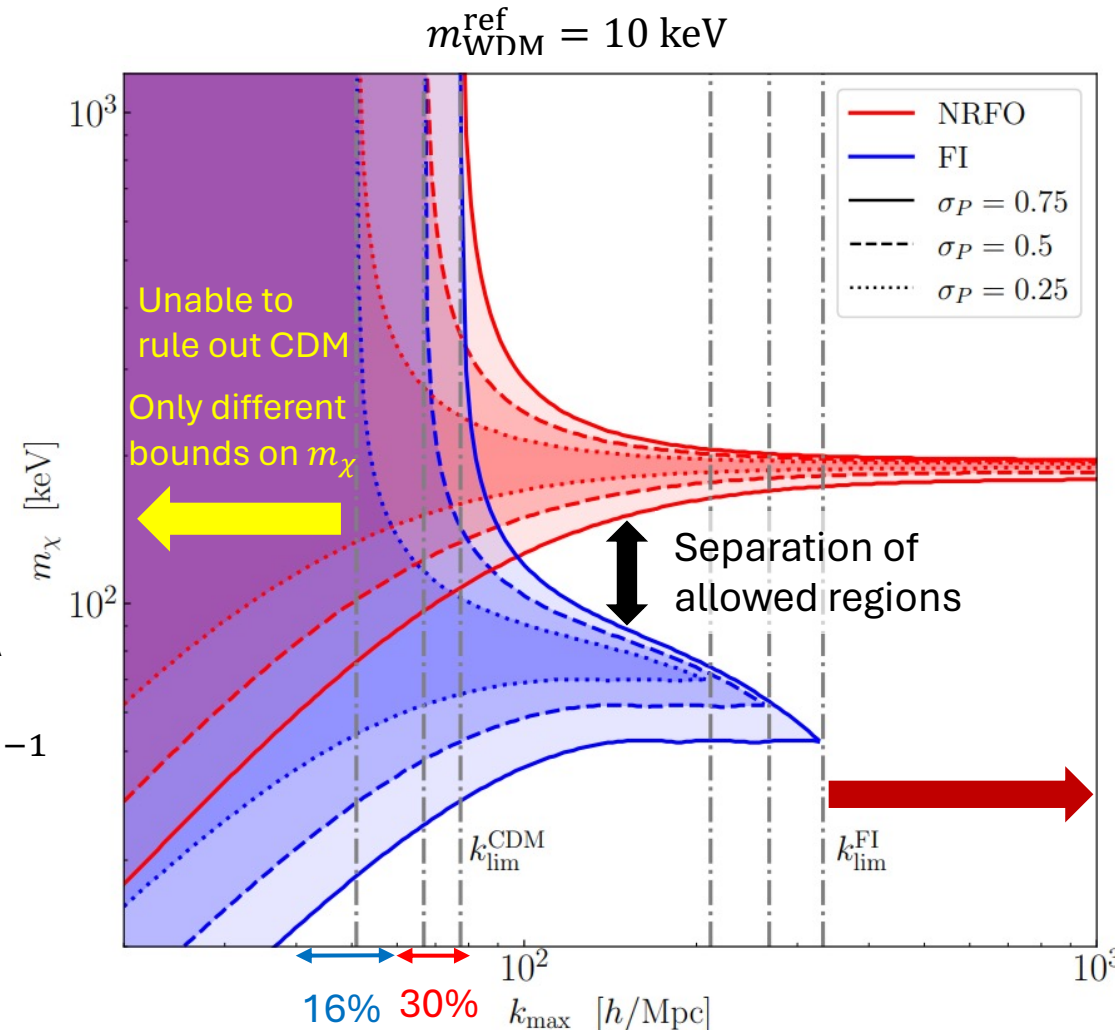
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constant symmetric relative errors on $P(k)$

Forecasts on 21 cm fluctuation measurements show that HERA can measure matter power spectrum up to $k \approx 40 \sim 80 \text{ Mpc}^{-1}$ with precision 16~30 %

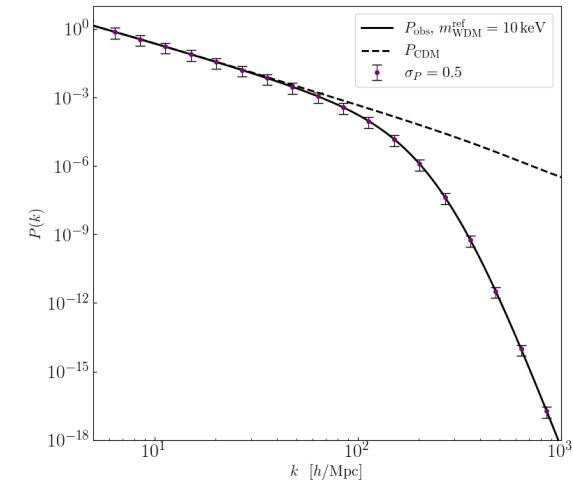
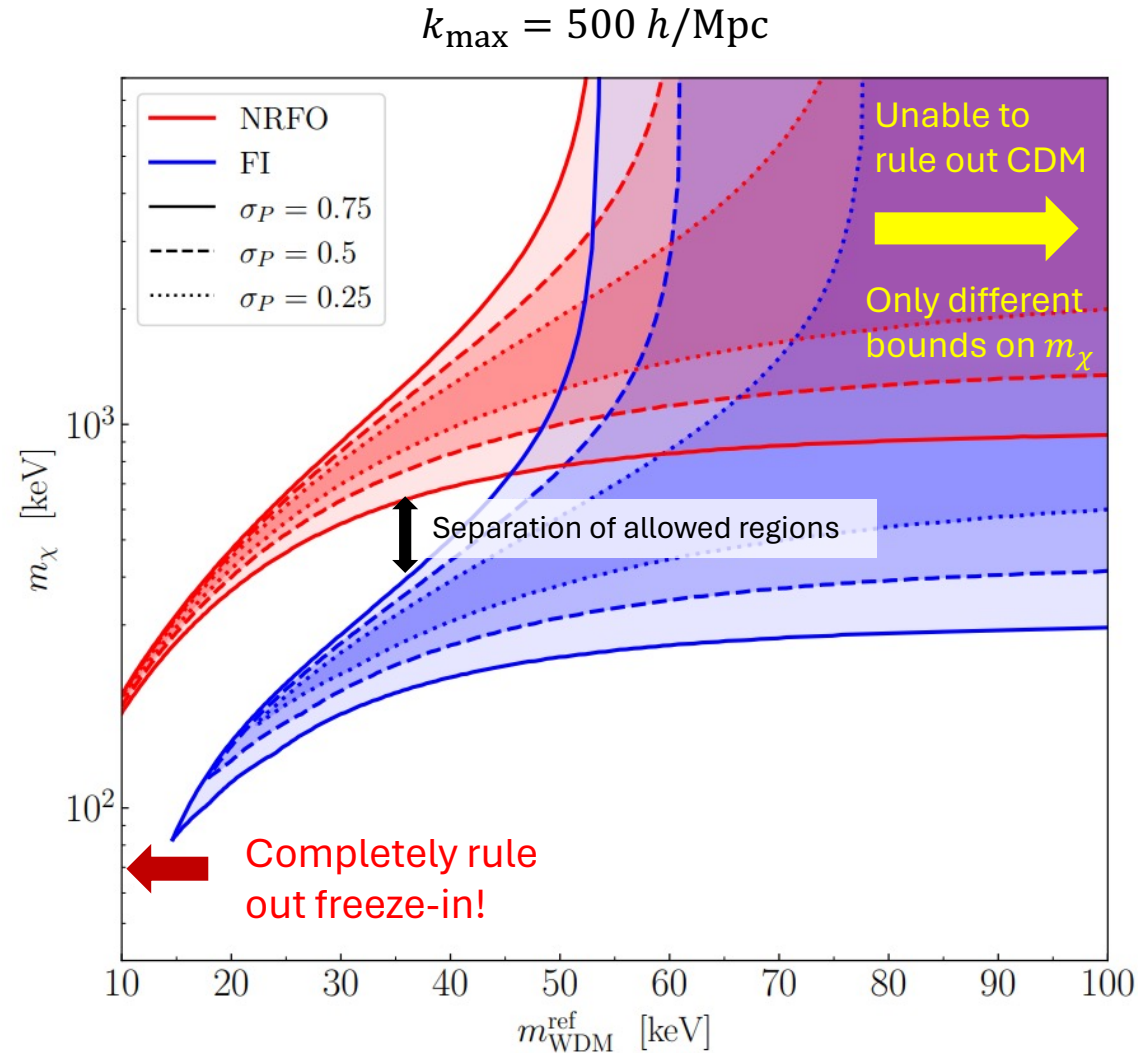
J. Muñoz, C. Dvorkin, F. Cyr-Racine
Phys.Rev.D 101 (2020) 6, 063526



Completely rule out one scenario!

Distinguishing different scenarios

- Assuming $N = 20$ data points evenly distributed on log-scale from $k_{\min} = 1 \text{ h/Mpc}$ to k_{\max}



constant symmetric
relative errors on $P(k)$

Conclusion

- The cosmic structure contains information about the thermal history of dark matter
- Current WDM bounds can be re-interpreted to place constraints on decoupling temperatures (physical quantities relevant for the early universe) in different scenarios
- For different scenarios, these constraints imply different bounds on DM mass for different scenarios.
- If future data observes deviation from CDM predictions, it is potentially possible to discriminate different production mechanisms if there is sufficient precision

