Degenerate n-nbar Oscillation
and
Neutron Star Heating

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Fu, **SFG**, Guo & Wang [arXiv:2405.08591]

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September 28, 2024
CPCS 2024 @ UESTC

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Outline

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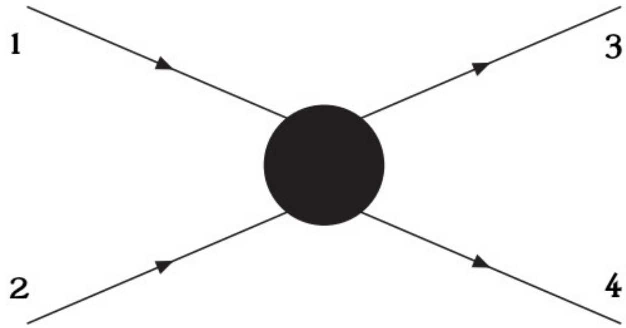
- Degenerate Oscillation
- Neutron-Antineutron Oscillation in NS
- Neutron Star Heating & GUT

Degeneracy in Boltzmann Eq.

Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{p}}\frac{d\mathbf{p}}{dt} = \mathbb{C}[f]$$

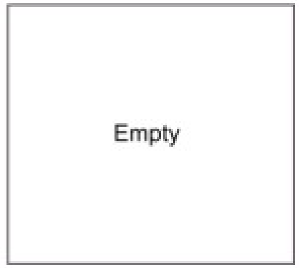


$$\mathbb{C}[f] = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

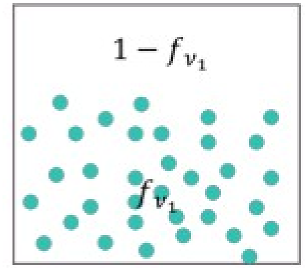
$$|\mathcal{M}|^2 \{ f_1 f_2 [1 \pm f_3][1 \pm f_4] - f_3 f_4 [1 \pm f_1][1 \pm f_2] \}$$

How the degeneracy factor $1 \pm f$ is derived?

Description with 2nd Quantization



vs



$$|N\rangle \equiv \frac{(a^\dagger)^N}{\sqrt{N!}}|0\rangle$$

$$a^\dagger|N\rangle = \sqrt{N+1}|N+1\rangle$$

$$a|N\rangle = \sqrt{N}|N-1\rangle$$

$$a^\dagger a|N\rangle = N|N\rangle$$

$$aa^\dagger|N\rangle = (1 \pm N)|N\rangle$$

$|\Omega\rangle$ $\quad f(\mathbf{x}, \mathbf{p})$

$$N \equiv \int f(\mathbf{x}, \mathbf{p}) d^3\mathbf{x} d^3\mathbf{p}$$

$$n_{\mathbf{p}} \equiv \int f(\mathbf{x}, \mathbf{p}) d^3\mathbf{x}$$

$a^\dagger_{\mathbf{p}}$ $\quad a_{\mathbf{p}}$ $\quad \hat{n}_{\mathbf{p}} = a^\dagger_{\mathbf{p}} a_{\mathbf{p}}$

$$a^\dagger_{\mathbf{p}} a_{\mathbf{p}}|\Omega\rangle = \int d^3\mathbf{x} f(\mathbf{x}, \mathbf{p})|\Omega\rangle$$

$$a_{\mathbf{p}} a^\dagger_{\mathbf{p}}|\Omega\rangle = \int d^3\mathbf{x}[1 \pm f(\mathbf{x}, \mathbf{p})]|\Omega\rangle$$

Degeneracy with External State

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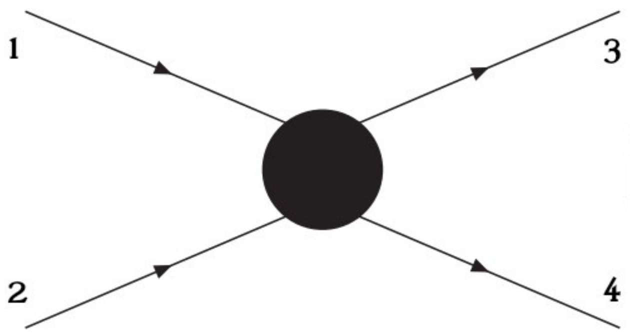

$$|\mathcal{M}|^2\{f_1 f_2[1\pm f_3][1\pm f_4] - f_3 f_4[1\pm f_1][1\pm f_2]\}$$

Initial State:

$$\mathcal{M} \sim \langle \Omega + f | \cdots a | i + \Omega \rangle$$

$$|\mathcal{M}|^2 \sim \mathcal{M}^* \mathcal{M}$$

$$= \langle i + \Omega | \cdots a^\dagger a | i + \Omega \rangle$$

$$\downarrow$$

$$f$$

Final State:

$$\mathcal{M} \sim \langle \Omega + f | a^\dagger \cdots | i + \Omega \rangle$$

$$|\mathcal{M}|^2 \sim \mathcal{M}^* \mathcal{M}$$

$$= \langle i + \Omega | a a^\dagger \cdots | i + \Omega \rangle$$

$$\downarrow$$

$$1 \pm f$$

Degeneracy @ Amplitude Level

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In QFT, fermion mixing is described as

$$\psi_i \equiv \sum_\alpha U^*_{\alpha i} \psi_\alpha \qquad \psi_i \sim a u e^{-i p_i \cdot x} + b^\dagger v e^{i p_i \cdot x}$$

SFG, Chui-Fan Kong, Pedro Pasquini [2310.04077]

Fermion oscillation needs to involve 3 parts:

$$\mathcal{M}_{\beta\alpha} \equiv \mathcal{M}_d \left[\sum_i U_{\beta i} U^*_{\alpha i} e^{i p_i \cdot (x-y)} \langle \Omega | a_{\mathbf{p} i} a^\dagger_{\mathbf{p} i} | \Omega \rangle \right] \mathcal{M}_p$$

Spinors u & v combined into M_p & M_d

$$\downarrow$$

$$1 - f_i$$

Pauli blocking factor already appears in amplitude!

On-Shell Renormalization

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$$\mathcal{M}_{\beta\alpha} \equiv \mathcal{M}_d \left[\sum_i U_{\beta i} U^*_{\alpha i} e^{i p_i \cdot (x-y)} \langle \Omega | a_{\mathbf{p}i} a^\dagger_{\mathbf{p}i} | \Omega \rangle \right] \mathcal{M}_p$$

$$1 - f_i$$

Pauli blocking factor already appears in amplitude!

$$S_F \propto \int \frac{d^4 p}{(2\pi)^4} e^{-ip\cdot(x-y)} \left[\frac{i}{p^2 - m^2 + i\epsilon} - (2\pi)\delta(p^2 - m^2)\Theta(p^0) f_+(\mathbf{p}) \right] (\not p + m_\nu)$$

$$= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_\mathbf{p}} e^{-ip\cdot(x-y)} (\not p + m_\nu) - \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{e^{-ip\cdot(x-y)}}{2E_\mathbf{p}} f_+(\mathbf{p}) (\not p + m_\nu)$$

$$= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_\mathbf{p}} e^{-ip\cdot(x-y)} [1 - f_+(\mathbf{p})] (\not p + m_\nu),$$

On-shell renormalization

$$\frac{i}{\not p - m_i + i\epsilon} \to \frac{i Z_i}{\not p - m_i + i\epsilon}$$

Can also lead to the 1+-f factors in Boltzmann eq.

Degenerate Oscillation

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$$\mathcal{M}_{\beta\alpha} = \mathcal{M}_d \left[\sum_i U_{\beta i} U^*_{\alpha i} e^{i p_i \cdot (x-y)} (1 - f_i) \right] \mathcal{M}_p$$

Events detected w/o oscillation

$$N_\alpha(x = y) \propto \sum_\beta \left| \sum_i U_{\beta i} U^*_{\alpha i} (1 - f_i) \right|^2$$

Events detected after oscillation

$$N_{\alpha\to\beta}(x - y) \propto \left| \sum_i U_{\beta i} U^*_{\alpha i} e^{i p_i \cdot (x-y)} (1 - f_i) \right|^2$$

Fraction of events in β flavor

$$P_{\alpha\beta}(x - y) \equiv N_{\alpha\to\beta}(x - y) / N_\alpha(x = y)$$

Neutrino Oscillation

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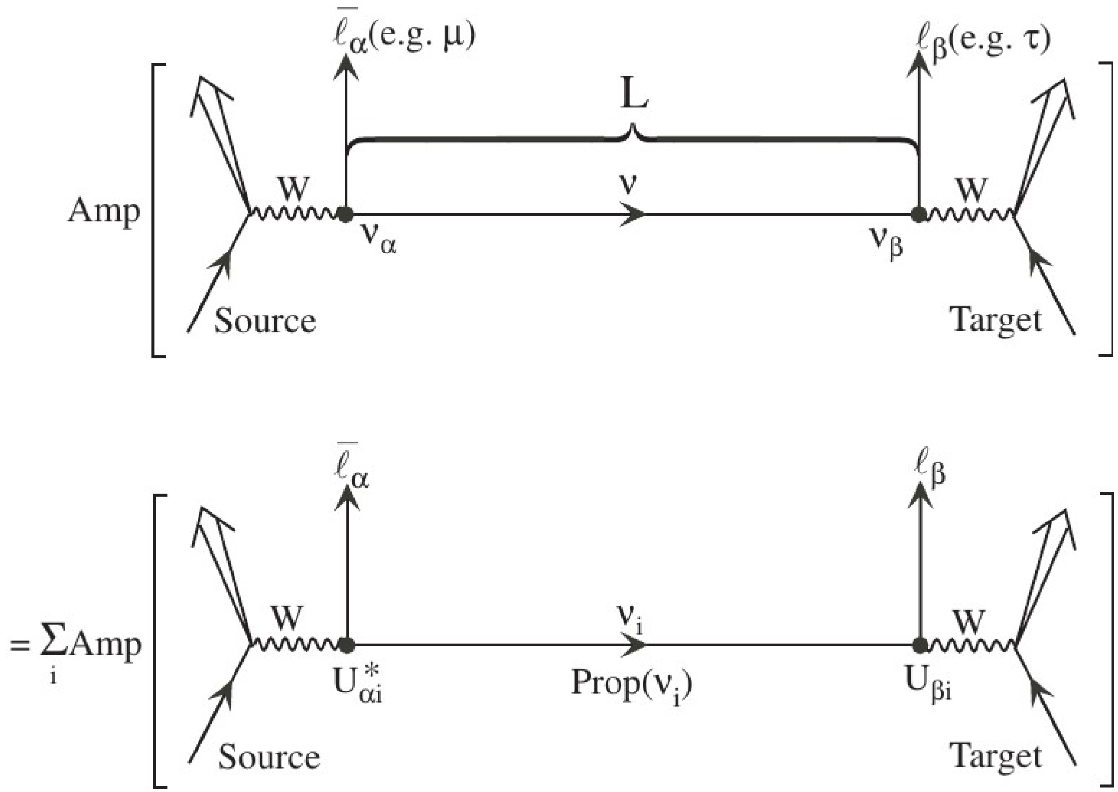

[Kayser, https://arxiv.org/abs/hep-ph/0506165]

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \rightarrow \boxed{\sum_i U_{\alpha i} e^{i(E_i t - \vec{P}_i \cdot \vec{x})} \nu_i} = \boxed{\sum_i U_{\alpha i} P_i U^\dagger_{\beta i} \nu_\beta} \equiv \sum_\beta A_{\alpha\beta} \nu_\beta$$

ν Oscillation in Matter

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PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

Neutrino oscillations in matter

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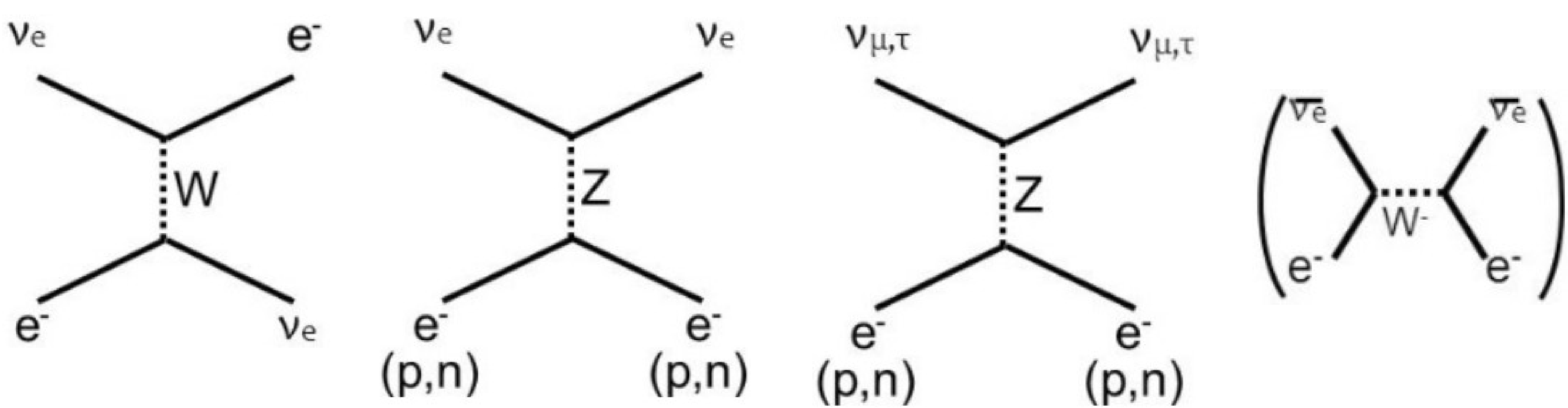
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

$$\mathcal{H} = \frac{M M^\dagger}{2 E_\nu} \pm \mathbf{V}$$



Collective Neutrino Oscillation

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$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\psi = \mathsf{H}\psi$$

$$H = \frac{\Delta m^2}{2E}B + \sqrt{2}G_F n_e \delta_{ee} + \sqrt{2}G_F \int d^3\mathbf{p}'(1-\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

Vacuum | **Matter potential** | **Neutrino potential**

$$\mathsf{B} = \mathsf{U}\left(\frac{1}{2}\mathrm{diag}[-1,\ 1]\right)\mathsf{U}^\dagger = \frac{1}{2}\begin{bmatrix} -\cos 2\theta_\mathrm{v} & \sin 2\theta_\mathrm{v} \\ \sin 2\theta_\mathrm{v} & \cos 2\theta_\mathrm{v} \end{bmatrix}$$

$$[\rho_{\mathbf{p}'}(t,\mathbf{x})]_{\alpha\beta} = \sum_{\nu'} n_{\nu',\mathbf{p}'}(t,\mathbf{x})\langle\nu_\alpha|\psi_{\nu',\mathbf{p}'}(t,\mathbf{x})\rangle\langle\psi_{\nu',\mathbf{p}'}(t,\mathbf{x})|\nu_\beta\rangle,$$

$$[\bar{\rho}_{\mathbf{p}'}(t,\mathbf{x})]_{\beta\alpha} = \sum_{\bar{\nu}'} n_{\bar{\nu}',\mathbf{p}'}(t,\mathbf{x})\langle\bar{\nu}_\alpha|\psi_{\bar{\nu}',\mathbf{p}'}(t,\mathbf{x})\rangle\langle\psi_{\bar{\nu}',\mathbf{p}'}(t,\mathbf{x})|\bar{\nu}_\beta\rangle,$$

Duan, Fuller & Qian [arXiv:1001.2799]

Outline

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- **Degenerate Oscillation**
- **Neutron-Antineutron Oscillation in NS**
- **Neutron Star Heating & GUT**

GUT & Baryon Number Violation

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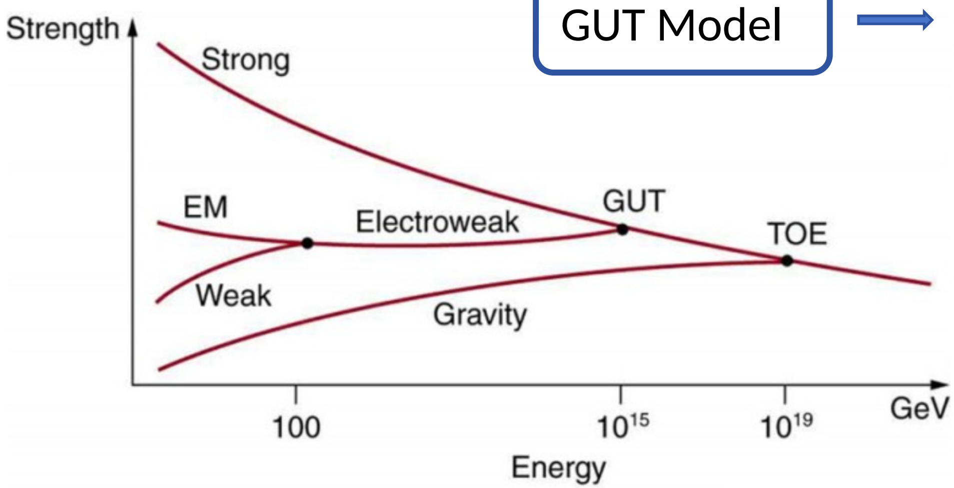

GUT Model → Baryon Number Violation

- Proton Decay
- Neutron-Antineutron Oscillation

Table 1

GUT model	Is $N-\bar{N}$ observable?	Implications
(NON SUSY)		
$SU(5)$	No	$\Delta(B-L)=0$
$SU(2)_L \times SU(2)_R \times SU(4)_c$	Yes	$M_c \simeq 10^5$ GeV
Minimal SO(10)	No	
E_6	No	
(SUSY GUT)		
$[SU(3)]^3$	Yes	Induced breaking of R-parity
$SO(10)$	No	

Table Caption: This table summarizes the observability of neutron-anti-neutron oscillation in various GUT models.

Degenerate n-nbar Oscillation

Being neutral, neutron can have Majorana mass term:

$$\mathrm{H} \approx \begin{pmatrix} H_{11} & \delta m \\ \delta m & H_{22} \end{pmatrix}$$

Mixing between neutron & antineutron

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \mathrm{U} \begin{pmatrix} n \\ \bar{n} \end{pmatrix} \qquad \mathrm{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \tan 2\theta = \frac{2\delta m}{H_{22} - H_{11}}$$

Neutron-antineutron oscillation!

$$P_{n\bar{n}} = \frac{c^2 s^2 (1-f_1)^2 + c^2 s^2 (1-f_2)^2}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2}$$

$$- \frac{2(1-f_1)(1-f_2) c^2 s^2 \cos(\Delta E t)}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2} \qquad \Delta E \equiv \sqrt{(H_{11} - H_{22})^2 + 4\delta m^2}$$

Degenerate n-nbar Oscillation

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$$P_{n\bar{n}} = \frac{c^2 s^2 (1-f_1)^2 + c^2 s^2 (1-f_2)^2}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2}$$

$$- \frac{2(1-f_1)(1-f_2) c^2 s^2 \cos(\Delta E t)}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2}$$

reduces to the usual one with $f_i \to 0$

$$P_{n\bar{n}} = 4 c^2 s^2 \sin^2 \left(\frac{\Delta E t}{2} \right)$$

in a dense neutron environment $f_1 \to 1, f_2 \to 0$

$1 - f_1 \ll s \ll 1$ $\quad P_{n\bar{n}} \to c^2 \sim 1$ ✗

$s \ll 1 - f_1 \ll 1$ $\quad P_{n\bar{n}} \to \frac{s^2}{(1-f_1)^2} \ll 1$ ✓

Degenerate n-nbar Oscillation

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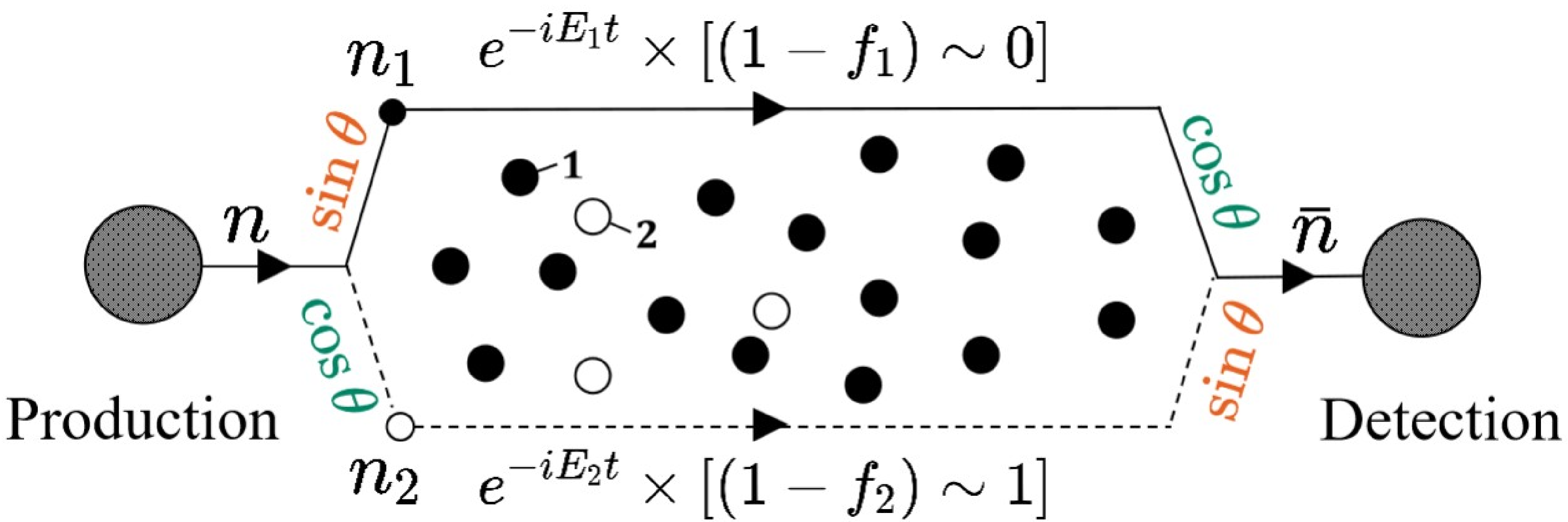
$$P_{n\bar{n}} \to \frac{s^2}{(1-f_1)^2}$$

Degeneracy can enhance the oscillation probability



Zero Distance Limit

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$$P_{n\bar{n}} \approx \frac{s^2}{(1-f_1)^2} \qquad P_{nn} \approx c^2$$

The degeneracy effect already appears at zero distance!

Standing fraction of antineutron in neutron star.

$$f_1(\mathbf{p}) = \frac{1}{e^{\frac{\varepsilon_n(\mathbf{p})-\mu}{T}}+1} \approx f_n$$

$$R(\mathbf{p}) \equiv \frac{f_{\bar{n}}}{f_n} = \frac{P_{n\bar{n}}}{P_{nn}} \approx \frac{\tan^2\theta}{[1-f_1(\mathbf{p})]^2} \qquad f_{\bar{n}} \approx \frac{s^2}{[1-f_1(\mathbf{p})]^2} f_1$$

Note that only neutron is in thermal equilibrium!

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Neutron Degeneracy in NS

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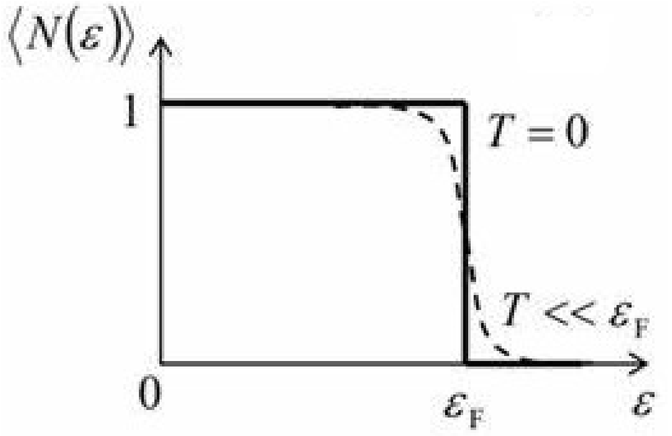
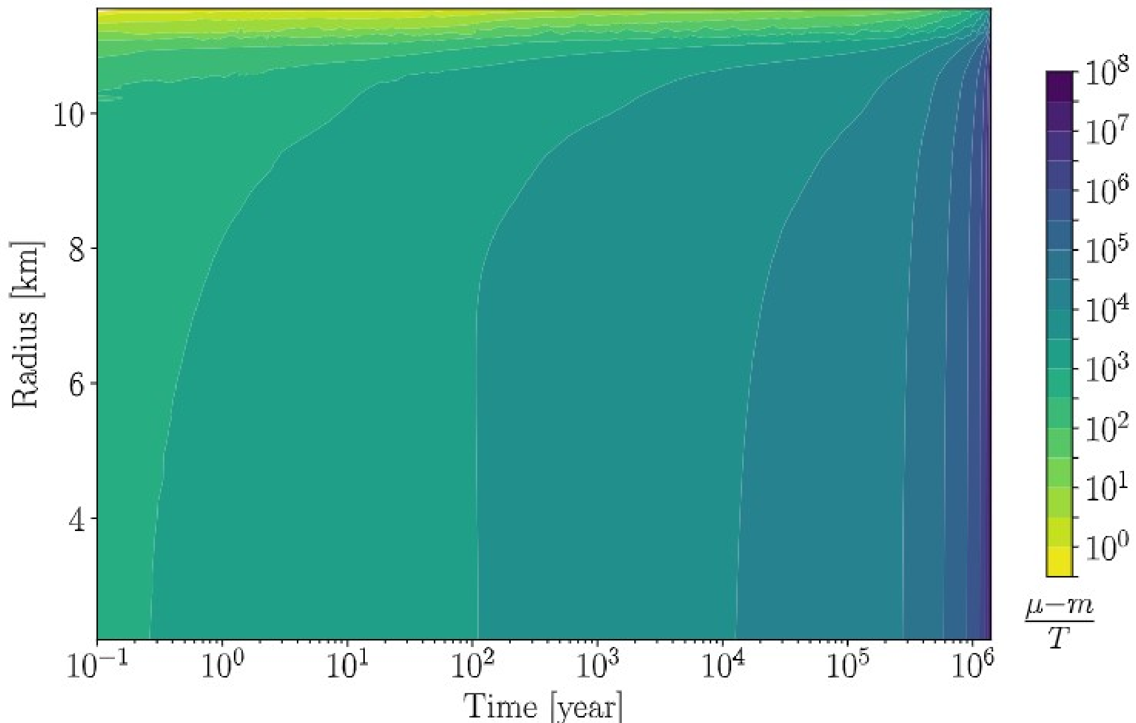
$$f_1(\mathbf{p}) = \frac{1}{e^{\frac{\varepsilon_n(\mathbf{p})-\mu}{T}} + 1} \qquad \varepsilon_n(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}$$

Degenerate fermi gas:

$$k_F = (3\pi^2 n_n)^{1/3} \qquad \mu = \sqrt{k_F^2 + m^2}$$





Fu, **SFG**, Guo & Wang [arXiv:2405.08591]

Antineutron Distribution

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$$f_{\bar{n}} \approx \frac{s^2}{[1 - f_1(p)]^2} f_1$$

$$n_{\bar{n}} = \frac{m^2 T}{2\pi^2} e^{\frac{\mu}{T}} \left[2K_2\left(\frac{m}{T}\right) + e^{\frac{\mu}{T}} K_2\left(\frac{2m}{T}\right)\right] s^2$$

Huge enhancement

$$\frac{\mu - m}{T} \sim \mathcal{O}(10^4) \quad \Rightarrow \quad e^{\frac{\mu - m}{T}} \sim 10^{\mathcal{O}(1000)}$$

Antineutron Annihilation

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During a single annihilation, nucleon number reduces by 2

$$\mathrm{d}N = -2\langle \sigma v \rangle n_n n_{\bar{n}} \mathrm{d}t \mathrm{d}V$$

Replacing the antineutron number density $n_{\bar{n}} = R n_n$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{2R}{1+R} n_n \langle \sigma v \rangle N \equiv -\Gamma N$$

$$n_n = 10^{38}\,\mathrm{cm}^{-3}$$

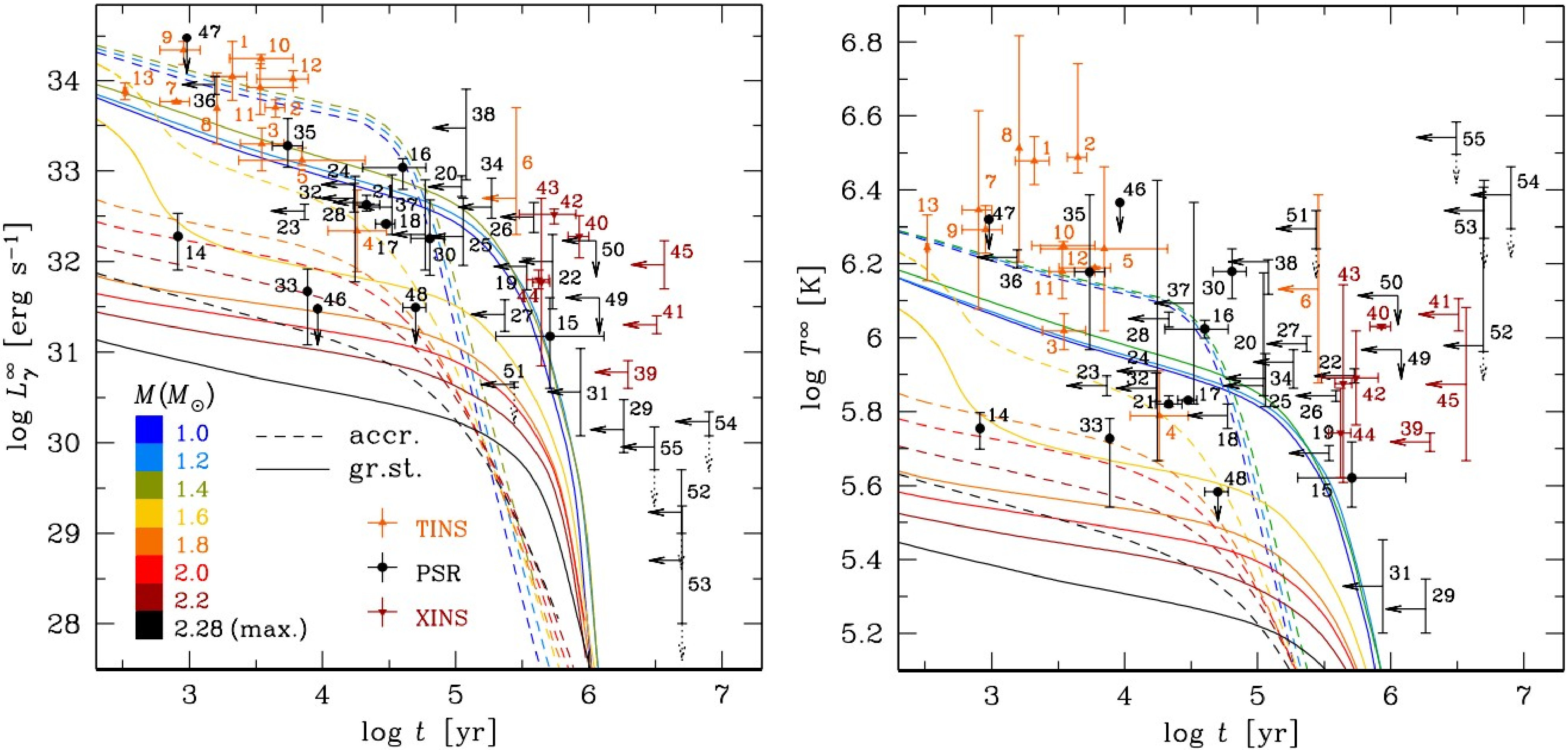
$$\langle \sigma v \rangle \approx 10^{-15}\,\mathrm{cm}^3/\mathrm{s}$$

$$\Gamma \approx 2R \times 10^{23}\,\mathrm{s}^{-1}$$

NS lifetime > 10^6 yrs

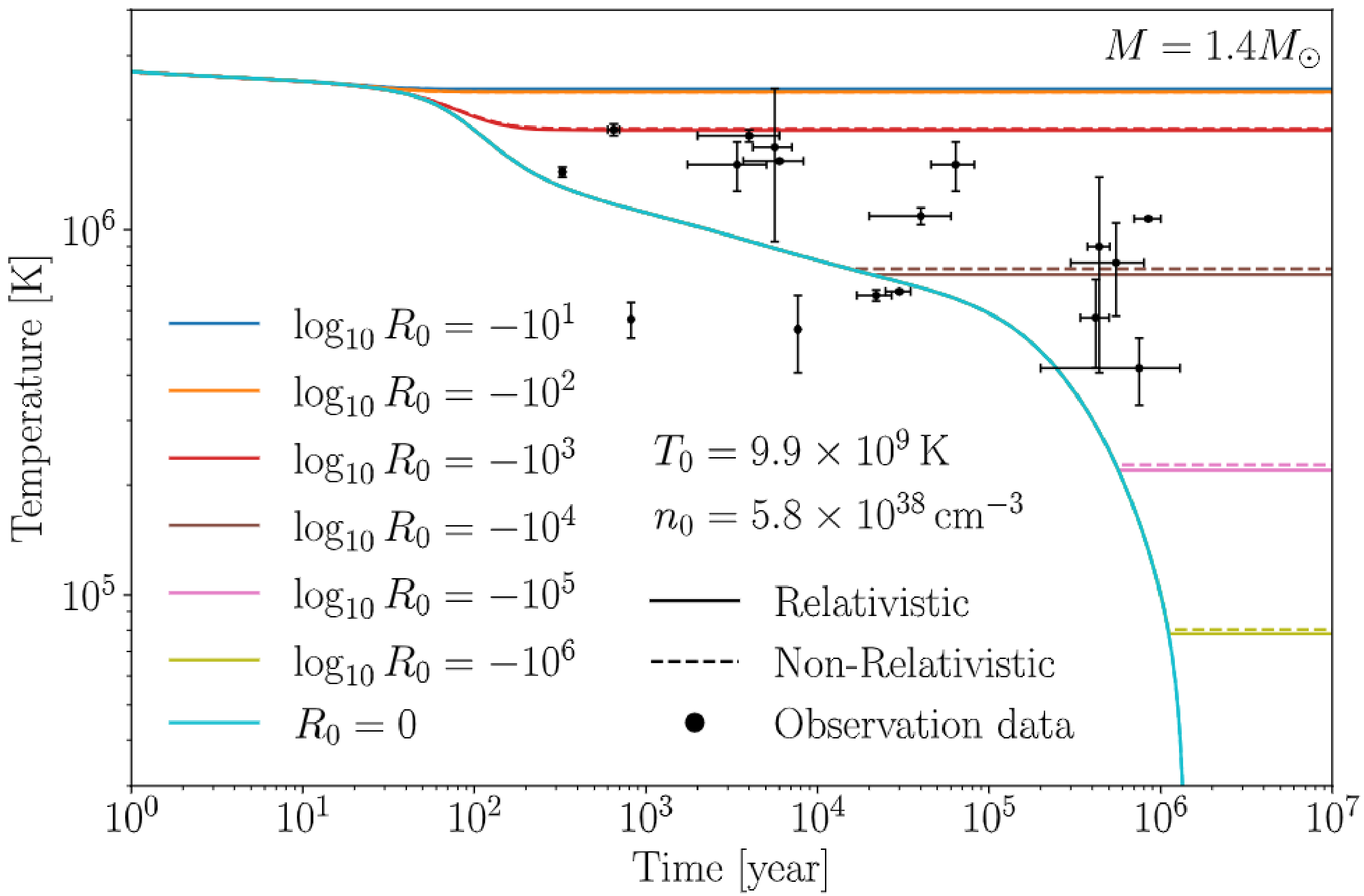
$$R \approx \frac{s^2}{[1 - f_1(\mathbf{p})]^2} \lesssim 10^{-37}$$

NS Lifetime & Temperature Data



Potekhin, Zyuzin, Yakovlev, Beznogov & Shibanov
MNRAS 496, 5052-5071 (2020) [arXiv:2006.15004]

NS Heating



$$R \approx \left(\frac{T}{T_0}\right)^{\frac{3}{2}} e^{\frac{2(\mu - m)}{T} - \frac{2(\mu_0 - m)}{T_0}} R_0 \lesssim 10^{-43}$$

NS Heating

$$R \approx \left(\frac{T}{T_0}\right)^{\frac{3}{2}} e^{\frac{2(\mu-m)}{T} - \frac{2(\mu_0-m)}{T_0}} R_0 \lesssim 10^{-43}$$

$$\log_{10} R_0 \lesssim -\mathcal{O}(10^4) \qquad R_0 \equiv R(T_0, n_0)$$

$$T_0 \equiv 9.9 \times 10^9 \mathrm{K}$$

$$s^2 \lesssim 10^{-\mathcal{O}(10^4)}$$

$$n_0 \equiv 5.8 \times 10^{38} \mathrm{cm}^{-3}$$

Heating power

$$2 R n_n^2 \langle \sigma v \rangle V m \quad \sim 10^{24} \mathrm{W}$$

Cooling power

$$4\pi R^2 \sigma T^4 \sim 10^{21}\, \mathrm{W}$$

Constraints on GUT

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$$\mathrm{H} \approx \begin{pmatrix} H_{11} & \delta m \\ \delta m & H_{22} \end{pmatrix} \qquad s \sim \frac{\delta m}{\Delta H}$$

$$\delta m \sim \frac{\Lambda_{\mathrm{QCD}}^6}{M_X^5} \qquad \Lambda_{\mathrm{QCD}} \sim 180\,\mathrm{MeV}$$

$$\Delta H = \mathcal{O}(\mathrm{MeV}) \sim \mathcal{O}(\mathrm{GeV})$$

$$s^2 \lesssim 10^{-\mathcal{O}(10^4)} \Rightarrow M_X > 10^{\mathcal{O}(1000)}\mathrm{GeV}$$

which is far beyond the Planck scale!

Summary

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- **Degenerate Oscillation**

  1. Consistent picture of degeneracy in external & intermediate states

- **Neutron-Antineutron Oscillation in NS**

  1. Concrete realization with n-nbar oscillation
  2. Standing fraction of antineutron

- **Neutron Star Heating & GUT**

  1. Degeneracy enhancement
  2. Very strong constraint

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Thank You

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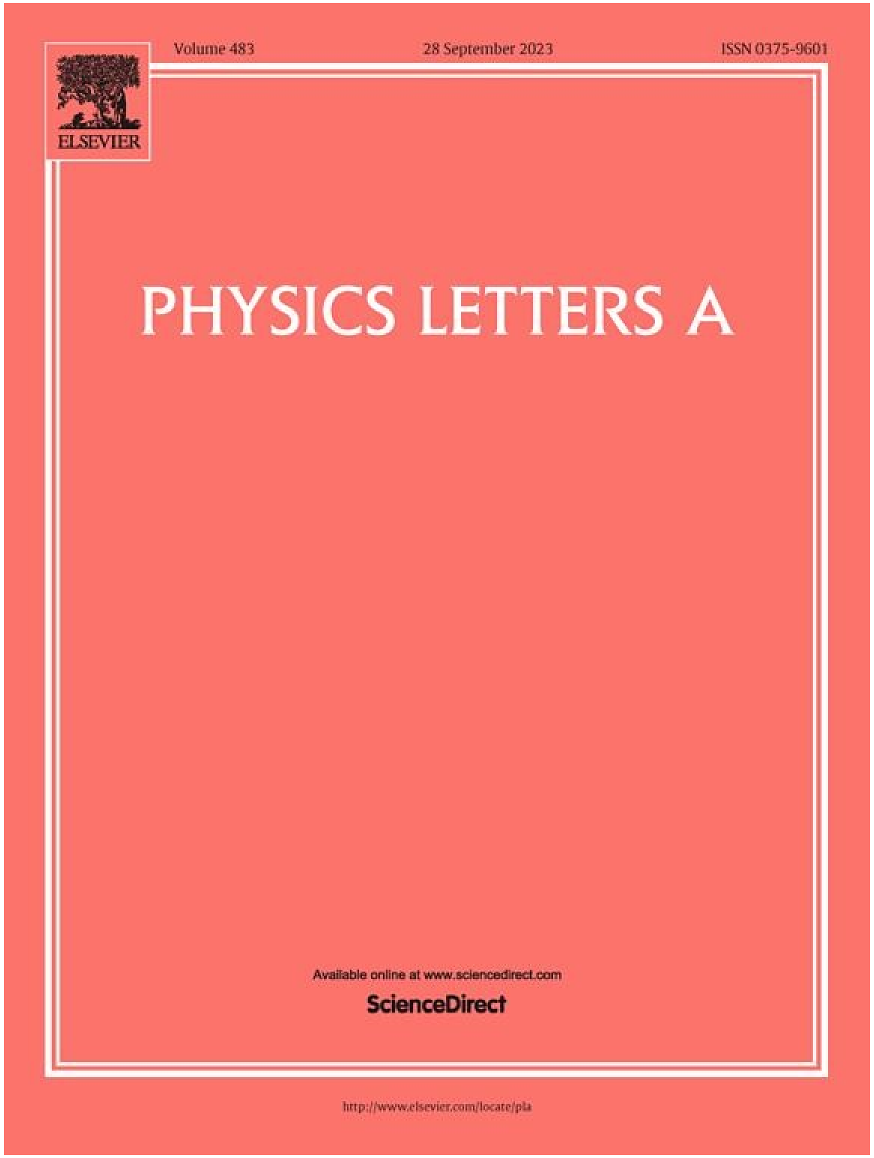

Aims & Scope

- Nonlinear science,
- Statistical physics,
- Mathematical and computational physics,
- AMO and physics of complex systems,
- Plasma and fluid physics,
- Optical physics,
- General and cross-disciplinary physics,
- Biological physics and nanoscience,
- Astrophysics, Particle physics and Cosmology.

Superfluidity in NS