Prospects of measuring recoil force from neutrino emission using micromechanical resonators

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Particle Physics on Tabletops

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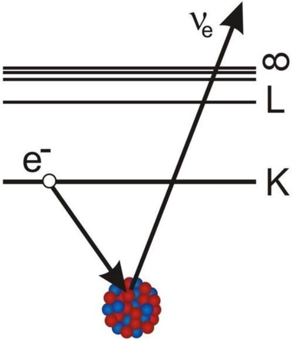
Electron capture

$$\underbrace{(A,Z)+e^-}_{\text{bound}} \to (A,Z-1)+\nu_e$$



EC from K shell

Typical contributions of atomic shells to decay rate

- K $\sim 90\%$
- $L \sim 10\%$
- $M \sim 1\%$

Kinematics

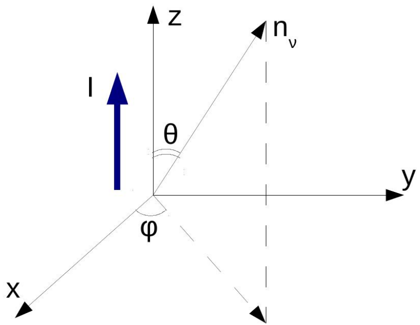
- $E_\nu \approx Q_{EC} \sim 1\,\text{MeV}$
- Nuclear recoil $T_N \lesssim 60\,\text{eV}$
- $|\boldsymbol{p}_N| = |\boldsymbol{p}_\nu| \approx E_\nu$

Recoil force from neutrino radiation in EC

Angular distribution

$$\frac{dw(\boldsymbol{n}_\nu)}{d\Omega} = \frac{w}{4\pi}\,(1 + BP\eta\cos\theta)$$

- B is the asymmetry coefficient
- P is the nuclear polarization
- η incorporates effects of m_ν
- $w = \ln 2/T_{1/2}$ is the total decay rate



Force acting on a radioactive sample

$$\boldsymbol{F} = -N \cdot p_\nu \int d\Omega \cdot \boldsymbol{n}_\nu \cdot \frac{dw(\boldsymbol{n}_\nu)}{d\Omega}$$

First study of the effect:

C. DeAngelis+ PRC **86**, 034615 (2012)

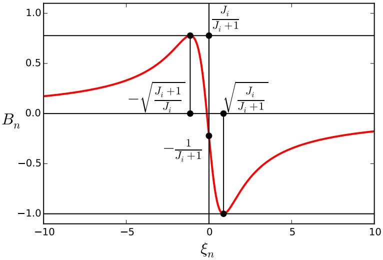
More detailed force calculation:
A. L. Barabanov & **OT**
PRC **99**, 045502 (2019)

Calculation of recoil force

Angular asymmetry (allowed transitions)

$$B = \begin{cases} \dfrac{J_i}{J_i+1}, & J_f = J_i + 1 \\ -1, & J_f = J_i - 1 \\ -\dfrac{1 + 2\sqrt{J_i(J_i+1)}\,\xi}{(J_i+1)(1+\xi^2)}, & J_f = J_i \neq 0 \end{cases}$$

$$\xi \equiv \frac{g_V M_F}{g_A M_{GT}}$$



Effect of m_ν on angular distribution

$$\eta = \frac{c \sum_x p_{\nu x}^2 |\psi_x(0)|^2}{\sum_x p_{\nu x} E_{\nu x} |\psi_x(0)|^2} \simeq 1 - \frac{1}{2}\left(\frac{m_\nu c^2}{E_\nu}\right)^2 \leq 1, \quad \eta = 1 \text{ for } m_\nu = 0$$

Calculation of recoil force

Nuclear polarization

$$P \simeq \frac{\beta(J_i+1)}{3J_i} \quad \text{for } \beta \equiv \frac{\mu B_0}{k_B T} \ll 1$$

Recoil force z-projection ($m_\nu = 0$)

$$F_z = -\frac{B}{3}\frac{\ln 2\, I_{EC}}{T_{1/2}}\frac{E_\nu}{c} N P = -\frac{B}{3}\frac{E_\nu}{c} I_{EC} P \alpha$$

- I_{EC} is the relative transition probability
- α is the source activity

Parametrization

$$F_z = -m\,\frac{B[\text{T}]}{T[\text{K}]}\cdot C\cdot f$$

$$f_n = \frac{\beta_0\, I_{EC} \ln 2}{9\,T_{1/2}} \cdot \frac{E_\nu}{m_a c} \cdot \frac{\mu}{\mu_N}$$

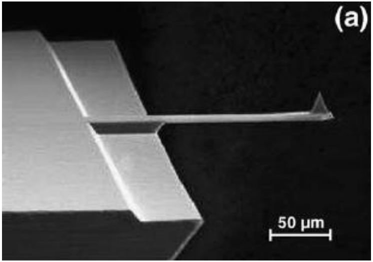
$$C = B(J_i+1)/J_i$$

Force measurement

Idea (C. DeAngelis+, 2012)

- Attach sample to cantilever
- Force is determined from Hooke's law: $z = F/k$
- Can measure $F > F_{min} \sim 10^{-12}$ N ($k = 0.2\,$N/m)
- High nuclear polarization is required $P \sim 100\%$ ($\mu B_0 \sim k_B T$)
- Source activity required $\alpha \geq 1$ GBq

- In practice $B_0 \lesssim 10$ T, $T \sim 1$ K $\Rightarrow P \lesssim 1\%$
- Required sample mass could be larger than cantilever mass



Typical commercial Si single cantilever. (A. Suter, 2004)

$V = 125 \times 35 \times 4\,\mu\text{m}^3$, $k = 40\,$N/m

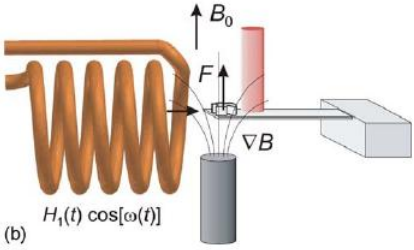
$$F_{\text{min}} = \frac{\sqrt{2kk_BT}}{Q}$$

Use of magnetic resonance force microscopy

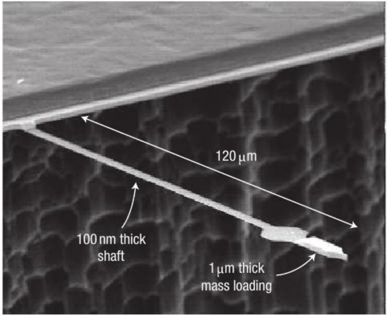
Possible improvements:

1. Increase recoil force (due to higher P)
2. Reduce measurement threshold

For lowering the threshold, one can use MRFM



$$F_z = M_z \nabla B(z)$$



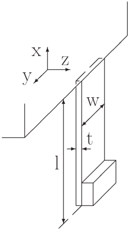
Use of magnetic resonance force microscopy

Static measurement

- Constant field B_0
- $z = F/k$
- $F > F_{\text{min}} \sim 10^{-12}$ N
- Polarization $P \sim 100\%$
- Activity $\alpha \geq 1$ GBq
- For ^{119}Sb $m = 4 \cdot 10^{-7}$ g

Resonance

- Constant field B_0 + oscillating field $b(t)$
- $z = FQ/k$, $Q \leq 10^5$
- $F > F_{\text{min}} \sim 10^{-19}$ N
- Polarization $P \sim 0.1 - 1\%$
- Activity $\alpha \sim 1$ MBq
- For ^{119}Sb $m = 7 \cdot 10^{-12}$ g



$$k = Ewt^3/4l^3$$
$$F_{\text{min}} = \sqrt{2kk_BT}/Q$$

Optimal source isotopes

Selection criteria

- $J_i \neq 0$
- Decay from ground state
- Allowed transition to a single state with $I_{EC} \geq 0.98$
- Force parameter
$$f_n = F/m \geq 10^{-13} \text{ N/g}$$
- Force fluctuations are small:
$$\frac{\alpha}{\nu_c} \geq 100$$
- Heating power due to secondary radiation should be small

Pure Gamow–Teller transitions to ground state

${}^{A}X_i \to {}^{A}X_f$ $J_i^\pi \to J_f^\pi$	$T_{1/2}$ I_{0EC}, %	μ/μ_N E_0^*, keV	Q_{0EC}, keV $E_{\nu 0}$, keV	f_n, N/g $m_{\min}$, g
^{163}Er $\to$ ^{163}Ho $5/2^- \to 7/2^-$	75.0 min 99.89	$+0.557$ 0	1211 1164	$8.0 \cdot 10^{-9}$ $1.3 \cdot 10^{-12}$
^{135}La $\to$ ^{135}Ba $5/2^+ \to 3/2^+$	19.5 h 98.1	$+3.70$ 0	1207 1175	$4.1 \cdot 10^{-9}$ $3.6 \cdot 10^{-12}$
^{165}Er $\to$ ^{165}Ho $5/2^- \to 7/2^-$	10.36 h 100	$+0.643$ 0	377 332	$3.1 \cdot 10^{-10}$ $3.2 \cdot 10^{-11}$
^{131}Cs $\to$ ^{131}Xe $5/2^+ \to 3/2^+$	9.69 d 100	$+3.543$ 0	355 325	$9.5 \cdot 10^{-11}$ $7.5 \cdot 10^{-11}$
^{71}Ge $\to$ ^{71}Ga $1/2^- \to 3/2^-$	11.43 d 100	$+0.547$ 0	233 223	$1.6 \cdot 10^{-11}$ $6.3 \cdot 10^{-10}$
^{55}Fe $\to$ ^{55}Mn $3/2^- \to 5/2^-$	2.74 y 100	$+2.7$ 0	231 225	$1.2 \cdot 10^{-12}$ $8.6 \cdot 10^{-9}$
^{179}Ta $\to$ ^{179}Hf $7/2^+ \to 9/2^+$	1.82 y 100	$+2.289$ 0	106 71	$1.4 \cdot 10^{-13}$ $7.0 \cdot 10^{-8}$

Mixed transitions to ground state

$^{A}X_i \to {}^{A}X_f$ $J_i^\pi \to J_f^\pi$	$T_{1/2}$ I_{nEC}, %	μ/μ_N E_n^*, keV	m, g Q_{nEC}, keV	N $E_{\nu n}$, keV	α, MBq f_n, N/g
^{37}Ar $\to$ ^{37}Cl	35.01 d	+1.145	$1.0 \cdot 10^{-10}$	$1.6 \cdot 10^{12}$	0.37
$3/2^+ \to 3/2^+$	100	0	814	811	$7.5 \cdot 10^{-11}$
^{49}V $\to$ ^{49}Ti	330 d	4.47	$1.0 \cdot 10^{-9}$	$1.2 \cdot 10^{13}$	0.30
$7/2^- \to 7/2^-$	100	0	602	597	$1.7 \cdot 10^{-11}$
^{7}Be $\to$ ^{7}Li*, ^{7}Li	53.22 d	−1.399	$1.0 \cdot 10^{-10}$	$8.6 \cdot 10^{12}$	1.29
$3/2^- \to 1/2^-$	10.44	477.6	384	384	$1.6 \cdot 10^{-11}$
$3/2^- \to 3/2^-$	89.56	0	862	862	$3.0 \cdot 10^{-10}$
^{51}Cr $\to$ ^{51}V*, ^{51}V	27.70 d	−0.93	$1.0 \cdot 10^{-9}$	$1.2 \cdot 10^{13}$	3.42
$7/2^- \to 5/2^-$	9.93	320.1	432	427	$3.0 \cdot 10^{-12}$
$7/2^- \to 7/2^-$	90.07	0	752	748	$4.7 \cdot 10^{-11}$
^{65}Zn $\to$ ^{65}Cu*, ^{65}Cu	243.9 d	+0.769	$1.0 \cdot 10^{-7}$	$9.3 \cdot 10^{14}$	30.50
$5/2^- \to 5/2^-$	50.04	1115.6	236	228	$5.8 \cdot 10^{-13}$
$5/2^- \to 3/2^-$	48.54	0	1352	1344	$3.3 \cdot 10^{-12}$

Possible applications

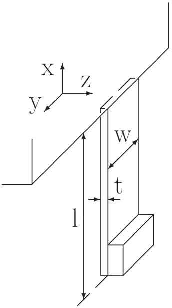
$$\boldsymbol{F} = -N \cdot p_\nu \int d\Omega \cdot \boldsymbol{n}_\nu \cdot \frac{dw(\boldsymbol{n}_\nu)}{d\Omega}$$

- Neutrino mass:

$$\frac{F(m_\nu \neq 0)}{F(m_\nu = 0)} \simeq 1 - \left(\frac{m_\nu c^2}{Q_{EC}}\right)^2, \quad \delta F \sim 10^{-10}$$

- BSM physics: extra terms in $dw(\boldsymbol{n}_\nu)/d\Omega$
  Example: Lorentz violation

$$\frac{dw_{nEC}}{d\Omega} = \frac{w_{nEC}}{4\pi}\left(1 + B_n P\left(\boldsymbol{n}_\nu \boldsymbol{n}_J + \chi_i^{l0}\left[\boldsymbol{n}_\nu \times \boldsymbol{n}_J\right]_l\right)\right)$$



Possible applications

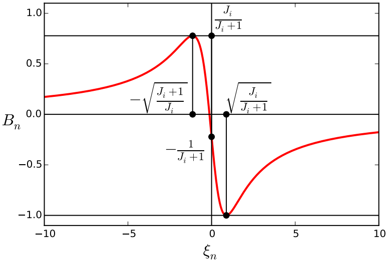
$$\boldsymbol{F} = -N \cdot p_\nu \int d\Omega \cdot \boldsymbol{n}_\nu \cdot \frac{dw(\boldsymbol{n}_\nu)}{d\Omega}$$

- Relative capture probabilities $P_K,\ P_L, \ldots$:

$$\begin{cases} P_K E_{\nu K} + P_L E_{\nu L} = E_\nu (= p_\nu c) \\ P_K + P_L = 1 \end{cases}$$

- Fermi and Gamow–Teller mixing:

$$\xi \equiv \frac{g_V M_F}{g_A M_{GT}}$$



Summary

- There is a recoil force caused by neutrino emission in EC
- Formula for the force is obtained for allowed nuclear transitions
- The force can be measured with micromechanical devices
- Applying methods of magnetic resonance force microscopy increases sensitivity
- Information about weak interactions, atomic and nuclear structure can be probed
- The proposed experiment can complement existing experiments

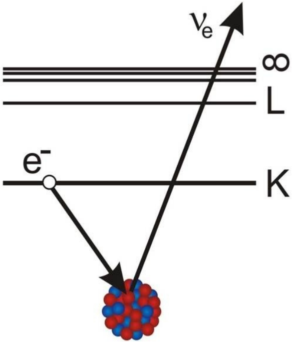
Thank you!

Backup slides

Electron capture

Main process

$$\underbrace{(A,Z)+e^-}_{\text{bound}} \rightarrow (A,Z-1)+\nu_e$$



Secondary processes

- Atomic de-excitation: X-rays + Auger e^-
- Nuclear de-excitation: γ + internal conversion e^-

Competing processes

- β^+ decay $(A,Z) \rightarrow (A,Z-1)+e^+ +\nu_e$
- Radiative EC
  $(A,Z)+e^- \rightarrow (A,Z-1)+\nu_e+\gamma$
- Internal ionization
  $(A,Z)+e^- \rightarrow (A,Z-1)+\nu_e+e^-$

Calculation of recoil force: details

$$|J_i\rangle = \sum_{M_i} a_{M_i}(J_i)|J_i M_i\rangle, \quad \sum_{M_i} |a_{M_i}(J_i)|^2 = 1$$

$$dw_{nEC}(\mathbf{n}_\nu) = \sum_x dw_{nx}(\mathbf{n}_\nu) = \frac{2\pi}{\hbar} \sum_{x M_f \sigma_e \sigma_\nu} \left| \langle n J_f M_f | \sum_j \hat{h}_j(\sigma_e, \sigma_\nu) | J_i \rangle \right|^2 \frac{pEd\Omega}{(2\pi\hbar)^3 c^2}$$

$$\hat{h}_j(\sigma_e, \sigma_\nu) = \frac{G_F V_{ud}}{\sqrt{2}} e^{-i\frac{\mathbf{p}_{\nu n x}\mathbf{r}_j}{\hbar}} \left(g_A \, \mathbf{j}(\sigma_e, \sigma_\nu) \boldsymbol{\sigma}_j + i g_V \, j_4(\sigma_e, \sigma_\nu) \right) \hat{\tau}_{j-}$$

$$\langle a_{M_i}(J_i) a^*_{M_i'}(J_i) \rangle = \langle |a_{M_i}(J_i)|^2 \rangle \, \delta_{M_i M_i'}, \quad P = \frac{\langle M_i \rangle}{J_i}, \quad \langle M_i \rangle = \sum_{M_i} M_i \langle |a_{M_i}(J_i)|^2 \rangle$$

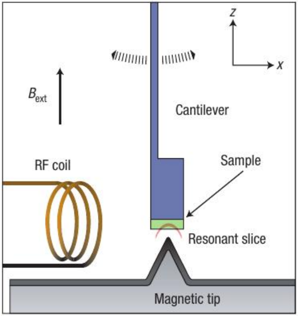
$$j_\lambda(\sigma_e, \sigma_\nu) = i u_\nu^\dagger(\sigma_\nu) \gamma_4 \gamma_\lambda (1 + \gamma_5) u_e(\sigma_e) \psi_x(0)$$

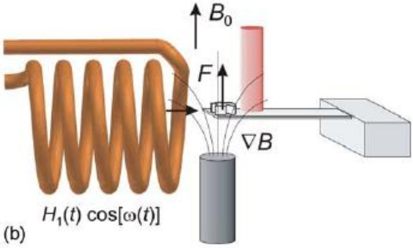
$$u_e(\sigma_e) = \begin{pmatrix} \varphi_e(\sigma_e) \\ 0 \end{pmatrix}, \quad u_\nu(\sigma_\nu) = \sqrt{\frac{E + m_\nu c^2}{2E}} \begin{pmatrix} \varphi_\nu(\sigma_\nu) \\ \dfrac{c\,\boldsymbol{\sigma}\mathbf{p}}{E + m_\nu c^2} \varphi_\nu(\sigma_\nu) \end{pmatrix}$$

$$\langle n J_f M_f | \textstyle\sum_j \sigma_{jq} \hat{\tau}_{j-} | J_i M_i \rangle = \sqrt{\frac{2J_i+1}{2J_f+1}} \, C^{J_f M_f}_{J_i M_i 1 q} \, M_{GT}$$

$$\langle n J_f M_f | \textstyle\sum_j \hat{\tau}_{j-} | J_i M_i \rangle = \delta_{J_f J_i} \delta_{M_f M_i} \, M_F$$

Magnetic resonance force microscopy





$$F_z = M_z \nabla B(z)$$

Magnetic resonance force microscopy

Cyclic adiabatic inversion