Effetive Field Theory (EFT)

- Why EFT ?
- SM (Ren + Nonren) as an EFT
- EFT for Dark Matter Physics

Why EFT ? (weak coupling case)

- We don't know what happens at energy higher than it is affordable
- High Energy physics can leave footprints in low energy regime, which can be adequately described by effective lagrangian with an infinite tower of local operators
- If new physics scale is much higher than experimental energy scale, the lowest dim nonrenormalizable operators will give the dominant corrections to the SM prdictions
  Fermi's theory of weak interaction is a good example
- One can do meaningful phenomenology with a few number of unknown parameters
- Existing proof : the very most successful SM down to $r \lesssim 10^{-18}$ m
- In any case, we are living with EFT any way in real life

Why EFT ? (strong coupling case)

- In a strongly coupled theory such as QCD where nonperturbative aspects are very important, it is ususally very difficult to solve a problem
- Very often physical dof is different from fields in the lagrangian
  (quarks and gluon vs. hadrons in QCD)
- Useful (often critical) to construct EFT based on the symmetries of the underlying strongly interacting theory, using the relevant dof only
- Most important to identify the relevant dof and relevant symmetries
- Many examples in QCD: chiral lagrangian [$+$ (axial) vector mesons, heavy hadrons], NRQCD for heavy quarkonium, HQET for heavy hadrons, SCET etc.

Naive Dimensional Analysis

- Natural Units in HEP:

$$c = \hbar = 1 \to [\vec{L} = \vec{r} \times \vec{p}] = 0$$

$$[L] = [T] = [\vec{p}]^{-1}$$

$$E = \sqrt{(pc)^2 + (mc^2)^2} \longrightarrow E = \sqrt{p^2 + m^2},$$

$$\text{QM Amp} \sim \int_{\text{path}} e^{iS/\hbar} \longrightarrow [\text{Action}] = 0 = [\int d^4x \mathcal{L}]$$

- $[E] = [p] = [M] = [L]^{-1} = [T]^{-1}$
- Everything will be in mass dimensions:

$$[\mathcal{L}] = 4, \quad [\sigma(= \text{Area})] = -2, \quad [\tau(= \Gamma^{-1})] = -1$$

- Both the decay rate ($\Gamma \equiv \tau^{-1}$) and the cross section (σ) are given by

  Fermi's Golden Rule

  with suitable flux facors

$$|\mathcal{M}|^2 \times \text{phase space} \left(\equiv \Pi_{i=1^n} \frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}) \right) \times (2\pi)^4 \delta(\sum_i p_i - \sum_f p_f)$$

- Note that $[\Gamma] = +1$ and $[\sigma] = -2$
- It is often enough to do the dimensional analysis for Γ and σ, when there is only one important mass scale from the phase space integration
- A number of easy examples will be given in this lecture

Scalar fields

- Lagrangian for a real scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \mu\phi^3 - \frac{\lambda}{4}\phi^4 + \sum_{i=1}^{\infty}\frac{C_{4+i}}{\Lambda^i}\phi^{4+i}$$

- $[\partial] = +1, [\mathcal{L}] = 4 \to [\phi] = 1$
- $[m] = [\mu] = +1$ and $[\lambda] = [C_i] = 0$
- C_i terms are nonrenormalizable interaction terms ($\phi^{d>4}$: Irrelevant operators $\rightarrow$ Will discuss shortly)
- Field op ϕ create or annihilate a particle of mass m:

$$\phi \sim a(p)e^{-ip\cdot x} + a^\dagger(p)e^{+ip\cdot x}$$

- Complex scalar $\phi \sim a + b^\dagger$ with a and b relevant to particle and antiparticle

Fermion fields

- Lagrangian for fermion fields :

$$\mathcal{L} = \overline{\psi}(i\partial\cdot\gamma - m_\psi)\psi + \frac{C}{\Lambda^2}(\overline{\psi}\psi)^2 + \ldots.$$

- $[\psi] = 3/2\ ,\ [m] = 1\ ,\ [C] = 0$
- C term: nonrenormalizable (irrelevant at low energy)
- Dirac field operator:

$$\begin{aligned}\psi &\sim bu + d^\dagger v \\ \overline{\psi} &\sim b^\dagger\overline{u} + d\overline{v}\end{aligned}$$

- Fermi's theory of weak interaction is the classic example

- Dimensional analysis for $\psi\overline{\psi}$ scattering

$$\mathcal{M}(\psi(p_1, s_1)\overline{\psi}(p_2, s_2) \to \psi(p_3, s_3)\overline{\psi}(p_4, s_4)) \sim \frac{1}{\Lambda^2}$$

$$\sigma \sim \left(\frac{1}{\Lambda^2}\right)^2 \times (phasespace) \sim \left(\frac{1}{\Lambda^2}\right)^2 \times s$$

- Mandelstam variables for $2 \to 2$ scattering:

$$s \equiv (p_1 + p_2)^2, t = (p_3 - p_1)^2, u = (p_4 - p_1)^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

- Cross section becomes zero as $s \to 0$: Irrelevant

Unitarity Violation

- What happen at high energy ?

$$\sigma \to \infty \to$$

Violation of perturbative Unitarity near $\sqrt{s} \sim \Lambda/\sqrt{C}$
$\to$ New dof's will come into play for cure (e.g., $W^\pm$ or Z^0)

- This is the wonder of Nature with special relativity and quantum mechanics
- In the SM, the pointlike interaction is replaced by the $W^\pm, Z^0$ propagator, which cuts off the bad high energy behavior
- $\sigma \sim 1/s$ at very high energy scale $\sqrt{s} \gg m_{W,Z}$

Vector fields

- Lagrangian for abelian gauge field with a charged particle (QED):

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(iD\cdot\gamma - m_\psi)\psi \\
F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \\
D_\mu\psi &\equiv (\partial_\mu + ieA_\mu)\,\psi
\end{aligned}$$

- $[A_\mu] = 1, [F_{\mu\nu}] = 2, [e] = 0$
- Dimensionless coupling $e \rightarrow$ Renormalizable interaction (marginal operator, meaning that it is important at all energy scales)
- RG equation for e may run into a Landau pole, above which the coupling diverge $\rightarrow$ Either new theory before/around Landau pole, or low energy theory is free field theory

Heavy Particle EFT

- If the energy scale is so low that the particle cannot be created or destroyed, the particle number will be conserved
- Heavy particle EFT

$$p^\mu = mv^\mu + k^\mu, \quad |k| << m$$

- Remove e^{-imt} factor from the field : $\phi = e^{-imv\cdot x}\psi_v(x)$
- Lagrangian (with Lorentz sym restore by v^μ) :

$$\mathcal{L}(\psi_v, v^\mu) = \psi_v^\dagger v \cdot D\psi_v + ...$$

- Can be applied to baryon ChPT, heavy meson ChPT, etc..

Renormalizable Opertors

- dim 0 : $I_{\rm op}$ (cosmological constant)
- dim 1 : S (scalar tadpole)
- dim 2 : S^2 , $A_\mu A^\mu$ (mass terms for bosons)
- dim 3 : $\overline{\psi}\psi$ (Fermion mass term) , S^3 (self interaction of singlet scalar)
- dim 4 : $S\overline{\psi}\psi$ (Yukawa interaction) , S^4 (Scalar self coupling) , A_μ^4 , $\partial_\mu A_\nu A^\mu A^\nu$ (self interactions of gauge fields)

NB: S, S^3 etc possible only for gauge singlet S

nabelian Gauge Symmetry and Renormalizability

- Renormalizable Interactions are only 3 types:

$$B^3,\ B^4,\ \overline{F}FB$$

- Power counting renormalizable interactions for spin-1:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + m^2_A \frac{1}{2} A_{\mu a} A^{\mu a} + \partial_\mu A^a_\nu A^{\mu b} A^{\nu c} + + A^a_\mu A^b_\nu A^{\mu c} A^{\nu}$$

(all possible contraction over group indices)

- Although this is power counting renormalizable, it is not
- Only special type of lagrangian consistent with local Nonabelian gauge symmetry is renormalizable
- Local gauge symmetry is really a powerful principle for a spin-1 object

Some remarks on QFT

- QFT is the basic framework for particle physics, and is a marriage of QM and Special Relativity
- Spin-Statistics theorem
  - Bosons : totally symmetric wavefunction
  - Fermions : totally antisymmetric wavefunction
  - Intrinsic $P(B, F) = (+B, -F)$
- CPT is a symmetry of any local QFT
  $\rightarrow CP$ violation implies T (time-reversal) violation
- CPT theorem: $m_n = m_{\bar{n}}$ and $\tau_n = \tau_{\bar{n}}$, $\mu_n = \mu_{\bar{n}}$
- However, a partial width of n and $\bar{n}$ can be different $\rightarrow$ Direct CP Violation :

$$\Gamma(n \rightarrow f) \neq \Gamma(\bar{n} \rightarrow \bar{f})$$

Heavy Quarknia Quantum Numbers

- Bound State of spin-1/2 Q and $\bar{Q}$ with ${}^{2S+1}L_J$:

$$P = (-1)^{L+1}, \quad C = (-1)^{L+S} \to 0^{-+}, 1^{--}, 1^{++}, 1^{+-},$$

- Bound State of spin-0 Q and $\bar{Q}$ with ${}^{2S+1}L_J$ (with $S = 0$ and $L = J$):

$$P = (-1)^{L}, \quad C = (-1)^{L} \to 0^{++}, 1^{--}, 2^{++}, \text{etc.}$$

- No place for π (with 0^{-+})
- Observed J^{PC} clearly says that quarks are spin-1/2 fermions, not scalars
- Exotic mesons don't follow the above assigment

Effective Lagrangian Approach

- If new physics scale is high enough, it is legitimate to integrate out the heavy d.o.f.
- The low energy physics can be described in terms of effective lagrangian :

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm ren} + \sum_{d=5}^{\infty} \frac{\mathcal{O}^{(d)}}{\Lambda_d^{d-4}}$$

  where all the operators in $\mathcal{L}_{\rm eff}$ are made of light d.o.f. with their local gauge symmetries
- Effects of the nonrenormalizable operators $\sim (E/\Lambda_d)^{d-4}$ relative to the amplitude from $\mathcal{L}_{\rm ren}$
- EFT is useful, as long as $E \ll \Lambda_d$, since we can keep only a few of the NR operators

SM as an EFT: Below e^+e^- Threshold

- Only relevant quantum dof is photon A_μ
- If E increases, we need to include more and more NR operators
- Eventually, unitarity will be broken $\rightarrow$ We have to include new d.o.f.'s in the EFT, and redefine the EFT with more d.o.f.
- QED at $E \ll 2m_e$: A_μ, local $U(1)$ and P, C

$$\mathcal{L}_{\rm EET} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^4}{(4\pi)^2 \Lambda^4} F^4 + \ldots$$

  where $\Lambda \sim m_e$
- This effective lagrangian describes $\gamma\gamma$ scattering, the cross section of which will break unitarity when E reaches $2m_e$

SM as an EFT: Below e^+e^- Threshold

- The cross section grows like $\sim s^3$:

$$\sigma(\gamma\gamma \to \gamma\gamma) \sim \frac{e^8}{\Lambda^8} s^3$$

and see at which energy scale unitarity is violated

- Unitarity will be restored due to a new process that opens up: $\gamma\gamma \to e^+e^-$
- One has to redefine the effective lagrangian near e^+e^- threshold, by including the electron/positron fields explicitly

Digress on Unitarity

- Unitarity is the most profound thing in QM
- Scattering Operator S is unitary:

$$\langle f|S|i\rangle = S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(p_i - p_f)T_{fi}$$

- Unitarity: $S^\dagger S = SS^\dagger = 1$

$$T_{fi} - T^*_{fi} = i(2\pi)^2 \sum_n \delta^4(p_f - f_n)T_{fn}T^*_{in}$$

- If interaction is weak, we can ignore the RH $\rightarrow$ T becomes Hermitian $T_{fi} = T^*_{if}$
- Optical theorem for $f = i$:

$$2\mathrm{Im}T_{ii} = (2\pi)^4 \sum_n |T_{in}|^2\delta^4(P_i - P_n)$$

Rayleigh Scattering: Why is Sky Blue ?

- Photon scattering with neutral atom A where

$$E_\gamma \ll \Delta E_{n1} \equiv E_n - E_1$$

$\rightarrow$ Elastic scattering of light on neutral atoms

- Atom is described by nonrelativistic Schrödinger wave function ψ_A with dim 3/2:

$$\mathcal{L} = \psi_A^\dagger \left(i\frac{\partial}{\partial t} - H \right) \psi_A + \frac{e^2}{\Lambda^3} \psi_A^\dagger \psi_A F_{\mu\nu} F^{\mu\nu} + \ldots.$$

- $\Lambda \sim \Delta E_{21}, r_0$??
- Note that photon couples to a neutral atom. How ???

- No coupling of photon to neutral objects only at renormalizable level
- Photon couples to neutral particle at nonrenormalizable level due to quantum fluctuation can involve charged particles in the loop
- Likewise, gluons can couple to photons
- γA scattering cross section :

$$\sigma(\gamma A \to \gamma A) \sim \frac{e^4}{\Lambda^6} E_\gamma^4 \sim \frac{1}{\lambda_\gamma^4}$$

  for $E_\gamma \ll \Delta E_{2,1}$
- Blue light scatters more than red light $\to$ Sky is blue, and we can enjoy the beautiful sunrise/sunset in red

Van der Waals Force

- Potential between neutral atoms are described by two-photon exchange diagrams from the previous lagrangian $\psi_A^\dagger \psi_A F^2$
- Additional contact interaction has to be considered:

$$\frac{1}{\Lambda^2}\left(\psi_A^\dagger \psi_A\right)^2$$

- Calculate the two contributions and discuss what is the form of the force between two neutral atoms (Van der Waals interaction) ?
- What is a in the exponent in $V(r) \sim r^a$?
- What if we consider the neutral atom relativistically ? (Itzykson and Zuber, QFT)

QED as an EFT below $\mu^+\mu^-$ threshold

- QED at $2m_e \leq E \ll 2m_\mu$: A_mu, e, $\bar{e}$, local $U(1)$ and P, C

$$\mathcal{L}_{\text{Eff}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{e}(iD - m_e)e + \frac{e^4}{(4\pi)^2\Lambda_1^4}F^4 + \frac{e}{(4\pi)^2\Lambda_2}\overline{e}\sigma^{\mu\nu}eF_{\mu\nu}$$

where $\Lambda_1 \sim m_\mu$, and $\Lambda_{2,3} \sim O(1)$ TeV or larger (see later discussions on these points)

- NP scale in each NR operator is independent (different from each other) in general, since the origin can be different
- Scale for F^4 is now $\sim m_\mu$, unlike the previous case

QED as an EFT below $\mu^+\mu^-$ threshold

- Additional $1/(4\pi)^2$ suppression for NR operators generated at one-loop level, compared with NR operators generated at tree level, even if their operator dim's are the same
- If we impose $SU(2)_L \times U(1)_Y$ instead of $U(1)_{\rm em}$, the Λ_2 term should be replaced by

$$\frac{e}{(4\pi)^2 \Lambda_2^2} \overline{e_L} \sigma^{\mu\nu} H e_R F_{\mu\nu} \rightarrow \frac{ev}{\sqrt{2}(4\pi)^2 \Lambda_2^2} \overline{e_L} \sigma^{\mu\nu} e_R F_{\mu\nu}$$

and the effect becomes smaller for the same Λ_2, or the bound on Λ_2 becomes stronger

- Chiraliry flip operator

QED as an EFT above $\mu^+\mu^-$ threshold

- QED at $E \ll 2m_\pi$: A_μ, e, $\bar{e}$, μ, $\bar{\mu}$, local $U(1)$ and P, C

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Eff}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{e}(iD - m_e)e + \overline{\mu}(iD - m_\mu)\mu \\
&+ \frac{e^4}{(4\pi)^2\Lambda_1^4}F^4 + \frac{e}{(4\pi)^2\Lambda_2}\overline{e}\sigma^{\mu\nu}eF_{\mu\nu} + \frac{e}{(4\pi)^2\Lambda_3}\overline{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \\
&+ \frac{e}{(4\pi)^2\Lambda_4}\overline{e}\sigma^{\mu\nu}\mu F_{\mu\nu} + \frac{e^2}{\Lambda_5^2}(\overline{e}e)(\overline{e}\mu) + H.c.
\end{aligned}
$$

where $\Lambda_1 \sim m_\pi$, $\Lambda_{2,3} \gtrsim XX$ TeV , and $\Lambda_{4,5} \gtrsim XX$ TeV or larger

- $\Lambda_{2,3}$ terms contribute to $(g-2)_{e,\mu}$
- $\Lambda_{4,5}$ generate $\mu \to e\gamma$ and $\mu \to 3e$

Muon Decay $\mu \to e\overline{\nu_e}\nu_\mu$

- Apply the Fermi's theory of weak interaction with replacing (p, n) by (ν_μ, μ)

$$\mathcal{L}_{CCweak} = -\frac{G_F}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\mu \mu)(\bar{e}\gamma_\mu \nu_e) + H.c.$$

- Muon lifetime :

$$\tau^{-1} = \Gamma_\mu = \frac{G_F^2}{2(4\pi)^3} m_\mu^5$$

cf. Compare with the exact expression:

$$\tau^{-1} = \Gamma_\mu \sim \frac{G_F^2}{192\pi^3} m_\mu^5 \propto m_\mu^5$$

- $\Gamma \propto m^5$ is a generic behavior of a fermion decaying through 4-fermion (dim 6) operators (τ, proton decays etc.)

Tau lepton decays

- $m_\tau = 1.777$ GeV $\sim (2m_p - m_\mu)$
- Similar behavior for τ lepton decays

$$\Gamma_{\tau\to e}/\Gamma_{\mu\to e} = (m_\tau/m_\mu)^5 = (1.777/0.105)^5 \sim 1.4 \times 10^6$$

$$e\overline{\nu_e} : \mu\overline{\nu_\mu} : (\overline{u}d + \overline{u}s) = 1:1:1 \to 1:1:N_c$$

- Data $= 17\% : 17\% : 66\%$
- Another evidence for $N_c = 3$:
- Including the QCD corrections to hadronic τ decays,

$$1:1:N_c(1 + \alpha_s/\pi + ...)$$

- We have to correct the Dirac structure from $V \times V$ to $(V-A) \times (V-A)$

EFT above $\pi^+\pi^-$ threshold

- Assume isospin symmetry for $N = (p, n)$ and $\vec{\pi} = (\pi^+, \pi^0, \pi^-)$

$$\mathcal{L}_{\rm new} = \frac{1}{2} D_\mu \vec{\pi} D^\mu \vec{\pi} - \frac{1}{2} m_\pi^2 \vec{\pi}^2 + \lambda_\pi (\vec{\pi}^2)^2 + \ldots.. + \overline{N}(\pi, \pi^2, \ldots) N + \ldots\ldots.$$

- Here

$$D_\mu \vec{\pi} = \partial_\mu \vec{\pi} - ieQ\vec{\pi}$$

$$\text{with} Q = \text{diag}(1, 0, -1)$$

- However, in experiments, $\pi\pi \to \pi\pi$ amplitude was energy-dependent, and soft-pion interactions were weak (???)

$\pi^\pm \to \mu^\pm \nu, e^\pm \nu$ decay

- Naive guess does not work. WHY ?

$$\mathcal{L} = y \pi \bar{l} \nu$$

- This works better.

$$\mathcal{L} = \frac{1}{\Lambda} \partial_\mu \pi \bar{l} \gamma^\mu (1 - \gamma_5) \nu$$

- This implies that the vector mediator between the leptonic current and the hadronic current
- Vector field $\sim$ gauge field couples to the conserved current and show the universality
- Note that $\tau(\pi^\pm) = 2.6 \times 10^{-8}$, vs. $\tau(\mu) = 2.2 \times 10^{-6}$
- Universality ?

$\pi^{\pm} \to \mu^{\pm}\nu, e^{\pm}\nu$ decay (cont'd)

- Eventually the correct answer is

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{u}\gamma^{\mu}(1-\gamma_5)d\bar{l}\gamma^{\mu}(1-\gamma_5)\nu$$

  with $< 0|\bar{u}\gamma^{\mu}\gamma_5 d|\pi(q) >= i f_{\pi} q^{\mu}$ ($f_{\pi} = 93$ MeV)

- Vector interaction : gauge interaction which has Universality
- But gauge fields can couple to conserved currents
- Then K, η were discovered below proton mass :$SU(2)_f \to SU(3)_f$
- And ρ mesons were discovered in the $I = J = 1$ channel in the $\pi\pi$ scattering

Derivative couplings of π's

- Lagrangian for π's can be organized as

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr} \left[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right] + O(p^4)$$

- Derivative expansions
- Mass terms given by

$$\frac{f_\pi^2}{2} \mathrm{Tr} \left[\mu m(\Sigma + \Sigma^\dagger)\right]$$

- In fact, pions are NG bosons for spontaneously broken global chiral symmetry of QCD: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$

$$\Sigma(x) \rightarrow L \Sigma(x) R^\dagger$$

L and R : global chiral transformations

Hidden local symmetry

- Another useful way is to introduce $\xi(x)$ defined as $\Sigma \equiv \xi\xi$ with

$$\xi(x) \to L\xi(x)U^\dagger(\pi(x)) = U(\pi(x))\xi(x)R^\dagger$$

$U(\pi(x)) \equiv U(x)$ belongs to $SU(3)_V$

- Two independent vector fields which transform as

$$\begin{aligned} A_\mu(x) &= \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right) \to U(x)A_\mu(x)U^\dagger(x) \\ V_\mu(x) &= \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right) \\ &\to U(x)V_\mu(x)U^\dagger(x) + U(x)\partial_\mu U^\dagger(x) \end{aligned}$$

- V_μ behaves like a gauge field for local $SU(3)$
- Covariant derivative $D_\mu \equiv \partial_\mu - V_\mu$

Hidden local symmetry

- One can introduce matter fields such as baryon octet $B(x)$ that transform as $B(x) \to U(x)B(x)U^\dagger(x)$, etc., and

$$\rho_\mu(x) \to U(x)\rho_\mu(x)U^\dagger(x) + U(x)\partial_\mu U^\dagger(x)$$

Then note that

$$(\rho_\mu - V_\mu) \to U(x)(\rho_\mu - V_\mu)U^\dagger(x)$$

- In particular one an write down the ρ-meson mass term in a chiaral invariant way:

$$m_\rho^2 \mathrm{Tr}(\rho_\mu - V_\mu)^2$$

- Lagrangian for the baryon octet B ($< \ldots >$: Trace):

$$\langle \overline{B}(iD - m_B)B\rangle - D\langle \overline{B}\{A, B\}\rangle - F\langle \overline{B}[A, B]\rangle$$

Coset space construction a la CCWZ

- Consider spontaneously broken theory $G \to H$
- Callan-Coleman-Wess-Zumino (CCWZ) prescription (1969)
- This is a general procedure to construct lagrangian with assumed symmetry, being manifest or hidden (spontaneously broken)
- This is also useful for describing strongly interacting EWSB without fundamental Higgs boson, or Higgs boson as a Nambu-Goldstone boson (Composite Higgs boson scenario)

Nucleons and neutron β decay

- proton + neutron known to make a nucleus of an atom
- $m_p \approx m_n \to$ approximate isospin symmetry
- β decay of $n \to pe\overline{\nu_e}$ is known
- Effective lagrangian for protons and neutrons

$$\begin{aligned}\mathcal{L} &= \overline{p}(iD\cdot\gamma - m_p)p + \frac{\kappa_p}{2m_p}\overline{p}\sigma^{\mu\nu}pF_{\mu\nu} + (p \to n) \\ &- \mathcal{L}(A_\mu) + \frac{G_F}{\sqrt{2}}(\overline{p}\gamma^\mu n)(\overline{e}\gamma_\mu\nu_e) + H.c.\end{aligned}$$

where $D_\mu p = (\partial_\mu + ie_pA_\mu)p$

- Dim 5 term generate the anomalous magnetic moments of p and n, in addition to the $g = 2$ for the pointlike g-factors for charged spin-1/2 fermions
- $\kappa_{p,n} \sim O(1)$ is needed to fit the data:

EM Polarizabilities of Nucleons

- Higher Dim operators with nucleons and em fields:

$$C_1 \frac{e^2}{\Lambda^3} F_{\mu\nu} F^{\mu\nu} \; \overline{N} N + C_2 \frac{e^2}{\Lambda^3} F_{\rho\mu} F^{\rho}_{\nu} \; \overline{N} \sigma^{\mu\nu} N + \ldots$$

- C_1 and C_2 related with the electric and magnetic polarizabilities of nucleons
- In particular, neutron couples to photons at nonrenormalizable level again
- There is no absolutely dark matter, namely which has absolutely no interactions with light at all
- Neutrinos and dark matters interact with photons, but their interaction rates are suppressed by $(E/\Lambda)^{\text{positive power}}$ and thus $\ll 1$
- Need higher energy to see these effects (or much shorter wavelength photon)

Neutron β decay

- Fermi's 4-fermion interaction theory describes the neutron β decay
- It is an irrelevant operator : $G_F m_p^2 \simeq 10^{-5}$
- Neutron life time for $n \to p e^- \overline{\nu_e}$

$$\Gamma_n = \tau_n^{-1} \sim \frac{G_F^2}{2(4\pi)^3} (\Delta m)^5 \sim (XX)^{-1}$$

where $\Delta m = m_n - m_p \simeq 1.3$ MeV

- $\tau_n^{\rm exp} = 881$ sec
- Fermi assumed parity conservation ($V \times V$)

Neutrino Oscillation $\nu_e \leftrightarrow \nu_\mu$

- Both ν_e and ν_μ are electrically neutral
$\rightarrow$ Both of them can have Majorana masses, including the mass mixing between the two
- Assume they are both LH particles (as observe in CC weak interaction processes)
- Mass terms for the two Majorana neutrinos:

$$\mathcal{L}_{\nu \mathrm{mass}} = \frac{1}{2} m_{\alpha\beta} \overline{\nu^{\,c}_{\alpha L}} \nu_{\beta L} + H.c.$$

- Two mass eigenvalues will be different in general: $\Delta m^2 \neq 0$, with a mixing angle θ

Neutrino Oscillation

- Neutrino oscillation probability:

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$
$$= \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 (\mathrm{eV}^2) L(\mathrm{km})}{4EGeV)} \right)$$

- For 3 active neutrinos, two Δm^2's and 3×3 mixing matrix (MNS matrix)
- Neutrino oscillations were in fact observed atmospheric and solar neutrino oscillations
- Write down the effective lagrangian for $\nu_\mu \to \nu_e \gamma$. Estimate the coefficient of this operator from the 1-loop diagram in the SM and the lifetime of ν_μ in this mode.

Weinberg operator for neutrino mass

- If we impose $SU(2)_L \times U(1)_Y$ local gauge symmetry instead of $U(1)_{\rm em}$, the above neutrino mass terms will be replaced by dim-5 Weinberg operator breaking with $\Delta L = 2$:
$$\frac{y_{\alpha\beta}}{\Lambda_{\alpha\beta}} (L_\alpha H)(J_\beta H) + H.c.$$
with $\Lambda_{\alpha\beta} \sim 10^{12-16}$ GeV $\sim M_N$ (RH Majorana mass scale in seesaw mechanism)
- This is the only dim-5 operator which is invariant under the full SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$
- This nonrenormalizable terms can be made renormalizable (UV complete) by introducing the RH singlet neutrinos (Type-I seesaw), or by triplet Higgs fields (Type-II seesaw)

Proton Decay

- These decays are kinematically allowed, but never been observed

$$\begin{aligned} \tau(p \to e^+\pi^0) &> 8.2 \times 10^{33}\text{yr} \\ \tau(p \to K^+\nu) &> 6.7 \times 10^{32}\text{yr} \end{aligned}$$

- Why proton is so stable ?

$$\tau_p > \tau_{\text{universe}} = 4 \times 10^{17} \text{ sec}$$

- Consider operators $\overline{e}p\pi^0$ (dim 4), and $\overline{e}\gamma^\mu p\partial_\mu\pi^0$(dim 5), both give dangerously short lifetime for proton

Proton Decay

- One possible way out: p and π are composite of quarks, and B and L violation occurs at very high energy scale, where proton is no longer a good description with the following dim-6 operators:

$$\frac{g^2}{\Lambda^2} uude$$

  (ignoring Dirac structure)
- SUch operators can be generated in (SUSY) GUT, or MSSM with $R-$parity violation
- Calculate the lower bound on the scale Λ from the lower bound on the proton lifetime.

$\Delta B = 2$ Process: $n - \overline{n}$ Oscillation

- This is possible since electric charge is conserved

$$\mathcal{L}_{n-\bar{n}} \sim \mu n n, \quad \text{or} \quad \frac{g^4}{\Lambda^5} uddudd$$

- Sensitive probe of new physics with $\Delta B = 2$
  e.g. Z_3 baryon parity in the MSSM
- Experimental bound:

$$\tau(\text{free}) > 8.6 \times 10^7 \text{sec}, \quad \tau(\text{bound}) > 8.6 \times 10^7 \text{sec}$$

Estimate the transition rate for $n - \overline{n}$ using the above 6-quark operators, and derive the bound on the scale Λ.

Massive Weak Gauge Bosons ?

- Giving mass to spin-1 object has a problem in high energy
- Breaks perturbative unitarity and the model becomes nonrenormalizable
- Higgs mechanism solves the problem

CP violation in $K_L \to \pi\pi$

- How to describe CP violation ?
- Wolfenstien (1964) proposed a superweak model :

$$\mathcal{L}_{\rm superweak} \sim a G_F^2 \left(\bar{d}\Gamma s\right)^2$$

Can accommodate $\epsilon_K = 2 \times 10^{-3}$, if $a \sim$
(Similar model was also proposed for $B_d - \overline{B_d}$ mixing)

- The story changed after Weinberg proposed the SM, and the renormalizability was proved
  - Two Higgs doublet model (2HDM) with spontaneous CP violation
  - Three or more families by Kobayashi-Maskawa (KM) $\to$ Current paradigm (SM with 3 generations), and has been very well verified in the B, K systems (superweak model excluded)

Why not n or e EDM's ?

- CPT conserved in QFT
- CP violated, P and C violated; so why not T violation ?
- n or e EDM's would break both P and T
  cf. Usually said to be CP violating (better not use)
- Effective lagrangian for EDM

$$\mathcal{L}_{\mathrm{EDM}} = i\frac{d_n}{2m_n}\overline{n} i\gamma_5\sigma^{\mu\nu} n\, F_{\mu\nu} + H.c.$$

  and similarly for e, μ, p .....
- EDM constraints ($d_n = e/\Lambda$):

$$d_e = (0.7 \pm 0.7) \times 10^{-26} e \cdot \mathrm{cm}\ ,\ \ d_n < 2.9 \times 10^{-26} e \cdot \mathrm{cm}$$

- Bounds on new physics: $\Lambda >$ Few TeV for $O(1)$ phase

SM predictions for EDM's and the data

- Why are they so small ?
- In QCD, there is P and T violating term with $G\tilde{G}$ due to the instanton effects that make the vacuum structure of QCD rather nontrivial under topological consideration

Implications on the new physics

- How to describe CP violation ?
- Most new physics models at TeV scale are strongly constrained by FCNC and EDM
- New phase should be very small (essentially zero), or new particles better be heavier than a few TeV (more than 10's of TeV) in order to evade these bounds from EDM's and FCNC's
- Severe fine tuning needed in the flavor and CPV sector
- Real fine tuning problem of generic BSM
- Hidden sector scenarios are less constrained by these however

FCNC and GIM

- If there were only three familes with

$$\begin{pmatrix} u_L \\ d_L \cos\theta_C + s\sin\theta_C \end{pmatrix}, u_R, d_R, s_R,$$

there would be huge contribution to $K^0 \to \mu^+\mu^-$ mediated by W^0 gauge boson of $SU(2)_L$

- Precition vs. Data:

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu\nu_\mu)} = O(1), \quad vs. \quad \sim 3\times 10^{-7}(\text{Data})$$

- In nature (in the kaon system), FCNC is highly suppressed
- What is wrong ? How to cure the theory ?

GIM

- GIM introduced another quark called "charm" ($\equiv c$) with the orthogonal coupling to the down type quarks

$$\begin{pmatrix} u_L \\ d_L \cos\theta_C + s \sin\theta_C \end{pmatrix}, \quad \begin{pmatrix} c_L \\ -d_L \sin\theta_C + s \cos\theta_C \end{pmatrix}, \quad u_R, d_R, s_R,$$

- Then W^0 coupling is flavor diagonal, and no tree level contribution to $K_L \to \mu\mu$
- FCNC processes can occur only at one-loop or higher loops
- $m_c \sim 1.5$ GeV will explain Δm_K (Gaillard, Lee, Rosner 1974)
- Charm quark discovered in 1975
$\to$ "Triumph of Theoretical Physics"

Kobayashi-Maskawa Model for CP violation

- KM considered n families of the Weinberg-Salam model (1974 ?)
- Counted the number of CPV phases which cannot be rotated away:

$$n_{\text{angle+cpphase}} = 2n^2 - n^2 - (2n-1) = (n-1)^2$$

$$n_{\text{cpphase}} = (\text{above}) - \frac{n(n-1)}{2} = \frac{(n-1)(n-2)}{2}$$

- $n = 3 \to 1$ CPV phase (KM phase)
- The 5th quark (bottom or beauty) was discovered in 197x in the $b\bar{b}$ bound system: $\Upsilon \to \mu^+\mu^-$
- And finally the 6th quark (top or truth) was discovered at Tevatron in 1995

Kobayashi-Maskawa Model for CP violation

- Now we have 3 generations of SM chiral fermions with GIM built in, and one CP violating phase, which can explain all the data from the laboratory
- Except for the baryogenesis, for which the KM phase gives too small effects because of the smallness of ligth quark masses (Probably the most supporting argument for an extra source of CPV phase beyond the KM phase)
- Any new physics with new sources of flavor and CP violation is strongly constrained
- Especially, the CP violation in the quark sector form new physics should be subdominant to the effects from the KM phase

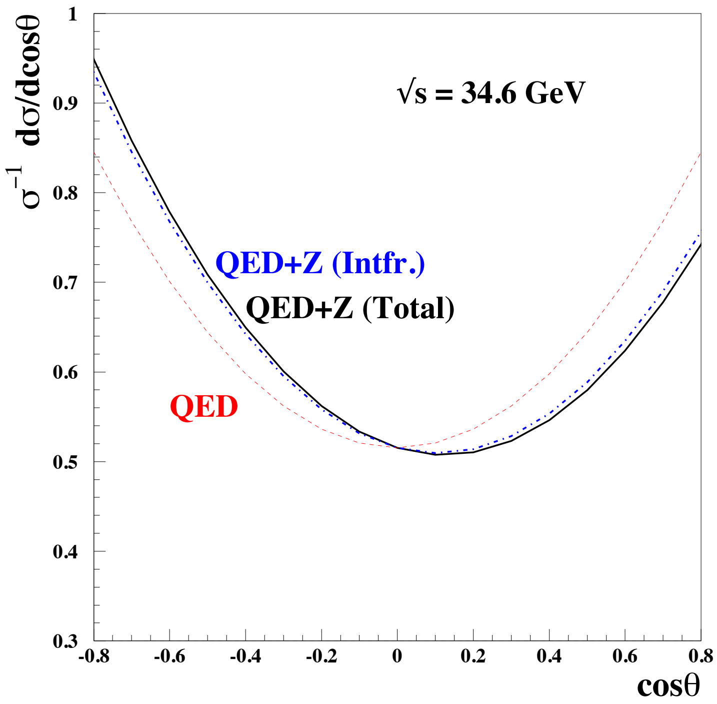
- Discovery of charmonium ($J/\psi, \psi^{'}, ...$) and Upsilon ($\Upsilon(nS)$.....) : new EFT called NRQCD was developed
- Discovery of B meson and Λ_b : new EFT called HQET developed, and somtimes combined with Heavy Quark Expansion (OPE)
- As energy was increased, new phenomena/particles were discovered, and EFT modified

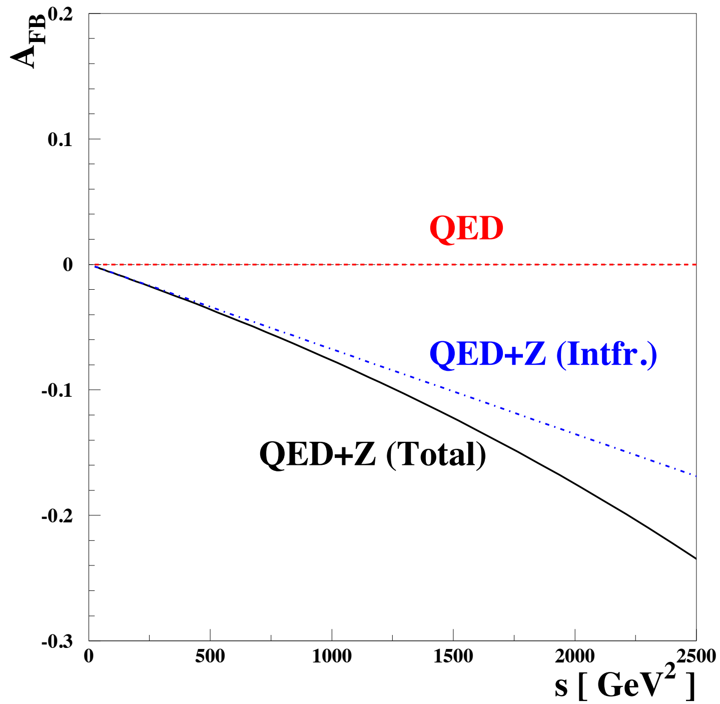
EFT below $W^\pm$ and Z^0

- SM with 3 families of chiral fermions with $W^\pm$ and Z^0
- Before the discovery of $W^\pm$ and Z^0, the EFT would be

$$\mathcal{L}_{\text{ren QED}} + \mathcal{L}_{\text{ren QCD}} + \frac{g^2}{\Lambda^2}\left(\overline{e}\Gamma e\right)\left(\overline{\mu}\Gamma \mu\right) + \ldots.$$

  where $\Gamma = 1, \gamma^5$, γ^μ, $\gamma^5\gamma^\mu$, $\sigma^{\mu\nu}$
- The first evidence of asymmetry was found in angular distribution of muons from e^+e^- collisions at PETRA in the 80's ($\sqrt{s} \sim 30$ GeV , well below the Z^0 pole)
- Source of A_{FB} is a term linear in $\cos\theta$ from interference between γ or Z vector coupling and the axial vector Z coupling.





- Since $\sqrt{s} \ll M_Z$, good approx. to assume 4 fermion interactions by integrating out Z boson
- $A_{\mathrm{FB}} \simeq -\frac{3G_F}{\sqrt{2}} \frac{s}{4\pi\alpha}(g_L - g_R)^2 \equiv kG_F s$
- $k \simeq -7$ from EFT, whereas $k = -5.78$ from the full expression

Top FB Asym at the Tevatron

- $t\bar{t}$ production at the Tevatron dominated by $q\bar{q}$ channel
- Enough to consider dimension-6 four-quark operators to describe the new physics effects assuming new physics scale is high enough:

$$\mathcal{L}_6 = \frac{g_s^2}{\Lambda^2} \sum_{A,B} [C_{1q}^{AB} (\bar{q}_A \gamma_\mu q_A)(\bar{t}_B \gamma^\mu t_B) + C_{8q}^{AB} (\bar{q}_A T^a \gamma_\mu q_A)(\bar{t}_B T^a \gamma^\mu t_B)$$

where

$$T^a = \lambda^a/2, \quad \{A,B\} = \{L,R\}, \quad L,R \equiv (1 \mp \gamma_5)/2 \quad (q = u,d,s,c,b)$$

- Other d=6 operators are all reducible to the above operators after Fierzing (Hill and Parke 1994)
- We ignore flavor changing dim-6 operators such as

- Our approach will be useful even if the $A_{\rm FB}$ approaches the SM prediction
- This contact term used to explore light quark substructures
- Similar analysis has been done for light quark and lepton systems using $\bar{q}q\bar{q}q$, $\bar{q}q\bar{l}l$, and $\bar{l}l\bar{l}l$, with various Dirac and color structures
- One can do exactly the same analysis for top compositeness scale
- The scale Λ in our effective lagrangian could be interpreted as
  Compositemess scale of a top quark seen by a light quark, or vice versa
- Bound on Λ can be derived from $M_{t\bar{t}}$ or p_T^t distributions at the Tevatron

- The squared helicity amplitude is given by

$$\overline{|\mathcal{M}(t_L\bar{t}_L + t_R\bar{t}_R)|^2} = \frac{4\,g_s^4}{9\,\hat{s}} m_t^2 \left[2 + \frac{\hat{s}}{\Lambda^2}(C_1 + C_2)\right] s_{\hat{\theta}}^2$$

$$\overline{|\mathcal{M} t_L\bar{t}_R + t_R\bar{t}_L)|^2} = \frac{2\,g_s^4}{9}[\left(1 + \frac{\hat{s}}{2\Lambda^2}(C_1 + C_2)\right)(1 + c_{\hat{\theta}}^2)$$

$$+ \quad \hat{\beta}_t \left(\frac{\hat{s}}{\Lambda^2}(C_1 - C_2)\right) c_{\hat{\theta}}]$$

where

$$C_1 \equiv C_{8q}^{LL} + C_{8q}^{RR}, \quad C_2 \equiv C_{8q}^{LR} + C_{8q}^{RL}$$

$$\hat{\beta}_t^2 = 1 - 4m_t^2/\hat{s}, \quad s_{\hat{\theta}} \equiv \sin\hat{\theta}, \quad c_{\hat{\theta}} \equiv \cos\hat{\theta}$$

- The term linear in $\cos\hat{\theta}$ could generate the FB asym

Dim 6 operators with SM gauge sym

- Buchmüller and Wyler [Nucl.Phys. B268 (1986) 621] made a catalogue of dim 6 operators that are invariant under the SM gauge group

$$G_{\rm SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$$

- We already studied some of them in $\mu \to e\gamma$, e/n EDM's, $\mu \to 3e$, etc., assuming $U(1)_{\rm em}$ symmetry, not the full $G_{\rm SM}$
- Assuming $G_{\rm SM}$ will introduce additional $1/\Lambda$ factor often, because LH and RH fermions are now different because of $G_{\rm SM}$
- For example,

$$\frac{1}{\Lambda} e\bar{e}\sigma^{\mu\nu} e F_{\mu\nu} \to \frac{1}{\Lambda^2} e\overline{e_L}\sigma^{\mu\nu} H e_R F_{\mu\nu}$$

Finally, EFT for CDM ?

- About 25% of the universe is made of nonbaryonic DM

$$p = \frac{1}{3}\rho \ \ (\mathrm{Rel}), \quad \mathrm{or} \quad p = 0 \ \ (\mathrm{Nonrel})$$

- The rest is the so-called Dark Energy $p = -\rho$
- No informations on the mass and the spin of the CDM
- $\tau_{DM} > 10^{26??}$ sec and no electric charge
- Many many possible models for the CDM
  - Some CDM models solve the hierarchy problems (neutralino, gravitino in SUSY models). strong CP problem (axion) or both (axino)
  - Simplest extension of the SM (real singlet scalar, Mojorana fermion, etc.)
  - Hidden sector CDM

EFT for CDM

- A number of study done with all possble Lorentz structures:

$$\frac{1}{\Lambda^2}\left(\overline{\chi}\chi\right)\mathcal{O}_{\rm SM}$$

$\mathcal{O}_{\rm SM}$ is the SM gauge singlet operator

  - Thermal relic density from $\chi\chi \to$ (SM particles)
  - Direct detection from $\chi N \to \chi N$
  - Collider signatures from $q\bar{q} \to \overline{\chi}\chi + g(\gamma)$
- Used for complementarity of light CDM scenarios vs. collider constraints
- However these three processes involve very different kinematic ranges, and very often the messengers are not very heavy
- Eventually EFT approach becomes not so useful quantitatively [See M. Drees' talk at Lepton Photon 2011]

EFT dictates that CDM decay

- Instead, EFT for CDM says that χ should decay into the SM particles by higher dim operators such as

$$\frac{1}{\Lambda^2}\left(\overline{e}e\right)\left(\overline{\nu}\chi\right), \text{etc.}$$

unless DM number is protected by some local gauge symmetry

- $\Lambda \sim 10^{16}$ GeV can make $\tau(\chi)$ long enough ($\gg 10^{26}$ sec)
- Could be used for positron excess observed by PAMELA
- What renormalizable interactions would generate such nonrenormalizable interactions that make χ decay ?

Stability/Longevity of CDM

- DM could be long lived if it is very light (axion, sterile neutrino, etc.)
- DM could be stable due to global symmetry
- Suppose global $Z_2 : S \to -S$ (Higgs-portal real scalar DM model):

$$\mathcal{L}_{\rm scalarDM} = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2} m_S^2 S^2 - \lambda_{HS} S^2 H^\dagger H - \frac{1}{4!}\lambda_S S^4$$

- Any global symmetry is supposed to be broken by (quantum) gravity effect:

$$\frac{1}{\Lambda} S O_{\rm SM}^{dim-4}, \quad (\frac{1}{\Lambda} S H^\dagger H??)$$

Fermionic DM case

- S with EW scale mass will decay immediately even for $\Lambda \sim M_{\text{Planck}}$

$$\Gamma(S) \sim \left(\frac{m_S}{100\text{GeV}}\right)^3 10^{-37} GeV$$

- Similar conclusion for fermioin DM too
- We have to avoid dim-5 operators that make DM decay

Any way out of this ?

- DM is stable due to unbroken local gauge symmetry
- DM S is a composite scalar made of N_F fermions or N_B bosons :
  - $$\frac{1}{\Lambda} S O_{\rm SM}^{dim-4} \rightarrow \frac{1}{\Lambda^{3N_F/2}} QQ...QQ \times O_{\rm SM}^{dim-4}, \quad \text{and}$$
    $$< 0|QQ...QQ|S > \sim \Lambda_{comp}^{3N_F/2-1}$$
  - Therefore the dangerous dim-5 operator is transformed into dim-10 operator for $N_F = 4$
  - Of course this assumes there is a new confining force (again dark gauge interaction, e.g. $SU(4)_h$) which makes S composite
  - Even the dim-3 $SH^\dagger H$ operator becomes OK for S longevity

Appraisal of dark gauge symmetry

- I argued that dark gauge symmetry could guarantee the absolute stability for a pointlike CDM particle, or longevity of composite CDM
- NB: Proton longevity is a consequence of baryon number being an accidental sym of the SM, which is broken only at dim-6 level because proton is composite of qqq
- If dark gauge symmetry is non confining and broken by dark Higgs mechanism, there naturally appear new force mediators such as dark gauge bosons (dark photon) and dark Higgs, both of which can affect DM phenomenology in many different and interesting ways (GC γ-ray excess, Heavy fermion DM decays into $h + \phi + \nu$, etc.)

How about 750 GeV diphoton excess

- One oprtion : $S(750)$ is a SM singlet ()scalar

$$\mathcal{L}_{|rmeff} = \frac{e^2}{(4\pi)^2 M} S F_{\mu\nu} F^{\mu\nu} + \frac{g_s^2}{(4\pi)^2 M} S G^a_{\mu\nu} G^{a\mu\nu}$$

  - Pure singlet scalar with mass 750 GeV ??? Why not very heavy ?
  - Why is there light vector-like fermions ?
  - A plausible answer :
    There is a new chiral $U(1)$ broken by new $U(1)$-charged Higgs, and S is a remnant of new Higgs mechanism, and vetor-lilke fermions are necessary to cancel all the gauge anomalies

How about 750 GeV diphoton excess

- Another option : Composite model

$$\mathcal{L}_{|rmeff} = \frac{e^2}{(4\pi)^2 M^3}\overline{Q}\Gamma Q F_{\mu\nu}F^{\mu\nu} + \frac{g_s^2}{(4\pi)^2 M^3}\overline{Q}\Gamma Q G^a_{\mu\nu}G^{a\mu\nu}$$

  with $< S|\overline{Q}\Gamma Q|0 > \sim \Lambda^2_{\rm new}$ (associated with new strong interactions)

- For the question of "why is there EW scale Q ?" can be answered