Spin Polarized Quasi-particle in Off-equilibrium Medium

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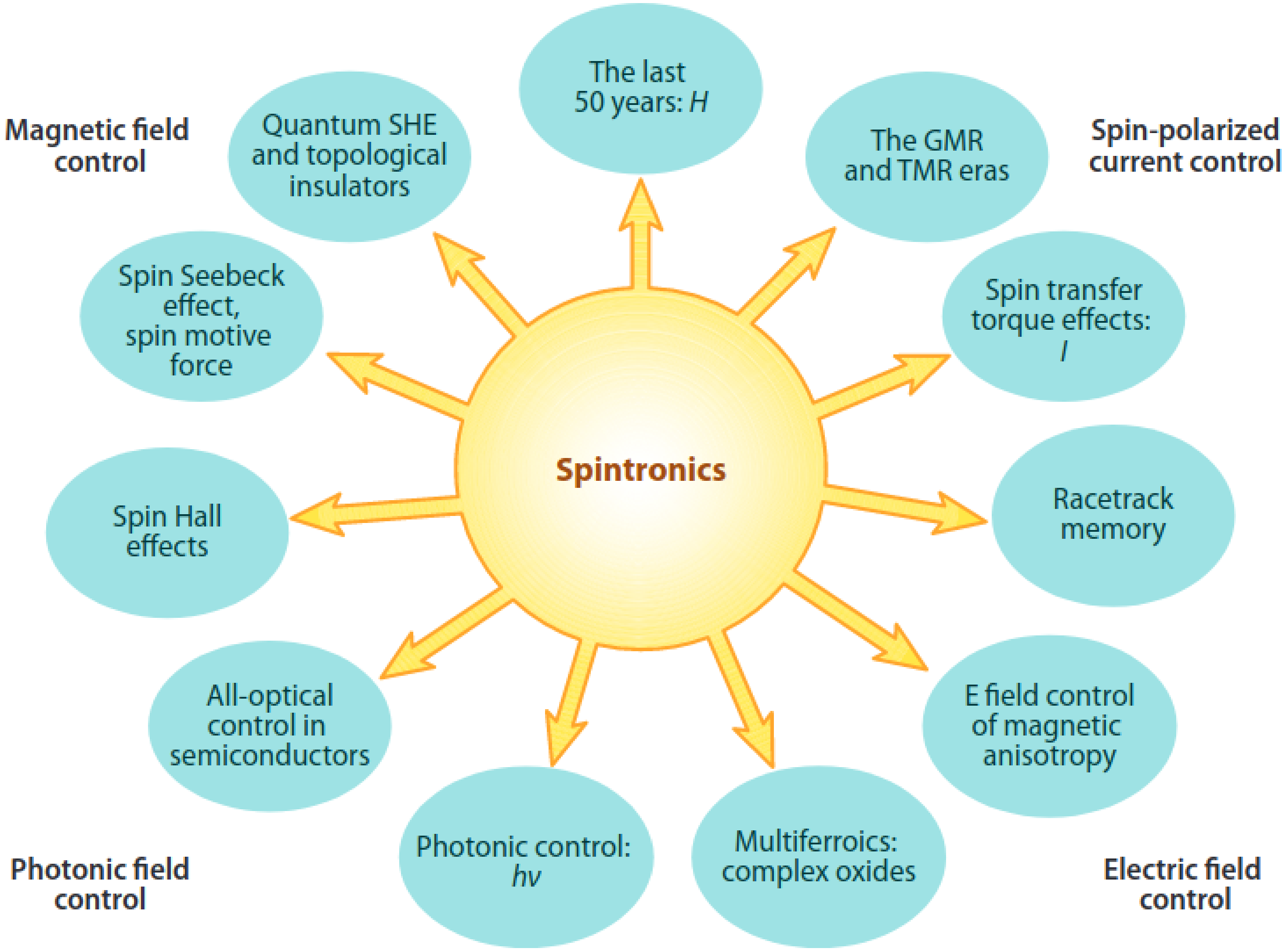
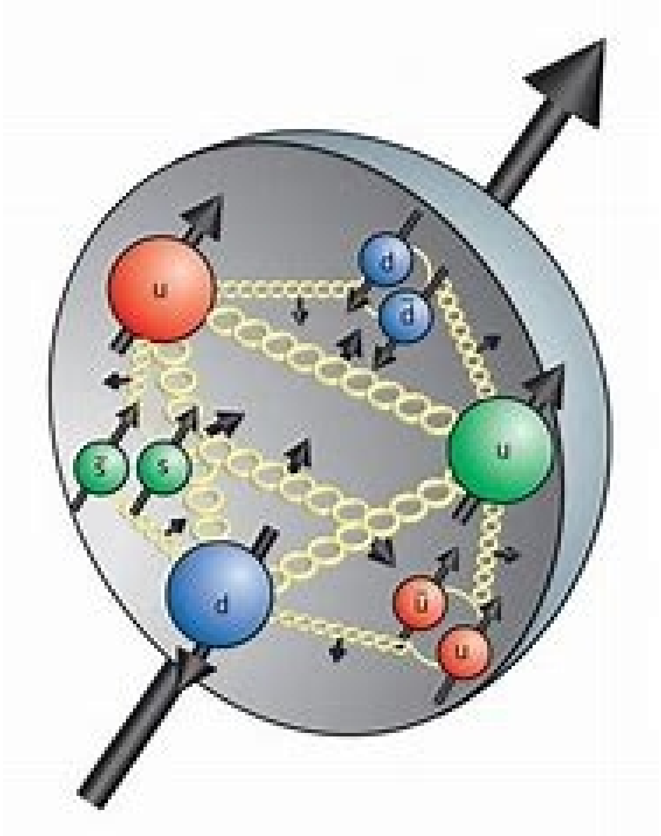
based on: 2306.14811, 2302.12450, 2410.XXXXX

Spintronics in condensed matter physics



Bader+Parkin
ARCMP 2010

Spin in particle physics



Proton spin puzzle
(1988-now)

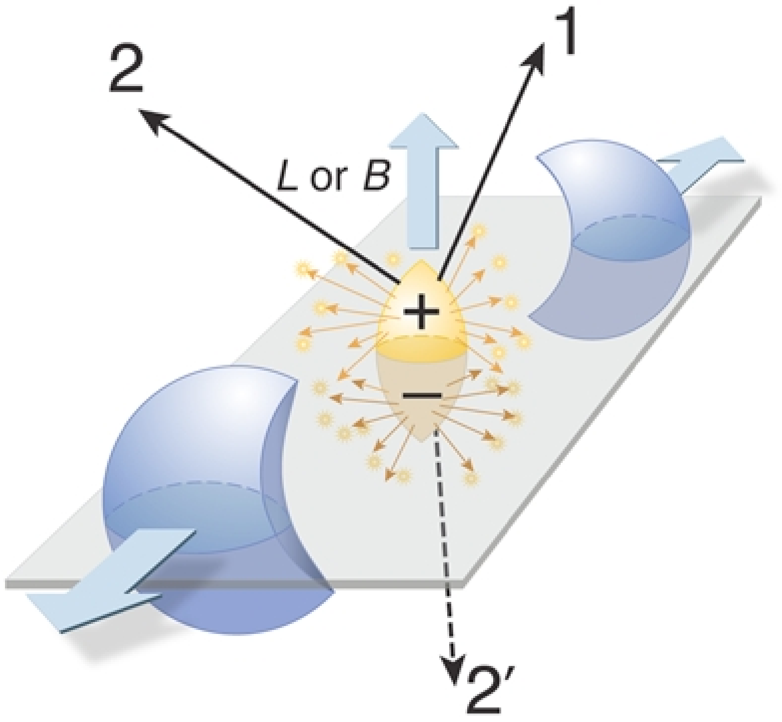
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Spin in high energy nuclear physics

- Spin not conserved, spin angular momentum exchange with orbital angular momentum
- Spin coupling to external field such as magnetic, vorticity etc

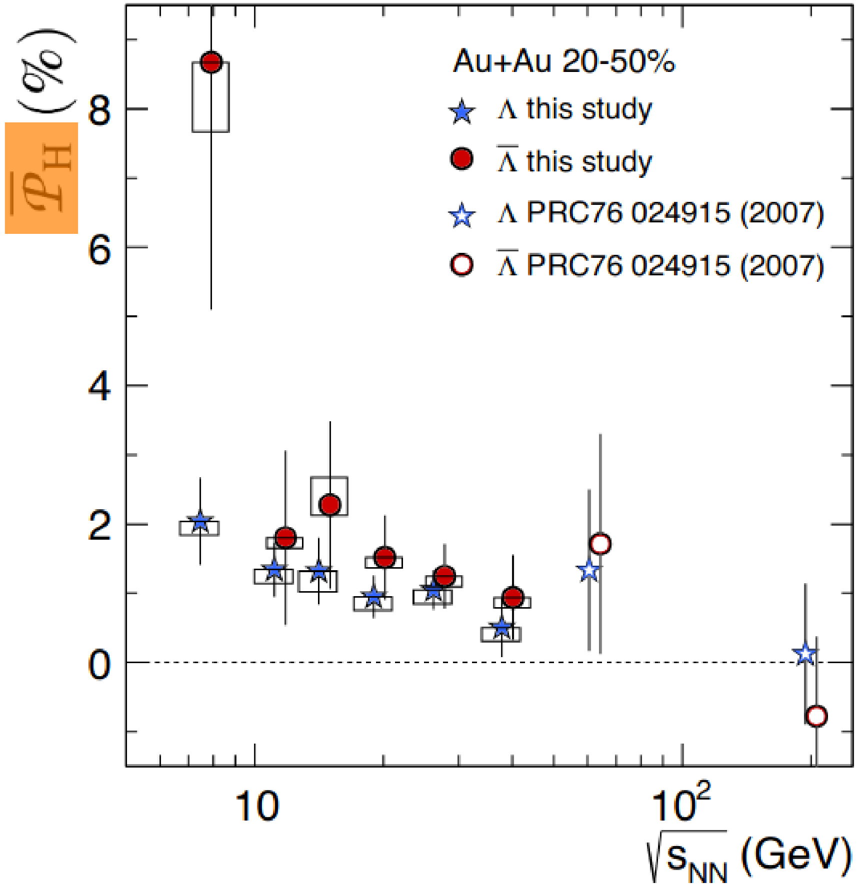
Offer a unique probe of spin property of QGP

global spin polarization in heavy ion collisions



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

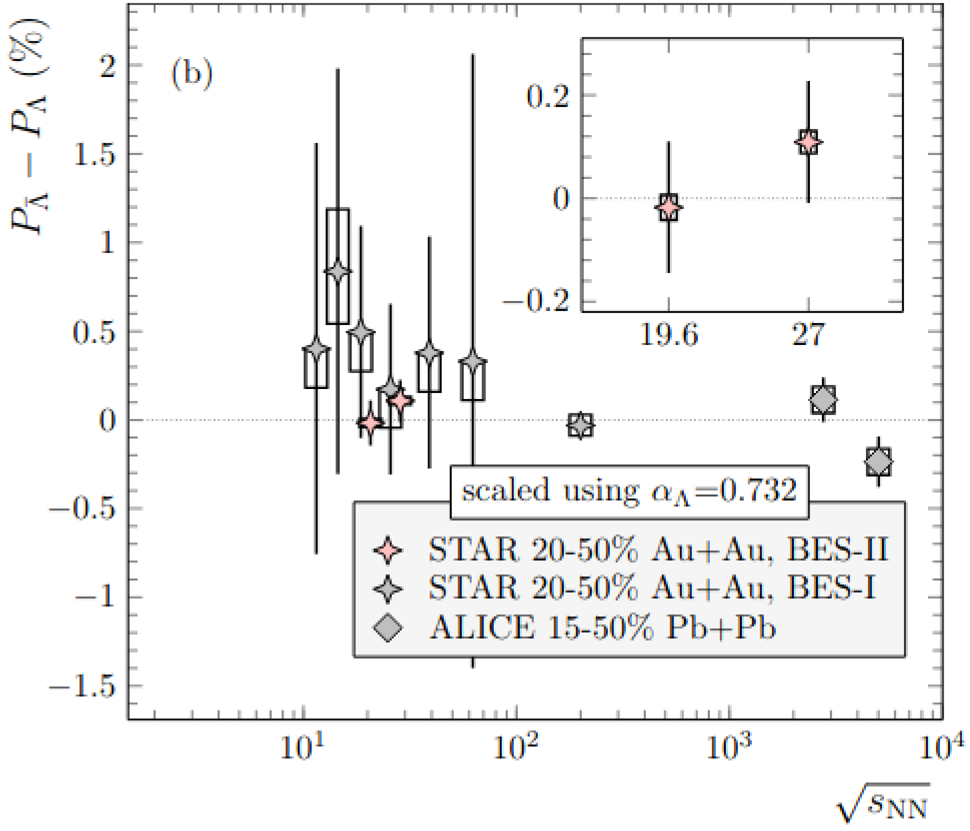
Liang, Wang, PRL 2005, PLB 2005



STAR collaboration, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Splitting in global spin polarization

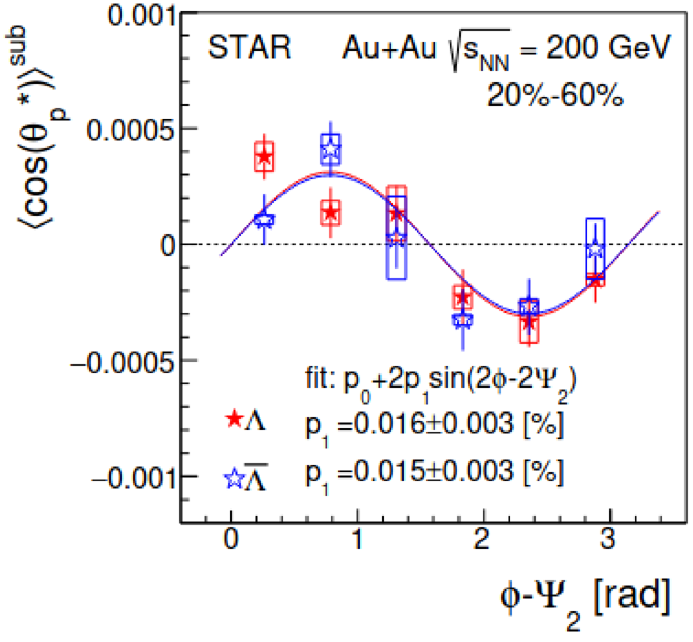


STAR collaboration, PRC 2023

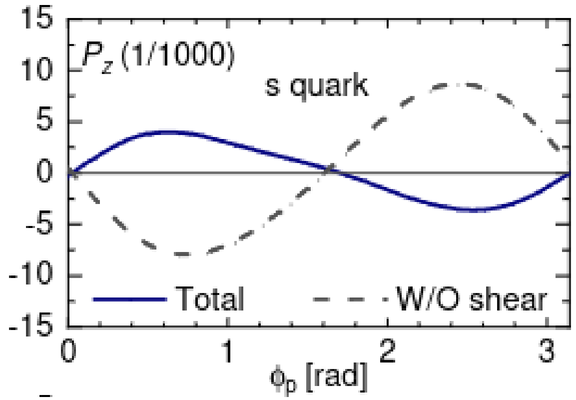
$$e^{-\beta(H_0 - \mathbf{S}\cdot\boldsymbol{\omega} - q\,\mathbf{S}\cdot\mathbf{B})}$$

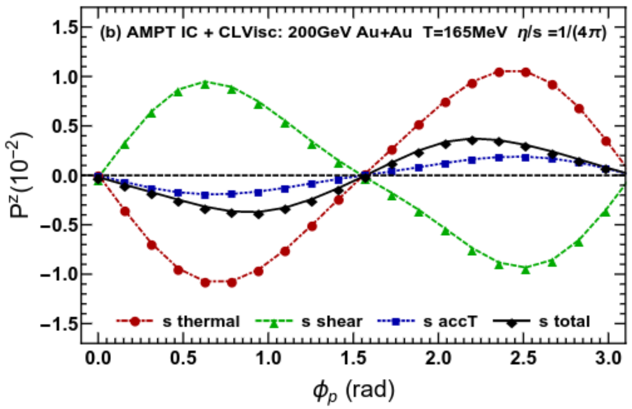
Existence of splitting inconclusive yet

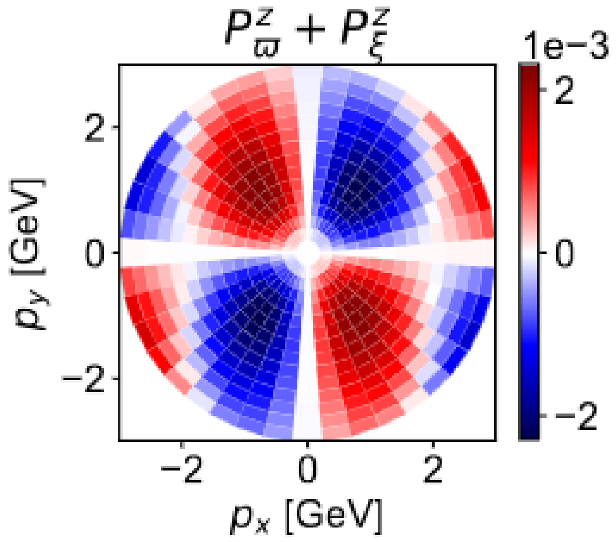
local spin polarization in heavy ion collisions



STAR collaboration, PRL 2019







Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

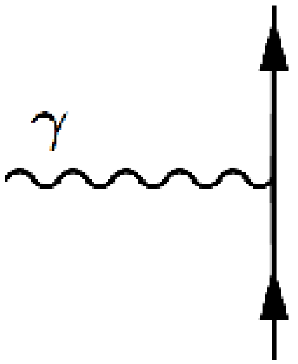
$\mathcal{P}^i \sim \omega^i \quad \mathcal{P}^i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}$ vorticity + shear

Spin polarization in heavy ion collisions

for $S = \frac{1}{2}$ particle

$$S_i \sim B_i$$

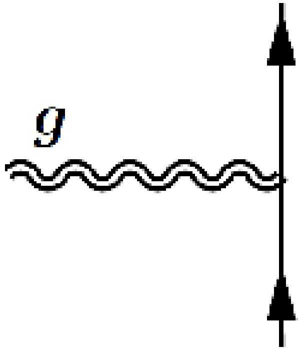
$$S_i \sim \epsilon^{ijk} \hat{p}_j E_k$$



external EM fields

$$S_i \sim \omega_i$$

$$S_i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$$



hydrodynamic gradient
(mimicked by metric)

How can radiative correction affect spin coupling to different sources?

Outline

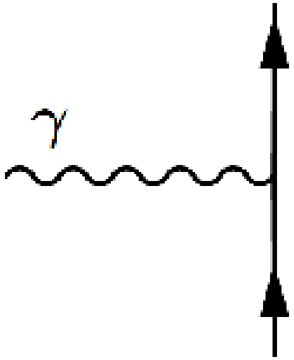
- Spin coupling to hydrodynamic gradient/EM fields ✓
- Spin polarization in EM fields from chiral kinetic theory
- Radiative corrections to spin polarization in EM fields
- Spin polarization in hydrodynamic state/metric perturbation from chiral kinetic theory
- (In)equivalence of off-equilibrium/metric perturbation
- Radiative corrections to spin polarization in hydrodynamic state
- Conclusion and outlook

Spin polarization from correlation functions

Wigner function

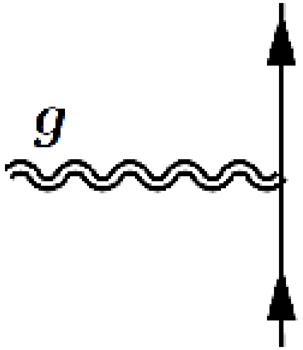
$$S^{<}_{\alpha\beta}(X = \frac{x+y}{2}, P) = \int d^4(x-y) e^{iP\cdot(x-y)/\hbar} \left(-\langle \bar{\psi}_\beta(y)\psi_\alpha(x)\rangle\right)$$

➢Spin polarization in EM fields



$$\langle S^{<}(X,P)\rangle_{\text{eq},A_\mu}$$

➢Spin polarization in hydrodynamic state

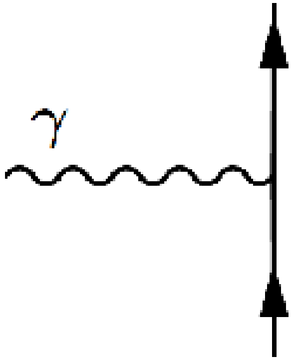


$$\langle S^{<}(X,P)\rangle_{\text{off-eq}} = \langle S^{<}(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$

$A_\mu,\ h_{\mu\nu}$ slow-varying $\partial_X \ll P$

Spin polarization in EM fields

$$\langle S^{<}(X,P)\rangle_{\mathrm{eq},A_\mu}$$



Chiral kinetic theory description

consider massless quark

$$\gamma_\mu \left(P^\mu + \frac{i}{2}\partial_X^\mu - \frac{i}{2}F^{\mu\nu}\partial_\nu^p \right) S^< = 0$$

Hidaka, Pu, Wang, Yang, PPNP 2022

$$S^< = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^\mu R_\mu + \left(1-\gamma^5\right)\gamma^\mu L_\mu \right]$$

$$\delta R^0 = 2\pi\, \mathbf{p}\cdot\mathbf{B}\, \delta'(P^2) f(p_0)$$

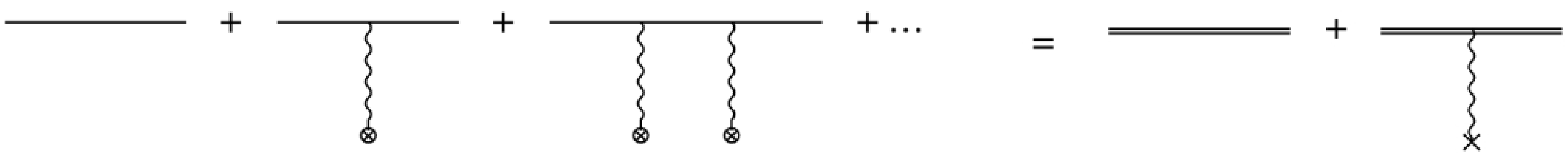
$$\delta R^i = 2\pi \left[\epsilon^{ijk} E_j p_k + p_0 B_i\right] \delta'(P^2) f(p_0)$$

Hidaka, Pu, Yang 2016

$$L_\mu = -R_\mu$$

Spin polarization from difference between R & L

Equivalent diagrammatic description: EM fields



gauge link

scattering on EM fields

$$\delta R^0 = 2\pi\, \mathbf{p}\cdot\mathbf{B}\,\delta'(P^2)\, f(p_0)$$

$$\delta R^i = 2\pi \left[\epsilon^{ijk} E_j p_k + p_0 B_i\right] \delta'(P^2)\, f(p_0)$$

modified spectral function

equilibrium distribution

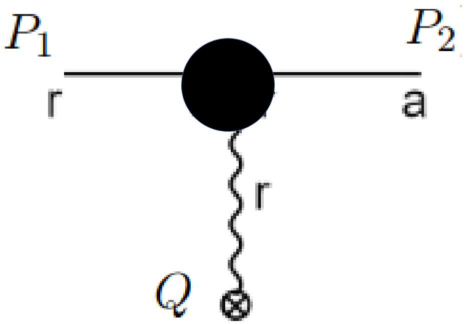
SL, Tian, 2023

Spin polarization = modified spectral function $\times$ equilbrium distibution

Standard KMS relation

In-medium electromagnetic form factors (FF)

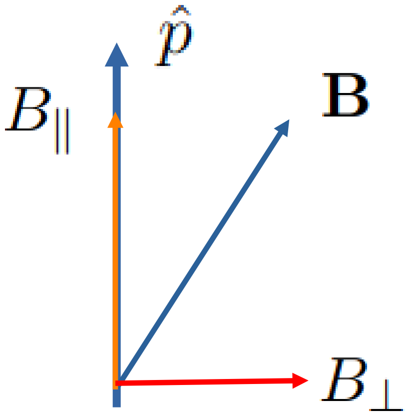
$$\Gamma^\mu = F_0 u^\mu + F_1 \hat{p}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho Q_\sigma}{2(P\cdot u)^2}$$



u^μ medium frame vector

$$S^{<0} = 2\pi F_2 p B_{||} \delta'(P^2) f(p_0)$$

$$S^{<i} = 2\pi \left[F_0 \epsilon^{ijk} E_j p_k + F_1 p_0 B_\perp^i + F_2 B_{||} p^i \right] \delta'(P^2) f(p_0)$$



spin Hall effect

spin-perpendicular magnetic coupling

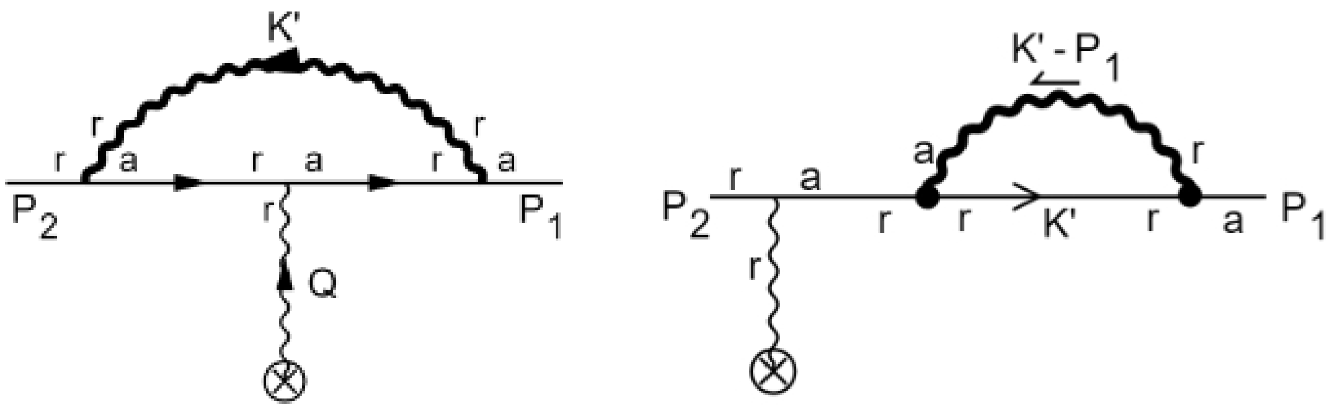
spin-parallel magnetic coupling

In vacuum $F_0 = F_1 = F_2 = 1$

In medium: lift of degeneracy expected

SL, Tian, 2023

Radiative correction to in-medium electromagnetic FF



$$\delta F_0 = \frac{2m_f^2}{p^2}X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin Hall effect

$$\delta F_1 = \frac{2m_f^2}{p^2}(X-1) + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-perpendicular magnetic coupling

$$\delta F_2 = \frac{2m_f^2}{p^2}X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-parallel magnetic coupling

SL, Tian, 2023

$$X(p,T)$$

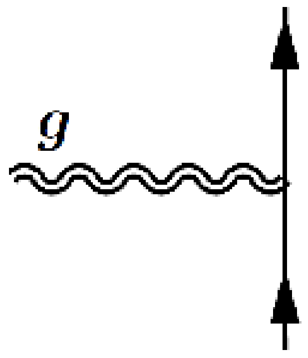
degeneracy partially lifted in in-medium FF

Spin polarization in hydrodynamic state

$$\langle S^{<}(X,P)\rangle_{\text{off-eq}} = \langle S^{<}(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$

CKT in
flat space

CKT in
curved space



CKT in flat space: off-equilibrium state

$$\frac{i}{2}\partial\!\!\!/ S^< + P\!\!\!/ S^< = 0$$

$$S^< = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^\mu R_\mu + \left(1-\gamma^5\right)\gamma^\mu L_\mu\right]$$

Hidaka, Pu, Yang 2016, 2017

$$R^\mu = -2\pi\delta(P^2)\left(P^\mu f_n + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho n_\sigma}{2P\cdot n}\partial_\nu f_n\right)$$

n^μ arbitrary frame vector $\rightarrow u^\mu$

$$\partial_i f\left(\frac{P\cdot u(X)}{T(X)}\right)$$ modified KMS relation (for axial component)

free theory dispersion + local equilibrium

$$f(p_0) \rightarrow f(p_0 - \frac{1}{2}\hat{p}\cdot\omega)$$ modified distribution

$$\Rightarrow \quad S^i \sim \left(\beta\omega^i + \epsilon^{ijk}\hat{p}_l\hat{p}_k\beta\sigma_{jl} + \partial_i\beta\right)$$

degenerate couplings to vorticity, shear, T-grad

CKT in curved space: equilibrium state

$$\gamma^\mu = e^\mu_a \gamma^a \qquad \bar{\psi} = \psi^\dagger \gamma^{\hat{0}}$$

μ curved index

a flat index

Gao, Huang, Mameda, Liu 2018

$$S^< = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^a R_a + \left(1-\gamma^5\right)\gamma^a L_a\right]$$

Clifford algebra in flat basis

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} n_d}{2P\cdot n} e^\mu_b D_\mu (P_c f_n)\right)$$

f_n : equilibrium distribution

$$D_\mu = \partial^X_\mu + \Gamma^\lambda_{\mu\nu}\frac{\partial}{\partial P_\nu}P_\lambda \qquad h_{0i} = -v^i \quad h_{00} = -2\frac{\delta T}{T}$$

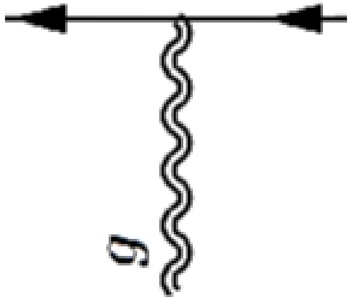
- Christoffel term can realize vorticity and T-grad, but not shear!

$$\langle S^<(X,P)\rangle_{\text{off-eq}} = \langle S^<(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$

Inequivalence?

Diagrammatic description of metric perturbation

Effect I: scattering of fermion



$$S^<(X,P) = \int d^4y\sqrt{-g(X)}e^{-iP\cdot y}\langle\bar{\psi}_\beta(X+\frac{y}{2})\psi_\alpha(X-\frac{y}{2})\rangle$$

Gao, Huang, Mameda, Liu 2018

$$\bar{\psi}(X+\frac{y}{2}) = \bar{\psi}(X)\exp(\frac{y}{2}\cdot\overleftarrow{D}) \quad \psi(X-\frac{y}{2}) = \exp(-\frac{y}{2}\cdot D)\psi(X)$$

gravitational gauge link

$$D_\mu = \partial_\mu^X + \frac{1}{4}\omega_{\mu,ab}\gamma^{ab} - \Gamma^\lambda_{\mu\nu}y^\nu\partial^y_\lambda$$

Effect II: rotation of spinor by spin connection

Summary of three approaches

Equilibrium + metric perturbation:
I. CKT;
II. diagrams

III. Off-equilibrium CKT

- II = I + spin-vorticity coupling
- Neither I nor II reproduces III

Need to work with off-equilibrium state

Off-equilibrium state from metric perturbation

static metric perturbation $h_{0i} = -v^i \quad h_{00} = -2\frac{\delta T}{T}$

equilibrium state in curved space $f(\frac{p_\mu u^\mu}{T}) \quad u^\mu = (g_{00}^{-1/2}, 0, 0, 0)$

$$f\left(\frac{p_a u^a}{T}\right) \qquad u^a = u^\mu e^a_\mu$$

choice of vielbein (local reference frame)

$$e^{\hat{0}}_0 = 1 + \frac{h_{00}}{2}, \quad e^{\hat{j}}_i = -\delta_{ij}, \quad e^{\hat{i}}_0 = -h_{0i}.$$

off-equilibrium state in flat space $u^{\hat{0}} = 1, \quad u^{\hat{i}} = -h_{0i}.$

SL, Tian, to appear

Types of radiative corrections

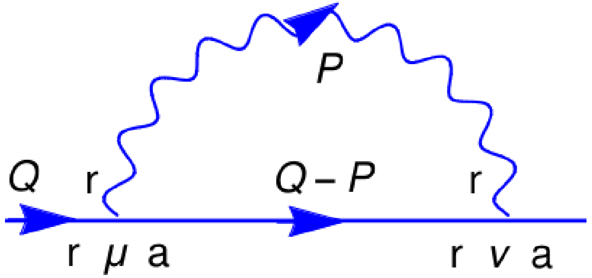
- spectral function
- distribution function
- KMS relation

At tree level, spin polarization from modified KMS + modified distribution

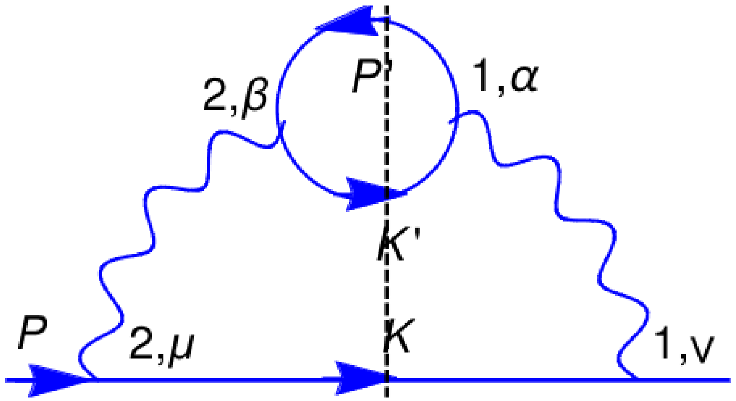
At loop level, radiative correction occurs in all three types

Focus on radiative correction to spectral function

How to include self-energy corrections?



correction to spectral function, usually ignored in kinetic theory



collision term in steady state

$$\delta f \sim O\left(\frac{\partial}{g^4}\right)$$

$$g^4 \times \delta f \sim O(\partial)$$

diagramatic resummation Gagnon, Jeon, 2006

consider correction to spectral function w/ off-equilibrium effect

collisional contribution to spin-shear coupling: SL, Wang, 2022, 2024

Spectral function representation

$$\rho_{\alpha\beta}(P) = \int d^4x e^{iP\cdot x}\langle\psi_\alpha(x)\bar{\psi}_\beta(0) + \bar{\psi}_\beta(0)\psi_\alpha(x)\rangle$$

$$S_{ra,\alpha\beta} = \int d^4x e^{iP\cdot x}\theta(x_0)\langle\psi_\alpha(x)\bar{\psi}_\beta(0) + \bar{\psi}_\beta(0)\psi_\alpha(x)\rangle$$

$$\rho(P) = 2\text{Re}[S_{ra}(P)] = 2\text{Im}\left[S_R\right]$$

valid for off-equilibrium state invariant under time-reversal

SL, Tian, to appear

Gradient corrections to retarded function

$$\frac{i}{2}\partial\!\!\!/ S_R(X,P) + P\!\!\!/ S_R(X,P) - \left(\Sigma_R(X,P)S_R(X,P) + \frac{i}{2}\{\Sigma_R(X,P), S_R(X,P)\}_{\rm PB}\right) = -1$$

$$S_R = S_R^{(0)} + S_R^{(1)} + \cdots,$$

$$\{A,B\}_{\rm PB} = \partial_P A\cdot\partial_X B - \partial_X A\cdot\partial_P B$$

$$S_R^{(0)} = -\frac{1}{P\!\!\!/} - \frac{1}{P\!\!\!/}\Sigma_R\frac{1}{P\!\!\!/}$$

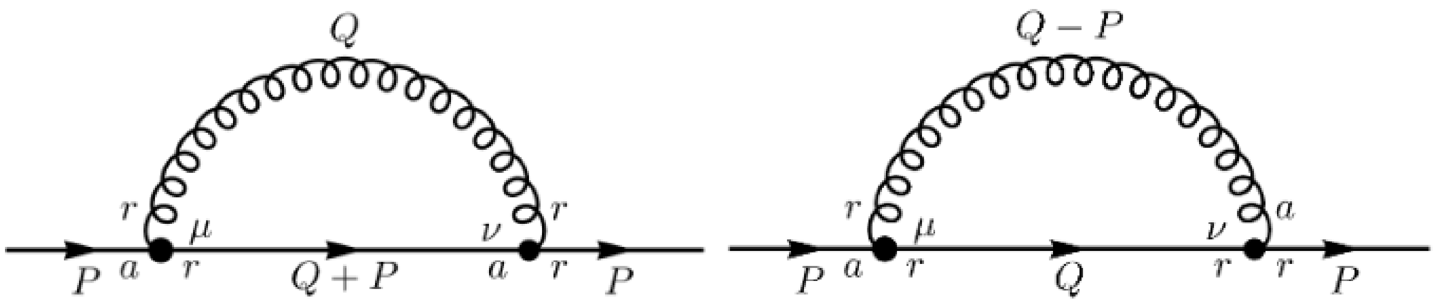
$$S_R^{(1)} = -\frac{1}{P\!\!\!/}\delta\Sigma_R\frac{1}{P\!\!\!/} + \gamma^5\gamma^\beta P^\nu T^{\mu\lambda}\epsilon_{\beta\lambda\mu\nu}\frac{-1}{(P^2)^2}$$

$$T_{\mu\lambda} = \partial_{[\mu}\Sigma^R_{\lambda]}$$

involves both equilibrium/off-equilibrium self-energy

SL, Tian, to appear

Equilibrium self-energy



$$\frac{\Sigma_{ar}}{g^2 C_F} = 2i\not{P}(A+B) + 4ip_0\gamma^0 A.$$

$P \gg T$, $P^2 \ll p/\beta$,

Energetic particle
close to mass shell

$$p_0 = p(1 + 8A)$$

$$A = \frac{1}{2(2\pi)^2}\frac{-i\pi}{2p\beta}.$$

modified dispersion:

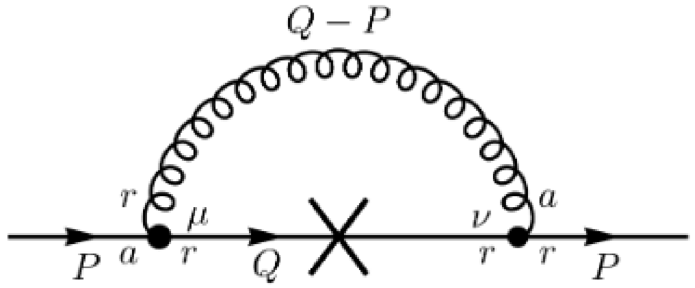
finite damping

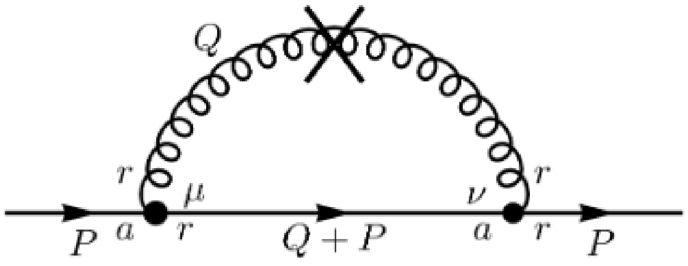
$$p_0 \to p\cdot u(X),\ \beta \to \beta(X)$$

No correction to
spectral function

SL, Tian, to appear

Off-equilibrium self-energy





off-equilibrium propagators for quark/gluon from CKT

Hidaka, Pu, Yang 2017

Huang, Mitkin, Sadofyev, Speranza 2020

Hattori, Hidaka, Yamamoto, Yang 2020

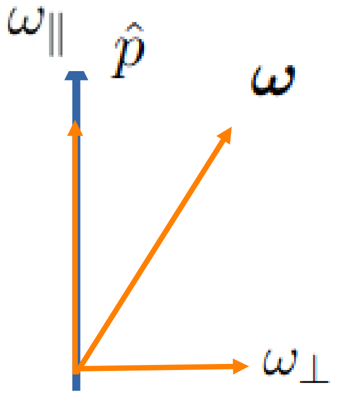
$$\frac{\delta\Sigma_{ar}}{g^2 C_F} = \gamma^5\gamma^\mu \mathcal{A}_\mu,$$

$$P \gg T \quad P^2 \ll p/\beta,$$

Energetic particle close to mass shell

$$\mathcal{A}^0 = i\omega^i p_i \beta(-4\delta A - 2\delta B),$$

$$\mathcal{A}^k = i\omega_{\parallel}^k \beta p(-4\delta A - 2\delta B) + \omega_{\perp}^k \beta p(-4\delta A - \delta B) + \epsilon^{ijk}\hat{p}_i\hat{p}_l\sigma_{jl}\beta p(-\delta B) + \epsilon^{ijk}p_i\partial_j\beta(-\delta C)$$



$$\delta C = \frac{1}{4(2\pi)^2}\left(\frac{-4C_a + 2C_b + i\pi C_a - 2C_a \ln\frac{p\beta(-1+a)}{2}}{2p\beta}\right)$$

$$a = p_0/p + i\eta.$$

similar expressions for δA δB

Polarized quasi-particle

tree level vs loop correction

loop

$$\int dp_0 \delta S^<(P) = \int dp_0 \delta\rho(P) f(p_0)$$

$$= \frac{g^2}{2(2\pi)^2}\frac{\pi}{2p}\gamma^5\gamma_i\Big[\omega^i_{\parallel}(2C_b+(-2+\ln 4)C_a)+\frac{2C_b+(2+\ln 4)C_a}{2}\omega^i_{\perp}+$$

$$\frac{2C_b+(-6+\ln 4)C_a}{2}\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}+\frac{2C_b+(-4+\ln 4)C_a}{2}\epsilon^{ijk}\hat{p}_j\frac{\partial_k\beta}{\beta}\Big]f(p)$$

$$C_a = 0.5 \qquad C_b \simeq 0.630,$$

tree

$$\int dp_0 S^<_{(0)}(P) = -\gamma^5\gamma_i\frac{2\pi}{2}\left(\omega^i+\epsilon^{ijk}\hat{p}_k\frac{\partial_j\beta}{\beta}+\epsilon^{ijk}\hat{p}_k\hat{p}_l\sigma_{lj}\right)f'(p)$$

degeneracy in couplings to
vorticity, shear, T-grad lifted

SL, Tian, to appear

Conclusion

- Spin polarization in off-equilibrium state can’t be fully mimicked by metric perturbation on equilibrium state
- Equilbrium state in curved space describes off-equilbrium state in flat space with suitable choice of local rest frame
- Sources of polarization: modified spectral; modified distribution; modified KMS
- Radiative correction to spectral function lifts degeneracy of spin coupling to vorticity, shear and T-grad

Outlook

- Radiative correction to distribution function a la gravitational FFs
- Radiative correction to modified KMS

Thank you!

Equilibration of hydro DOF

$$G^R_{\pi_i\pi_j} = \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right)\frac{\eta k^2}{i\omega - \gamma_\eta k^2} + \hat{k}_i\hat{k}_j \frac{(\epsilon+p)(k^2c_s^2 - i\omega\gamma_s k^2)}{\omega^2 - k^2c_s^2 + i\omega\gamma_s k^2}$$

$$G^R_{\epsilon\epsilon} = \frac{(\epsilon+p)k^2}{\omega^2 - k^2c_s^2 + i\omega\gamma_s k^2}$$

$\omega \to 0$ $\quad G^R_{\pi_i\pi_j} \to -\delta_{ij}(\epsilon+p) \quad G^R_{\epsilon\epsilon} \to -\frac{\epsilon+p}{c_s^2}$

$$\delta\pi_i = h_{0j}G^R_{\pi_i\pi_j} = -(\epsilon+p)\delta_{ij}h_{0j} \qquad \delta\epsilon = \frac{1}{2}h_{00}G^R_{\epsilon\epsilon} = -\frac{(\epsilon+p)}{2c_s^2}h_{00}$$

$\longrightarrow$ $\quad h_{0i} = -v^i \quad h_{00} = -2\frac{\delta T}{T}$

Equilibration of hydro DOF in static metric perturbation