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Phase transition dynamics and its implications for dark matter and gravitational wave

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Sun Yat-sen university

INPAC/TDLI Joint Seminar, 2024.11.13

Outline

1. Motivation

2. Electroweak strong first-order phase transition (SFOPT) and phase transition gravitational wave (GW) in a nutshell

3.Case I: Dark matter (DM) induced SFOPT (wall velocity)

4.Case II: anti-filtered (gauged)Q-ball DM

5.Case III: filtered DM

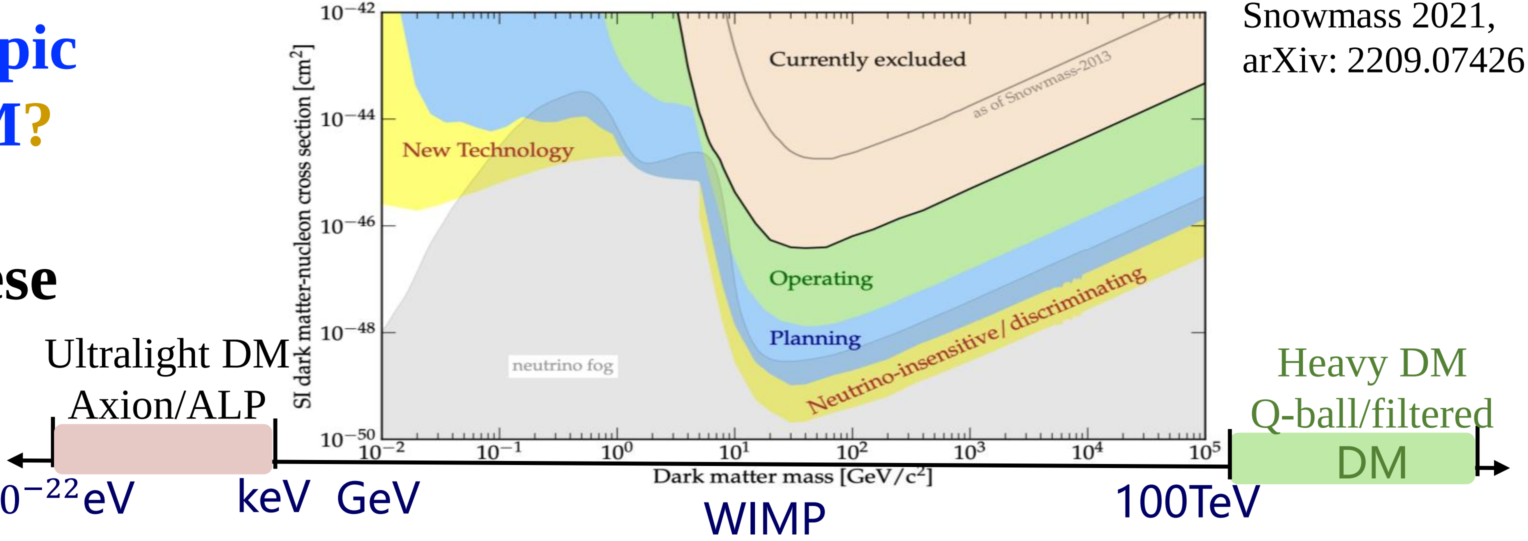
6.Summary and outlook

Motivation DM theory and experiments status

What is the microscopic nature of DM?

No expected signals in these region

arXiv: 1904:07915
Snowmass 2021,
arXiv: 2209.07426



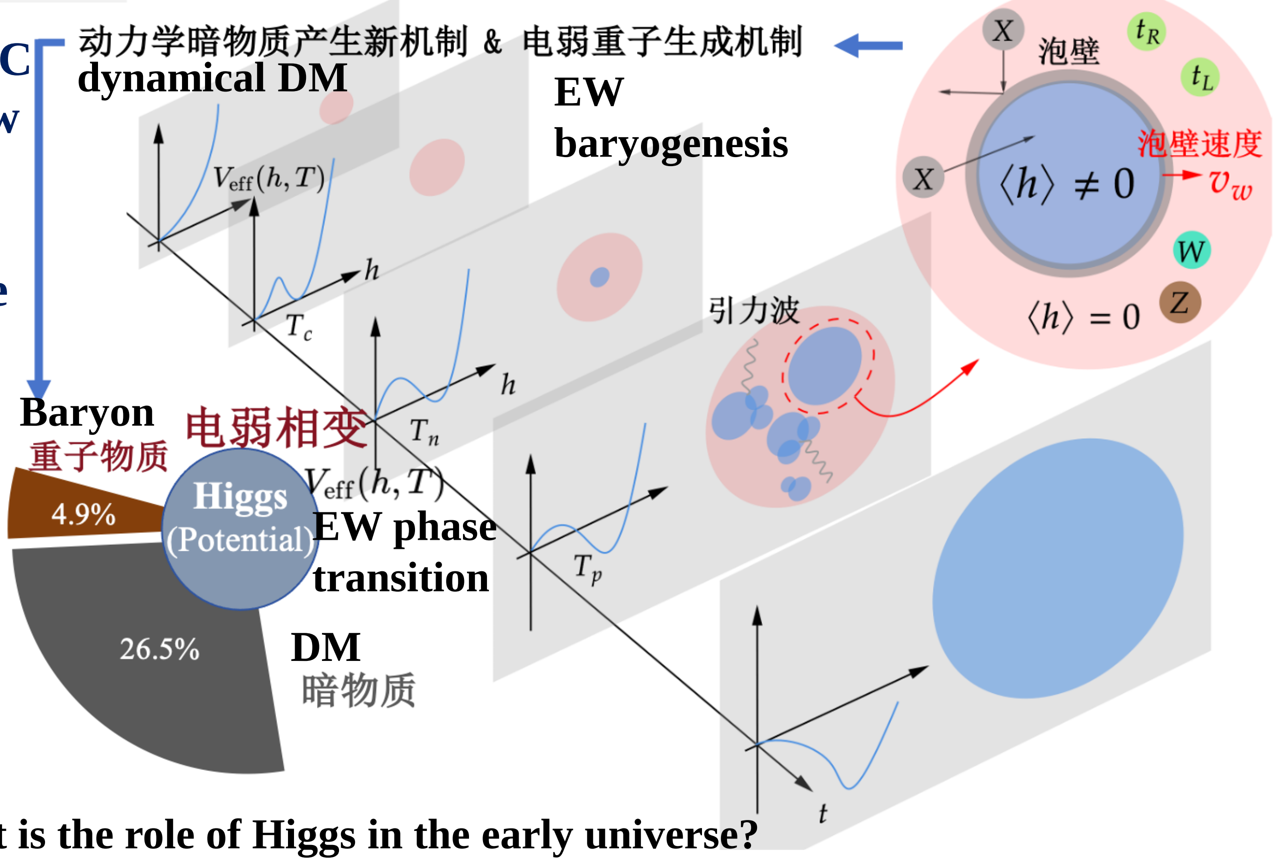
- **new DM mechanism beyond freeze out: cosmic phase transition**
- **new detection method: GW detector (LISA, TianQin, Taiji, aLIGO, FAST, SKA, NanoGrav, Cosmic Explorer…)**

Motivation DM in post-Higgs and GW Era

The observation of Higgs@LHC and GW@LIGO initiates a new era of exploring DM by GW.

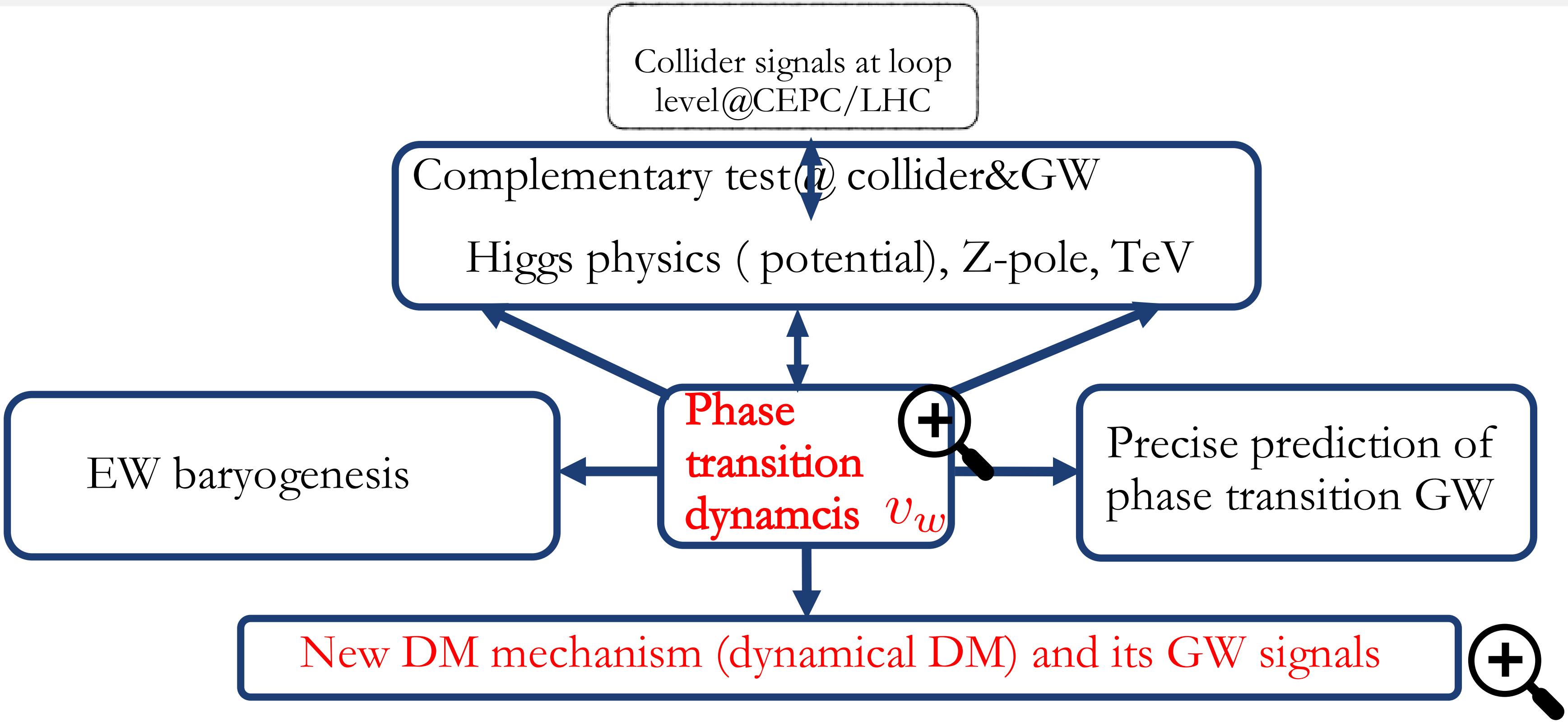
SFOPT by Higgs could provide a new approach for DM production.

Higgs' deep connections to cosmology, such as Higgs inflation, EW baryogenesis, and DM testable by GW signals.



What is the role of Higgs in the early universe?

Motivation **DM in post-Higgs and GW Era**



Particle physics、(finite temperature) QFT、hydrodynamics、GR

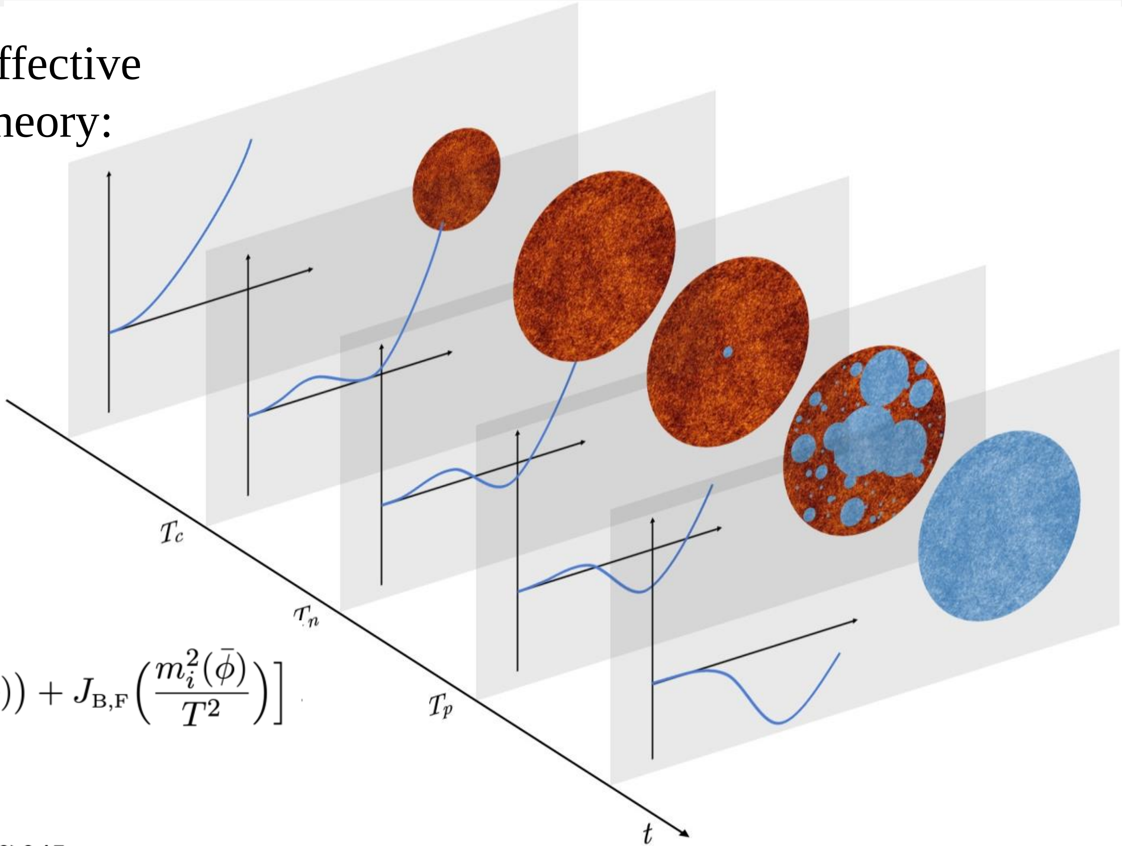
Phase transition in a nutshell

Calculate the finite-temperature effective potential using the thermal field theory:

Quantum tunnel $\Gamma = \Gamma_0 e^{-S(T)}$

$$S(T) = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\text{eff}} (\phi, T) \right]$$

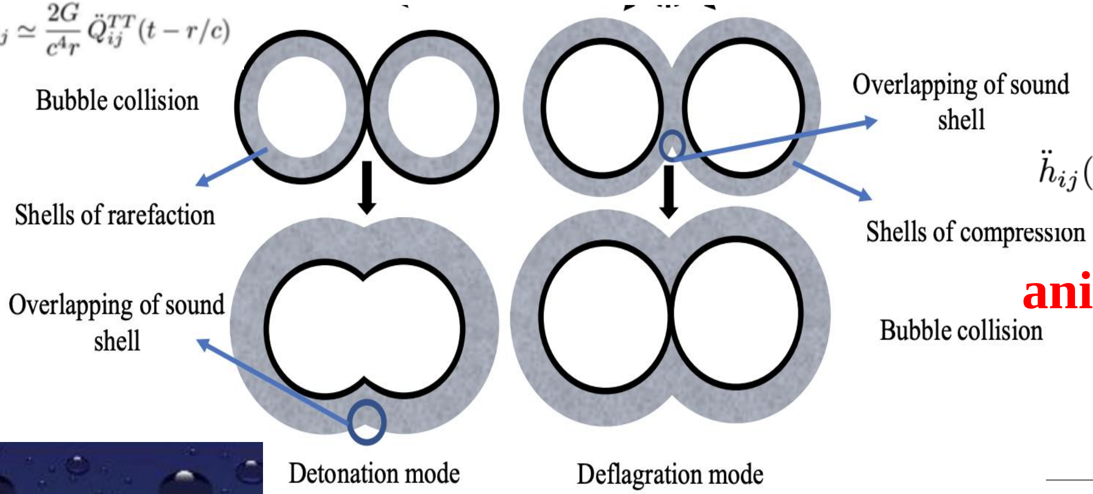
$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \Big[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\big(p^2 + m_i^2(\bar{\phi})\big) + J_{\mathrm{B,F}} \Big(\frac{m_i^2(\bar{\phi})}{T^2} \Big) \Big]$$



Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

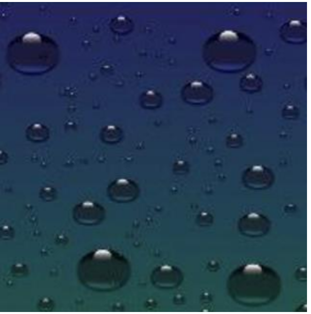
$$h_{ij} \simeq \frac{2G}{c^4 r}\ddot{Q}_{ij}^{TT}(t-r/c)$$

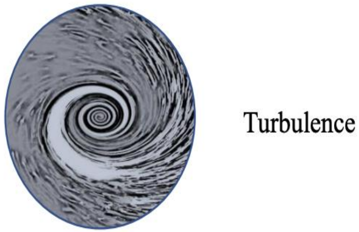


$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2} h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

anisotropic stress tenor: source of GW





General form Π_{ij}
$[\partial_i\phi\partial_j\phi]^{TT}$
$[\gamma^2(\rho+p)v_iv_j]^{TT}$
$[-E_iE_j - B_iB_j]^{TT}$
$\partial_i\Psi, \partial_i\Phi$

E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

characteristic frequency of the GW signal

$$f_* = \frac{1}{\ell_*} \geq H_*$$

$$\epsilon_* = \ell_* H_*$$

Ratio of the typical length-scale of the GW sourcing process (size of the anisotropic stresses) and the Hubble scale at the generation time

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\text{GeV}} \text{Hz}$$

Peak frequency of EW phase transition GW is around mHz detecatable by LISA、TianQin、Taiji

Phase transition dynamics

Theory: The most important and difficult phase transition parameter for GW, dynamical DM, baryogenesis is bubble wall velocity v_w

Finite-temperature effective potential

$V_{eff}(\phi, T)$

α T_p $R_* H_*$

(1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
(2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
(3). No perturbative calculations: lattice calculations and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity this talk v_w

Energy budget

κ

Experiment: GW experiment is most sensitive to bubble wall velocity v_w

arXiv: 2404.18703
Aidi Yang, **FPH**

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang ， arXiv:2007.10343，
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith， arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang,
Phys.Rev.D 107 (2023) 9, 095005

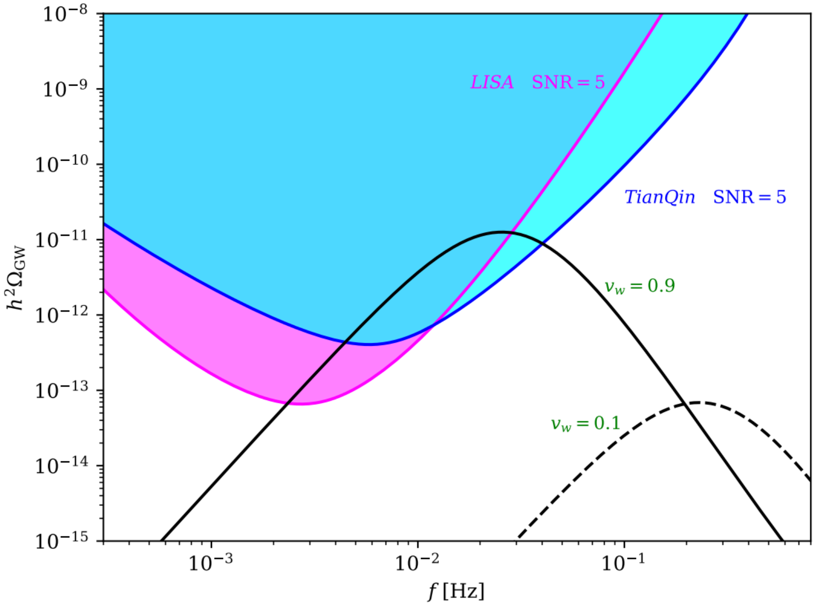
F. Giese, T. Konstandin, K. Schmitz and J. van de ,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang,
Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, JCAP 07 (2023) 006

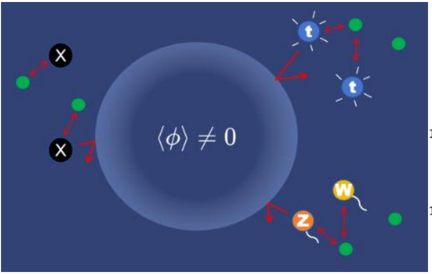
Bubble wall is essential (like a filter)

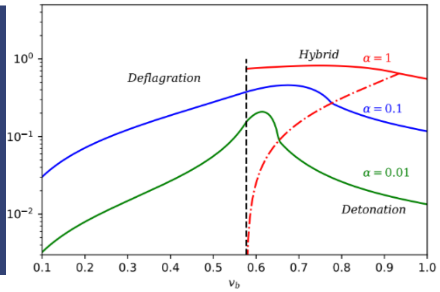
The most essential parameter for phase transition GW, phase transition DM, baryogenesis v_w

GW detection favor lager v_w
EW baryogenesis favor smaller v_w
Dynamical DM is sensitive to v_w

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang , arXiv:2007.10343 ,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith ,
arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005







$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Case I:DM induced SFOPT (wall velocity)

Inert Doublet Models (example)

$$V_0 = M_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 \Phi^\dagger \Phi D^\dagger D + \lambda_4 |\Phi^\dagger D|^2 + (\lambda_5/2)[(\Phi^\dagger D)^2 + h.c.],$$

mixed singlet-doublet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_D^2 H_2^\dagger H_2 + \frac{1}{2}\lambda_S S^2 |\Phi|^2 + \lambda_3 \Phi^\dagger \Phi H_2^\dagger H_2 + \lambda_4 |\Phi^\dagger H_2|^2 + \frac{\lambda_5}{2}[(\Phi^\dagger H_2)^2 + \text{H.c.}] + A[S\Phi H_2^\dagger + \text{H.c.}].$$

mixed singlet-triplet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_\Sigma^2 \text{Tr}(H_3^2) + \kappa_\Sigma \Phi^\dagger \Phi \text{Tr}(H_3^2) + \frac{\kappa}{2} |\Phi|^2 S^2 + \xi S \Phi^\dagger H_3 \Phi.$$

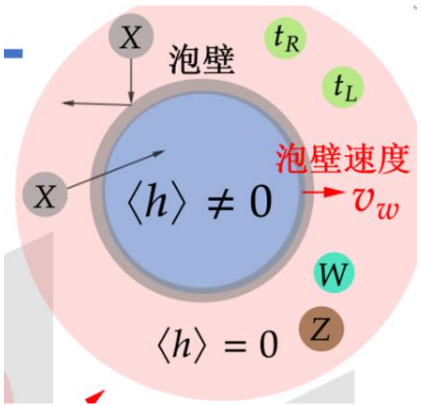
provide natural DM candidate

produce SFOPT and phase transition GW

FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022

Yan Wang, Chong Sheng Li, and **FPH**, *Phys.Rev.D* 104 (2021) 5, 053004;

How to calculate wall velocity?



Equation of Motion (EoM) of the Higgs field

$$(1-v_w^2)h'' + \sum_i \frac{dm_i^2}{dh} \int \frac{d^3p}{(2\pi)^3 2E_i} \delta f_i(x,p) + \frac{\partial V_{\text{eff}}(h,T)}{\partial h} = 0$$

Boltzmann equation

$$\frac{d}{dt}(f_i^{\text{eq}} + \delta f_i) = \left(\left(v_w + \frac{p_z}{E} \right) \frac{\partial}{\partial z} - \frac{(m_i^2)'}{2E} \frac{\partial}{\partial p_z} \right) (f_i^{\text{eq}} + \delta f_i) = -C[(f_i^{\text{eq}} + \delta f_i)]$$

Main task:

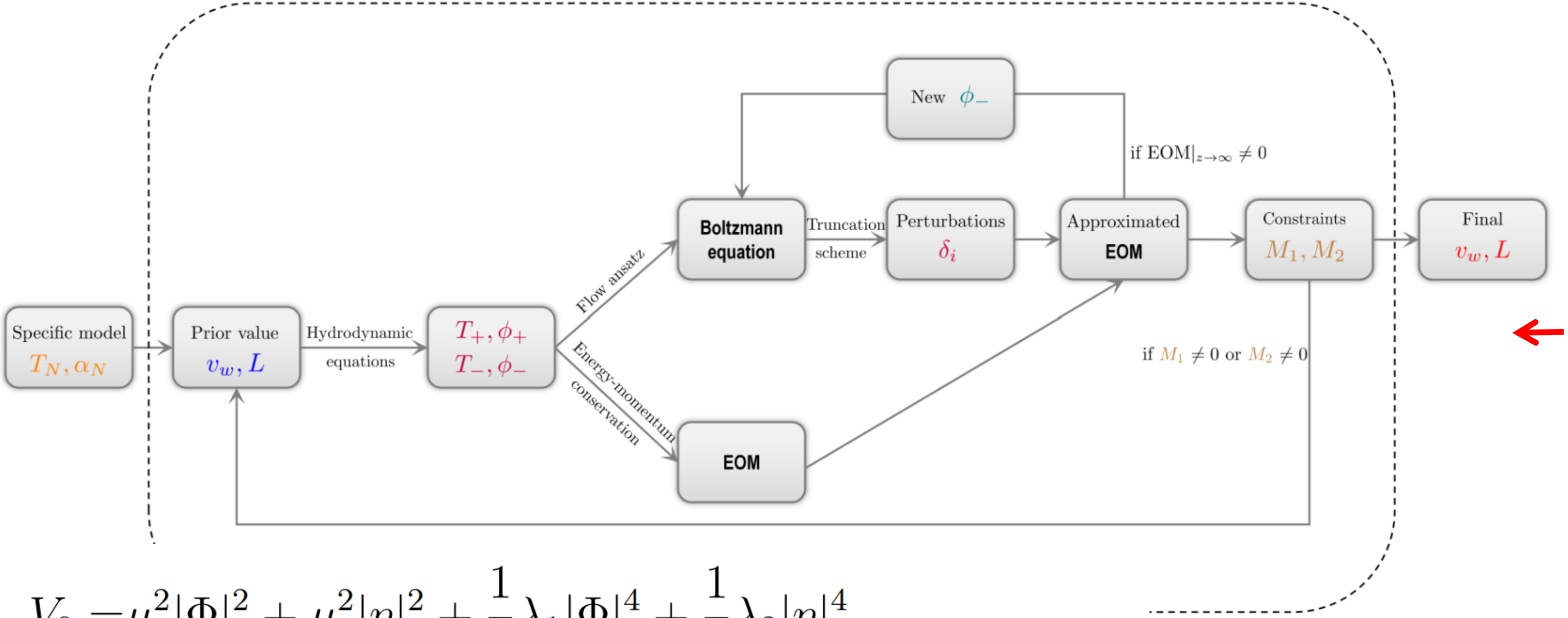
- Solving the Boltzmann equations for perturbations by using flow ansatz
- Calculating the collision terms analytically and numerically

δf_i

Substituting the perturbations into EoM and finally get bubble wall velocity and width v_w, L_w

Phase transition dynamics

A simple DM Model: **Bubble wall velocity in inert doublet model**



← More accurate processes

$$V_0 = \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4 + \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \mathrm{H.c.}\} \ ,$$

Siyu Jiang, **FPH**, Xiao Wang,
Phys.Rev.D 107 (2023) no.9, 095005

Higgs EoM in thermal plasma

The energy-momentum tensor of the scalar field

$$T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - V_{T=0}(\phi)\right)$$

The energy-momentum tensor of the plasma

$$T_{\rm pl}^{\mu\nu} = \sum_i \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{E_i}f_i(x,p)$$

Using energy-momentum conservation $\nabla_\mu(T_\phi^{\mu\nu} + T_{\rm pl}^{\mu\nu}) = 0$, $V_{\rm eff}(\phi,T) = V_{T=0}(\phi) + V_T(\phi)$

$$\rightarrow \Box\phi + \frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} + \underbrace{\sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3p}{(2\pi)^3 2E_i}\delta f_i(x,p)}_{\text{friction term}} = 0 \quad \text{(EOM)}$$

Higgs EoM in thermal plasma

Higgs EOM in the plasma frame

$$\left(1-v_w^2\right)\phi'' + \underbrace{\sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3p}{(2\pi)^3 2E_i}\delta f_i(x,p)}_{friction\ \ term} + \frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} = 0$$

Key point to calculate bubble wall velocity is to obtain the distribution function for the massive particles or the friction term.

How to calculate the distribution function or the friction term?
Boltzmann equation !

Friction term

The distribution function for each particle can be described by Boltzmann equation under WKB approximation

$$\frac{d}{dt}f = \left(\frac{\partial}{\partial t} + \dot{z}\frac{\partial}{\partial z} + \dot{p}_z\frac{\partial}{\partial p_z}\right) f = -C[f]$$

$\dot{z} = p_z/E$ and $\dot{p}_z = -\partial_z E = -(m^2)'/(2E)$ (external force from bubble wall)

Parameterization of the distribution function $f = \frac{1}{e^{(E+\delta)/T} \pm 1}$

taking reasonable flow ansatz

$$\delta = -\mu - \mu_{bg} - \frac{E}{T}(\delta T + \delta T_{bg}) - p_z(\delta v + \delta v_{bg})$$

The light particles contributes to the background.

Friction term

求解玻尔兹曼方程得到粒子偏离热平衡的扰动分布

$$\frac{d}{dt}(f_i^{\mathrm{eq}} + \delta f_i) = \left(\left(v_w + \frac{p_z}{E}\right)\frac{\partial}{\partial z} - \frac{(m_i^2)'}{2E}\frac{\partial}{\partial p_z}\right)(f_i^{\mathrm{eq}} + \delta f_i) = -C[(f_i^{\mathrm{eq}} + \delta f_i)]$$

Boltzmann equation for each particle species:

$$(-f_0')\left(\frac{p_z}{E}\left[\partial_z\mu + \frac{E}{T}\partial_z\left(\delta T + \delta T_{bg}\right) + p_z\partial_z\left(\delta v + \delta v_{bg}\right)\right] + \partial_t\mu \right.$$
$$\left. + \frac{E}{T}\partial_t\left(\delta T + \delta T_{bg}\right) + p_z\partial_t\left(\delta v + \delta v_{bg}\right)\right) + TC[\mu, \delta T, \delta v] = (-f_0')\frac{\partial_t\left(m^2\right)}{2E}$$

How to solve this equation?

Friction term

Use truncation scheme to solve the perturbations

After integration by $\int d^3p/(2\pi)^3$, $\int Ed^3p/(2\pi)^2$, and $\int p_z d^3p/(2\pi)^3$, (truncation scheme)

$$v_w c_2^i \left(\mu_i' + \mu_{bg}'\right) + v_w c_3^i \left(\delta T_i' + \delta T_{bg}'\right) + \frac{c_3^i T}{3}\left(\delta v_i' + \delta v_{bg}'\right) + \mu_i \Gamma_{\mu 1,i} + \delta T_i \Gamma_{T1,i} = \frac{v_w c_1^i}{2T}(m_i^2)' ,$$

$$v_w c_3^i \left(\mu_i' + \mu_{bg}'\right) + v_w c_4^i \left(\delta T_i' + \delta T_{bg}'\right) + \frac{c_4^i T}{3}\left(\delta v_i' + \delta v_{bg}'\right) + \mu_i \Gamma_{\mu 2,i} + \delta T_i \Gamma_{T2,i} = \frac{v_w c_2^i}{2T}(m_i^2)' ,$$

$$\frac{c_3^i}{3}\left(\mu_i' + \mu_{bg}'\right) + \frac{c_4^i}{3}\left(\delta T_i' + \delta T_{bg}'\right) + \frac{v_w c_4^i T}{3}\left(\delta v_i' + \delta v_{bg}'\right) + \delta v_i T \Gamma_{v,i} = 0$$

Phase transition dynamics

$$f = \frac{1}{e^{(E+\delta)/T} \pm 1}$$

$$(-f_0')\left(\frac{p_z}{E}\left[\partial_z \mu + \frac{E}{T}\partial_z\left(\delta T + \delta T_{bg}\right) + p_z \partial_z\left(\delta v + \delta v_{bg}\right)\right] + \partial_t \mu + \frac{E}{T}\partial_t\left(\delta T + \delta T_{bg}\right) + p_z \partial_t\left(\delta v + \delta v_{bg}\right)\right) + TC[\mu, \delta T, \delta v] = (-f_0')\frac{\partial_t\left(m^2\right)}{2E}$$

truncation scheme →

$$\hat{A}\delta' + \Gamma\delta = \Sigma,$$

source term: Σ

collision term: Γ

$$\delta = \left(\mu_t, \delta T_t, T\delta v_t, \mu_W, \delta T_W, T\delta v_W, \mu_A, \delta T_A, T\delta v_A\right),$$

$$\Sigma = \frac{v_w}{2T}\left(c_1^t (m_t^2)', c_2^t (m_t^2)', 0, c_1^W (m_W^2)', c_2^W (m_W^2)', 0, c_1^A (m_A^2)', c_2^A (m_A^2)', 0\right),$$

$$\hat{A} = \begin{pmatrix} \hat{A}_t & 0 & 0 \\ 0 & \hat{A}_W & 0 \\ 0 & 0 & \hat{A}_A \end{pmatrix}, \quad \text{where} \quad \hat{A}_i = \begin{pmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3}c_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3}c_4^i \\ \frac{1}{3}c_3^i & \frac{1}{3}c_4^i & \frac{1}{3}v_w c_4^i \end{pmatrix},$$

Phase transition dynamics

Collision terms (Monte Carlo integration)

$$\Gamma_{\mu1,t} \simeq \left(5.0\times10^{-4} g_s^4 + 5.8\times10^{-4} g_s^2 y_t^2\right) T\ ,$$

$$\Gamma_{T1,t} \simeq \Gamma_{\mu2,t} \simeq \left(1.1\times10^{-3} g_s^4 + 1.3\times10^{-3} g_s^2 y_t^2\right) T\ ,$$

$$\Gamma_{T2,t} \simeq \left(1.1\times10^{-2} g_s^4 + 4.0\times10^{-3} g_s^2 y_t^2\right) T\ ,$$

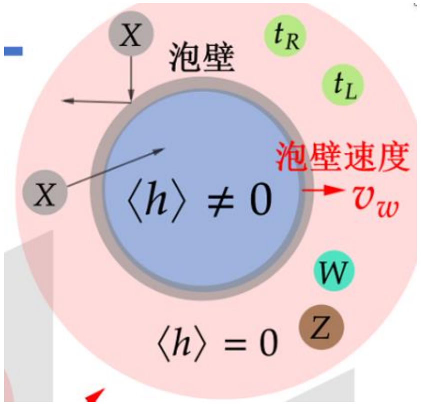
$$\Gamma_{v,t} \simeq \left(2.0\times10^{-2} g_s^4 + 1.8\times10^{-3} g_s^2 y_t^2\right) T\ ,$$

$$\Gamma_{\mu1,W} \simeq \left(2.3\times10^{-3} g_s^2 g_w^2 + 2.0\times10^{-3} g_w^4\right) T\ ,$$

$$\Gamma_{T1,W} \simeq \Gamma_{\mu2,W} \simeq \left(4.7\times10^{-3} g_s^2 g_w^2 + 4.1\times10^{-3} g_w^4\right) T$$

$$\Gamma_{T2,W} \simeq \left(1.5\times10^{-2} g_s^2 g_w^2 + 1.5\times10^{-2} g_w^4\right) T\ ,$$

$$\Gamma_{v,W} \simeq \left(5.7\times10^{-2} g_s^2 g_w^2 + 1.5\times10^{-2} g_w^4\right) T\ ,$$



$$\Gamma_{\mu1,A} \simeq 1.0\times10^{-2}\lambda_3^4 T\ ,$$

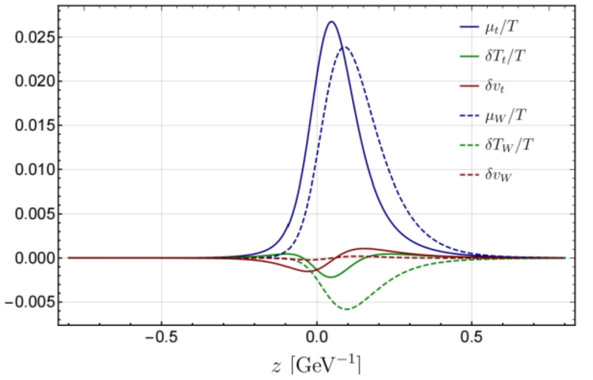
$$\Gamma_{T1,A} \simeq \Gamma_{\mu2,A} \simeq 4.9\times10^{-3}\lambda_3^4 T\ ,$$

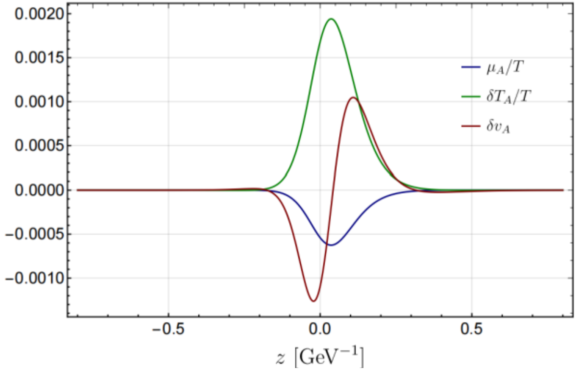
$$\Gamma_{T2,A} \simeq 5.1\times10^{-3}\lambda_3^4 T\ ,$$

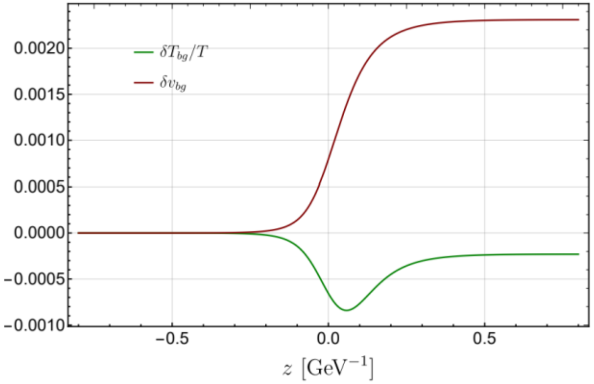
$$\Gamma_{v,A} \simeq 1.8\times10^{-3}\lambda_3^4 T\ .$$

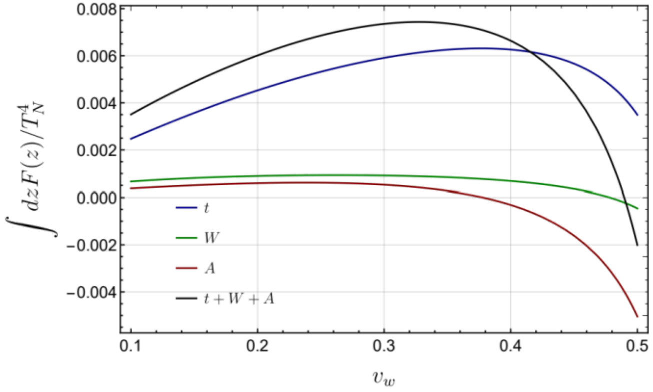
Phase transition dynamics

Solving perturbation equations:
 Green Function Method







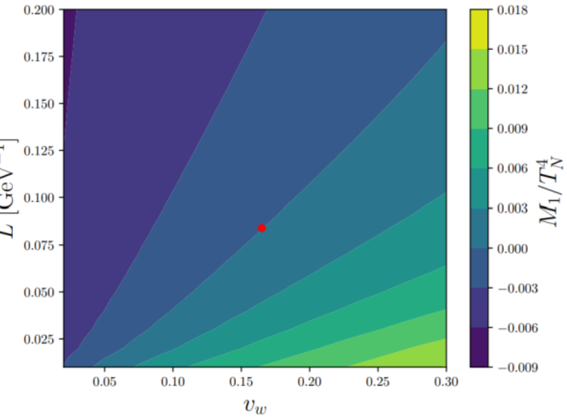


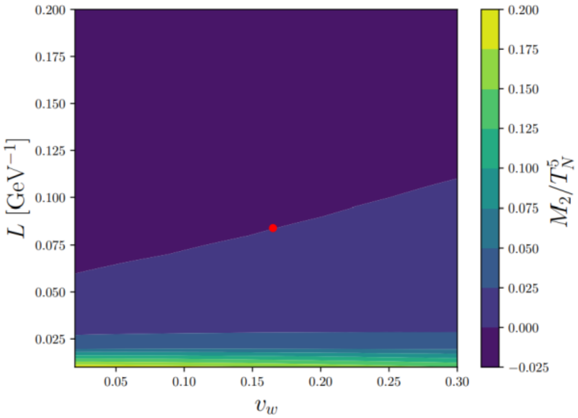
Phase transition dynamics

Solving the EOM: bubble wall pressure difference is 0; bubble wall thickness fixed

$$S_{\rm EOM} \equiv (1 - v_w^2)\,\phi'' + \frac{\partial V_{\rm eff}(\phi, T_+)}{\partial \phi} + \frac{N_t T_+}{2}\frac{dm_t^2}{d\phi} \times \left(c_1^t \mu_t + c_2^t\,(\delta T_t + \delta T_{bg})\right)$$
$$+ \sum_b \frac{N_b T_+}{2}\frac{dm_b^2}{d\phi}\left(c_1^b \mu_b + c_2^b\,(\delta T_b + \delta T_{bg})\right) = 0\ ,$$

$$M_1 = \int S_{\rm EOM}\,\phi' dz = 0, \quad M_2 = \int S_{\rm EOM}(2\phi - \phi_-)\phi' dz = 0\ .$$





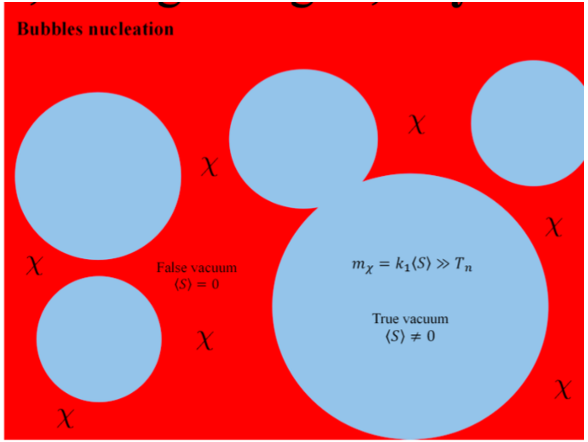
In the allowed parameter spaces, the wall velocity is around 0.165. The basic procedure in this work can also be used for any other SFOPT and dynamical DM model.

Case II: anti-filtered Q-ball DM

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Gauged Q-ball dark matter through a cosmological first-order phase transition, Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509, JHEP 07 (2024) 053

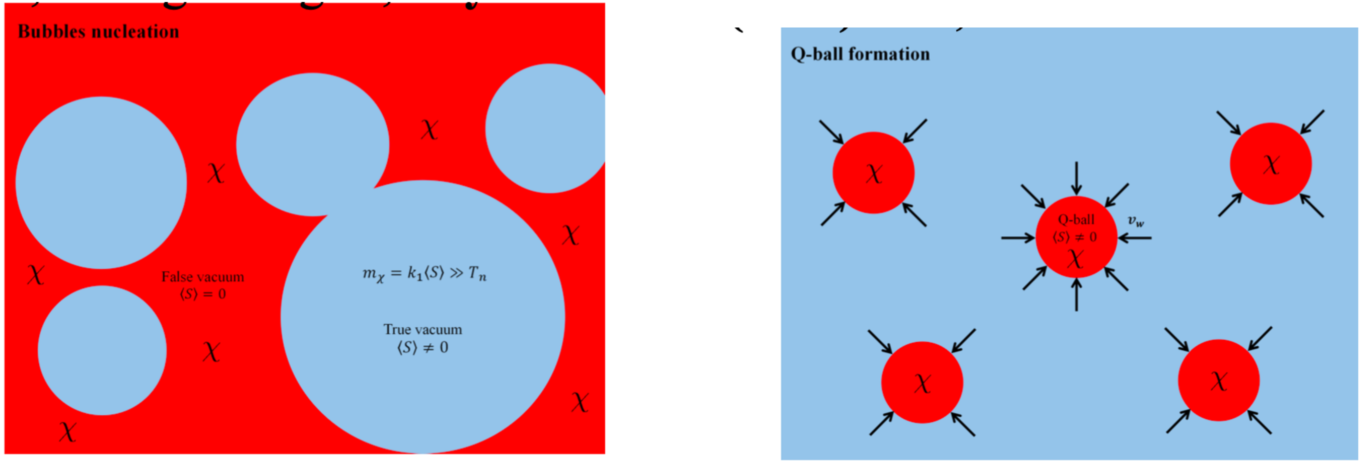
DM



Case II: anti-filtered Q-ball DM

Global Q-ball DM: **The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously..**

$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$



(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression

(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

New DM production scenario filtered by the bubbles

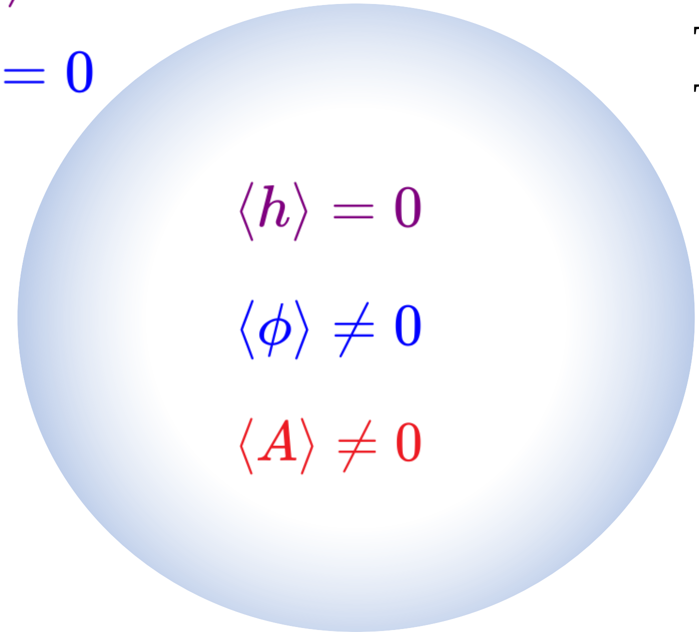
The global Q-ball model proposed by T.D. Lee

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Gauged Q-ball DM

$\langle h \rangle \neq 0$

$\langle \phi \rangle = 0$



When the conseved U(1) symmetry is local,
This introduces an extra gauge field A.
The minimal model achieving gauged Q-ball formation

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - V(\phi, h)$$

$$V(\phi, h) = \frac{\lambda_{\phi h}}{2} h^2 |\phi|^2 + \frac{\lambda_h}{4}\left(h^2 - v_0^2\right)^2$$

Interestingly, this portal coupling also naturally induces strong electroweak phase transition.

$$J_\mu = i\left(\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + 2i\tilde{g}\tilde{A}_\mu |\phi|^2\right)$$

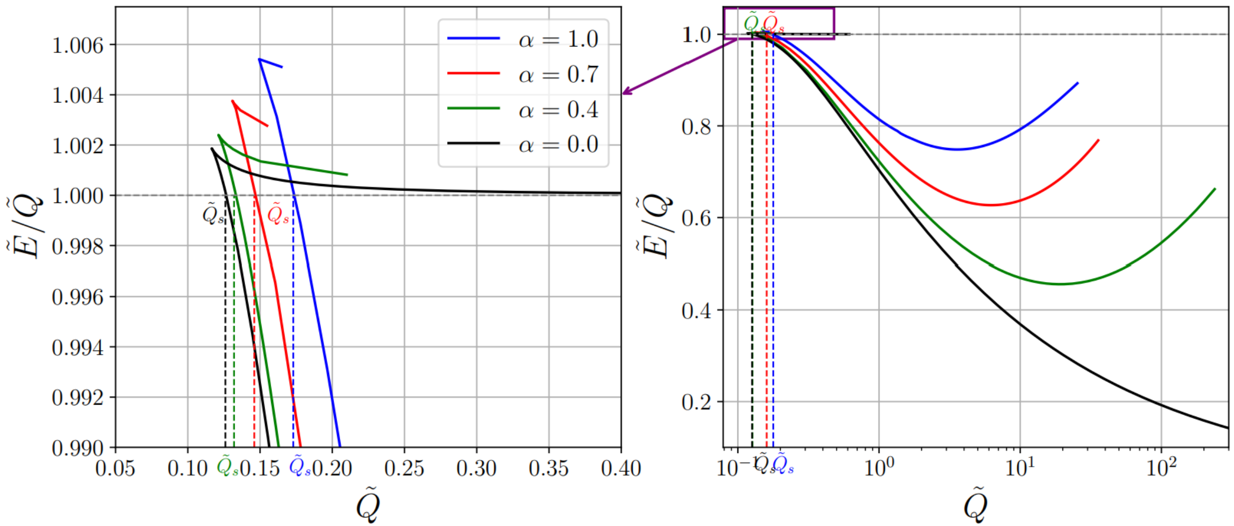
Conserved charge $Q = \int d^3x J^0$

Gauged Q-ball dark matter through a cosmological first-order phase transition, Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509

Gauged Q-ball DM

The stability of the Q-ball is an important criterion to judge whether it can serve as the DM candidate. Unlike the global Q-balls, the stability of gauged Q-balls is still being discussed.

a. quantum stability $$E < m_\phi Q \quad \text{or} \quad \tilde{E}/\tilde{Q} < 1 \; .$$

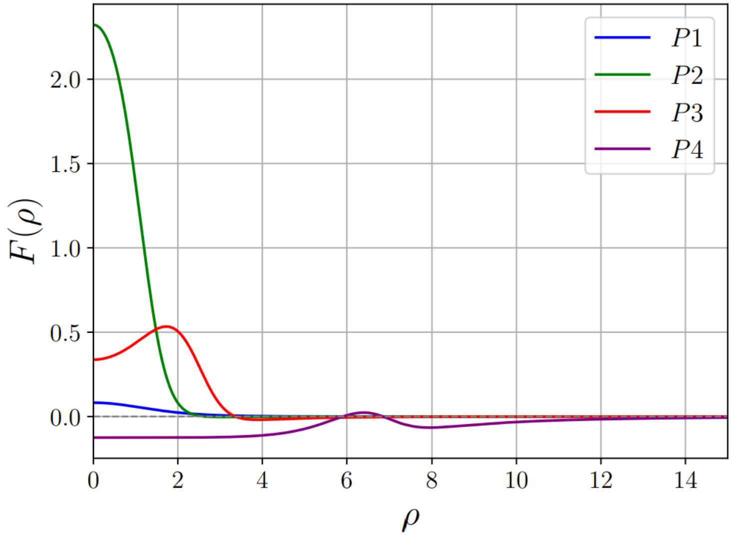


Gauged Q-ball DM

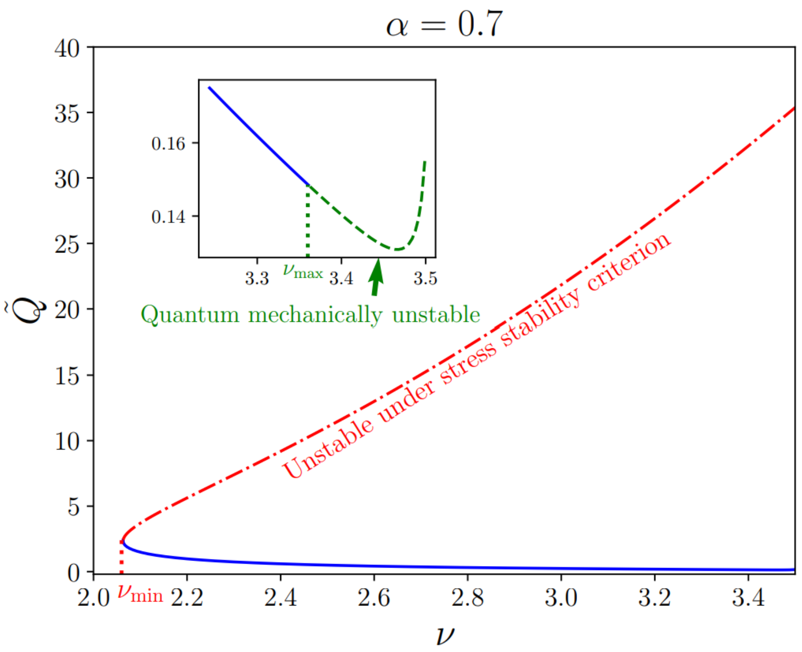
b. stress stability

$$F(r) = \frac{2}{3}s(r) + p(r) > 0 \ .$$

$$F(\rho) = 2\lambda_h v_0^4 \left[\frac{1}{2}(\partial_\rho \Phi)^2 + \frac{1}{2}(\partial_\rho \mathcal{H})^2 - \frac{\alpha^2}{2}(\partial_\rho \mathcal{A})^2 + \frac{1}{2}(\nu - \alpha^2 \mathcal{A})^2 \Phi^2 - \frac{k^2}{2}\Phi^2 \mathcal{H}^2 - \frac{1}{8}(\mathcal{H}^2 - 1)^2\right]$$



Gauged Q-ball DM



The red line represents the region where the gauged Q-ball is dominated by the gauge field such that it is unstable under stress stability criterion. The green line represents the region where the gauged Q-ball is unstable under quantum stability criterion.

We can conclude that the gauged Q-ball can survive only in

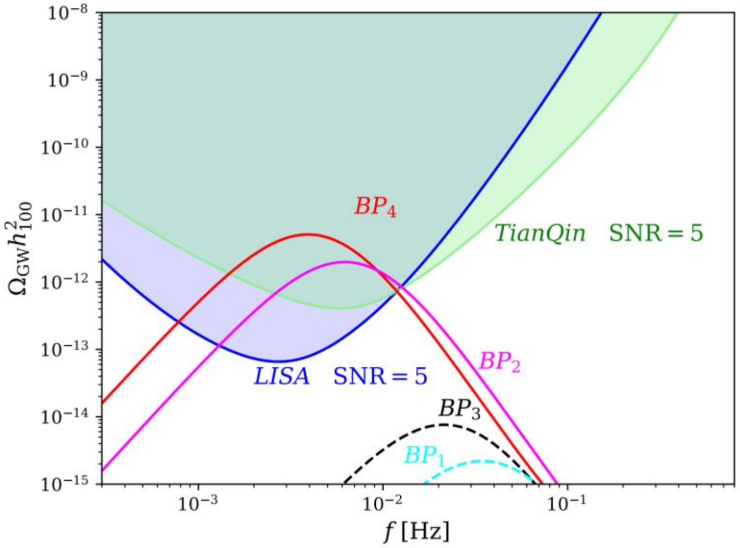
$$\nu \in [\nu_{\min}, \nu_{\max}]$$

or

$$Q \in [Q_{\min}, Q_{\max}]$$

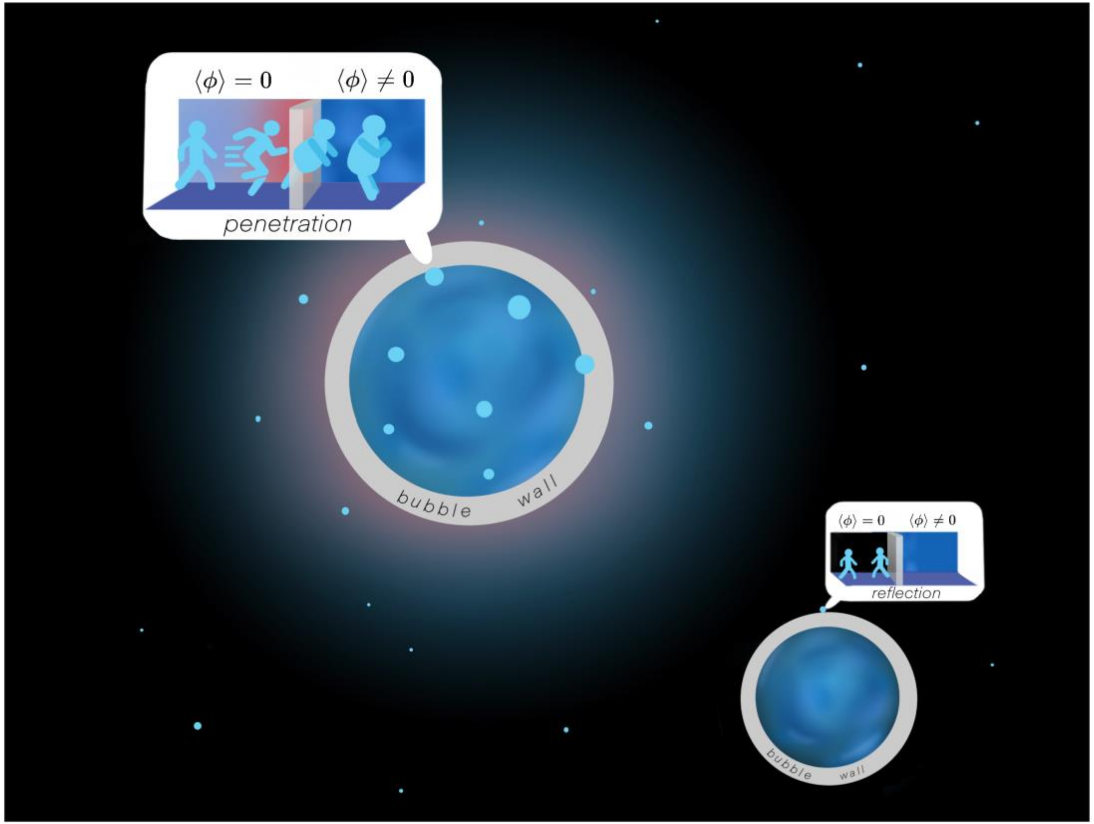
Gauged Q-ball DM

	$\lambda_{\phi h}$	T_p [GeV]	α_p	β/H_p	v_w	$F_\phi^{\rm trap}$	η_ϕ/η_L	$\delta\sigma_{Zh}$	GW
BP_1	6.8	69.8	0.12	540	0.1	0.932	0.48	-0.36%	●
BP_2	6.8	70.4	0.12	578	0.6	0.805	3.0	-0.36%	●
BP_3	7.0	63.0	0.15	372	0.1	0.965	3.4	-0.37%	●
BP_4	7.0	63.9	0.15	403	0.6	0.858	20.8	-0.37%	●



Gauged Q-ball dark matter through a cosmological first-order phase transition, Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509

Case III: filtered DM

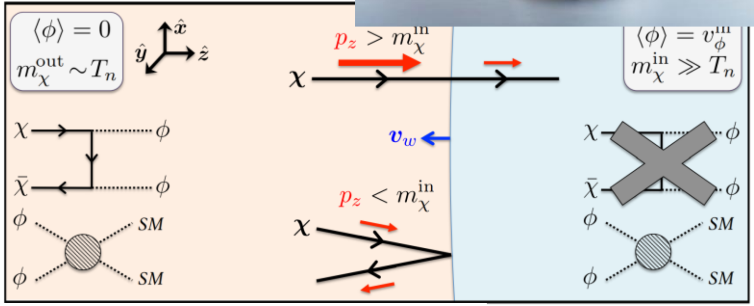


Bubble wall dynamics plays an essential role in the filtered DM mechanism.

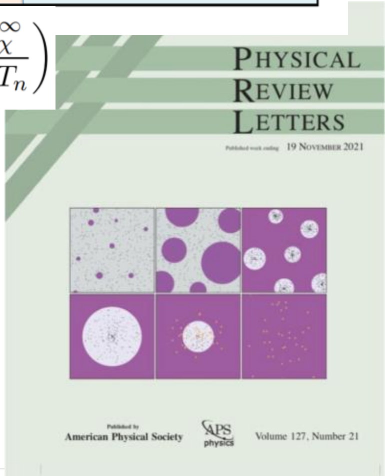
Siyu Jiang, FPH, Chong Sheng Li, Phys.Rev.D 108 (2023) 6, 063508

Case III: filtered DM

In recent years, this dynamical DM formed by phase transition has became a new idea and attracted more and more attentions. Namely, bubble in SFOPT can be the "filter" to packet the needed heavy DM.



$$\Omega_{\rm DM} h^2 \approx 0.17 \left(\frac{T_n}{\rm TeV}\right)\left(\frac{m_\chi^\infty}{30T_n}\right)^{-\frac{5}{2}} \exp\left(-\frac{m_\chi^\infty}{30T_n}\right)$$



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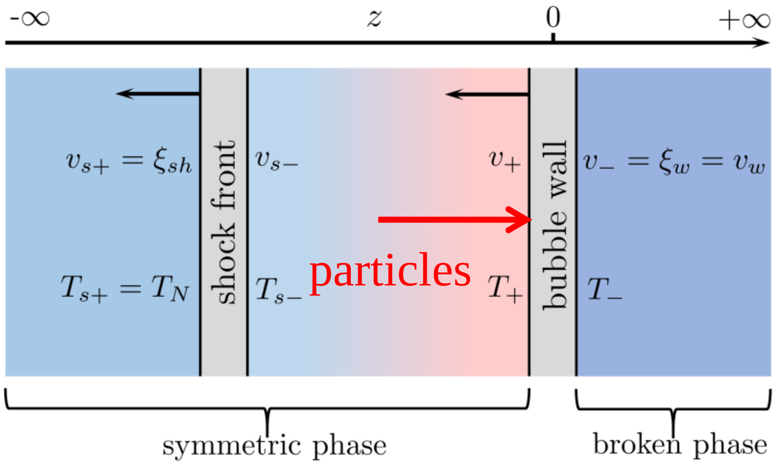
Haipeng An, et.al, arXiv: 2208.14857
Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218
more and more new works…

Case III: filtered DM

Original work:

$$\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n$$



$$\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)}\ .$$

$$J_w^{\rm in} = \frac{g_\chi}{(2\pi)^2}\int_0^{-1} d\cos\theta\cos\theta\int_{-\frac{m_\chi^{\rm in}}{\cos\theta}}^{\infty} dp\frac{p^2}{e^{\tilde{\gamma}_+(1+\tilde{v}_+\cos\theta)p/T_+}} = \frac{g_\chi T_+^3\left(1+\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+\right)}{4\pi^2\tilde{\gamma}_+^3(1-\tilde{v}_+)^2}e^{-\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+}.$$

$$n_\chi^{\rm in} = \frac{J_w^{\rm in}}{\gamma_w v_w} \qquad \Omega_{\rm DM}^{\rm (hy)}h^2 = \frac{m_\chi^{\rm in}(n_\chi^{\rm in}+n_{\bar\chi}^{\rm in})}{\rho_c/h^2}\frac{g_{\star0}T_0^3}{g_\star(T_-)T_-^3} \simeq 6.29\times10^8\frac{m_\chi^{\rm in}}{\rm GeV}\frac{(n_\chi^{\rm in}+n_{\bar\chi}^{\rm in})}{g_\star(T_-)T_-^3}$$

Case III: filtered DM

$$T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V_{T=0}(\phi)\right]$$

Energy-momentum tensor of scalar field

$$T_{\rm pl}^{\mu\nu} = \sum_i \int \frac{d^3k}{(2\pi)^3 E_i} k^\mu k^\nu f_i^{\rm eq}(k)$$

Energy-momentum tensor of fluid

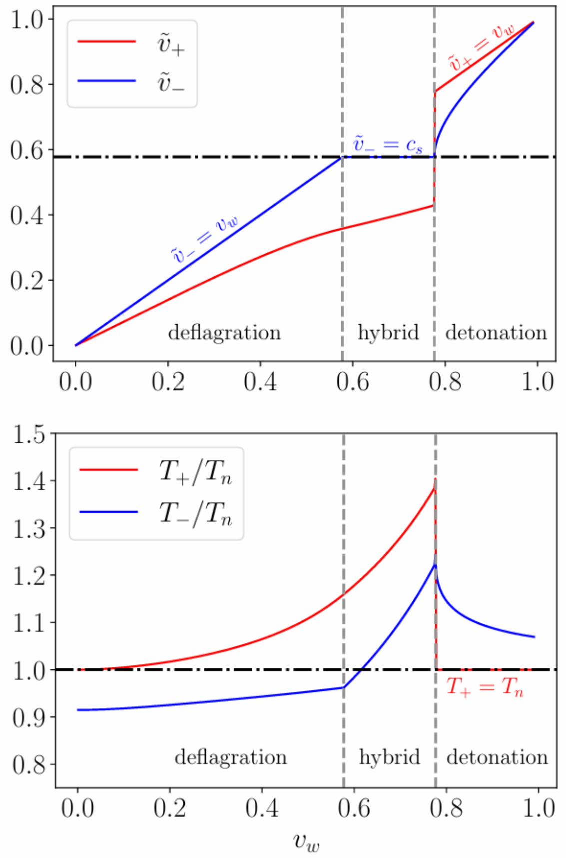
$$T_{\rm fl}^{\mu\nu} = T_\phi^{\mu\nu} + T_{\rm pl}^{\mu\nu} = \omega u^\mu u^\nu - p g^{\mu\nu}$$

Energy-momentum conservation

$$\omega_+\tilde{v}_+^2\tilde{\gamma}_+^2 + p_+ = \omega_-\tilde{v}_-^2\tilde{\gamma}_-^2 + p_-, \quad \omega_+\tilde{v}_+\tilde{\gamma}_+^2 = \omega_-\tilde{v}_-\tilde{\gamma}_-^2$$

$$\alpha_+ \equiv \epsilon/\left(a_+T_+^4\right) \qquad r_\omega = \omega_+/\omega_- = (a_+T_+^4)/(a_-T_-^4)$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longleftrightarrow \quad j\frac{v}{\xi} = \gamma^2(1-v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v, \quad \frac{\partial_\xi \omega}{\omega} = \left(1 + \frac{1}{c_s^2}\right)\gamma^2\mu\partial_\xi v\,.$$



Case III: filtered DM

General phase-transition model

$$V_{\rm eff}(\phi,T)=\frac{\mu^2+DT^2}{2}\phi^2-CT\phi^3+\frac{\lambda}{4}\phi^4-\frac{g_\star\pi^2T^4}{90}$$

$$\langle\phi\rangle=0,\quad \frac{3CT}{2\lambda}\left[1+\sqrt{1-\frac{4\lambda\left(\mu^2+DT^2\right)}{9C^2T^2}}\right]$$

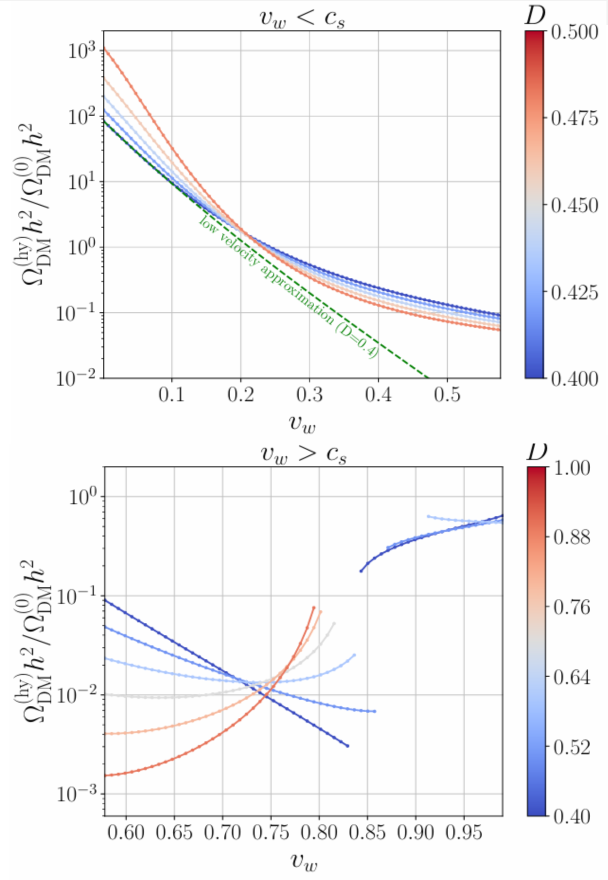
$$m_\chi^{\rm in}=\frac{y_\chi\phi_-}{\sqrt{2}} \qquad \phi_-=\phi(T_-)$$

$$n_\chi^{\rm in}\simeq\frac{g_\chi T_+^3}{\gamma_w v_w}\left(\frac{\tilde\gamma_+\left(1-\tilde v_+\right)m_\chi^{\rm in}/T_++1}{4\pi^2\tilde\gamma_+^3\left(1-\tilde v_+\right)^2}\right)e^{-\tilde\gamma_+(1-\tilde v_+)m_\chi^{\rm in}(T_-)/T_+}$$

We found:

(a) for $v_w \lesssim 0.2$: the DM relic density is enhanced

(b) for $v_w \gtrsim 0.2$: the DM relic density is reduced



Case III: filtered DM

Boltzmann equation

$$\mathbf{L}\left[f_\chi\right] = \mathbf{C}\left[f_\chi\right]$$

$$f_\chi = \mathcal{A}\left(z, p_z\right) f^{\rm eq}_{\chi,+} = \mathcal{A}\left(z, p_z\right) \exp\left(-\frac{\tilde{\gamma}_+(E - \tilde{v}_+ p_z)}{T_+}\right)$$

$$\mathbf{L}\left[f_\chi\right] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z} \qquad m_\chi(z) \equiv \frac{m^{\rm in}_\chi(\phi_-)}{2}\left(1 + \tanh\frac{2z}{L_w}\right)$$

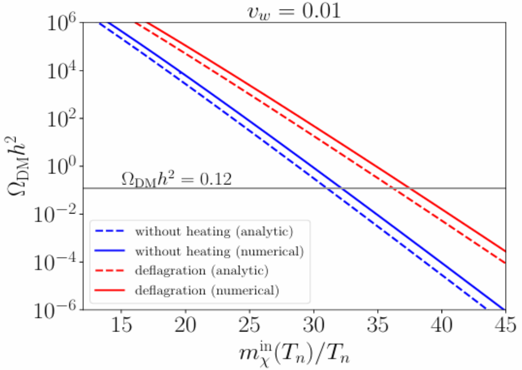
$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}\left[f_\chi\right] \approx \left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde{\gamma}_+\tilde{v}_+}{T_+}\right)\mathcal{A}\left(z, p_z\right)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde{\gamma}_+} e^{\tilde{\gamma}_+\left(\tilde{v}_+ p_z - \sqrt{m_\chi^2 + p_z^2}\right)/T_+}$$

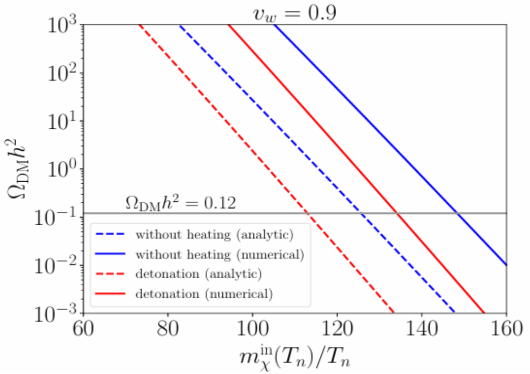
including $\chi\bar{\chi} \leftrightarrow \phi\phi, \chi\phi \leftrightarrow \chi\phi, \chi\chi \leftrightarrow \chi\chi, \chi\bar{\chi} \leftrightarrow \chi\bar{\chi}, \cdots$

$$\begin{aligned} g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{C}\left[f_\chi\right] &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E^{\mathcal{P}}_p} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[f_{\chi_p} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\ &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E^{\mathcal{P}}_p} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[\mathcal{A} f^{\rm eq}_{\chi_p,+} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\ &\equiv \Gamma_{\rm P}(z, p_z)\mathcal{A}\left(z, p_z\right) - \Gamma_{\rm I}(z, p_z)\ , \end{aligned}$$

Case III: filtered DM

$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right)/T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$





$v_w = 0.01$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2/\Omega_{\rm DM}^{(0)} h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2/\Omega_{\rm DM}^{(0)} h^2$
BP_1	31	66	32	71
BP_2	31.1	7.9	32.2	8.1
BP_3	30.8	778.8	31.9	858.5
BP_4	$\star$	$\star$	$\star$	$\star$

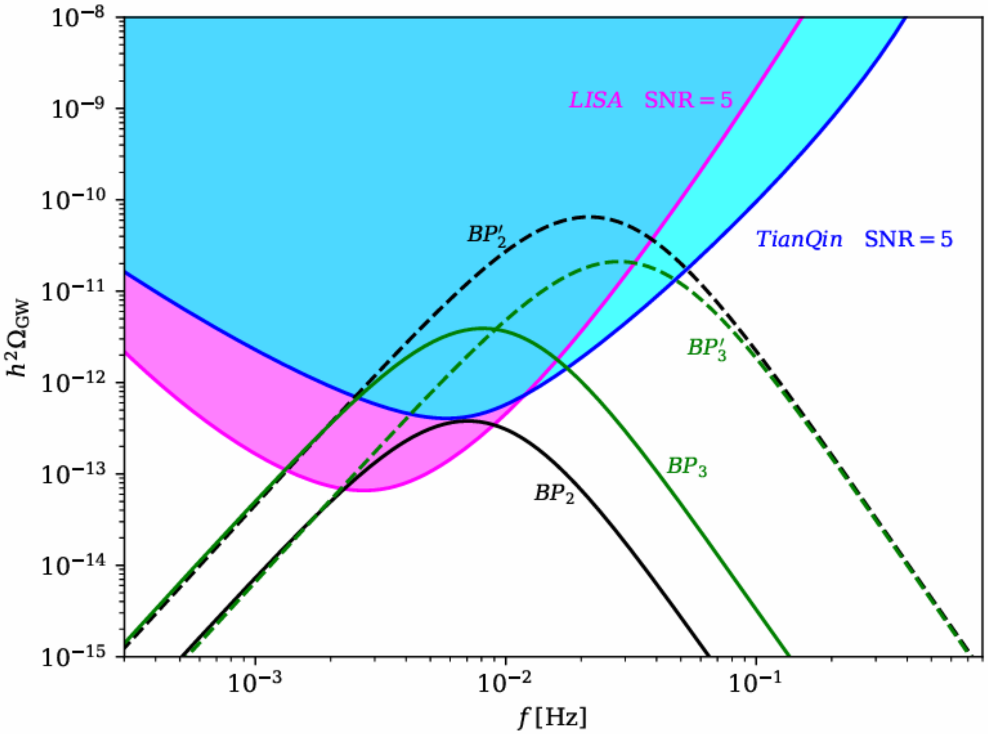
$v_w = 0.9$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2/\Omega_{\rm DM}^{(0)} h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2/\Omega_{\rm DM}^{(0)} h^2$
BP_1	125.3	1/19	147.8	1/27
BP_2	125.9	1/7	148.7	1/9
BP_3	124.6	1/10	147.3	1/12
BP_4	123.8	$1/(1.2\times10^{13})$	146.5	$1/(2.2\times10^{15})$

Case III: filtered DM

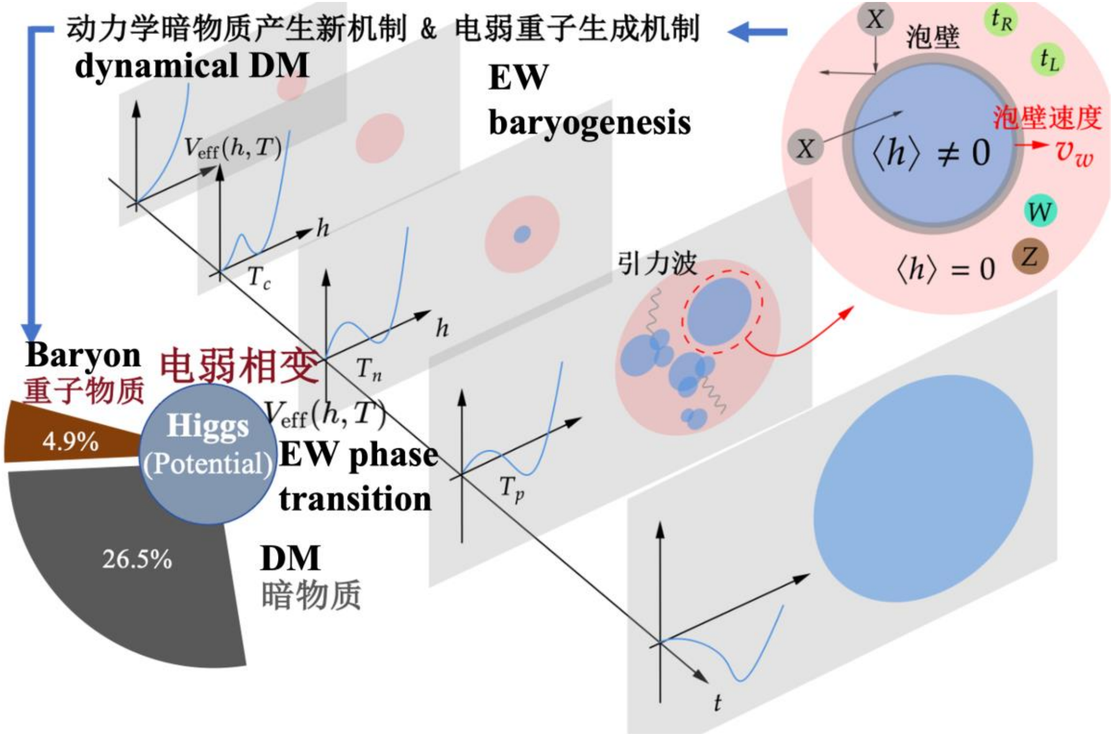
The hydrodynamic effects play essential roles in the filtered DM mechanism. For the deflagration mode with low bubble wall velocity, the hydrodynamic effects significantly enhance the relic density. In contrast, for the detonation mode, the relic density is obviously reduced. For the hybrid mode, the hydrodynamic correction is extremely large.

Precise calculation of filtered DM relic density can help to decide the phase-transition parameters precisely. This gives more accurate phase-transition GW spectra.



Summary and outlook

- **Bubble wall can be a natural filter to produce DM.**
- **GW provides new approaches to explore the nature of DM.**



Thanks! Comments and collaborations are welcome!

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