Challenge Gravitational Wave Measurement with a Novel Mössbauer Spectrometer

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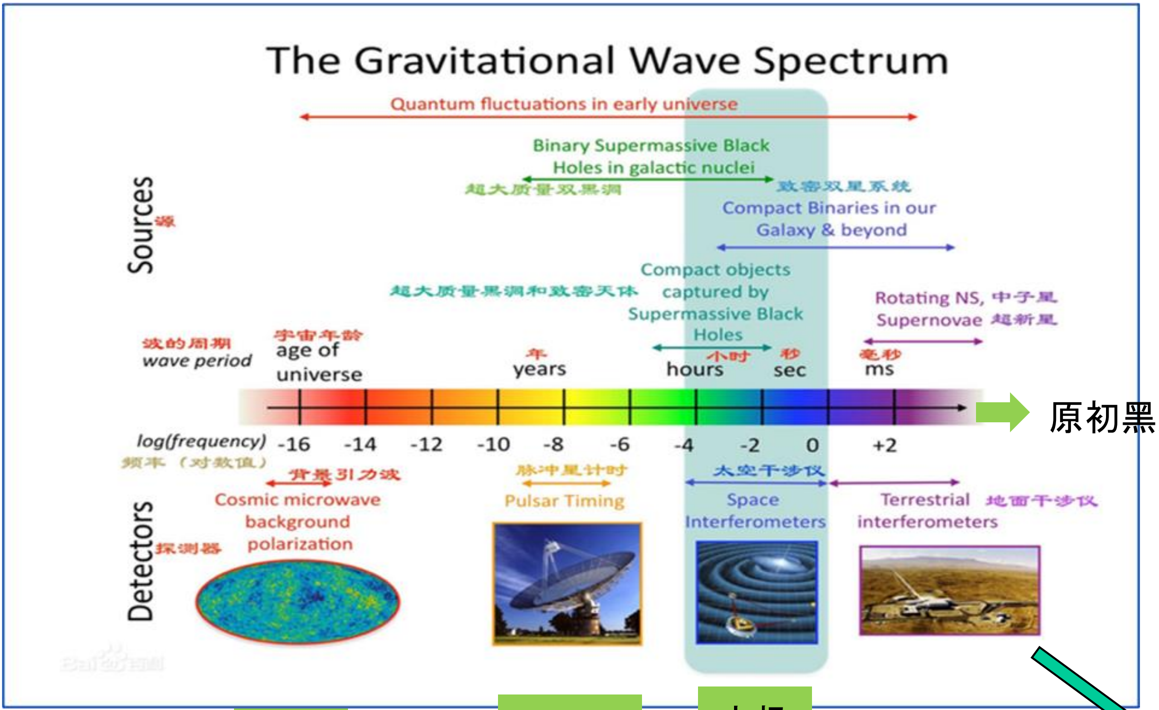
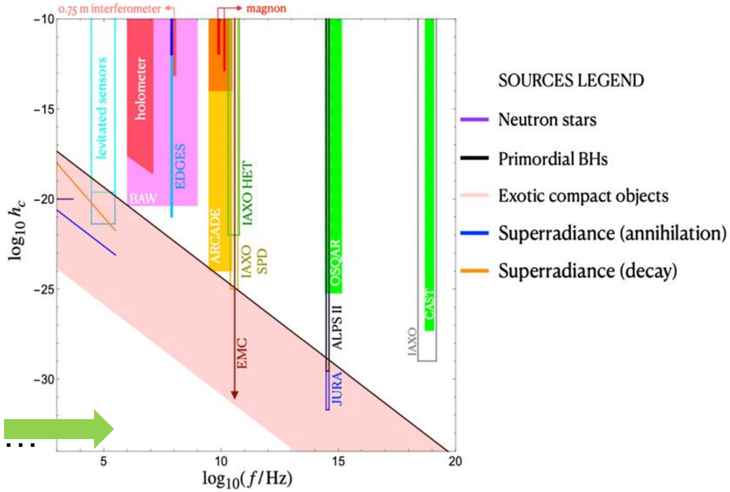
Science Bulletin 2024; 69(18): 2795-2798
10.1016/j.scib.2024.07.038

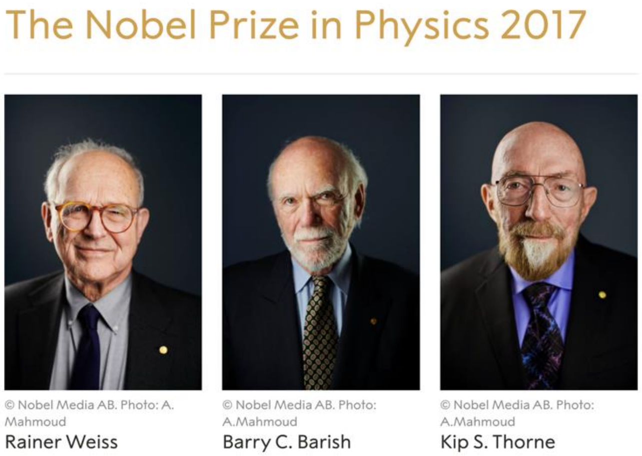
Gravitational Wave

GW：a new way to explore our universe



原初黑洞…





国内实验

阿里

PTA
(Fast)

太极,
天琴

微波背景涨落

计数频率变化

激光干涉

原子干涉

Thickness of a paper over the diameter of solar system

Mössbauer effect

- Recoil-less emission and resonant absorption of nuclear transition photons
  - Mössbauer active nucleus bound to solid state lattice
  - Thermal vibration(~10^{13} Hz) much faster than nuclear transitions (lifetime ~>10^{-7} s)
    - Effect averages to zero, no 1st Doppler broadening
    - Recoil to the whole crystallite(>10^{14} atom)
    - Without phonon excitations → recoil free

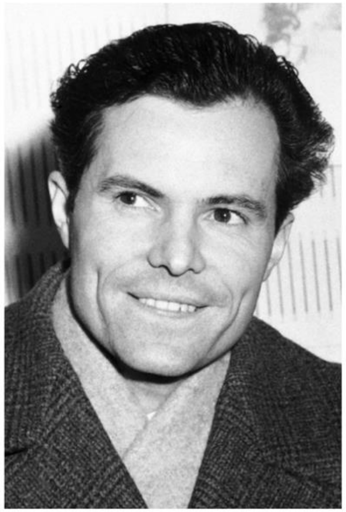
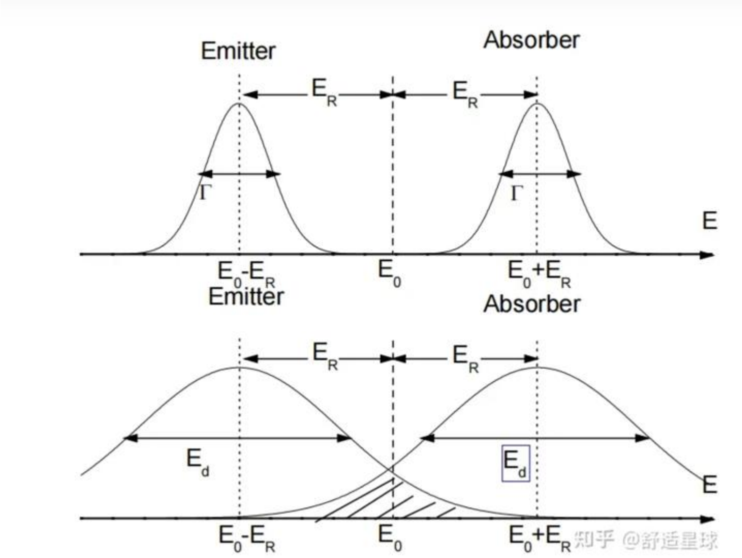
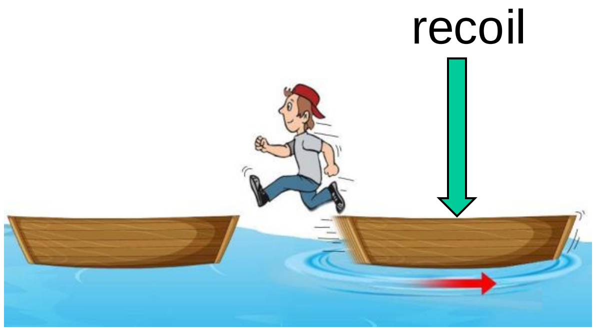


Photo from the Nobel Foundation archive.

Rudolf Ludwig Mössbauer

Prize share: 1/2



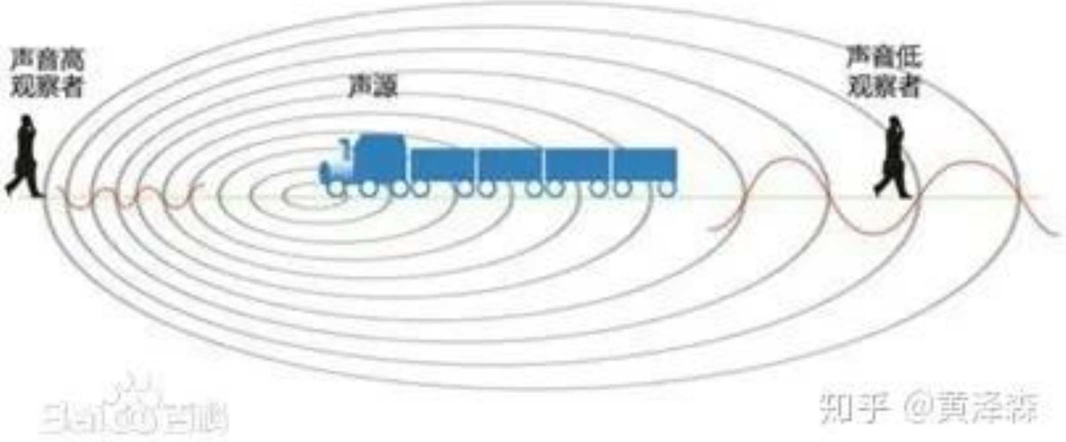


"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"

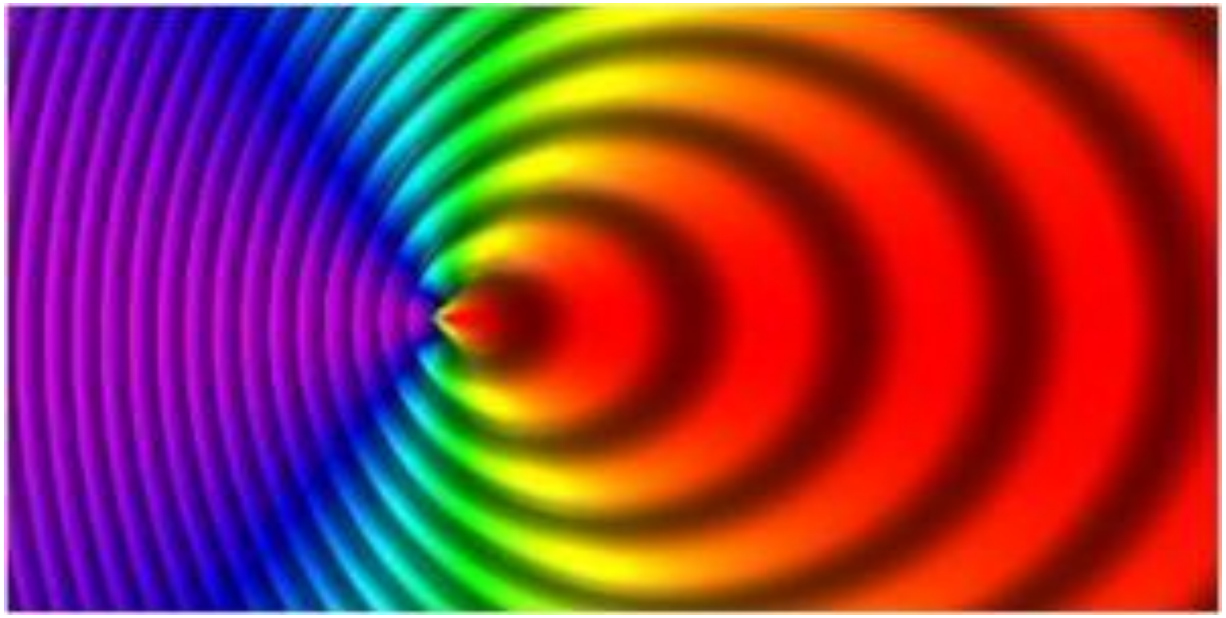
Solid, liquid, gas

Doppler shift

- Sound

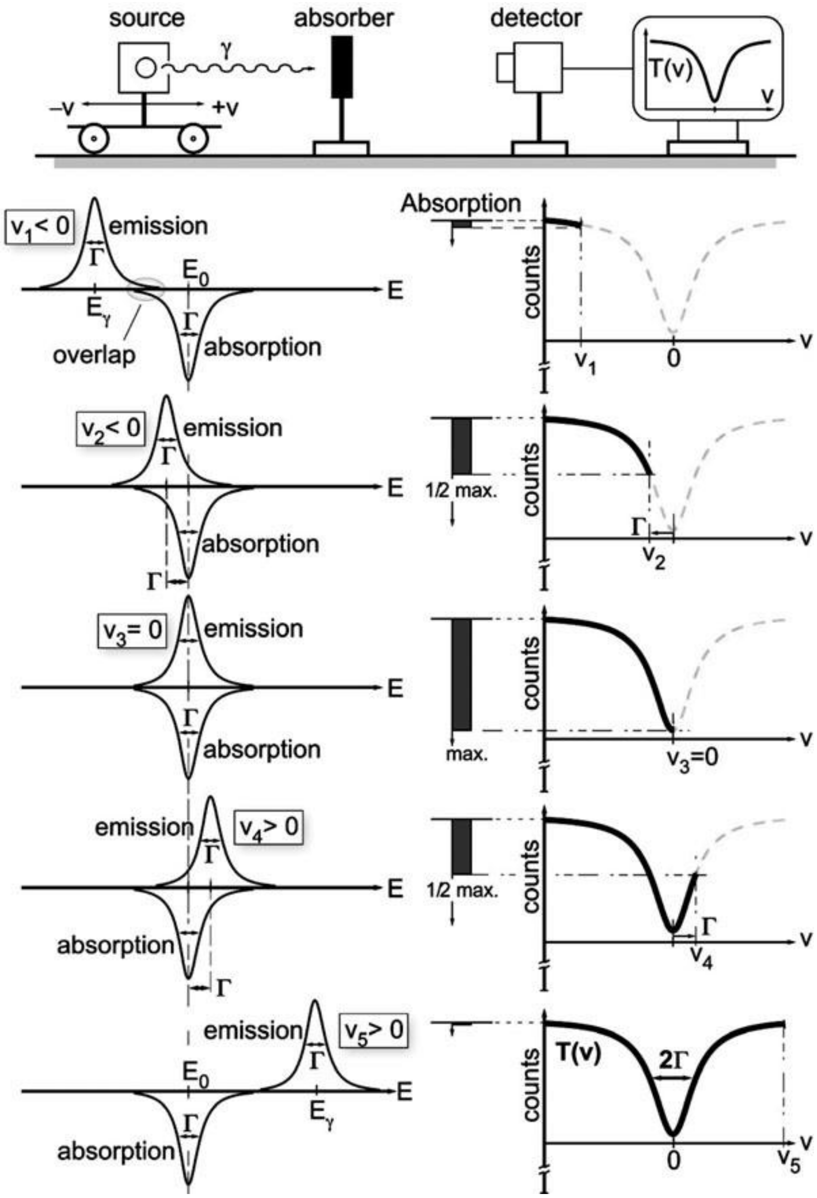


- Photon：E=hv



Red shift

"Traditional" Mössbauer Spectrometer

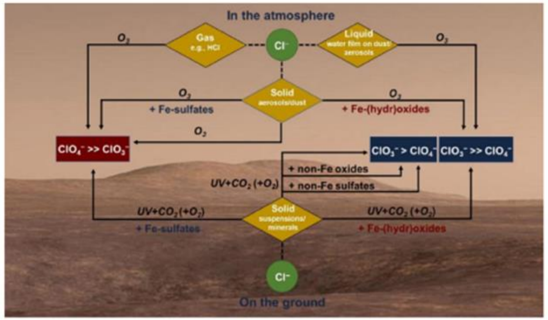


- Change the recoil-less emission and absorption overlap with doppler shift
- Strength of recoilless nuclear resonant absorption determined by the "overlap" of emission and absorption lines when the emission line is shifted by Doppler modulation
- High accuracy of the resonant absorption width w.r.t. photon energy

^{57}Fe ~ 10^{-13} (mostly used)
^{65}Zn ~ 10^{-15}
^{109}Ag ~ 10^{-22}
+ many others.

- Widely used in material analysis

Ex: mars



Relativity test with Mössbauer effect

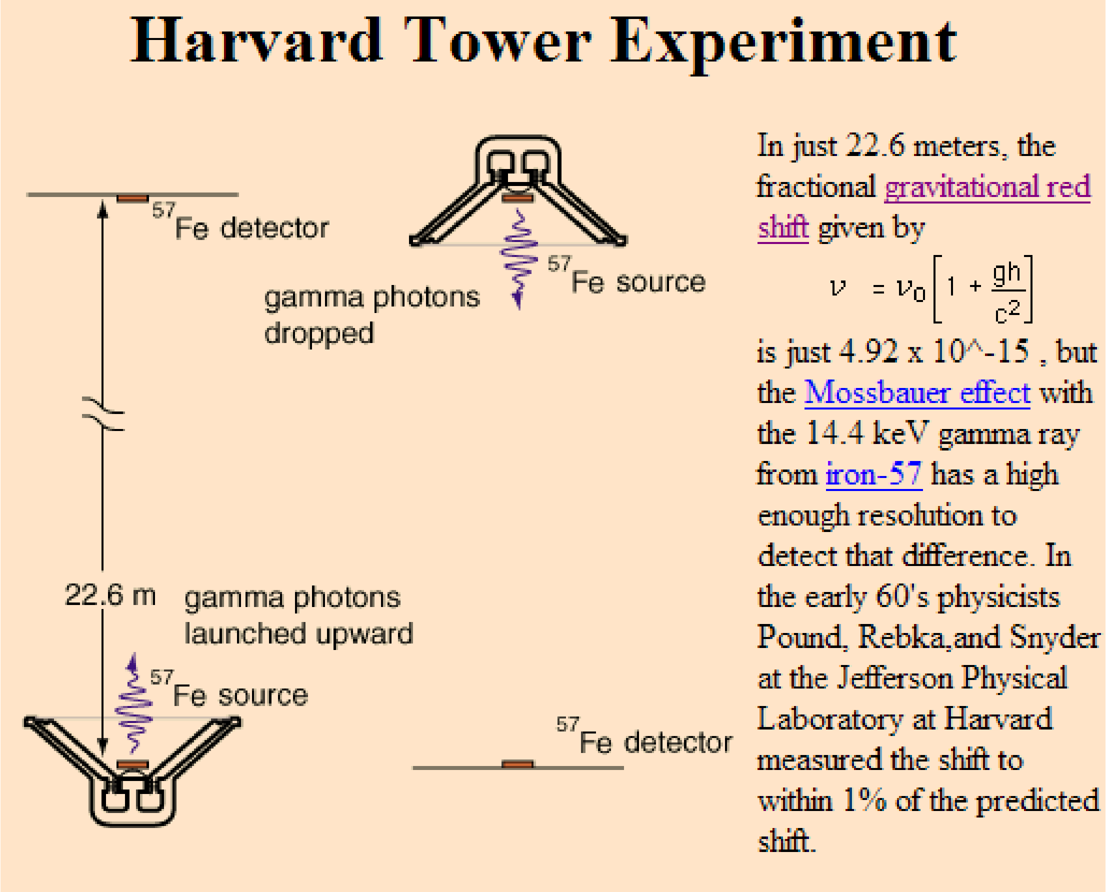
- Gravitational redshift test with Mössbauer effect

Jefferson laboratory at Harvard University. The experiment occurred in the left "tower". The attic was later extended in 2004.

Pound, Rebka & Snyder (1960-1965)

Observation of a height-induced frequency shift

$ghc^{-2} \sim 4.92 \times 10^{-15}$



Harvard Tower Experiment

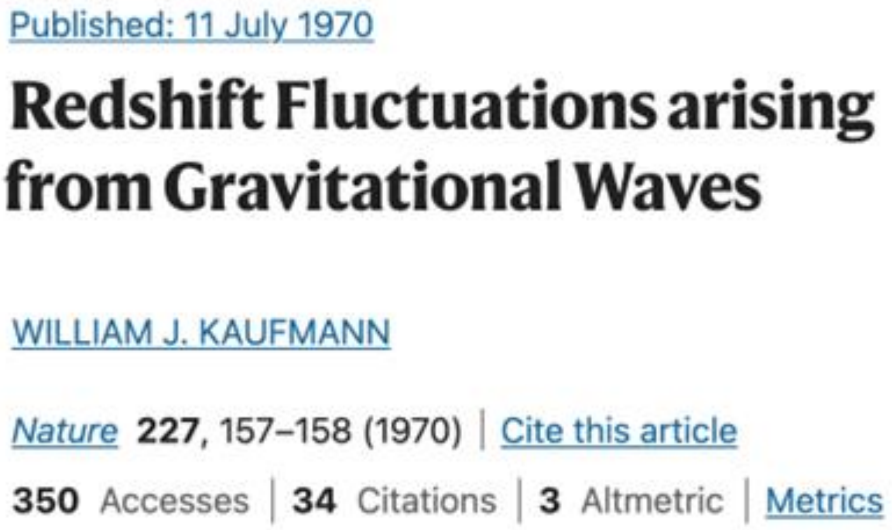
In just 22.6 meters, the fractional gravitational red shift given by

$$\nu = \nu_0\left[1 + \frac{gh}{c^2}\right]$$

is just 4.92 x 10^-15 , but the Mossbauer effect with the 14.4 keV gamma ray from iron-57 has a high enough resolution to detect that difference. In the early 60's physicists Pound, Rebka,and Snyder at the Jefferson Physical Laboratory at Harvard measured the shift to within 1% of the predicted shift.

Mössbauer for Gradational Wave

- Idea since 1970
  - Photon frequency varies when it propagates in an un-even space-time background.
- Gave way to clock-based experiments in later tests of “static” gravity.
  - See K. Hentschel, *Annals of Science* 53, 269–295 (1996)



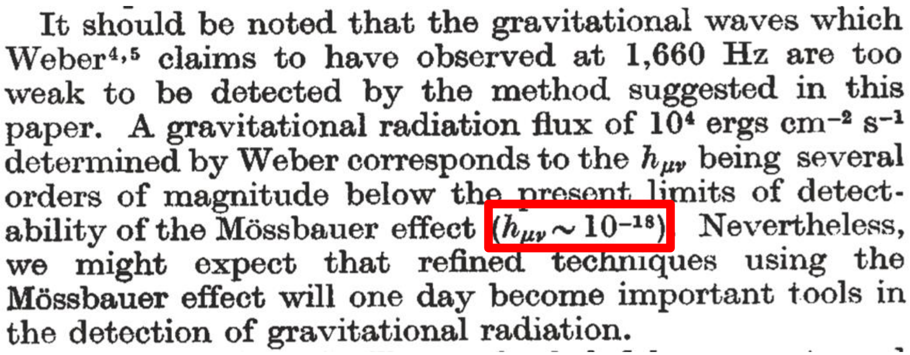
Published: 11 July 1970

Redshift Fluctuations arising from Gravitational Waves

WILLIAM J. KAUFMANN

Nature **227**, 157–158 (1970) | Cite this article

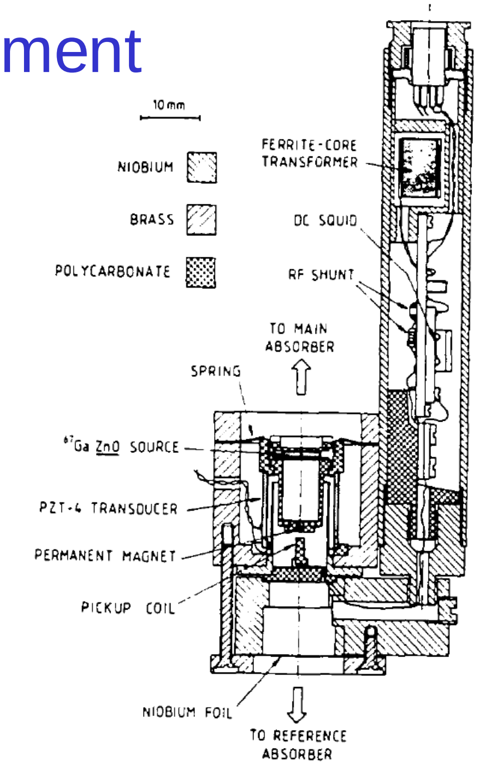
350 Accesses | **34** Citations | **3** Altmetric | Metrics

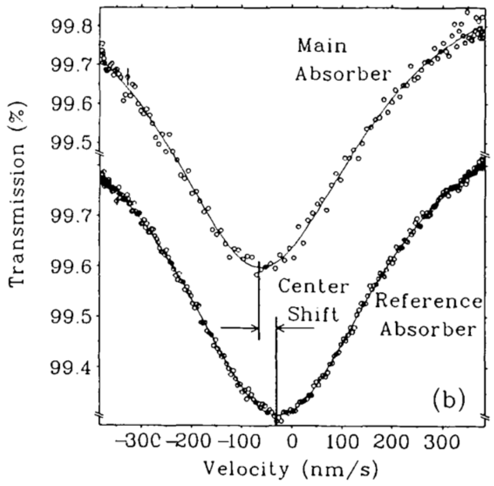


It should be noted that the gravitational waves which Weber^{4,5} claims to have observed at 1,660 Hz are too weak to be detected by the method suggested in this paper. A gravitational radiation flux of 10^4 ergs cm^{-2} s^{-1} determined by Weber corresponds to the $h_{\mu\nu}$ being several orders of magnitude below the present limits of detectability of the Mössbauer effect $\boxed{(h_{\mu\nu} \sim 10^{-18})}$. Nevertheless, we might expect that refined techniques using the Mössbauer effect will one day become important tools in the detection of gravitational radiation.

Issues in traditional Mössbauer Gravity tests

- A cryogenic ^{65}Zn measurement of the local *g*-value (Potzel et.al. 1992)
- Differential measurement of the resonance with a sinusoidal oscillator
  - Resonance is achieved
  - g-value is off, possibly due to various line-shifts





“such solid-state effects might be difficult …… there might exist two exceptions. The first are null redshift experiments, in particular measurements with stationary source and absorber”.

Can be improved with a stationary scheme

Idea of the stationary measurement

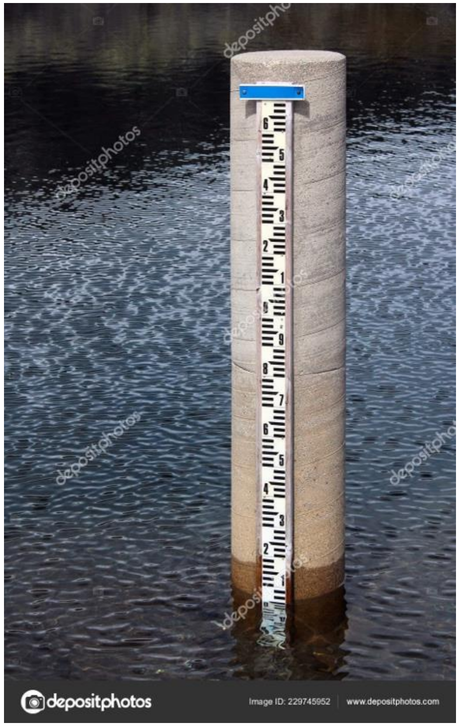
- Doppler shift ==> Gravitational shift

$$\boxed{1+z=\frac{1+v\cos(\theta)/c}{\sqrt{1-v^2/c^2}}}$$

$$\boxed{1+z=\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}},}\xrightarrow{\Delta h \ll r}\boxed{z=\frac{g}{c^2}*\Delta h}$$

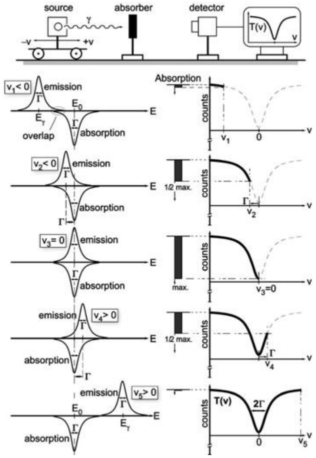
V=1mm/s ➔ ~10^{-12} Δh = 1mm ➔ ~10^{-19}

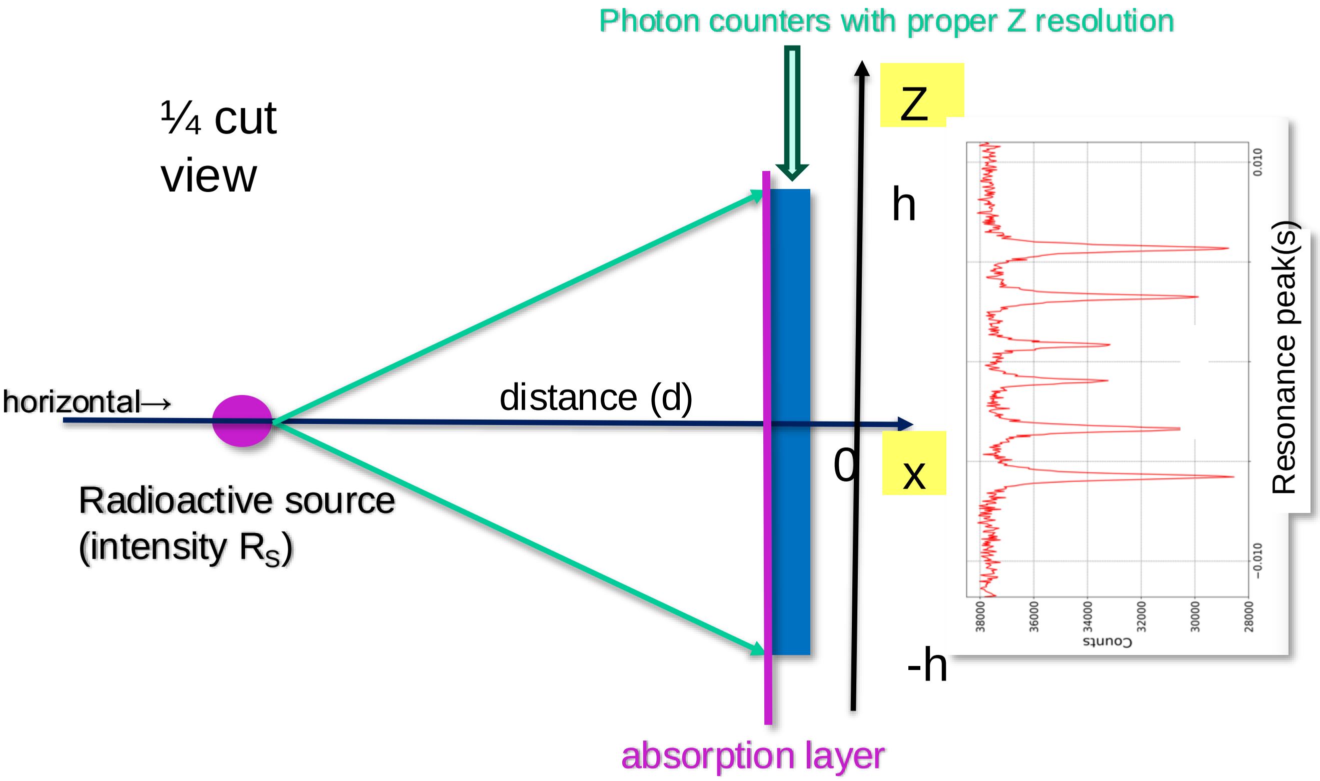
  - Improve on sensitivity to energy shift



Design of a gravitational shift Mössbauer Spectrometer

- Replace Doppler shift with gravitational shift
- Parallel readout the resonant absorption shape
- Time dependent of resonance’s ***height-shift*** *instead of absolute height*




Photon counters with proper Z resolution

¼ cut view

horizontal→

distance (d)

Radioactive source (intensity R_S)

absorption layer

Energy loss E_R is compensated by a slight height difference between absorber and source: *can be calibrated in advance.*

--- the absolute height of resonance (Z_0) is affected by large systematics: 2nd Doppler, chemical composition, etc.

--- but its time-dependent shift under GW is *not* affected.

The transmission integral

- Photon emission at source
  - Fractional recoil-less : f_s,
  - Fractional recoiled:1-f_s

$$f(T) = \exp\left[\frac{-3E_\lambda^2}{k_\mathrm{B}\Theta_\mathrm{D}Mc^2}\left\{\frac{1}{4} + \left(\frac{T}{\Theta_\mathrm{D}}\right)^2 \int_0^{\Theta/T}\frac{x}{\mathrm{e}^x - 1}\mathrm{d}x\right\}\right]$$

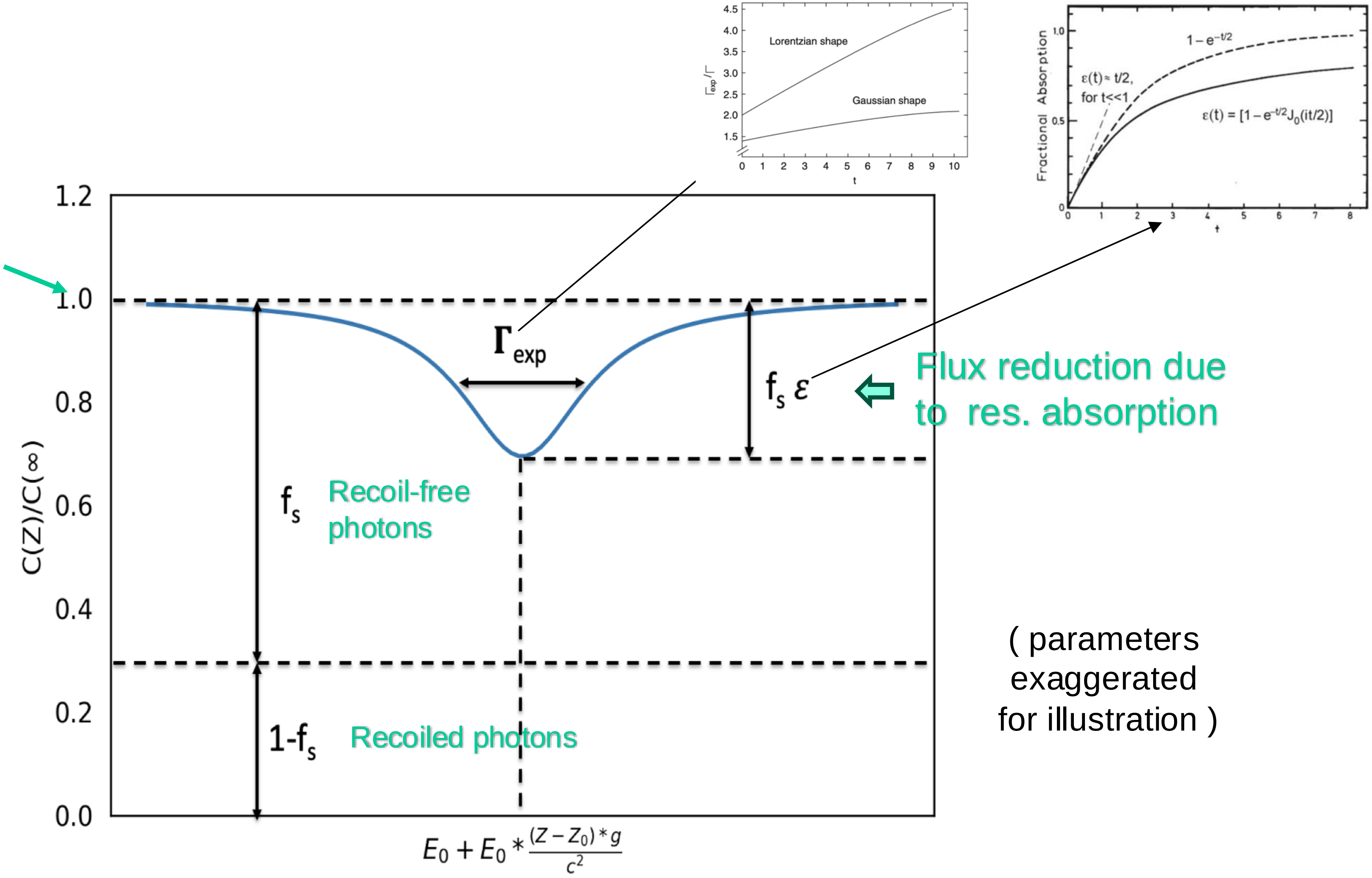
- Photon absorption at absorber
  - Mass attenuation effect: all photons
  - Resonant absorption: only resonant emitted photons
- The transmission integral (textbook)
  - Replaced the doppler shift with gravitational shift

$$C(Z) \; = \; \dot{N}_0 \, e^{-\mu_e t'} \cdot \left[(1 - f_S) + \int_{-\infty}^{\infty} f_S \xi(Z_S, E_0) \cdot \mathrm{e}^{-t\xi(Z, E_0 + \Delta E_0)\Gamma/2\pi} \mathrm{d}E\right]$$

$$\xi(Z, E_0) \; \equiv \; \frac{\Gamma/2\pi}{[E - g(Z - Z_S)E - E_0]^2 + (\Gamma/2)^2},$$

The simplified resonance line-shape

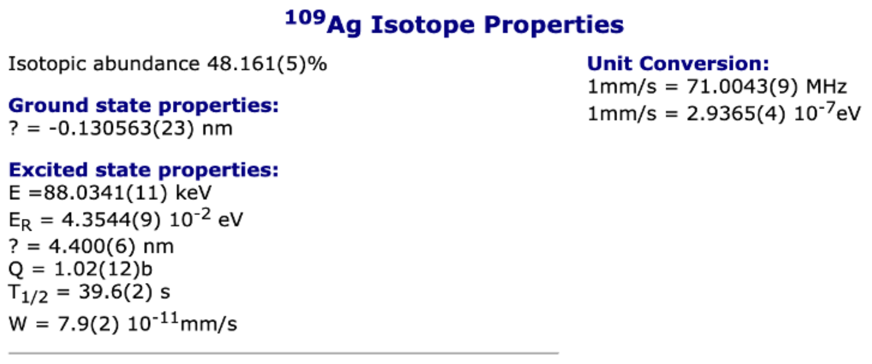
C(∞)
w/o res.



Flux reduction due to res. absorption

(parameters
exaggerated
for illustration)

The choice of isotope source: ^{109}Ag



^{109}Ag Isotope Properties

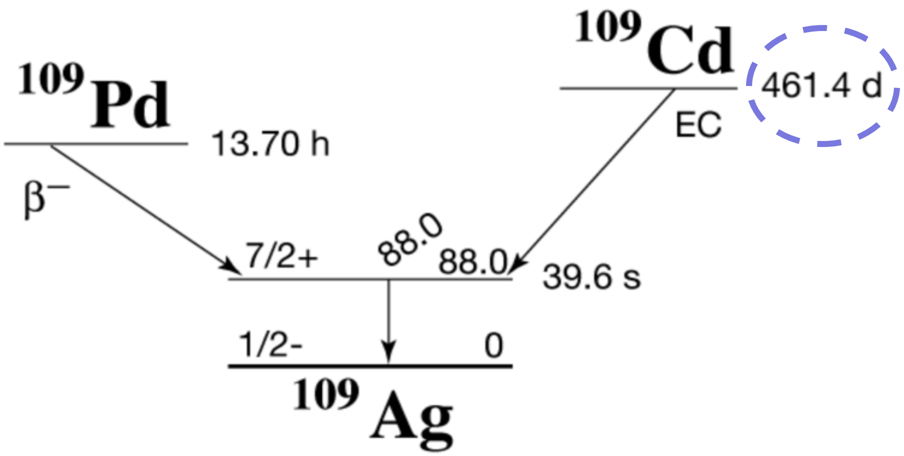
Isotopic abundance 48.161(5)%

Ground state properties:
? = -0.130563(23) nm

Excited state properties:
E =88.0341(11) keV
E_R = 4.3544(9) 10^{-2} eV
? = 4.400(6) nm
Q = 1.02(12)b
$T_{1/2}$ = 39.6(2) s
W = 7.9(2) 10^{-11}mm/s

Unit Conversion:
1mm/s = 71.0043(9) MHz
1mm/s = 2.9365(4) 10^{-7}eV

Decay Diagram



Mössbauer database (DICP, CAS)

- Long parent nuclei lifetime: 461 days allow for sufficient operation time
- Narrow 88 keV linewidth: $O(10^{-22})$ sensitivity
- Workable ΔZ ~ 10μm under terrestrial (1 g) gravity field for $\Gamma_{exp} = 4.1\Gamma$

The quest of the ^{109}Ag resonance:

$\Gamma_{exp} \sim 30\Gamma$, (W. Wildner and U. Gonser, 1979)

Improved resonance resolution, w broadening factors down to 16 (US)

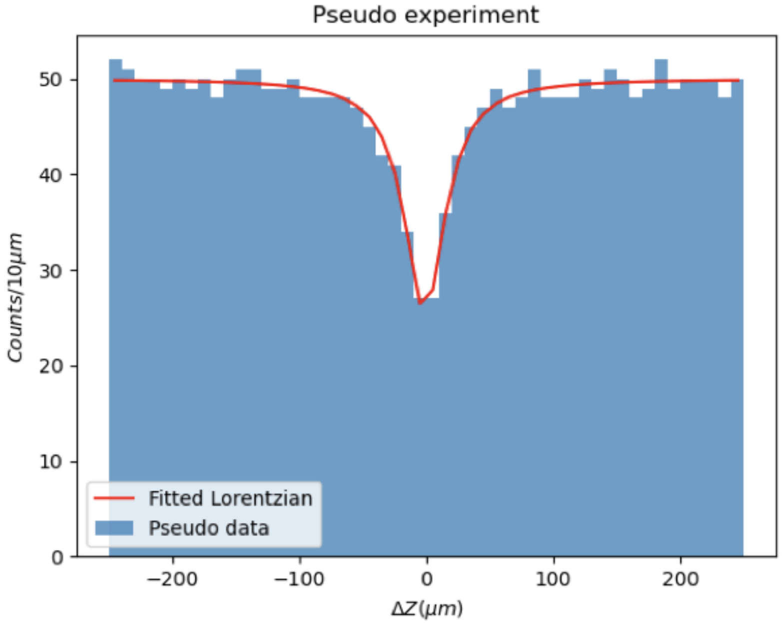
R. D. Taylor and G. R. Hoy, SPIE **875**, 126 (1988).
S.RezaieSerej, G. R. Hoy, and R. D. Taylor, Laser Phys. **5**, 240 (1995).

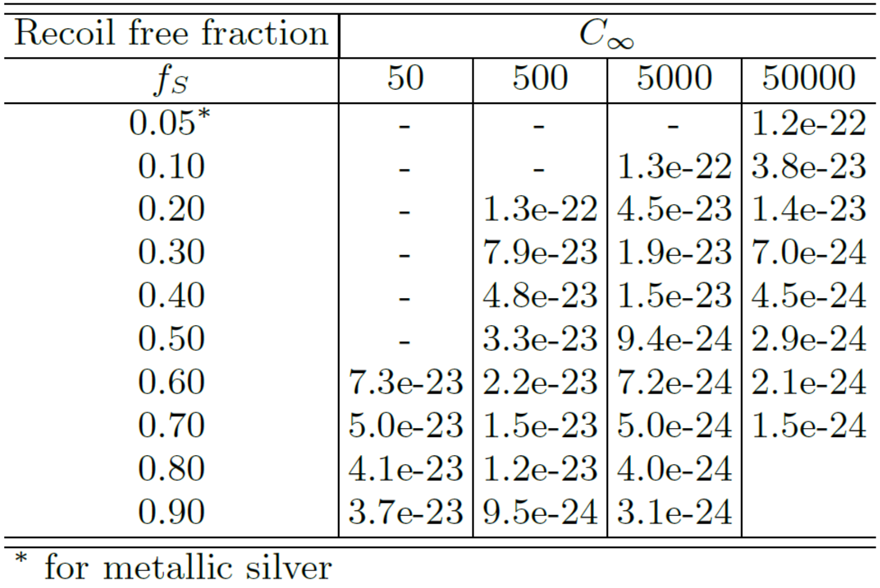
Russian group: improvements with Grav. Effects

V. G. Alpatov, et.al. Laser Physics 17, 1067–1072 (2007).
Yu. D. Bayukov, et.al. JETP Letters 90, 499–503 (2009).

Accuracy from Pseudo experiments

- Pseudo experiment simulation with reasonable parameter
  - Ex: z-position resolution 10 μm, detection efficiency ~100%
  - Readout frequency ~10 f_{GW}
- Sensitivity extracted by fitting the resonance absorption peak





Recoil free fraction	C_∞			
f_S	50	500	5000	50000
0.05*	-	-	-	1.2e-22
0.10	-	-	1.3e-22	3.8e-23
0.20	-	1.3e-22	4.5e-23	1.4e-23
0.30	-	7.9e-23	1.9e-23	7.0e-24
0.40	-	4.8e-23	1.5e-23	4.5e-24
0.50	-	3.3e-23	9.4e-24	2.9e-24
0.60	7.3e-23	2.2e-23	7.2e-24	2.1e-24
0.70	5.0e-23	1.5e-23	5.0e-24	1.5e-24
0.80	4.1e-23	1.2e-23	4.0e-24	
0.90	3.7e-23	9.5e-24	3.1e-24	

* for metallic silver

Silver alloy/compound with higher T_{deBye} helps improve f_S
e.g. AgB_2 has a higher T_{deBye} and f_S =0.2 (@ 4K)

Accuracy compatible to the accuracy needed for GW strain

Parallel measure of resonance absorption shape: fast enough for GW

The GW signal: frequency shift

Frequency shift due to different space-time matrix for photon emission and absorption

Estabrook and Wahlquist, Gen. Relat. Gravit. 6, 439–447 (1975);
Hellings, Phys. Rev. D 23, 832–843 (1981).

Consider a plain-wave strain perturbation

$$h(\boldsymbol{x},t) = h_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

$$\mathrm{ds^2 = c^2dt^2 - [1+h]dx^2 - [1-h]dy^2 - dz^2}$$

A particle's 4-momentum response to GW strain after one-way propagation:

$$\frac{\Delta f}{f_\gamma} = \frac{\ell^\mu \ell^\nu}{1-\cos\theta}\left[h^{\mathrm{D}}_{\mu\nu} - h^{\mathrm{E}}_{\mu\nu}\right]$$

$$\ell^\mu = f_\gamma(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\boxed{\begin{aligned}\frac{\Delta f}{f_\gamma} &= 2h_0\cos^2\frac{\theta}{2}\cos 2\phi \sin\left(\omega d \sin^2\frac{\theta}{2}\right)\\ &\cdot \sin\left(\omega t - \omega d\cos^2\frac{\theta}{2}\right),\end{aligned}}$$

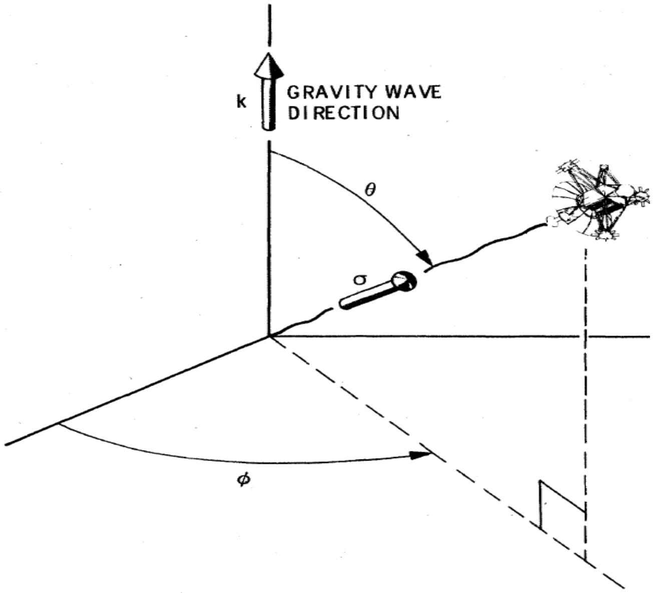


FIG. 1. Tracking geometry.
(figure from Hellings paper)

Energy diff. between $E(t_E,\vec{0})$ and $D(t_E+\frac{d}{c},\frac{\vec{d}}{c})$

When the baseline distance approaches to the GW's wavelength scale, a particle starts to see the strain difference.

$$\frac{\Delta f}{f_\gamma} = 2h_0\cos^2\frac{\theta}{2}\cos 2\phi\,\sin\left(\omega d\sin^2\frac{\theta}{2}\right) \cdot \sin\left(\omega t - \omega d\cos^2\frac{\theta}{2}\right),$$

Spin-2

* Requires a perpendicular *h* component.
* Extra complication w baseline at high freq.
* Vanishes when (anti) parallel to GW direction

Maximal shift with GW frequency:

$$\left.\frac{\Delta f}{f_\gamma}\right|_{\rm max.} = \begin{cases} \frac{\omega d}{2}h_0, & \omega d \ll 1 \ \&\ \theta \to \frac{\pi}{2}, \\ \eta(\omega d)\cdot h_0, & \omega d > 1,\ 1^{\rm st}\ \max. \end{cases}$$

η → 2 at high frequency, with multiple maxima.
Angular patterns becomes very complicated for $\omega d > O(10)$
Angular pattern allows for GW direction reconstruction
At low-freq, freq. shift decreases *linearly* with ωd

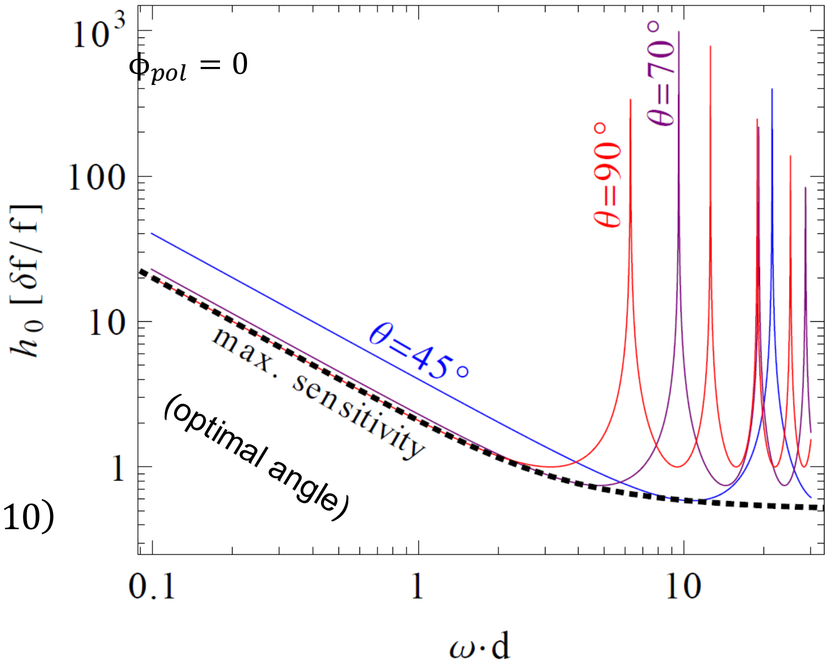
Non-trivial angular pattern with the incident GW direction:

Low GW freq: max. at 90.
High GW freq: modulated btw $0 < \theta < 2\pi$

"blind directions"

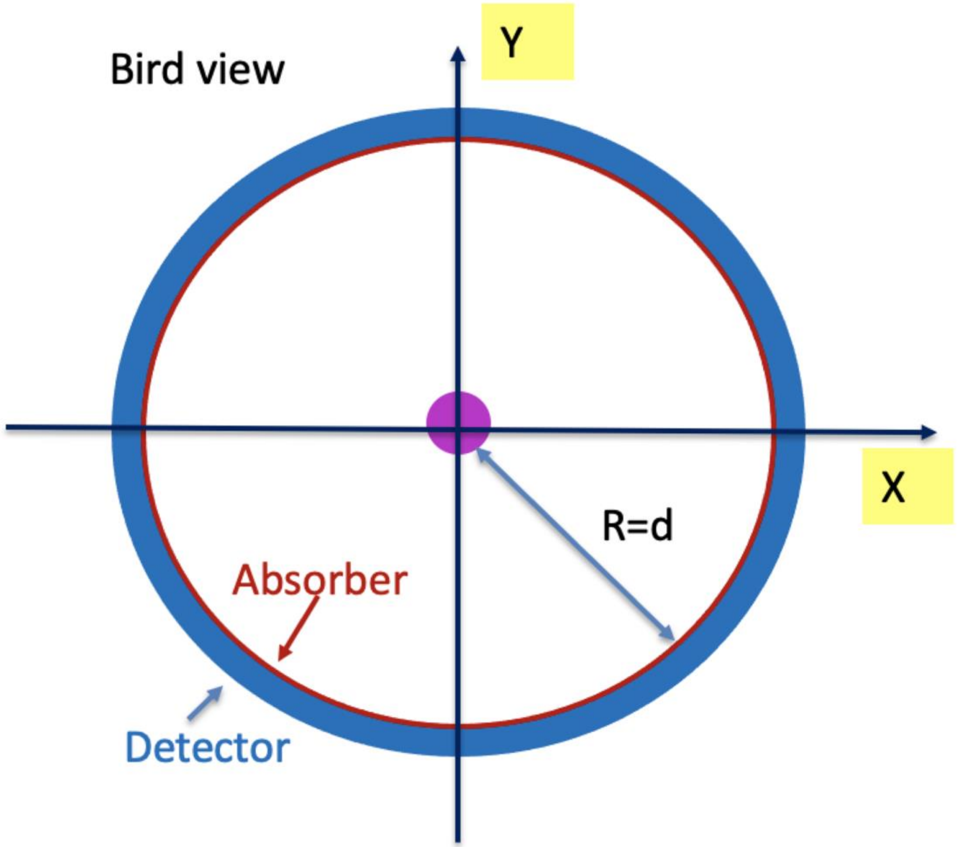
$$\omega d\sin^2\frac{\theta}{2} = n\pi, \quad n = 1, 2, 3...$$

Multiple directions can compensate for others' insensitive directions.



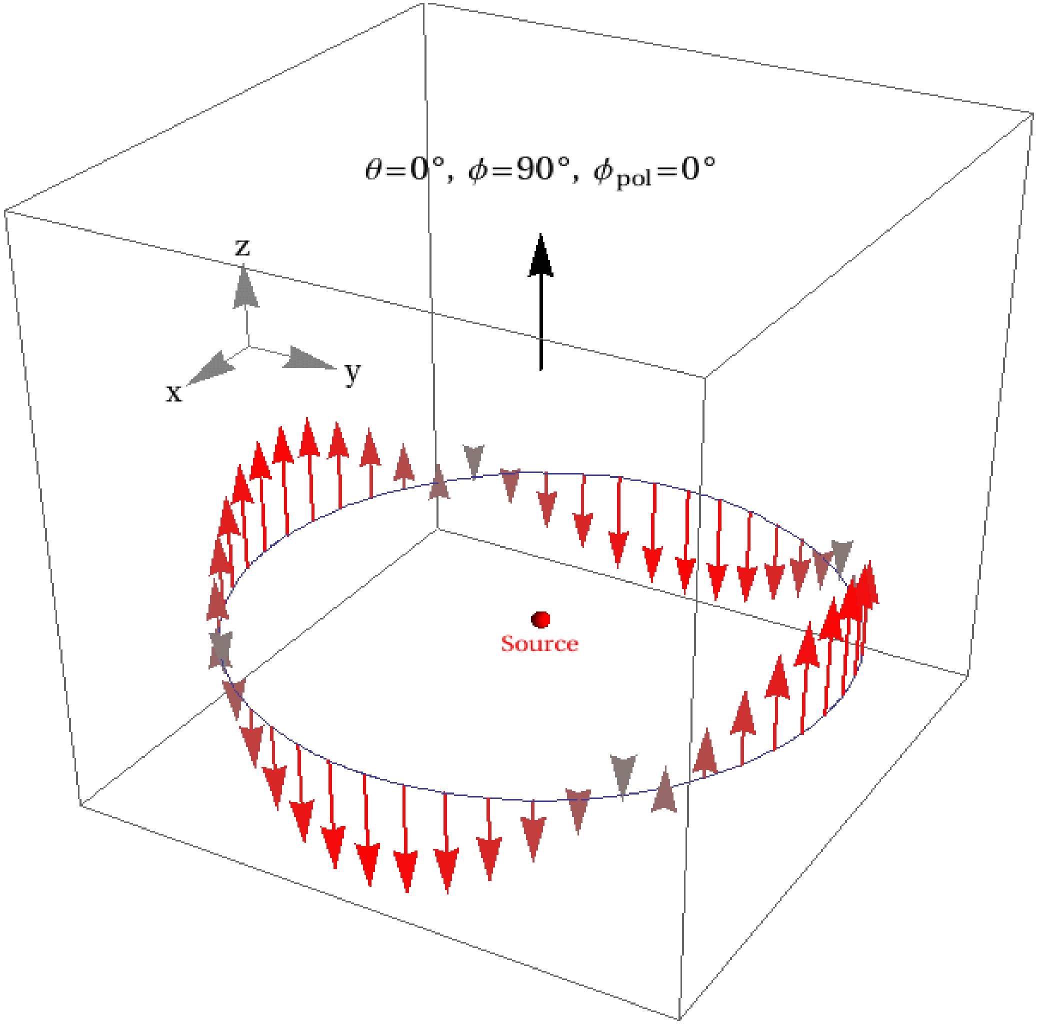
Stationary Mössbauer GW setup

- Absorber+detectors are in the resonance plane around source
  - At least two perpendicular directions relative to any GW incident θ angle.
  - Enhance acceptance of resonances photons



Hypothetical GW signal “seen” by our experiment

- Modulation of resonance peak shifts in the ring
  - Allow direction and polarization reconstruction



Benchmark scenarios

An estimate on the required source intensity: (R_S for an isotropic source)

$$
\begin{aligned}
R_s &= \frac{\omega}{2\pi}\frac{C_\infty\, 2d}{\Delta Z f_\phi f_t} = \frac{2\omega d g \xi^2}{f_\phi f_t}\left(\frac{\Gamma_{\mathrm{exp}}}{E_0}\right)\left(\frac{\delta f}{f}\right)^{-2} \\
&\approx 10^{14}\ \mathrm{Bq}\cdot\left(\frac{\omega/2\pi}{\mathrm{MHz}}\right)\left(\frac{d}{1\ \mathrm{m}}\right)\left(\frac{g}{g_\oplus}\right) \\
&\quad\cdot\left[\frac{\eta(\epsilon f_S)}{12.4}\right]^2\left(\frac{4\times10^{-21}}{\delta f/f}\right)^2
\end{aligned}
$$

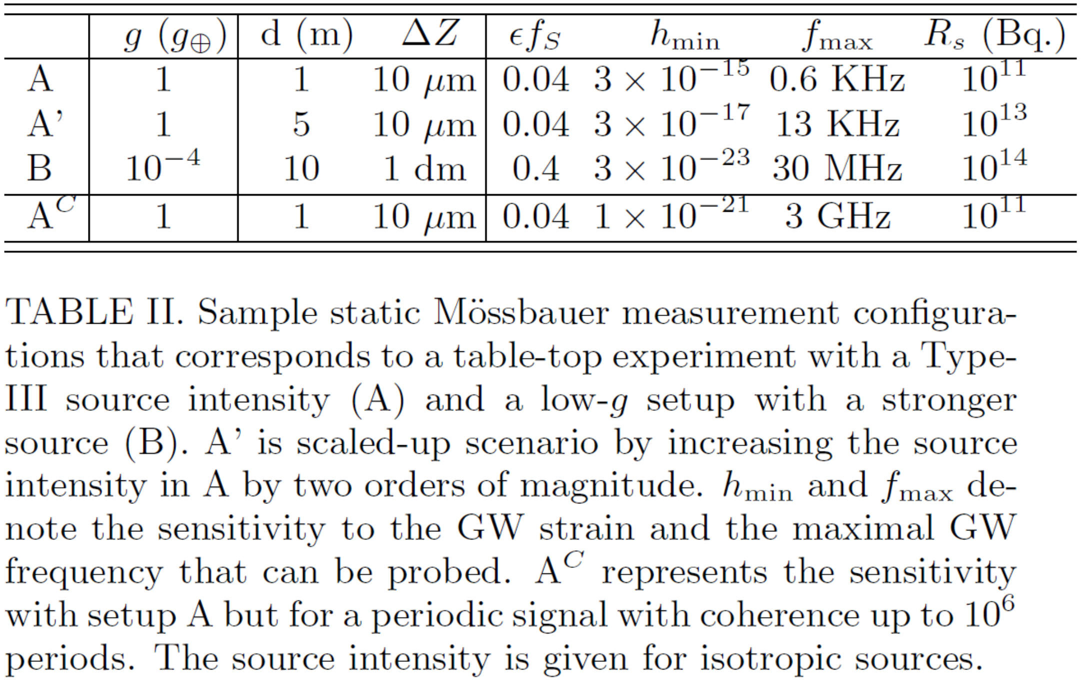
Beware: pars on the 2nd line do not scale independently.

Benchmarks:

A: table-top experiment.

B: 10-meter radius in low-*g*

- Source intensity scales linearly with baseline length and inversely with the local gravitational acceleration.
- Need to balance between resonance shift length, detector size, and practical sources.
- Non-isotropic source / focusing would immensely enhance efficiency.
- Coherently repeated signals can boost statistics: N_{90} -> N_{90}*Q



	g $(g_\oplus)$	d (m)	ΔZ	ϵf_S	h_{min}	f_{max}	R_s (Bq.)
A	1	1	10 μm	0.04	3×10^{-15}	0.6 KHz	10^{11}
A'	1	5	10 μm	0.04	3×10^{-17}	13 KHz	10^{13}
B	10^{-4}	10	1 dm	0.4	3×10^{-23}	30 MHz	10^{14}
A^C	1	1	10 μm	0.04	1×10^{-21}	3 GHz	10^{11}

TABLE II. Sample static Mössbauer measurement configurations that corresponds to a table-top experiment with a Type-III source intensity (A) and a low-g setup with a stronger source (B). A' is scaled-up scenario by increasing the source intensity in A by two orders of magnitude. h_{min} and f_{max} denote the sensitivity to the GW strain and the maximal GW frequency that can be probed. A^C represents the sensitivity with setup A but for a periodic signal with coherence up to 10^6 periods. The source intensity is given for isotropic sources.

No IC included

Expected sensitive on GWs

Science Bulletin 2024; 69(18): 2795-2798

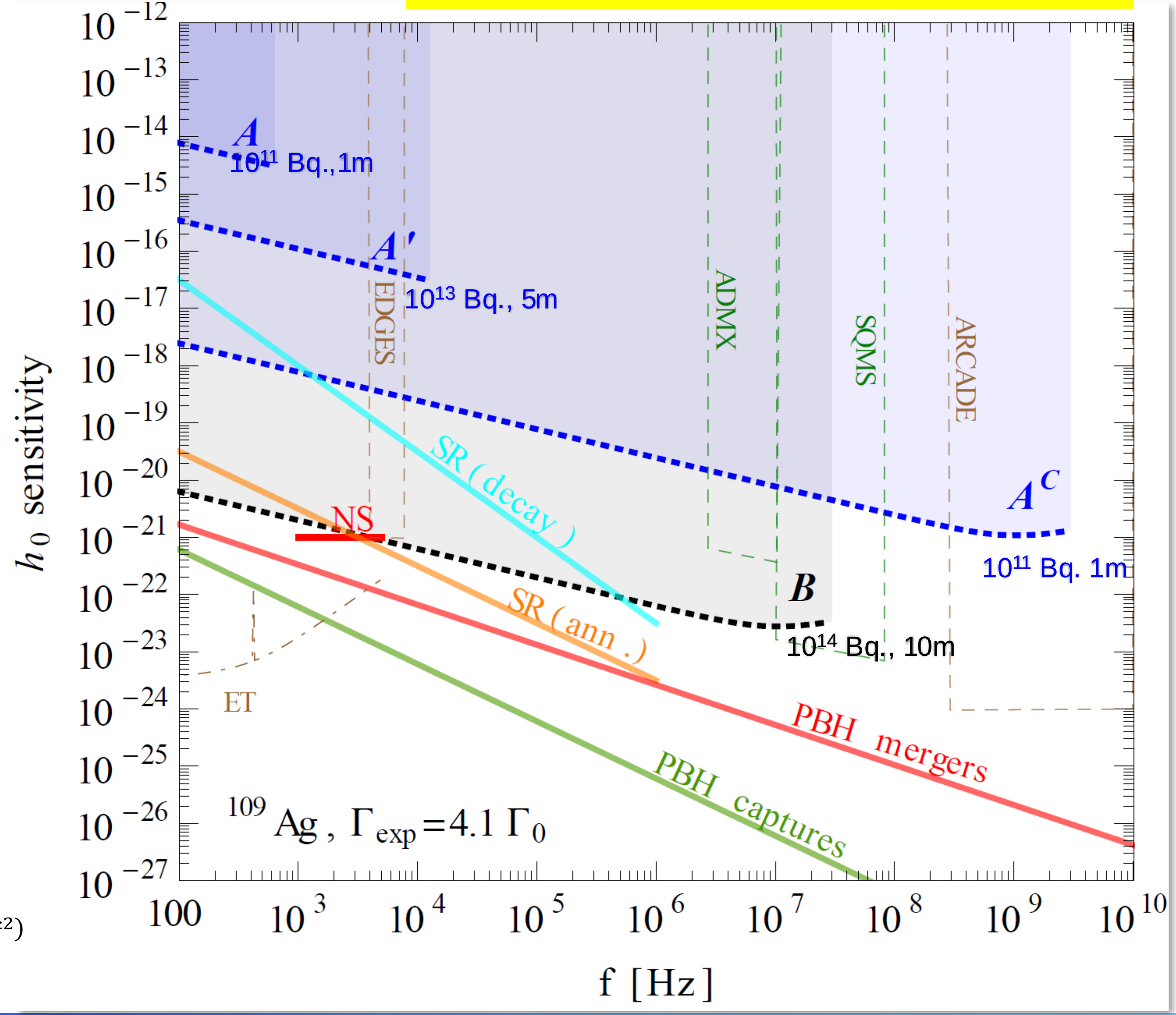
A, A' & B:
single period

A^C:
10^6 periods

Max Freq. Reach:
cutoffs at:
*3σ peak with C_∞,
$C_\infty > (\sigma)^2/(f_S\epsilon)^2$
* $2\pi f d < O(10)$

Coherent GWs from NS, SR, PBHs, see
N.Aggarwal et al., Living Rev. Rel. 24, 4 (2021)

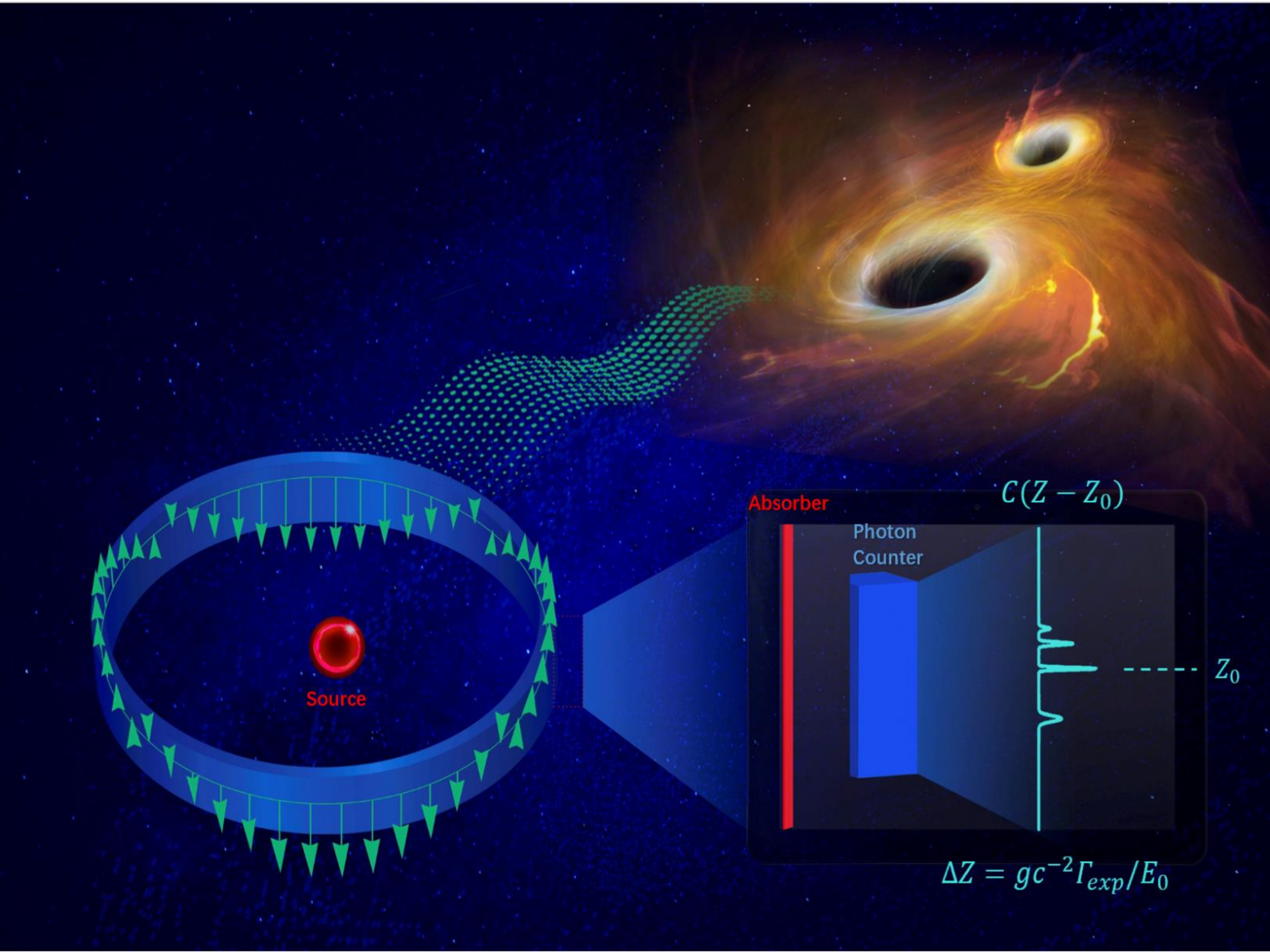
Inverse Gentsenshtein:
$\Delta F = F_0\, h * (form\ factor{\sim}\omega d^{n.\geq 2})$
also see: 2305.00877



Summary

- Conceptual design use Mössbauer effect for GW measurement:
  - Stationary scheme（from table size to multi-kilometers）
    - Measure the spatial resonance absorption peak shift.
    - Allow direction and polarization reconstruction.
    - Relatively small-scale setup (meter – 10 meter)
    - ^{109}Ag source gives 10^{-22} strain sensitivity, long lived mother particle.
    - Multiband sensitivity（*Sensitive to* f_{GW} > kHz）
  - Possible extension on sensitivity
    - low-g, focusing of photons(Laue lens)…
  - Larger diameter between source and absorber:
    - Aiming for lower frequency GWs

- Ground based experiment
- Moon/Asteroid based experiment?
- Lagrangian point based experiment？


Absorber
Photon Counter
Source
$C(Z - Z_0)$
Z_0
$\Delta Z = g c^{-2} \Gamma_{exp} / E_0$