Ab initio nuclear physics on the lattice

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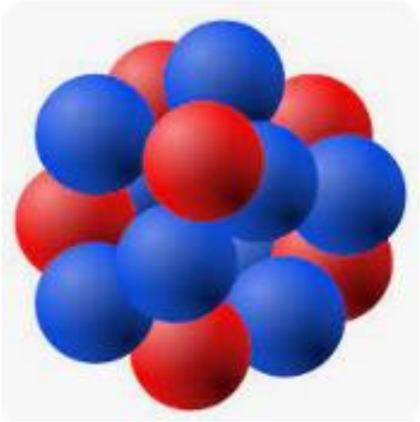


ShangHai JiaoTong University

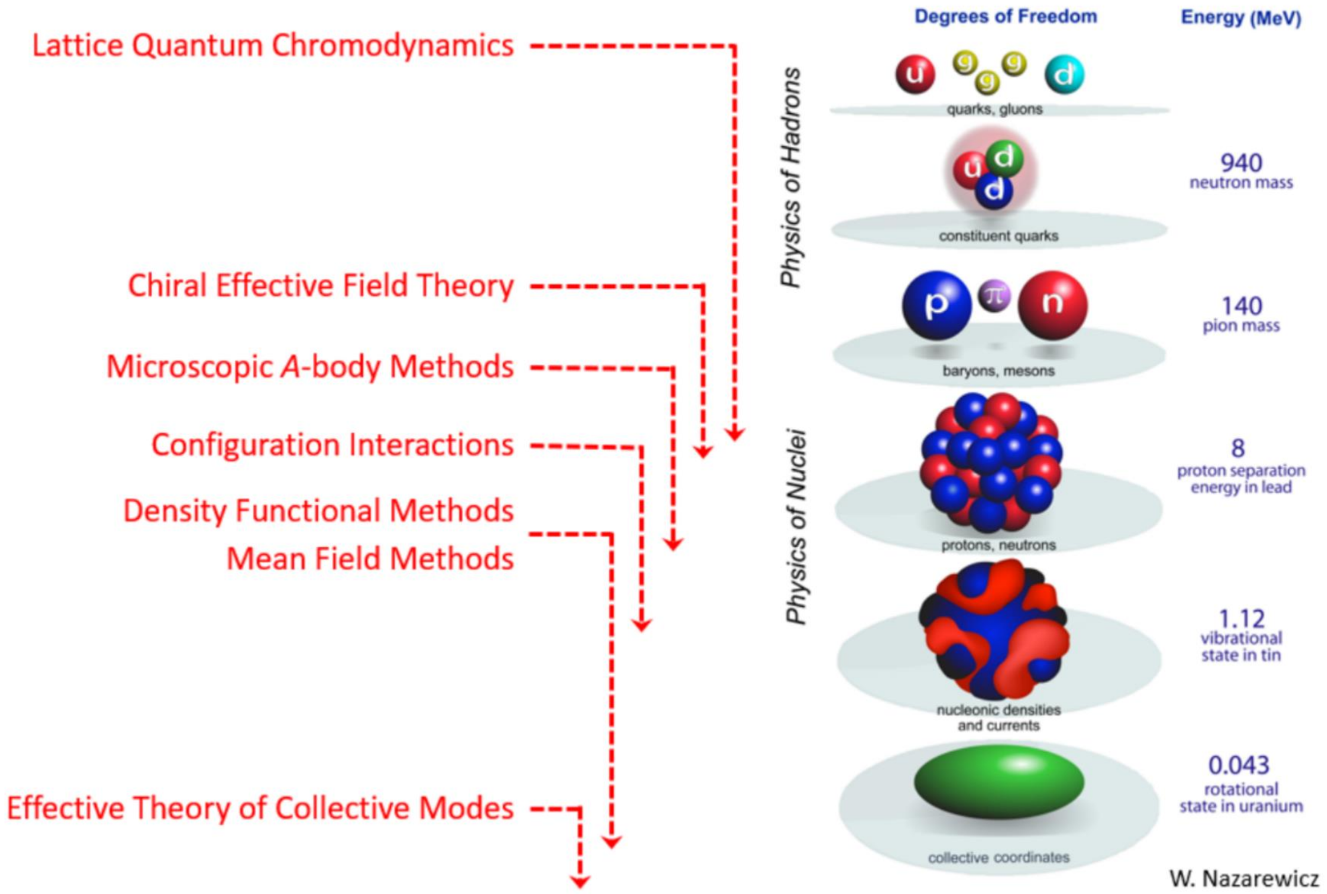
2024-DEC-11, ShangHai

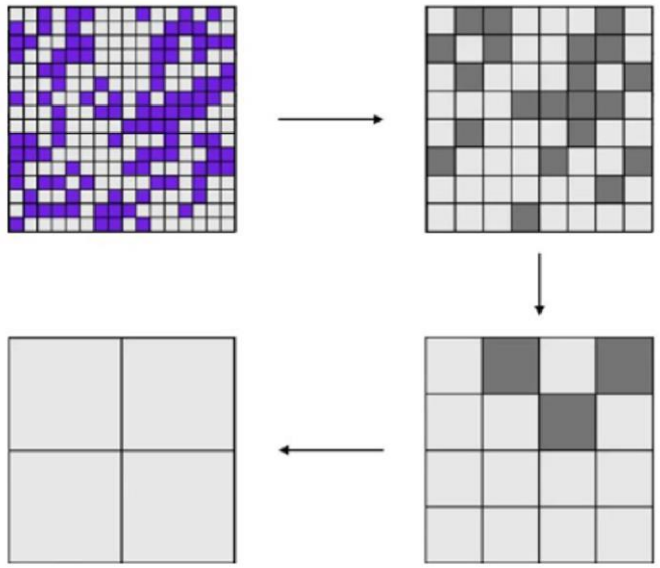
Contents

- Brief introduction to nuclear lattice EFT
- Nuclear force problem with NLEFT
- Strong correlation I: nuclear clustering
- Strong correlation II: nuclear thermodynamics
- Summary and perspective

What is a nuclear EFT?

- **Modern nuclear force constructions are based on the Effective Field Theory**
- **Theoretical foundation of EFT is the Wilsonian renormalization group:**
  - **High-momentum** details can be integrated out & hidden in LECs
  - **Low-momentum** physics kept invariant under ren. group transformations



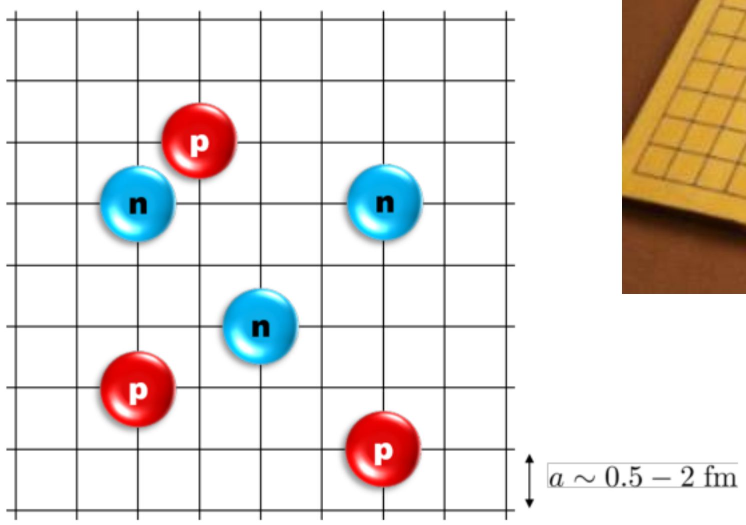


Lattice EFT: A many-body EFT solver

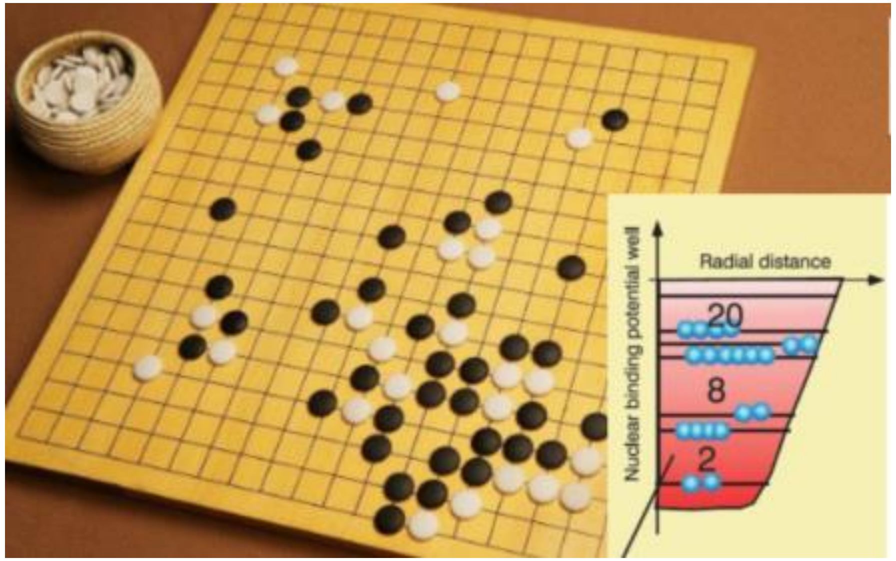
Lattice EFT = **Chiral EFT** + **Lattice** + **Monte Carlo**

Review: Dean Lee, Prog. Part. Nucl. Phys. 63, 117 (2009),
Lähde, Meißner, "Nuclear Lattice Effective Field Theory", Springer (2019)

- Discretized chiral nuclear force
- Lattice spacing $a \approx 1$ fm = 620 MeV (~chiral symmetry breaking scale)
- Protons & neutrons interacting via **short-range, δ-like** and **long-range, pion-exchange** interactions
- Exact method, polynomial scaling ($\sim A^2$)



Lattice adapted for nucleus



- Solve the non-perturbative nuclear many-body problem by sampling all configurations

Lattice EFT: A many-body EFT solver

- Get *interacting g. s.* from imaginary time projection:

$$|\Psi_{g.s.}\rangle \propto \lim_{\tau\to\infty} \exp(-\tau H)|\Psi_A\rangle$$

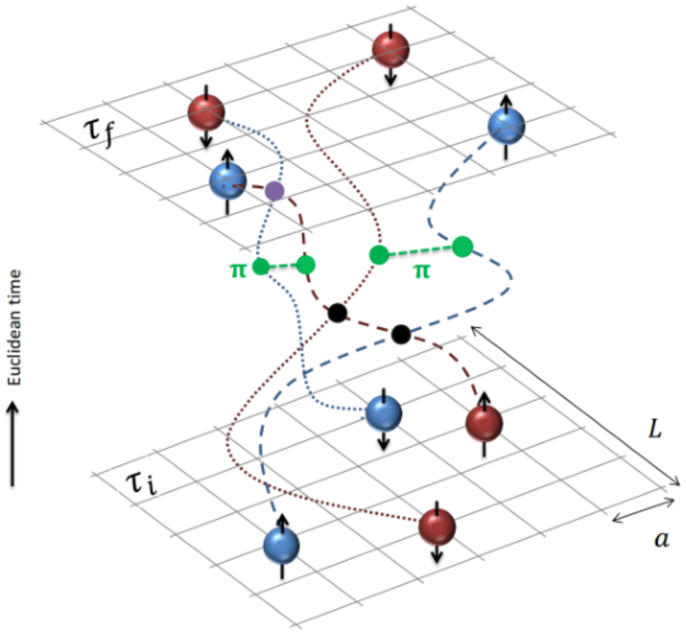
with $|\Psi_A\rangle$ representing A *free* nucleons.

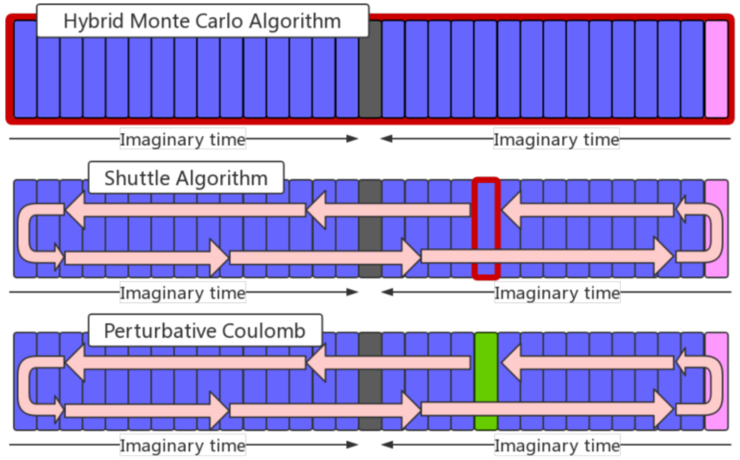
- Expectation value of any operator $\mathcal{O}$:

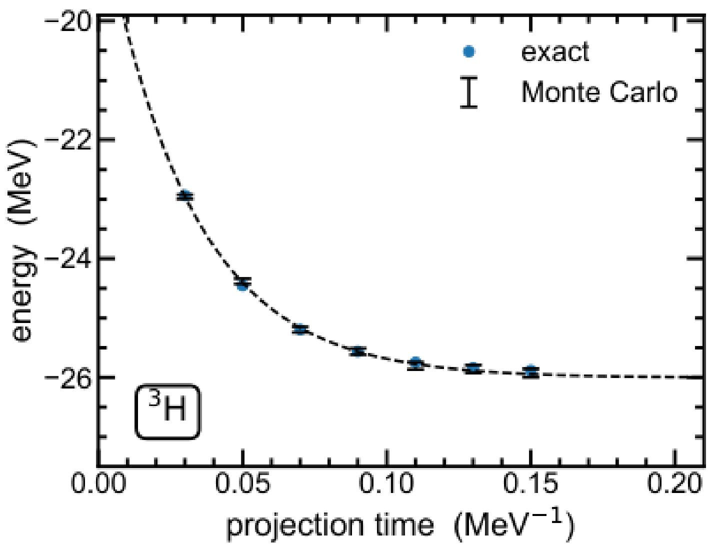
$$\langle O\rangle = \lim_{\tau\to\infty} \frac{\langle\Psi_A|\exp(-\tau H/2)\mathcal{O}\exp(-\tau H/2)|\Psi_A\rangle}{\langle\Psi_A|\exp(-\tau H)|\Psi_A\rangle}$$

- τ is discretized into time slices:

$$\exp(-\tau H) \simeq \left[: \exp(-\frac{\tau}{L_t}H) : \right]^{L_t}$$







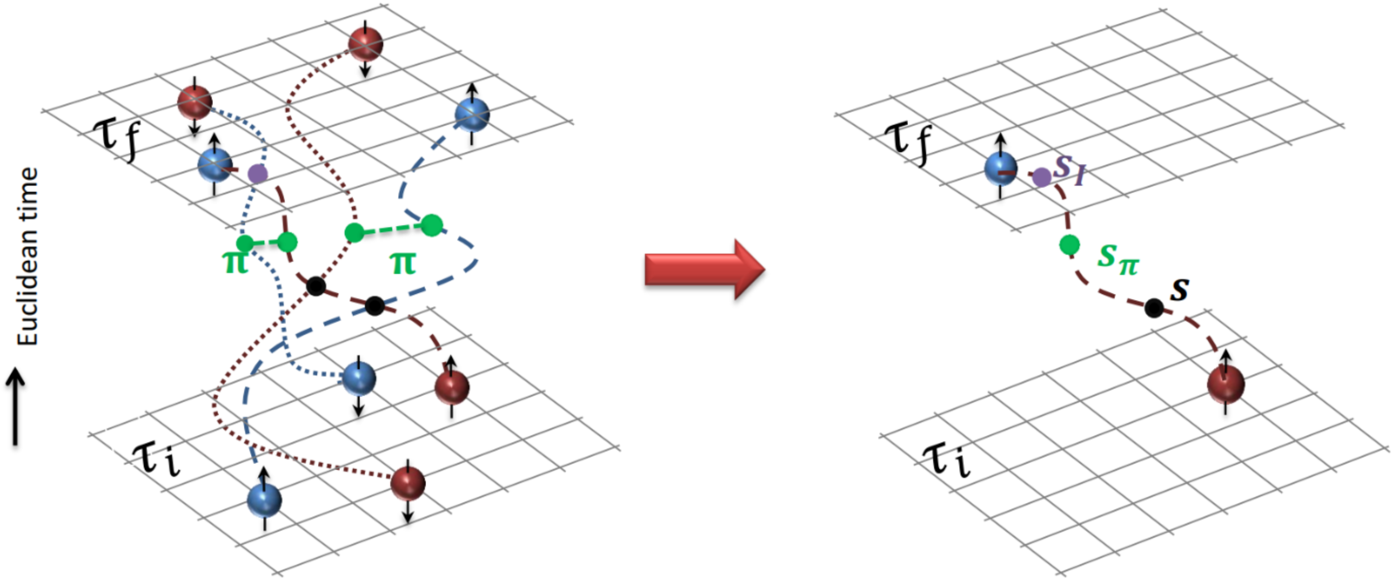
All possible configurations in $\tau \in [\tau_i, \quad \tau_f]$ are sampled.
Complex structures like nucleon clustering emerges naturally.

Lattice EFT: A many-body EFT solver

- Quantum correlations between nucleons are represented by fluctuations of the auxiliary fields.

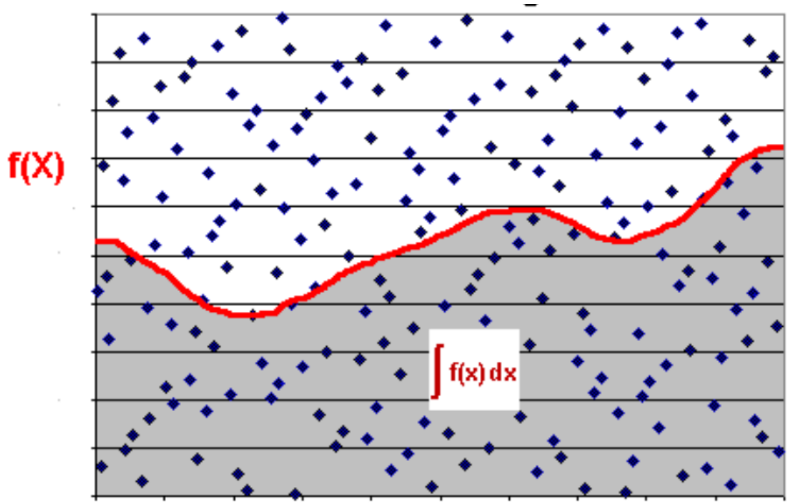
$$:\exp\left[-\frac{a_t C}{2}(\psi^\dagger\psi)^2\right] := \frac{1}{\sqrt{2\pi}}\int ds : \exp\left[-\frac{s^2}{2}+\sqrt{-a_t C}s(\psi^\dagger\psi)\right]:$$

- Long-range interactions such as OPEP or more complex interactions can be represented similarly.
- For fixed aux. fields, product of s.p. states (e.g., Slater determinant) keep the form of product of s.p. states in propagations. $\Longleftarrow$**No N-N interaction**



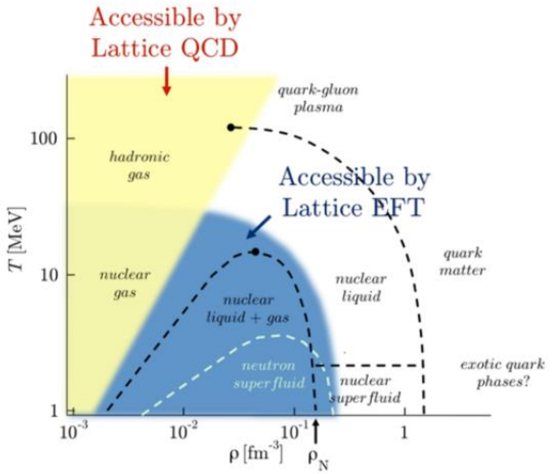
In lattice EFT, solving a general Hamiltonian consists of 5 steps:

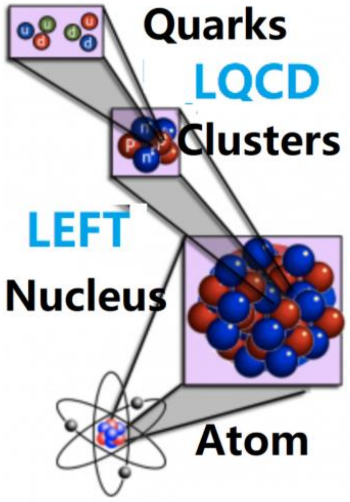
1. Rewrite expectation value as a path integral using auxiliary field transformation.
2. For each field configuration, calculate the amplitude.
3. Integrate over the field variables using Monte Carlo algorithms.
4. Take the limit $\tau \to \infty$ to find the true ground state.
5. Take the limit $L \to \infty$ to eliminate the finite volume effects.



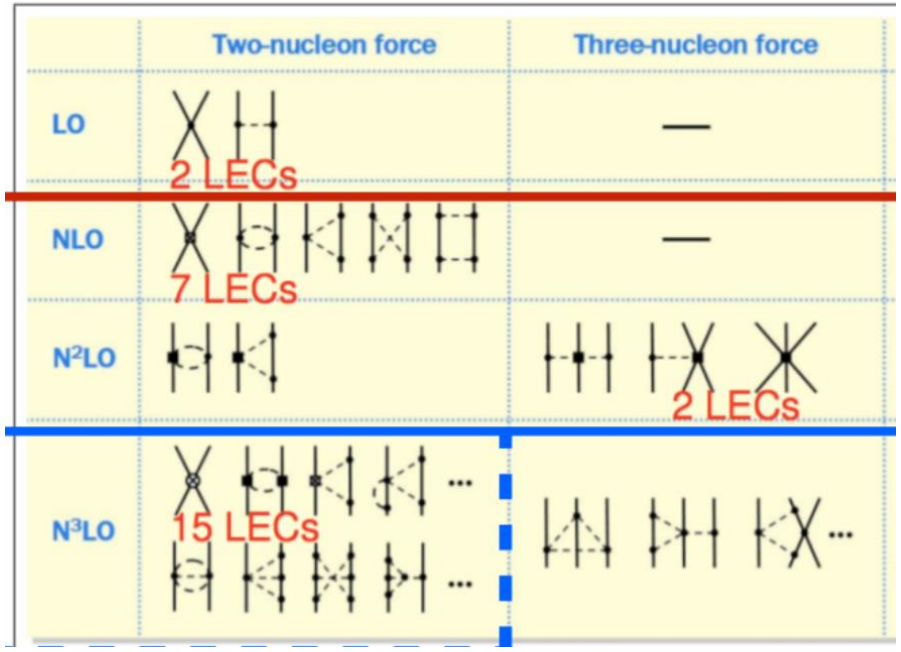
Compare Lattice EFT and Lattice QCD

	LQCD	LEFT
degree of freedom	quarks & gluons	nucleons and pions
lattice spacing	$\sim$0.1 fm	$\sim$1 fm
dispersion relation	relativistic	non-relativistic
renormalizability	renormalizable	effective field theory
continuum limit	yes	no
Coulomb	difficult	easy
accessibility	high T / low ρ	low T / ρ_{sat}
sign problem	severe for $\mu > 0$	moderate





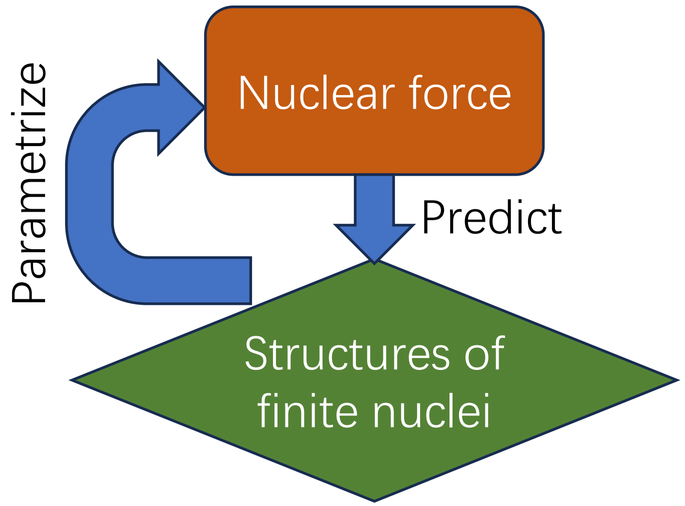
- Lattice EFT share a lot of common features with Lattice QCD. However,
  - Non-rel. → particle number conservation
  - Quadratic dispersion relation
    → no Fermion doubling problem
  - EFT contains non-renormalizable terms
    → no continuum limit



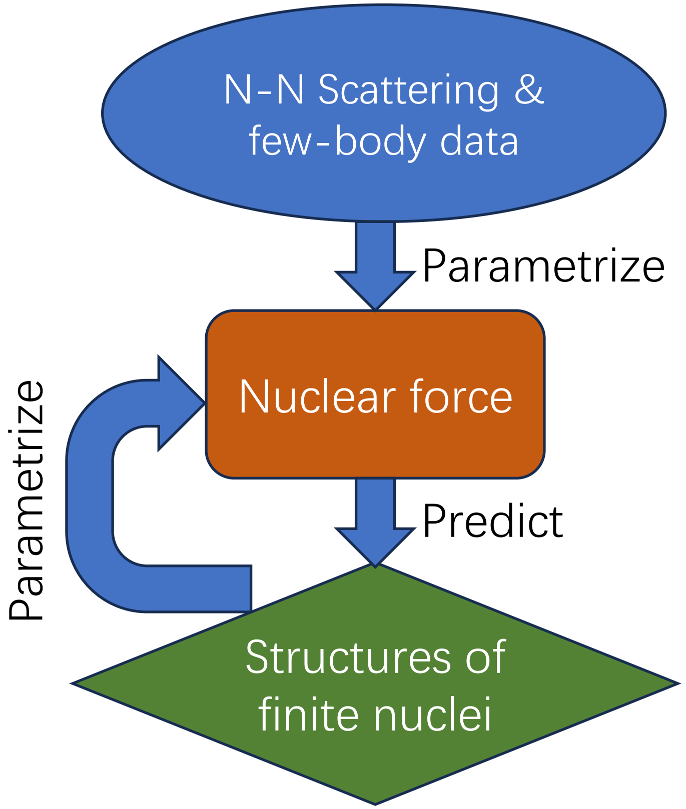
Nuclear Force Problem

Nuclear Force Problem: Can the nuclear force calibrated with the **N-N scattering and few-body data** uniquely and correctly predict the **structures of finite nuclei**?

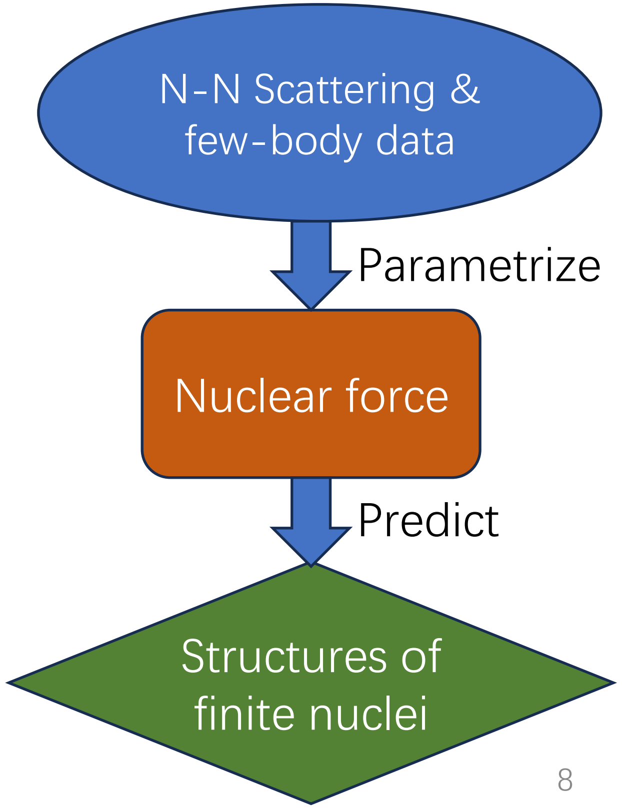
Effective nuclear forces
(Skyrme, RMF, shell model, etc.)



Ab initio (Weak form)

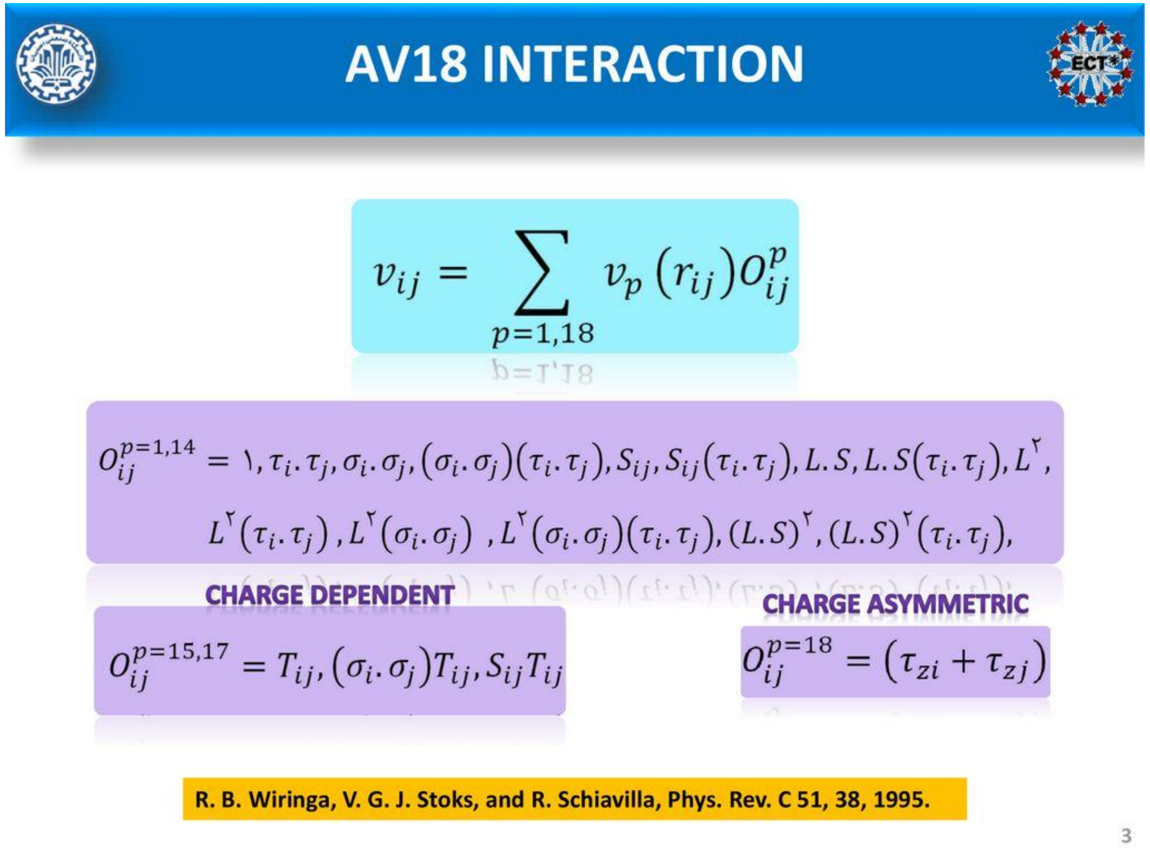


Ab initio (Strong form)

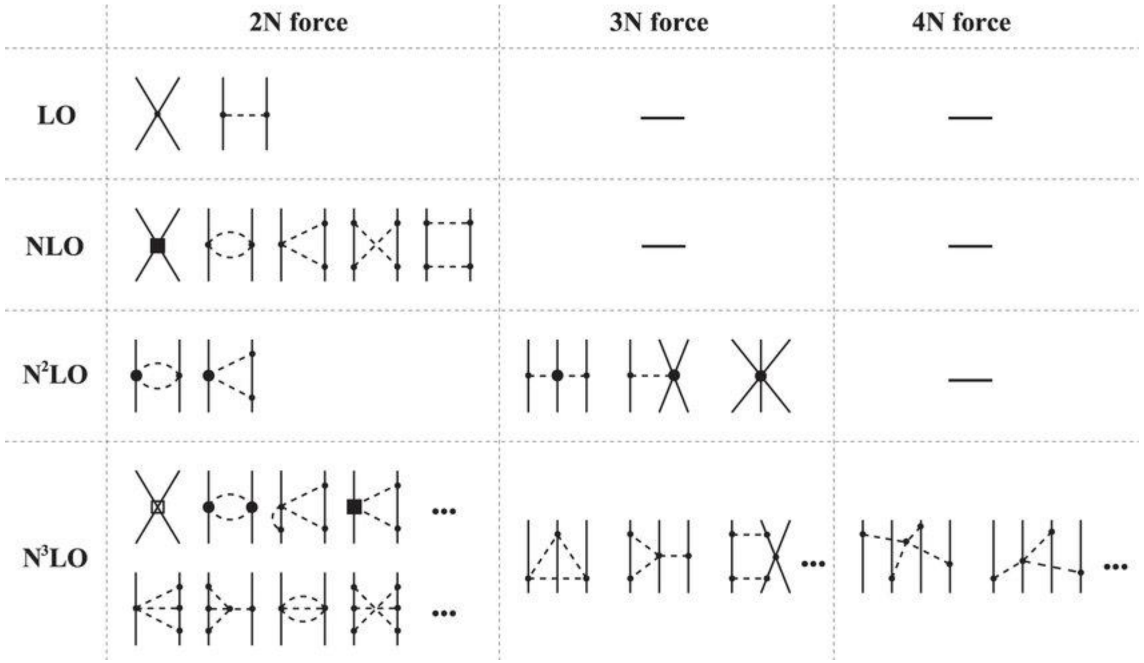


Typical nuclear forces

Nuclear force has strong **spin-isospin dependence**
→ Reflected by complicated **operator structures**



Nuclear chiral EFT



Symmetries of realistic nuclear forces

- A N-N two-body interaction can be uniquely fixed by its matrix elements in a complete basis
- In momentum basis $|\boldsymbol{p}_1 s_1 t_1 \boldsymbol{p}_2 s_2 t_2\rangle$ with $s_1, s_2 = \pm 1/2$ the spins, $t_1, t_2 = \pm 1/2$ the isospins, we have the two-by-two matrix elements

$$\langle \boldsymbol{p}_1' s_1' t_1' \boldsymbol{p}_2' s_2' t_2' | V | \boldsymbol{p}_1 s_1 t_1 \boldsymbol{p}_2 s_2 t_2 \rangle$$

- These matrix elements are strictly constrained by symmetries
  - Hermicity
  - Exchange symmetry
  - Translational invariance
  - Galilean invariance
  - Parity
  - Time reversal
  - Spatial rotation
  - Isospin symmetry

- **The QCD respects**
  - Discrete symmetries of PCT (exact)
  - Symmetries of Poincaré group (exact)
    - Spatial translation
    - Temporal translation → Energy conservation
    - Spatial rotation
    - Boosts → Galilean invariance
  - Isospin symmetry (approx.)

Spirit of EFT: Use symmetries to write down the most general interactions

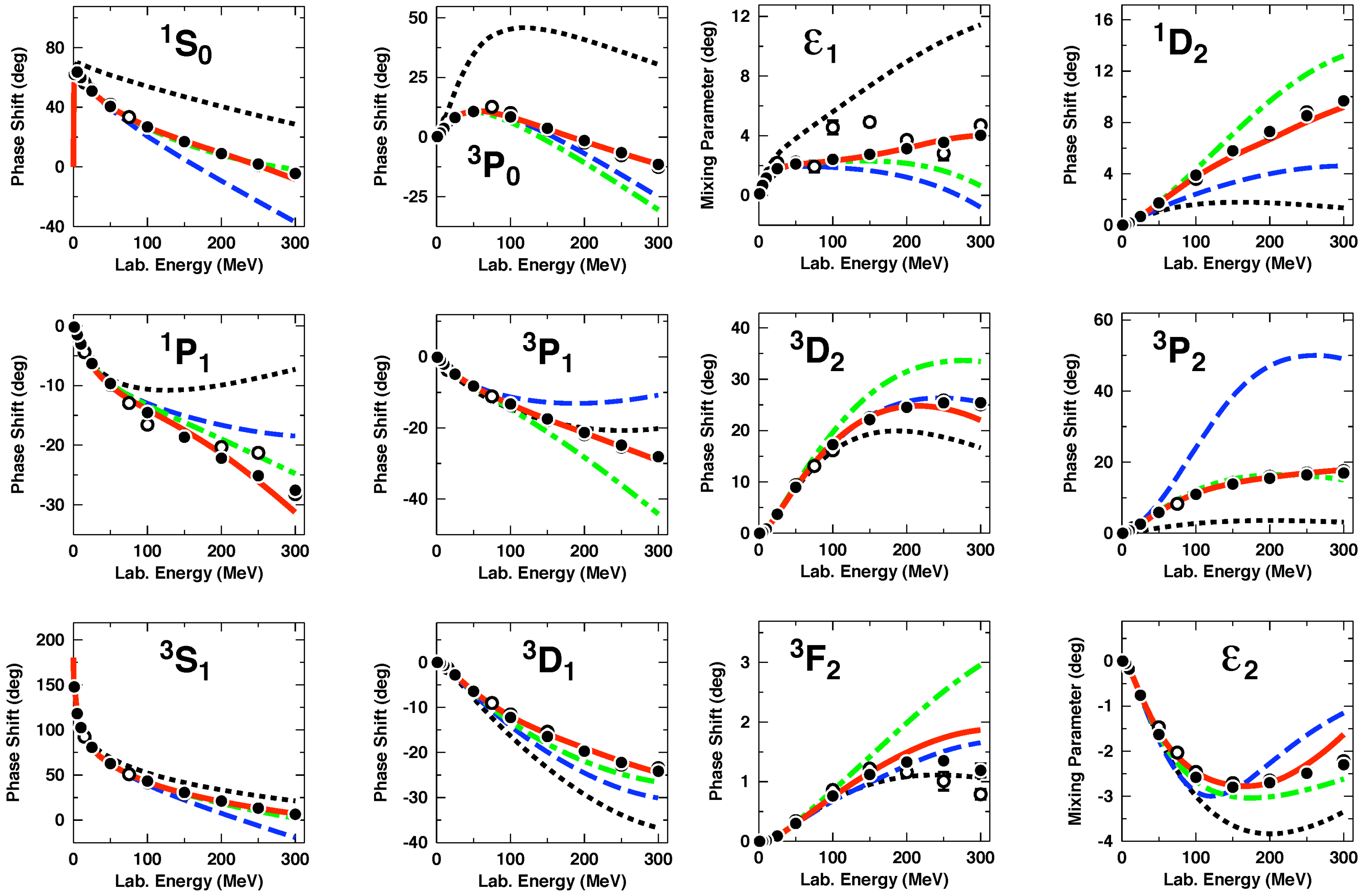
Parametrization of the nuclear force

- It is conventional to parametrize the nuclear force by matching to the low-energy **nucleon-nucleon scattering data**

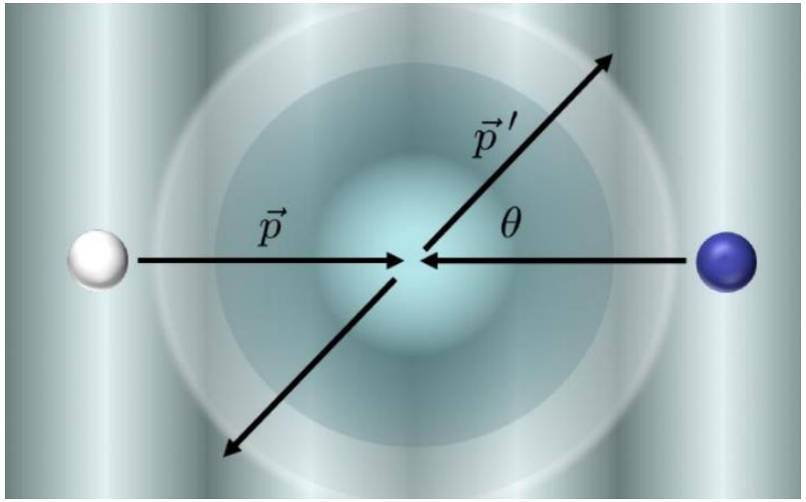
- Spin-1/2 + Spin-1/2 + Orbital
- We first couple the spins
  1/2 + 1/2 = 0 + 1
  Then couple the total spin S=s1+s2
  With the angular momentum L

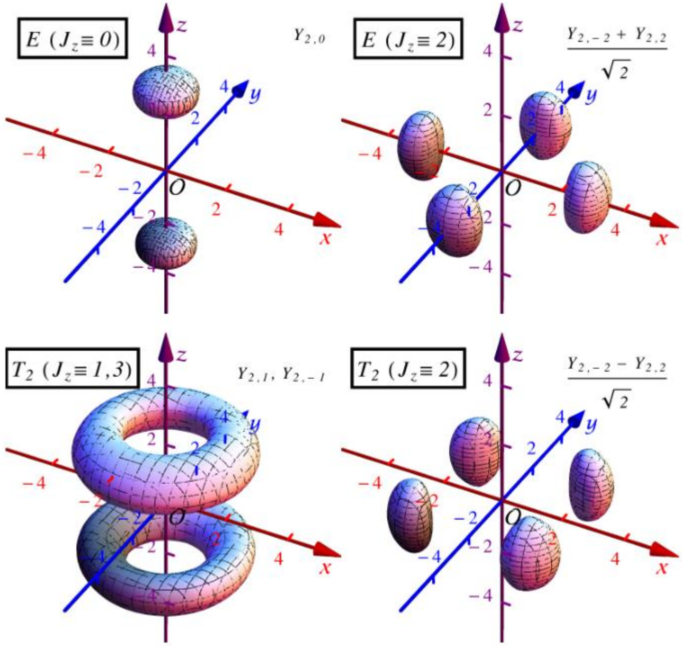
- Partial wave channels $^{2S+1}L_J$

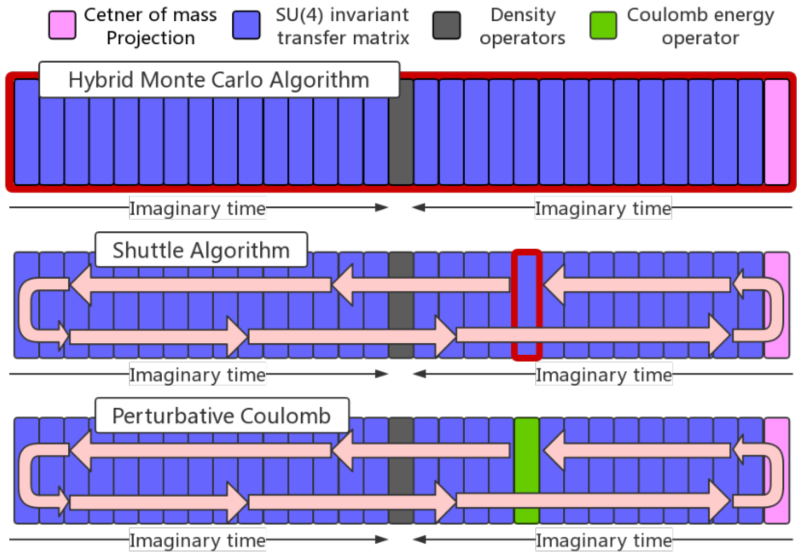
  S = 0, 1
  L = 0, 1, 2, 3, ⋯ (S, P, D, F, ⋯)
  J = |L – S|, |L – S| + 1, ⋯, L + S

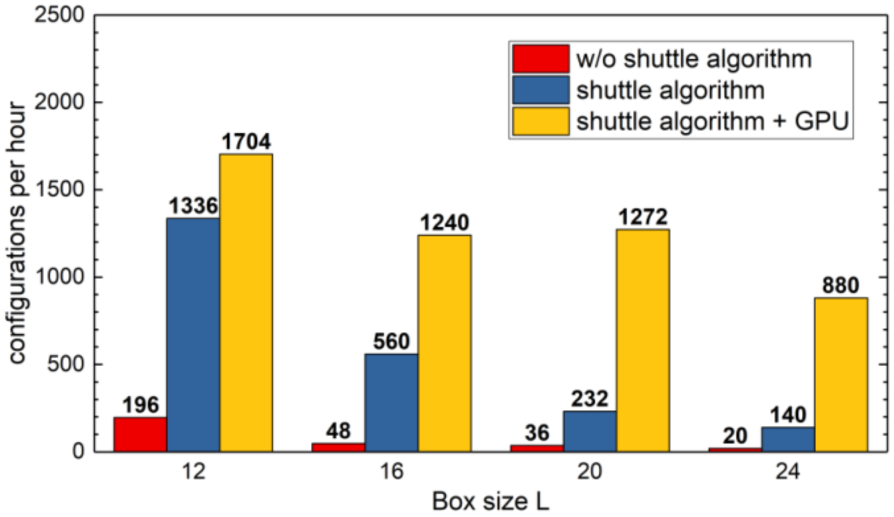


Effective Field Theory on the Lattice







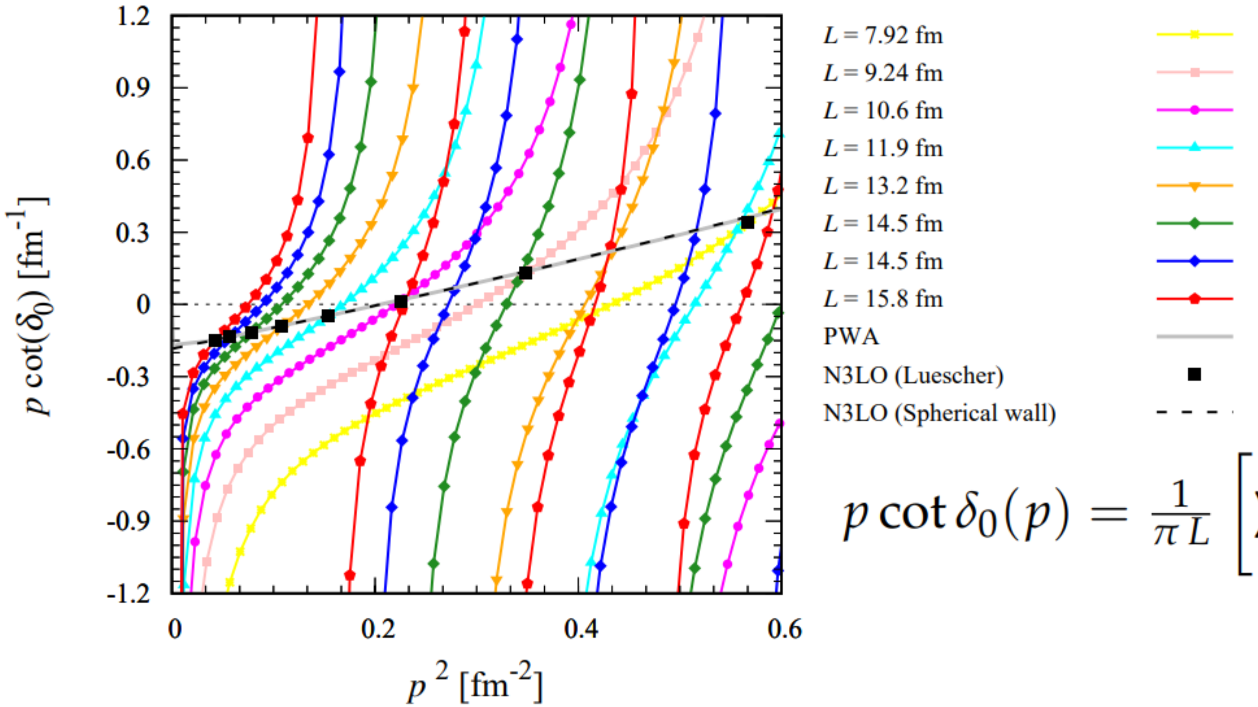


Lattice regularization breaks the **rotational symmetry** and **Galilean invariance**, which must be restored

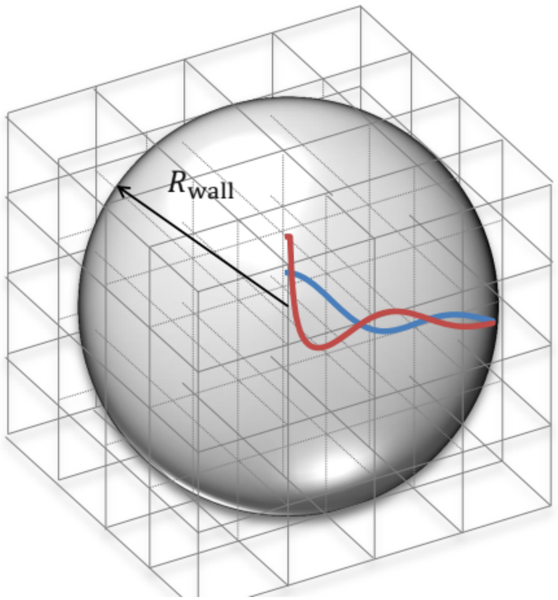
PLB 760, 309 (2016):
Restoration of rotational symm.
EPJA 53, 83 (2017):
N^2LO chiral force on lattice
PRC 98, 044002 (2018):
N^3LO chiral force on lattice

Shuttle Algorithm is 5-10 times faster than conventional algorithms
Combined with GPU, can speed up by 40-50 times
PLB 797, 134863 (2019)

Scattering on the lattice



$$p \cot \delta_0(p) = \frac{1}{\pi L} \left[\sum_{\vec{n}}^{\Lambda} \frac{\theta\left(\Lambda^2 - \vec{n}^2\right)}{\vec{n}^2 - (Lp/2\pi)^2} - 4\pi \Lambda \right]$$



Lüscher's finite volume method:

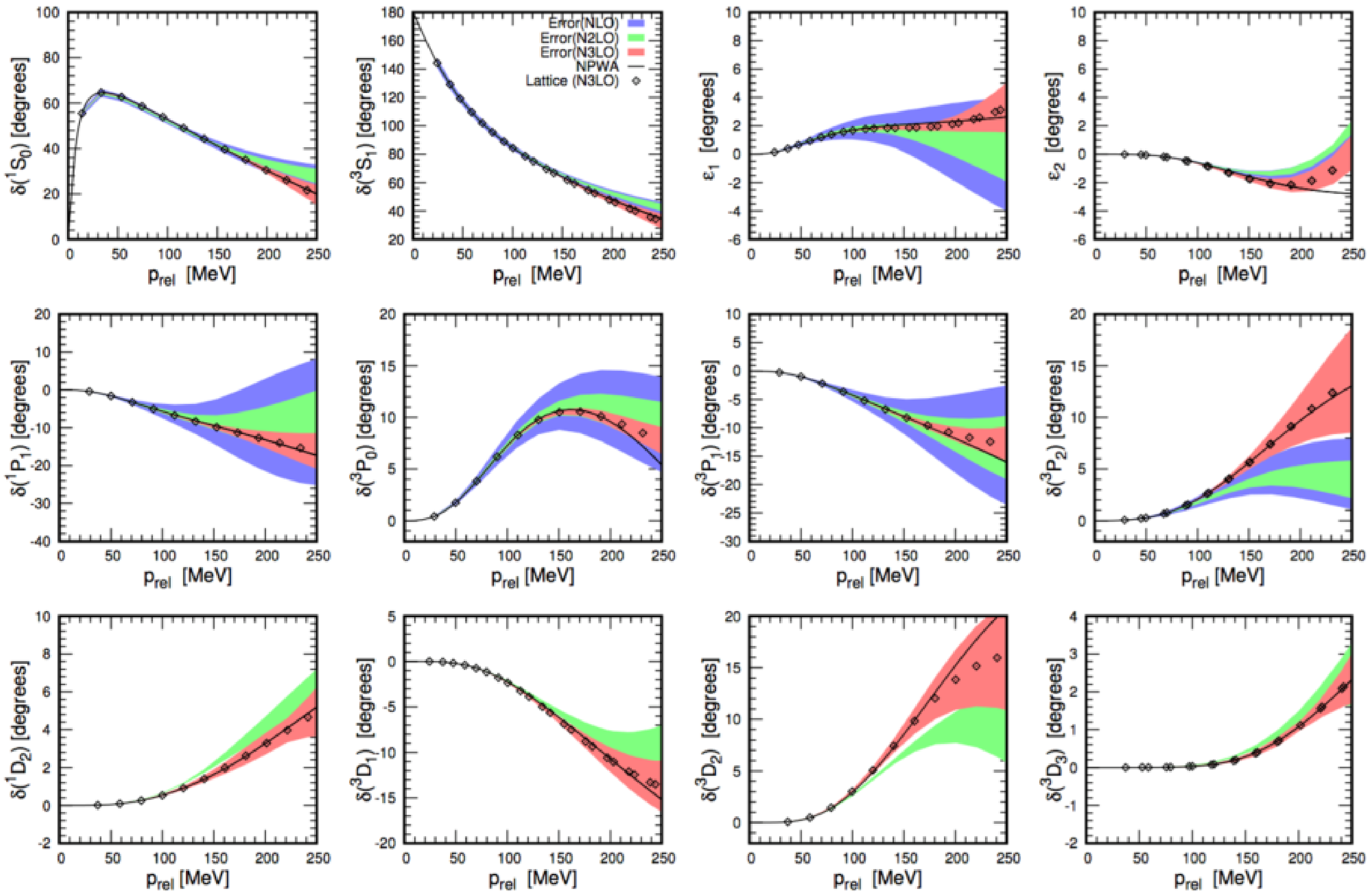
Lüscher, Comm. Math. Phys. 105 (1986) 153; NPB 354 (1991) 531

Spherical wall method:

$$R_\ell^{(p)}(r) = N_\ell(p) \times \begin{cases} \cot \delta_\ell(p)\, j_\ell(p\,r) - n_\ell(p\,r) \\ \cot \delta_\ell(p)\, F_\ell(p\,r) + G_\ell(p\,r) \end{cases}$$

Nucl. Phys. A 424, 47-59 (1984), Eur. Phys. J. A 34, 185-196 (2007).

Chiral nuclear force up to N^{3}LO: fit on the lattice



fit to N^{2}LO: Alarcon, Du, Klein, Lahde, Lee, Ning Li, B.L., Luu, Meissner, EPJA 53, 83 (2017)

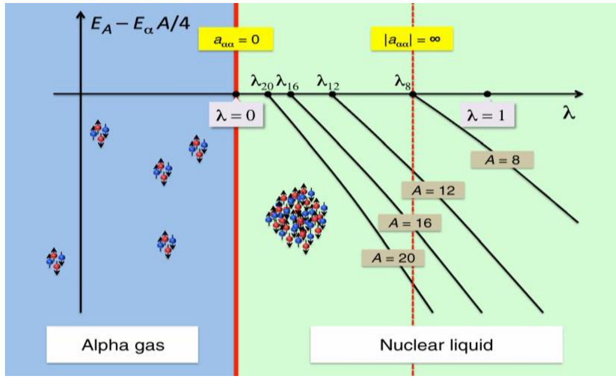
fit to N^{3}LO: Ning Li, Elhatisari, Epelbaum, Lee, B.L., Meissner, PRC 98, 044002 (2018)

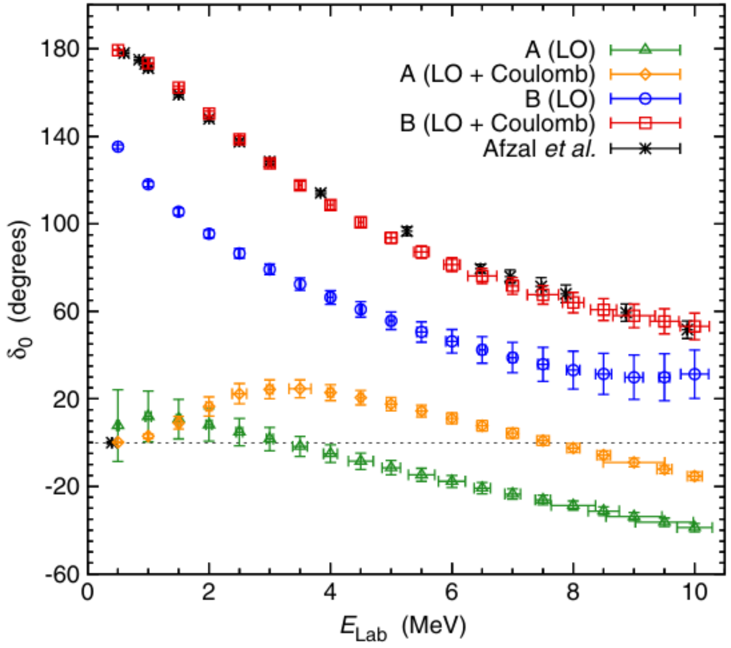
Nuclear binding near a quantum phase transition

PRL **117,** 132501 (2016)
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS
week ending 23 SEPTEMBER 2016

Nuclear Binding Near a Quantum Phase Transition

Serdar Elhatisari,[1] Ning Li,[2] Alexander Rokash,[3] Jose Manuel Alarcón,[1] Dechuan Du,[2] Nico Klein,[1] Bing-nan Lu,[2] Ulf-G. Meißner,[1,2,4] Evgeny Epelbaum,[3] Hermann Krebs,[3] Timo A. Lähde,[2] Dean Lee,[5] and Gautam Rupak[6]





Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^{3}H	−7.82(5)	−7.78(12)	−7.82(5)	−7.78(12)	−8.482
^{3}He	−7.82(5)	−7.78(12)	−7.08(5)	−7.09(12)	−7.718
^{4}He	−29.36(4)	−29.19(6)	−28.62(4)	−28.45(6)	−28.296
^{8}Be	−58.61(14)	−59.73(6)	−56.51(14)	−57.29(7)	−56.591
^{12}C	−88.2(3)	−95.0(5)	−84.0(3)	−89.9(5)	−92.162
^{16}O	−117.5(6)	−135.4(7)	−110.5(6)	−126.0(7)	−127.619
^{20}Ne	−148(1)	−178(1)	−137(1)	−164(1)	−160.645

- The nuclear force can be either local (position-dependent) or non-local (velocity-dependent).
- Locality is an essential element for nuclear binding.

Zeroth order Hamiltonian (perturbative order)

We use a zeroth order lattice Hamiltonian that respects the Wigner-SU(4) symmetry

$$H_0 = K + \frac{1}{2} C_{\mathrm{SU4}} \sum_{\boldsymbol{n}} : \tilde{\rho}^2(\boldsymbol{n}) :$$

The smeared density operator $\tilde{\rho}(\boldsymbol{n})$ is defined as

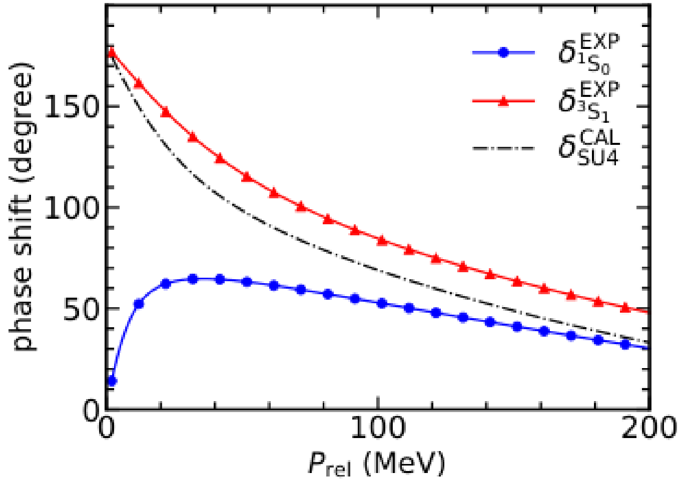
$$\tilde{\rho}(\boldsymbol{n}) = \sum_i \tilde{a}_i^\dagger(\boldsymbol{n}) \tilde{a}_i(\boldsymbol{n}) + s_L \sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} \sum_i \tilde{a}_i^\dagger(\boldsymbol{n}') \tilde{a}_i(\boldsymbol{n}'), \tag{1}$$

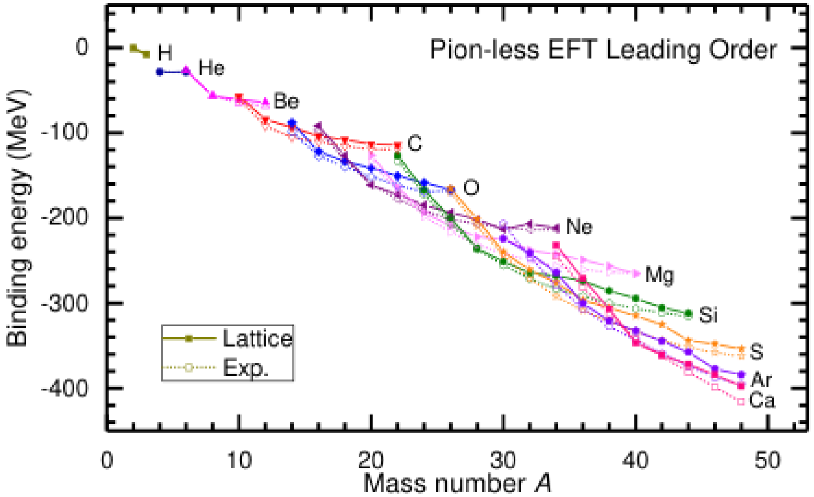
where i is the joint spin-isospin index

$$\tilde{a}_i(\boldsymbol{n}) = a_i(\boldsymbol{n}) + s_{NL} \sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} a_i(\boldsymbol{n}'). \tag{2}$$

In this work we use a lattice spacing $a = 1.32$ fm and the parameter set

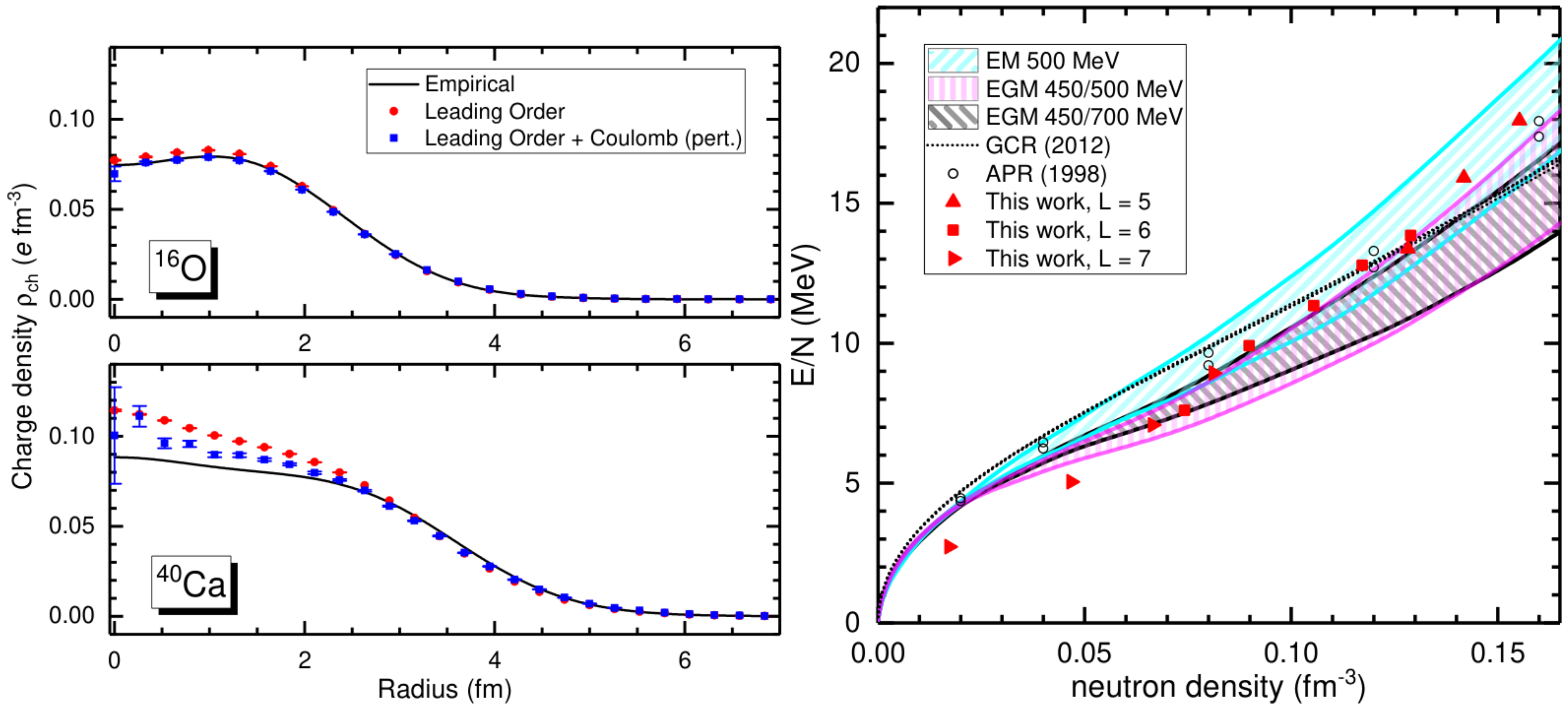
$C_{\mathrm{SU4}} = -3.41 \times 10^{-7}\ \mathrm{MeV}^{-2}$, $s_L = 0.061$ and $s_{NL} = 0.5$.





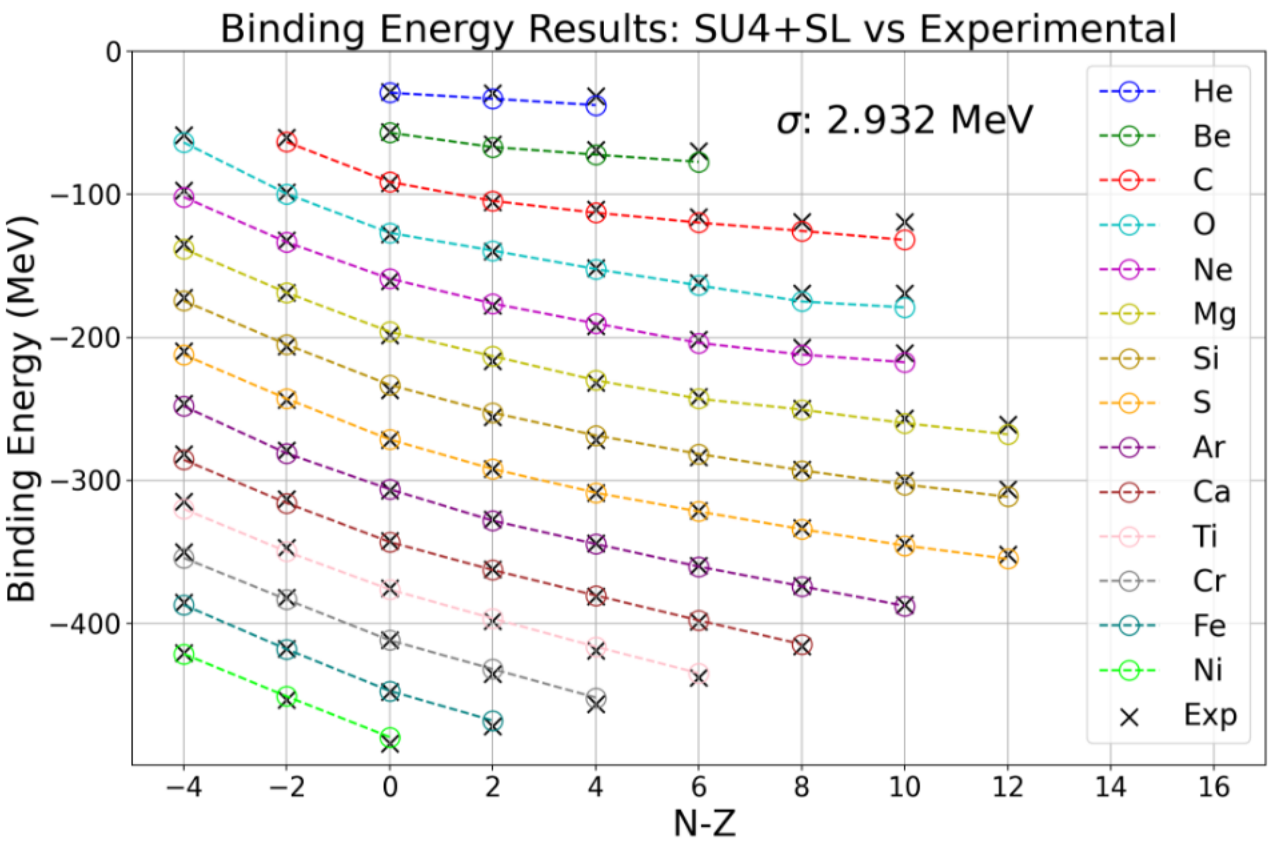
Essential elements for nuclear binding

Charge density and neutron matter equation of state
are impotant in element creation, neutron star merger, etc.



Lu, Li, Elhatisari, Lee, Epelbaum, Meissner, Phys. Lett. B 797 (2019) 134863

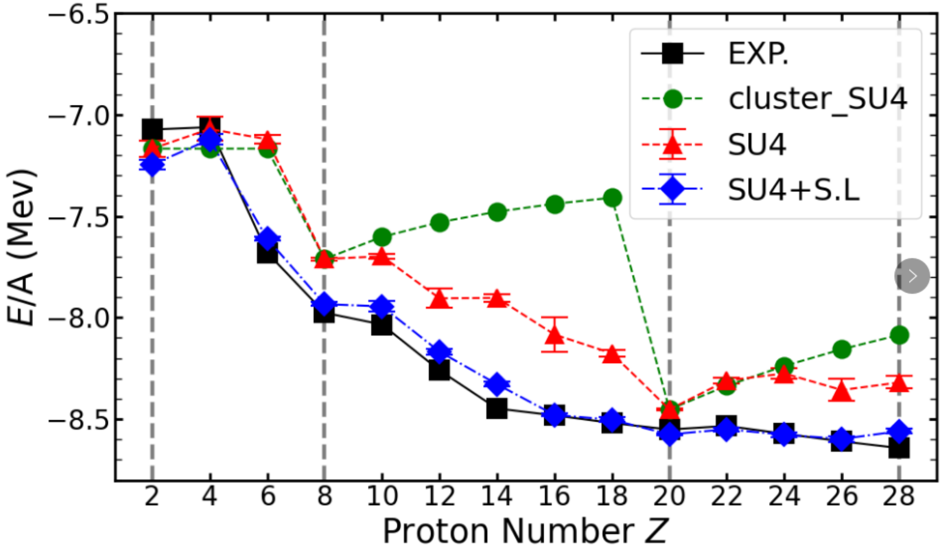
Nuclear binding energies with spin-orbit term (preliminary)



- Average error for **76** even-even nuclei: **2.932 MeV**
  Applicable to light/medium mass nuclei
  Zhong-Wang Niu et al., in preparation

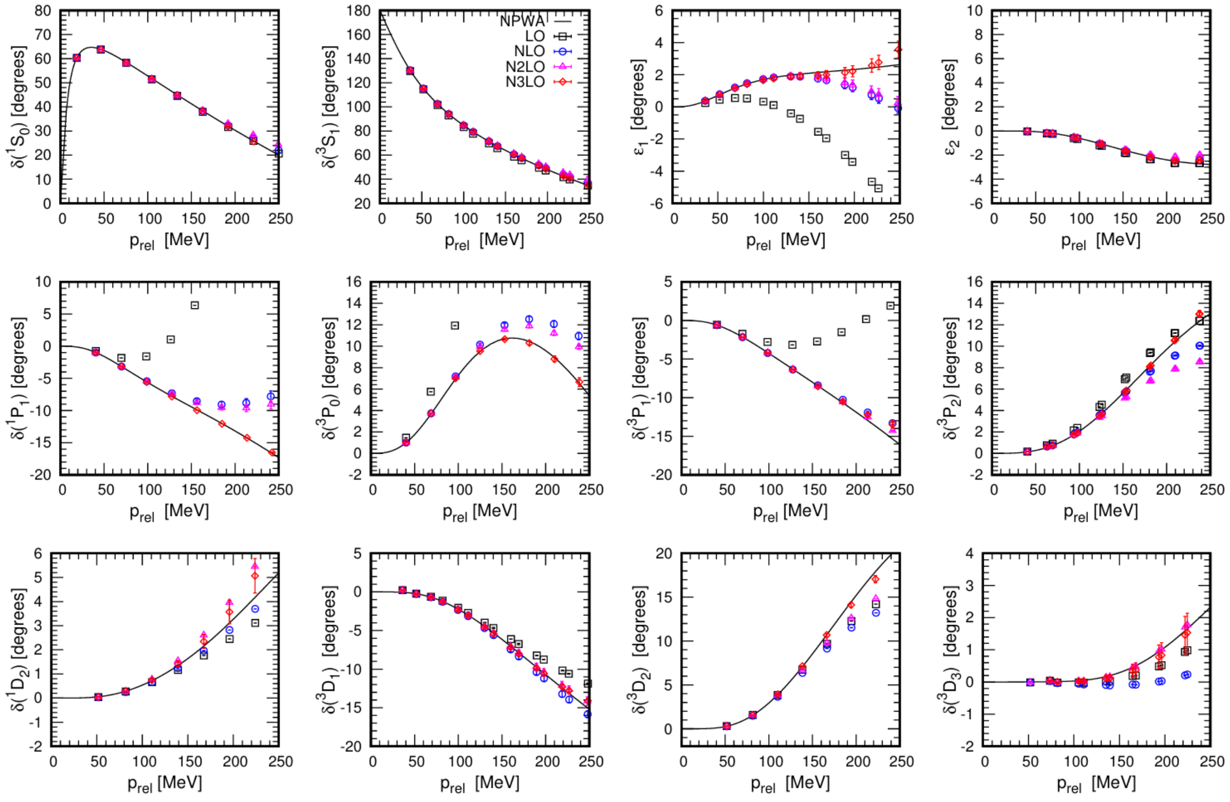
- Errors in other models
  - Relativistic mean field (PC-PK1): **2.258 MeV**
    Peng-Wei Zhao et al., PRC82, 054319 (2010)
  - Non-rel. mean field (UNDEF1): **3.380 MeV**
    Kortelainen et al., PRC 85, 024304 (2012).
  - Finite range droplet model: **1.142 MeV**
    P. Moller et al., Atom. Data Nucl. Data Tables 109, 1 (2016)

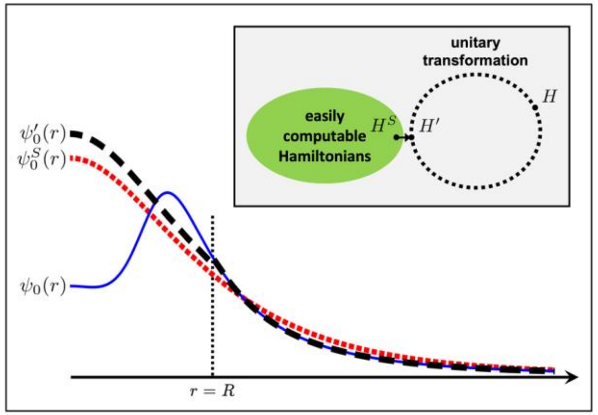
- **Spin-orbit term** is essential for **shell evolutions**.
  (proper SL term **do not induce sign problem**)
- SU(4) + SL Hamiltonian, **5 parameters** optimized with masses of ^{4}He, ^{16}O, ^{24}Mg, ^{28}Si, ^{40}Ca, etc.

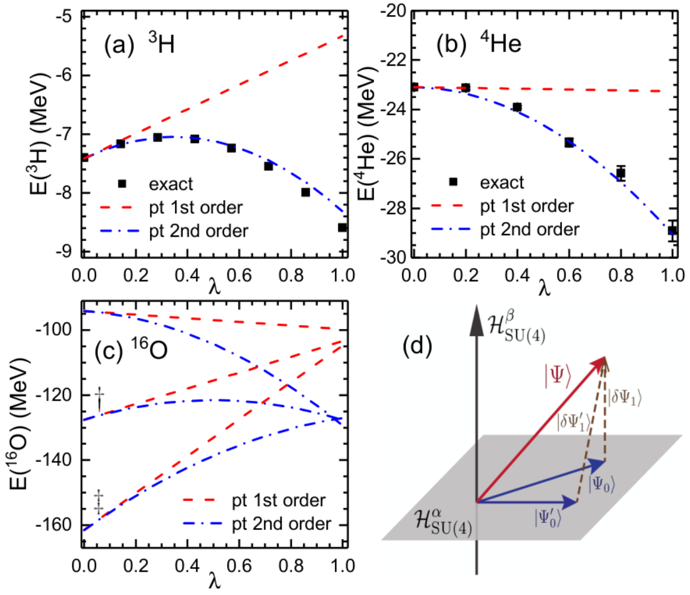


Precision lattice chiral nuclear forces

- High-precision fit to N-N scattering phase shifts at N^3LO
  Alarcon et al., EPJA 53, 83 (2017)
  Li et al., PRC 98, 044002 (2018)







Implementation in lattice calculations:

- Wave function matching method
  Elhatisari et al., Nature 630, 59 (2024)
- Perturbative quantum Monte Carlo method
  Lu et al., PRL 128, 242501 (2022)
- Rank-one operator method
  Ma et al., PRL 132, 232502 (2024)

Pinhole algorithm: Schematic

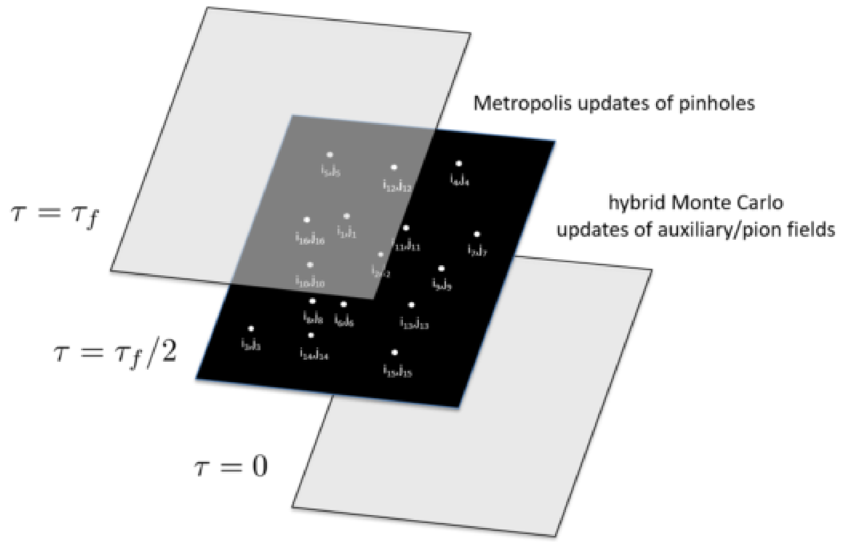
In terms of auxiliary fields, the amplitude Z can be written as a path-integral,

$$Z_{f,i}(i_1, j_1, \cdots, i_A, j_a; \boldsymbol{n}_!, \cdots \boldsymbol{n}_A; L_t)$$
$$= \int \mathscr{D}s\mathscr{D}\pi \langle \Psi_f(s,\pi)|\rho_{i_1,j_1,\cdots,i_A,j_A}(\boldsymbol{n}_1, \cdots, \boldsymbol{n}_A)|\Psi_i(s,\pi)\rangle.$$

We generate a combined probability distribution

$$P(s,\pi,i_1,j_1,\cdots,i_A,j_a;\boldsymbol{n}_!,\cdots \boldsymbol{n}_A) = |\langle \Psi_f(s,\pi)|\rho_{i_1,j_1,\cdots,i_A,j_A}(\boldsymbol{n}_1,\cdots,\boldsymbol{n}_A)|\Psi_i(s,\pi)\rangle|$$

by updating both the auxiliary fields and the pinhole quantum numbers.

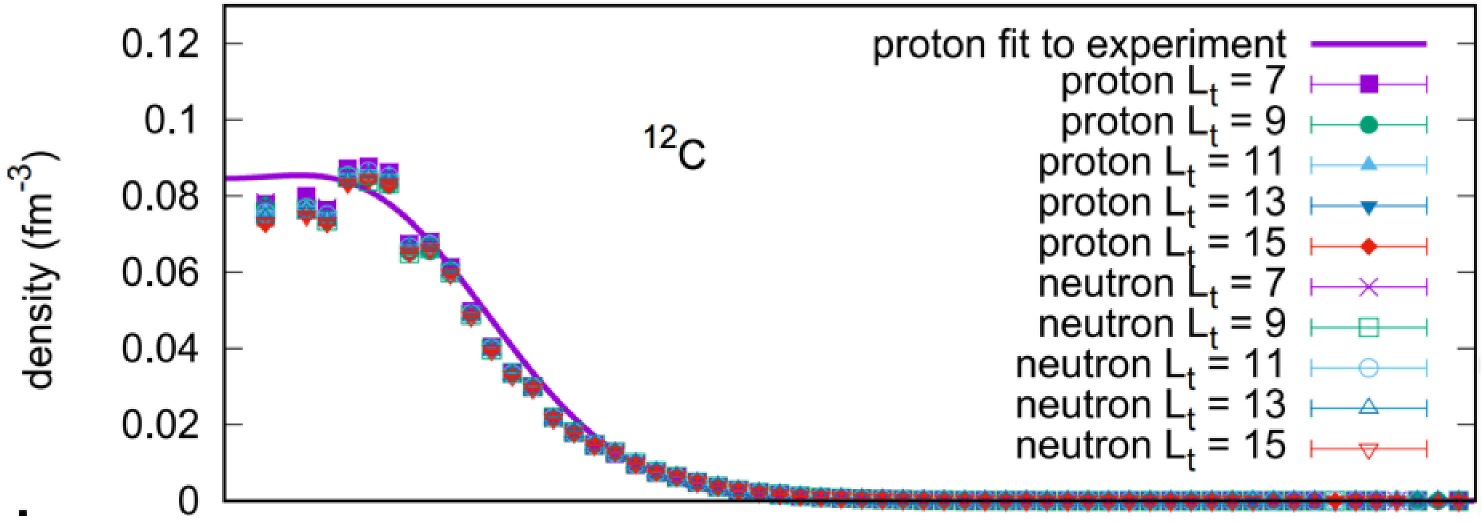


Pinhole algorithm: Intrinsic density distributions

- Densities relative to the center of mass:

$$\rho_{\mathrm{c.m.}}(r) = \sum_{\boldsymbol{n}_1,\cdots,\boldsymbol{n}_A} |\Phi(\boldsymbol{n}_1,\cdots\boldsymbol{n}_A)|^2 \sum_{i=1}^{A} \delta(r - |\boldsymbol{r}_i - R_{\mathrm{c.m.}}|)$$

- First LEFT calculation of **nuclear intrinsic densities**.
- **Proton radius** is included by **numerical convolution** $\rho(\boldsymbol{r}) = \int \rho_{\mathrm{Point}}(\boldsymbol{r}')e^{-(\boldsymbol{r}-\boldsymbol{r}')/(2a^2)}d^3r'$, proton radius $a \approx 0.84$ fm.

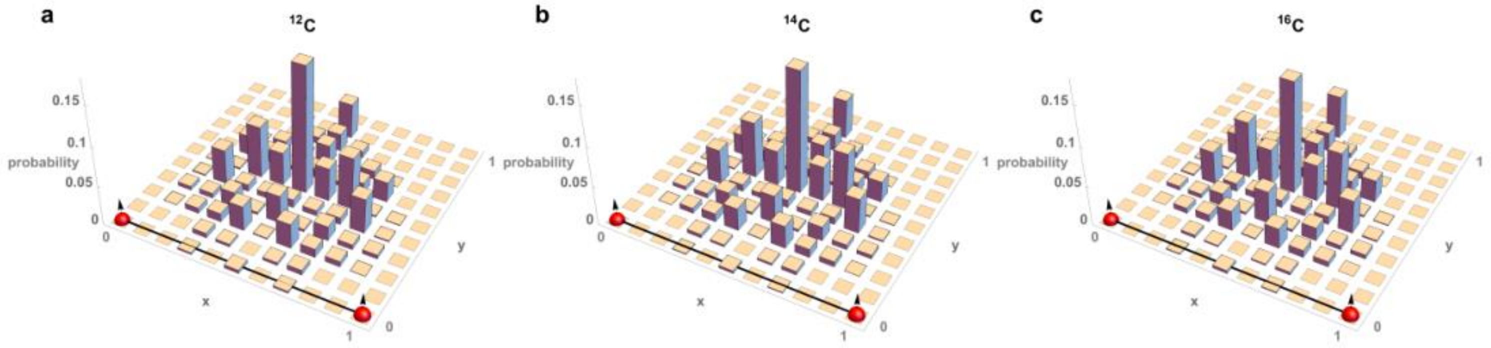


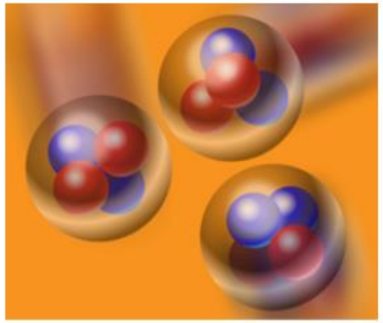
- Independent of projection time $L_t \Longleftrightarrow$ In ground state
- Sign problem suppressed $\longrightarrow$ Small errorbars

Elhatisari et al., Phys. Rev. Lett. 119, 222505 (2017)

Alpha clustering in nucleus

Positions of 3rd proton relative to the other two in $^{12,14,16}C$





- **Hoyle state**: Triple-α resonance, essential for creating ^{12}C in stars (Hoyle, 1954). *Fine-tuning for life?*
  Epelbaum et al., Phys. Rev. Lett. 106, 192501 (2011)
- **Perspective:** important many-body correlations, understand **internal structures** of ground and excited states by ***ab initio* calculations**.
- **Next step:** high-precision chiral interaction ⟶ EM form factors, shape coexistence, clustering, ...
  Elhatisari et al., Phys. Rev. Lett. 119, 222505 (2017)

Alpha clusters seem to be **fundamental unit** in several important scenarios.

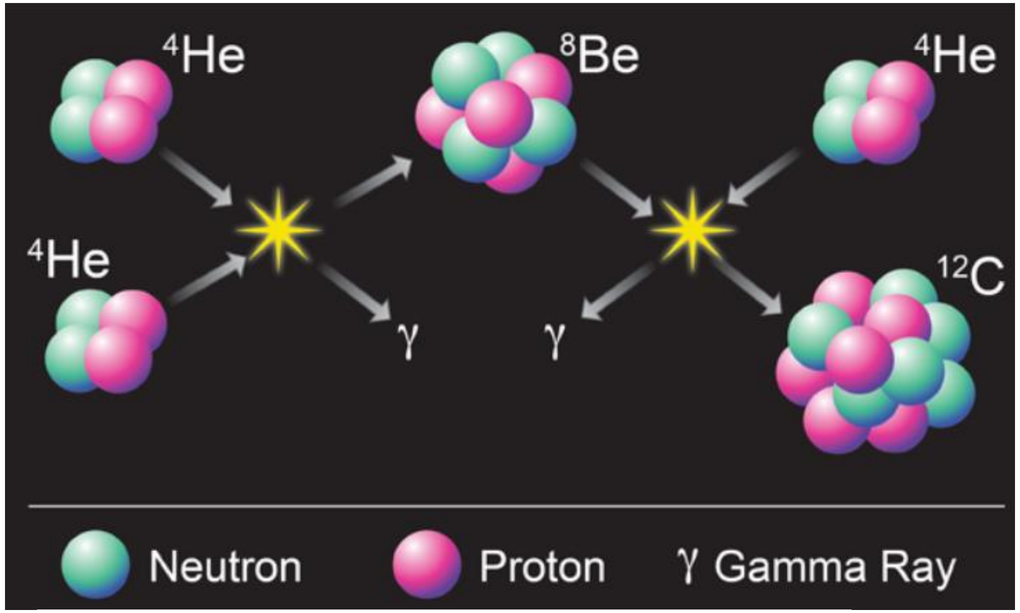
E.g., the 2^{nd} 0+ state of C12, or the **Hoyle state**, sensitively determines the abundance of Carbon in our universe.

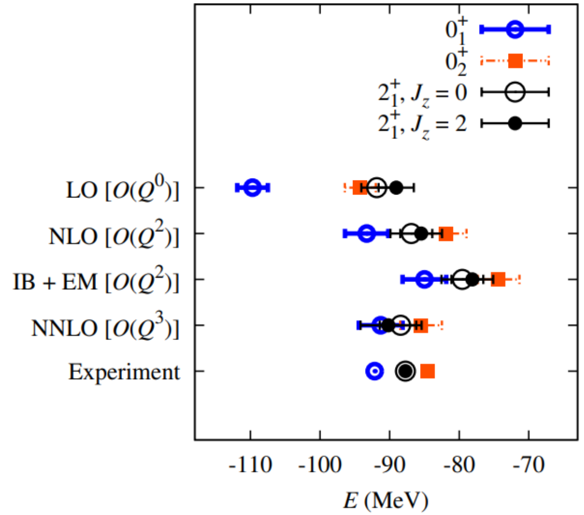
Many evidences suggest that the Hoyle state is a **clustering state**.

Can be explored from ab initio lattice EFT calculations.

Recent review for nuclear clustering:
M. Freer, H. Horiuchi, Y. Kanada-En'yo, Dean Lee, U.-G. Meißner, Rev. Mod. Phys. 90, 035004 (2018).

Alpha clustering in nucleus





Lattice EFT can capture the essential many-body correlations In clustering states, e.g, Hoyle state

Beyond-mean-field effects: Clustering states are difficult for mean-field calculations -> call for precision ab initio methods

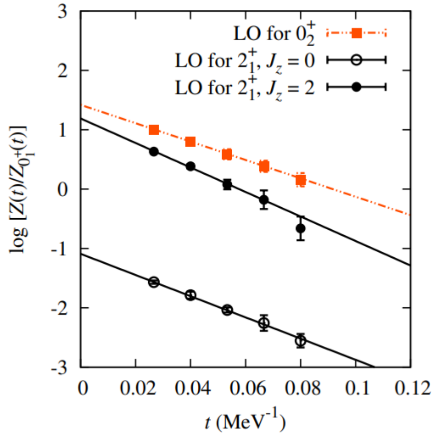


FIG. 1: Extraction of the excited states of ^{12}C from the time dependence of the projection amplitude at LO. The slope of the logarithm of $Z(t)/Z_{0_1^+}(t)$ at large t determines the energy relative to the ground state.

Ab initio calculation of the Hoyle state — 424 — 2011
E Epelbaum, H Krebs, D Lee, UG Meißner
Phys. Rev. Lett 106, 192501

Lattice simulations for few-and many-body systems — 327 — 2009
D Lee
Progress in Particle and Nuclear Physics 63 (1), 117-154

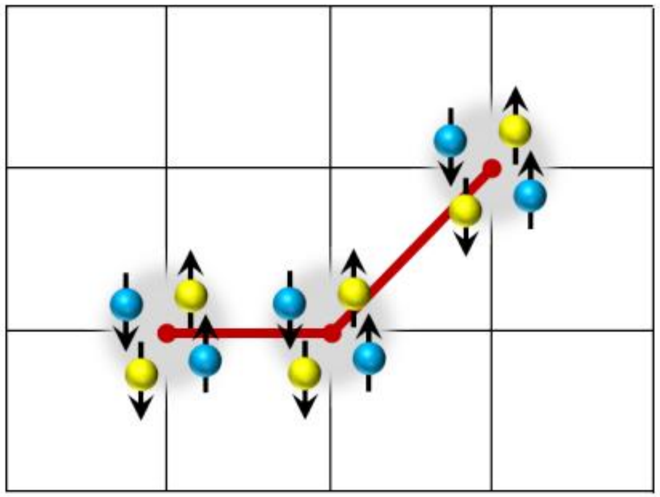
Structure and rotations of the Hoyle state — 312 — 2012
E Epelbaum, H Krebs, TA Lähde, D Lee, UG Meißner
Physical Review Letters 109 (25), 252501

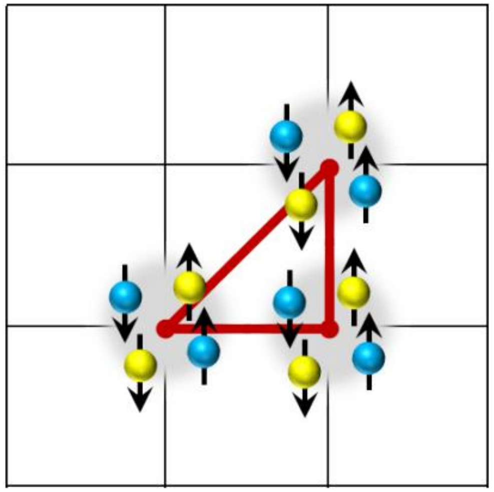
Microscopic clustering in light nuclei — 274 — 2018
M Freer, H Horiuchi, Y Kanada-En'yo, D Lee, UG Meißner
Review of Modern Physics 90, 035004

Ab initio alpha–alpha scattering — 171 — 2015
S Elhatisari, D Lee, G Rupak, E Epelbaum, H Krebs, TA Lähde, T Luu, ...
Nature 528 (7580), 111-114

Ab Initio Calculation of the Spectrum and Structure of O 16 — 168 — 2014
E Epelbaum, H Krebs, TA Lähde, D Lee, UG Meißner, G Rupak
Physical review letters 112 (10), 102501

Intrinsic structures of alpha clustering states

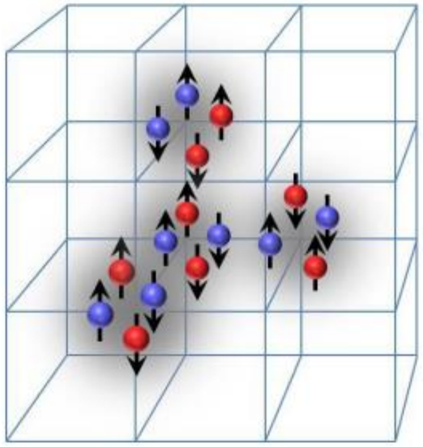




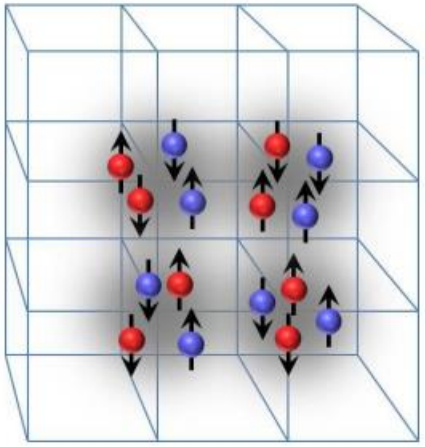
- Structure of clustering states can be inferred by preparing different initial state configurations.

- Educated guess for the shape of the state:
  Best guess has largest overlap with true eigenstate
  → Converges quickly

- Provides indirect evidences of the nuclear shapes



(a) Initial state “A”, 8 equivalent orientations.



(b) Initial states “B” and “C”, 3 equivalent orientations.

Structure and rotations of the Hoyle state,
Evgeny Epelbaum, Hermann Krebs, Timo A Lähde, Dean Lee, Ulf-G Meißner, PRL 109, 252501 (2012)

Ab Initio Calculation of the Spectrum and Structure of O 16
Evgeny Epelbaum, Hermann Krebs, Timo A Lähde, Dean Lee, Ulf-G Meißner, Gautam Rupak, PRL112, 102501 (2014)

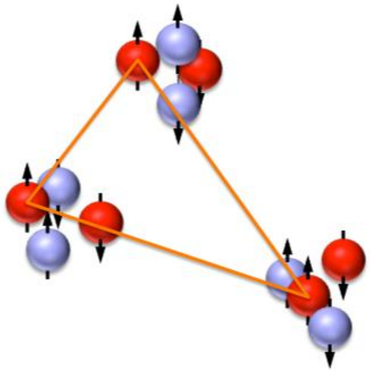
Clustering state tomography

- With **pinhole algorithm**, the clustering geometry can be extracted from the true ab initio eigenstates. E.g., relative configurations of alpha clusters in Carbon-12.

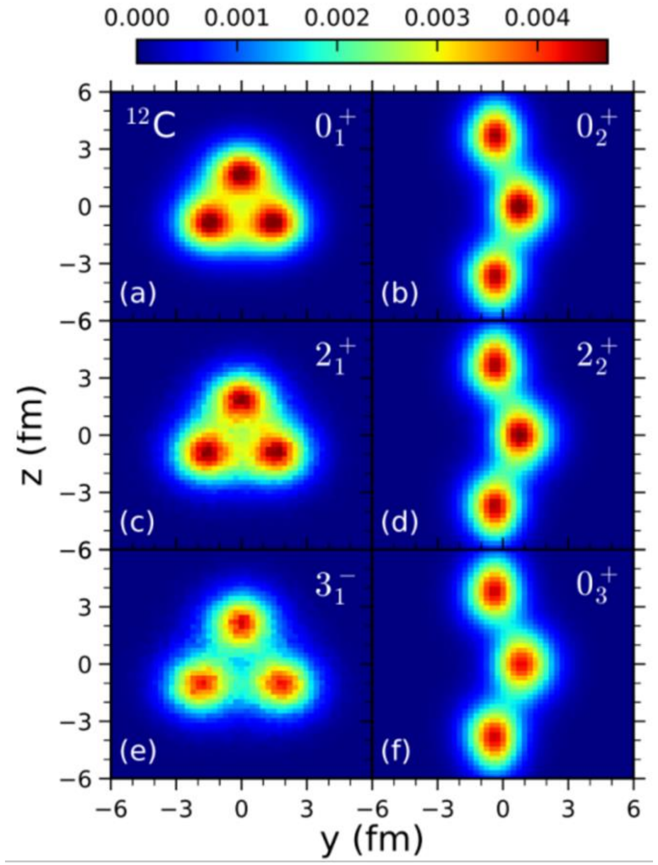
Tomography of nuclear clustering
Shen et al.,
Nat. Commun. 14, 2777 (2023)

We always align the longest edge with the x-axis and keep the triangle in the x-y plane.

$$\rho(d_1,d_2,d_3) = \sum_{j_1,j_2,j_3} \sum_{\mathbf{n}_1,\mathbf{n}_2,\mathbf{n}_3} |\Phi_{\uparrow j_1,\uparrow j_2,\uparrow j_3}(\mathbf{n}_1,\mathbf{n}_2,\mathbf{n}_3)|^2 \times \sum_{P(123)} \delta(|\mathbf{n}_1-\mathbf{n}_2|-d_3)\delta(|\mathbf{n}_1-\mathbf{n}_3|-d_2)\delta(|\mathbf{n}_2-\mathbf{n}_3|-d_1),$$

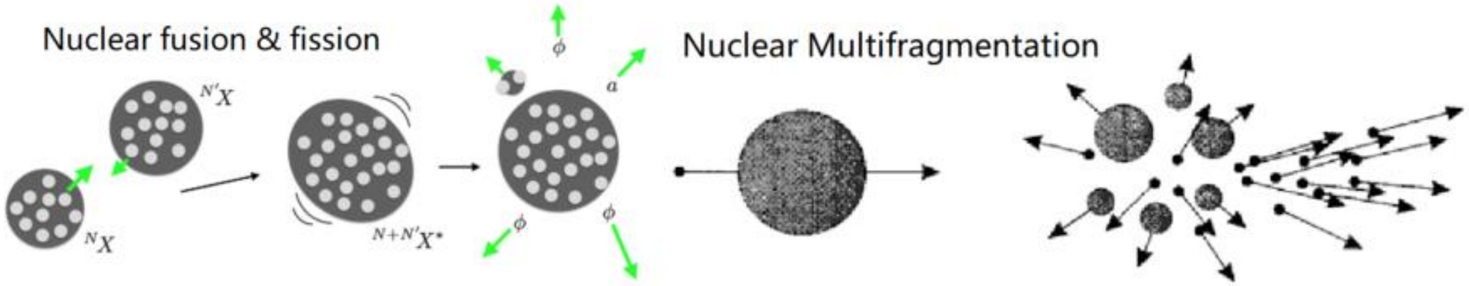


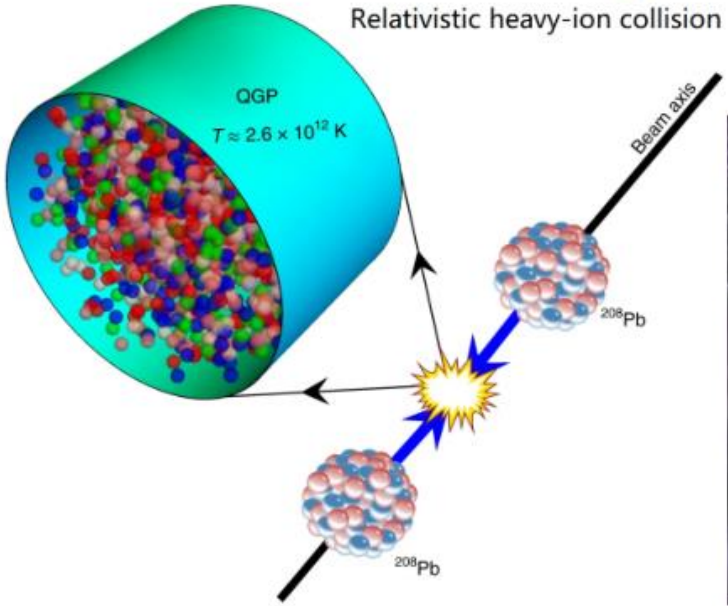
Three-body correlation in Carbon isotopes
Elhatisari et al., PRL 119, 222505 (2017)

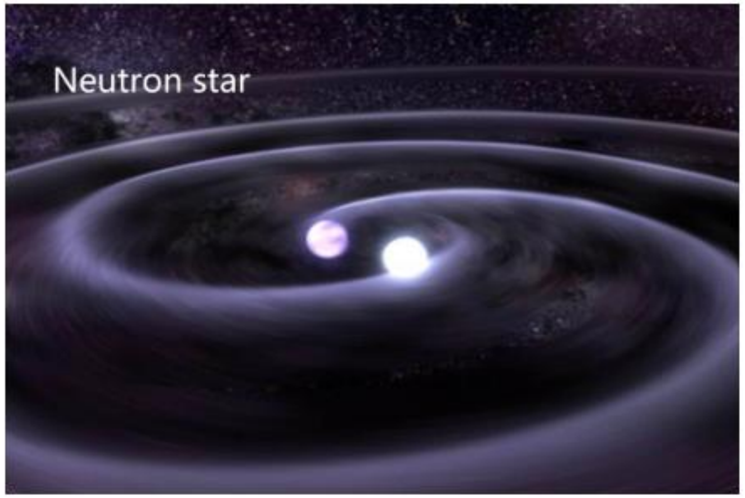


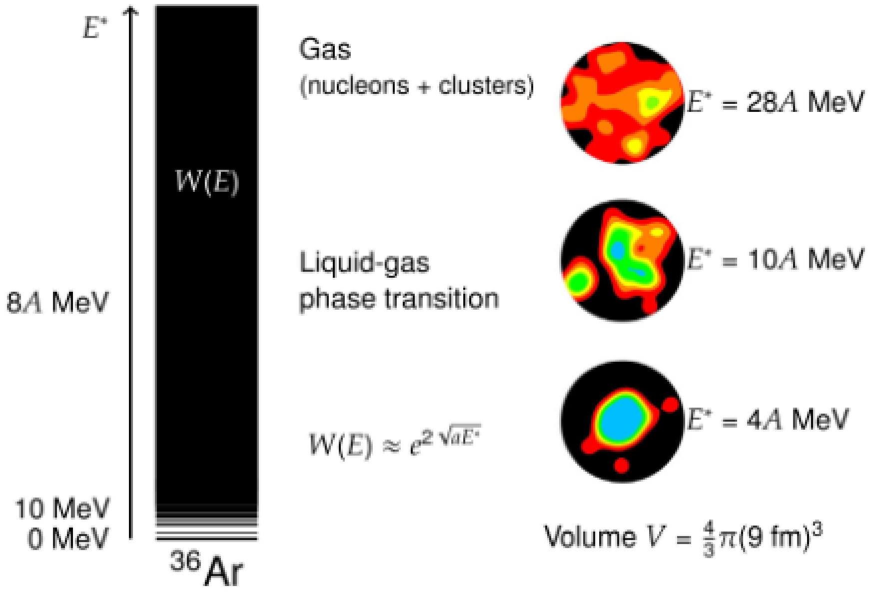
- Hoyle state is composed of a “bent-arm” or obtuse triangular arrangement of alpha clusters.
- The low-lying states of ^{12}C are either equilateral triangle or obtuse triangle.
- The states with the equilateral triangle formation also have a dual description in terms of particle-hole excitations in the mean-field picture.

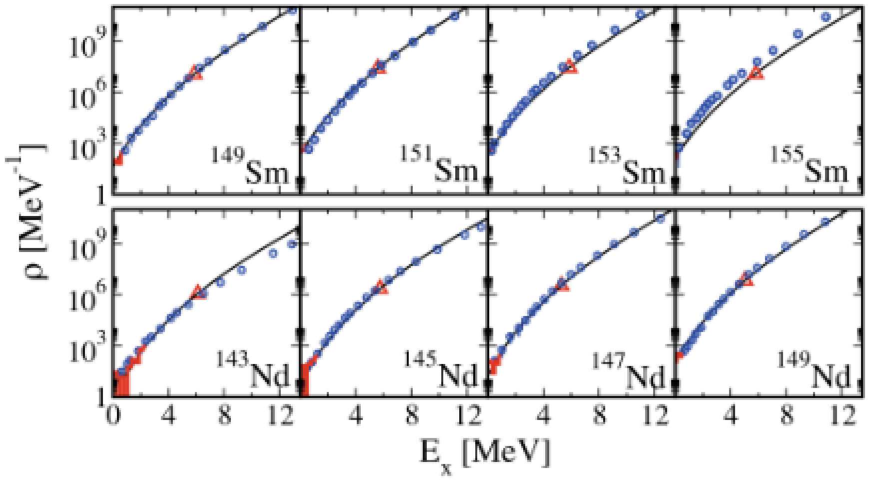
Nuclear thermodynamics



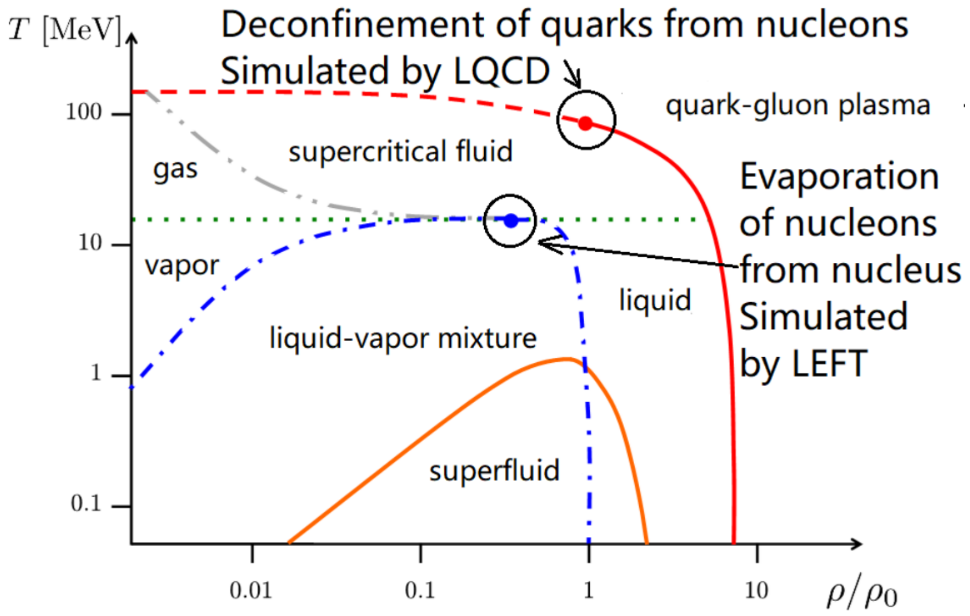


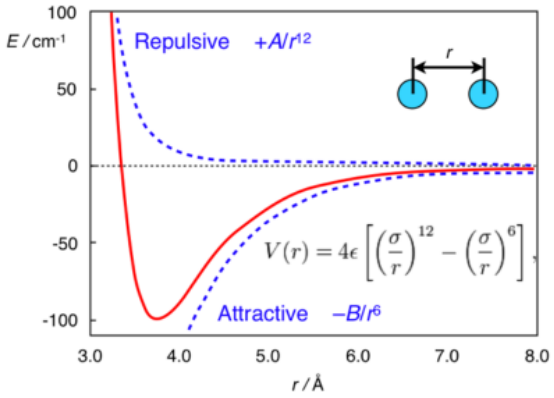


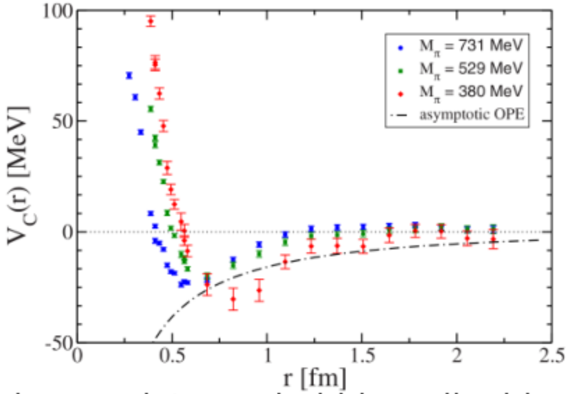


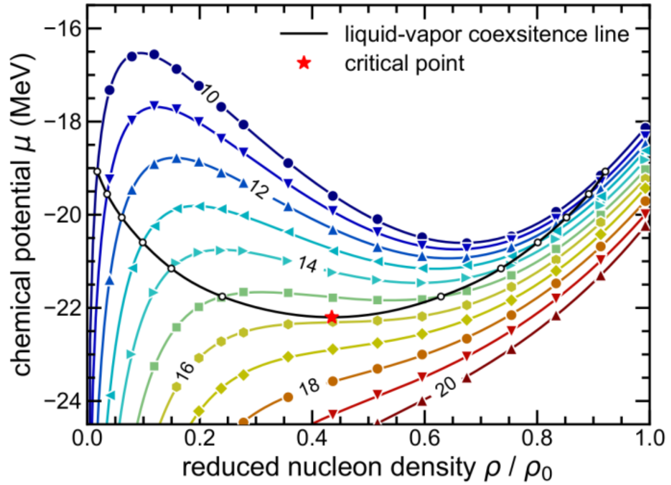


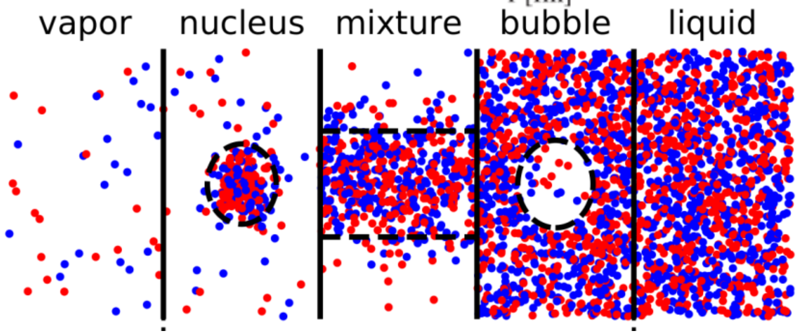
Nuclear thermodynamics from lattice EFT







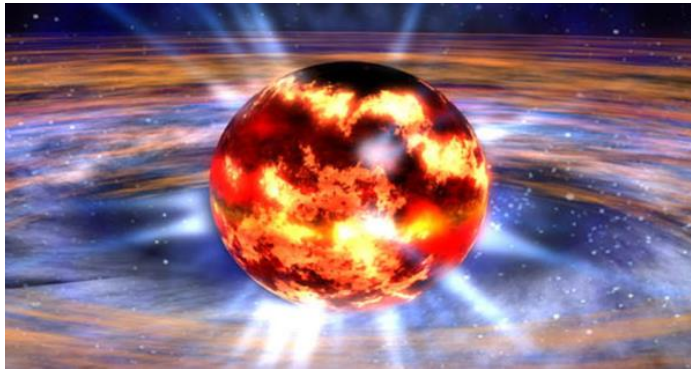




nuclear liquid v.s. normal liquid
nuclear force v.s. van der Waals force
nuclear liquid-gas phase transition
v.s. water liquid-gas phase transition

Ab initio calculation for liquid-gas phase transition in symmetric nuclear matter based on lattice EFT
Lu et al., PRL 125, 192502 (2020)

10~1000 times speed up with new Pinhole-trace algorithm

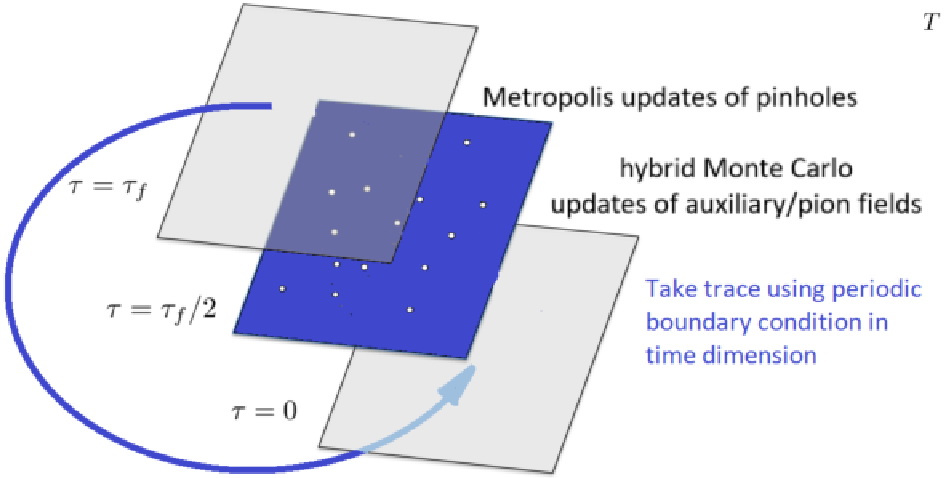


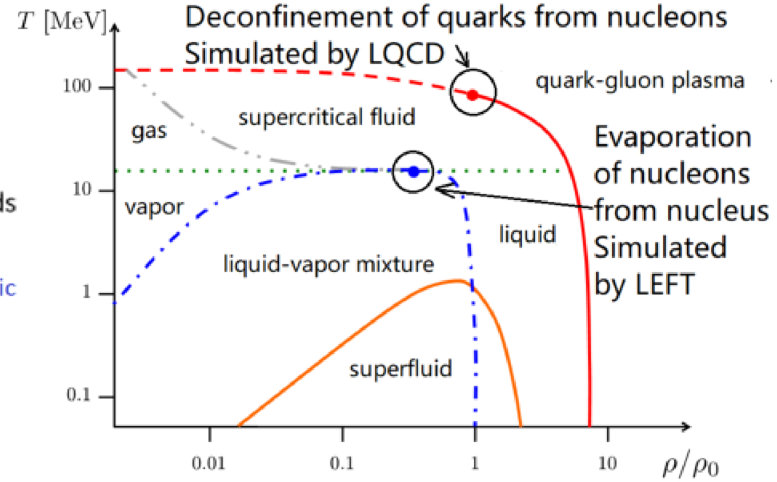
Simulate canonical ensemble with pinhole trace algorithm

- All we need: **partition function** $Z(T,V,A) = \sum_k \langle \exp(-\beta H) \rangle_k$, sum over all othonormal states in Hilbert space $\mathscr{H}(V,A)$.
- The basis states $|\boldsymbol{n}_1, \boldsymbol{n}_2, \cdots, \boldsymbol{n}_A\rangle$ span the whole A-body Hilbert space. $\boldsymbol{n}_i = (\boldsymbol{r}_i, s_i \sigma_i)$ consists of **coordinate, spin, isospin** of i-th nucleon.
- Cannonical partition function can be expressed in this complete basis:

$$Z_A = \mathrm{Tr}_A[\exp(-\beta H)] = \sum_{\boldsymbol{n}_1,\cdots,\boldsymbol{n}_A} \int \mathscr{D}s\mathscr{D}\pi \langle \boldsymbol{n}_1,\cdots,\boldsymbol{n}_A | \exp[-\beta H(s,\pi)] | \boldsymbol{n}_1,\cdots,\boldsymbol{n}_A \rangle$$

- **Pinhole algorithm** + **periodicity in β** = **Pinhole trace**
- Apply **twisted boundary condition** in 3 spatial dimensions to remove finite volume effects. Twist angle θ averaged with MC.



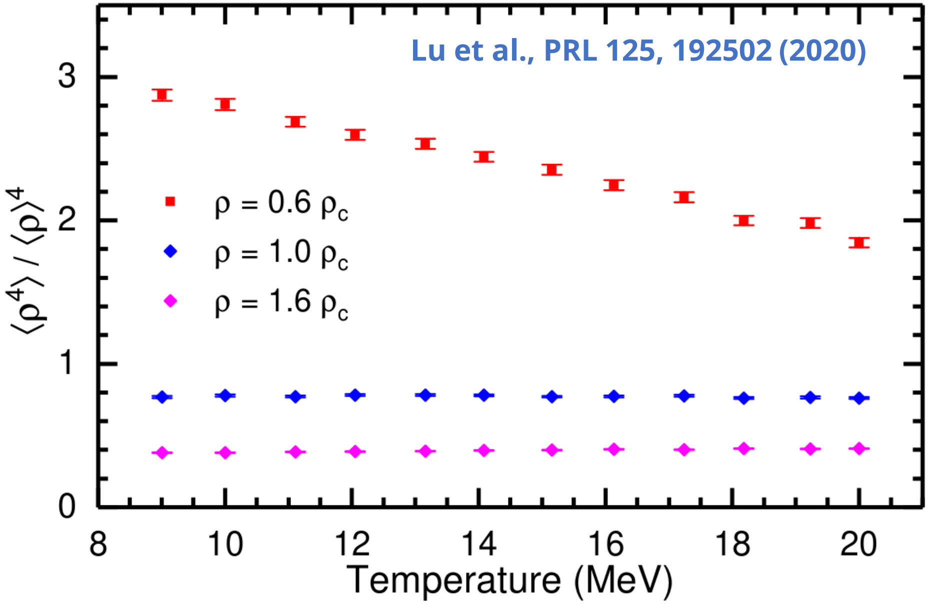


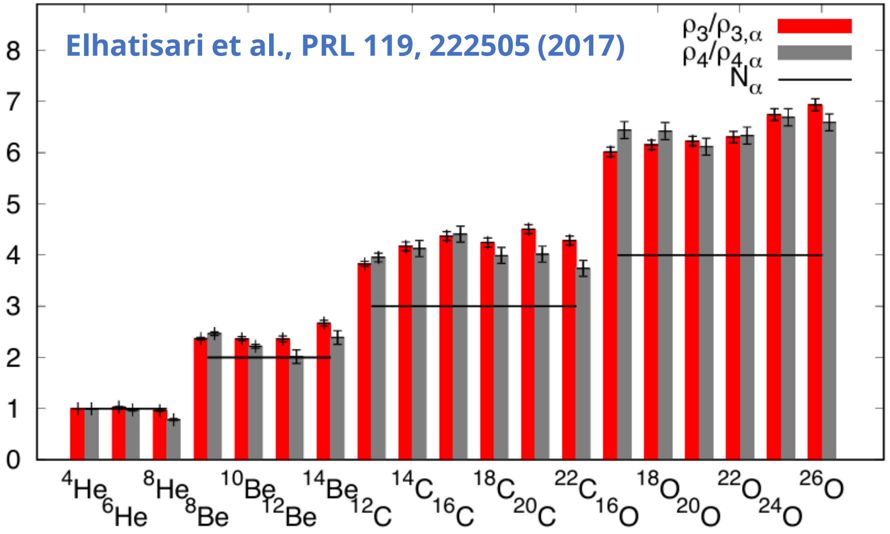
PRL 125, 192502 (2020)

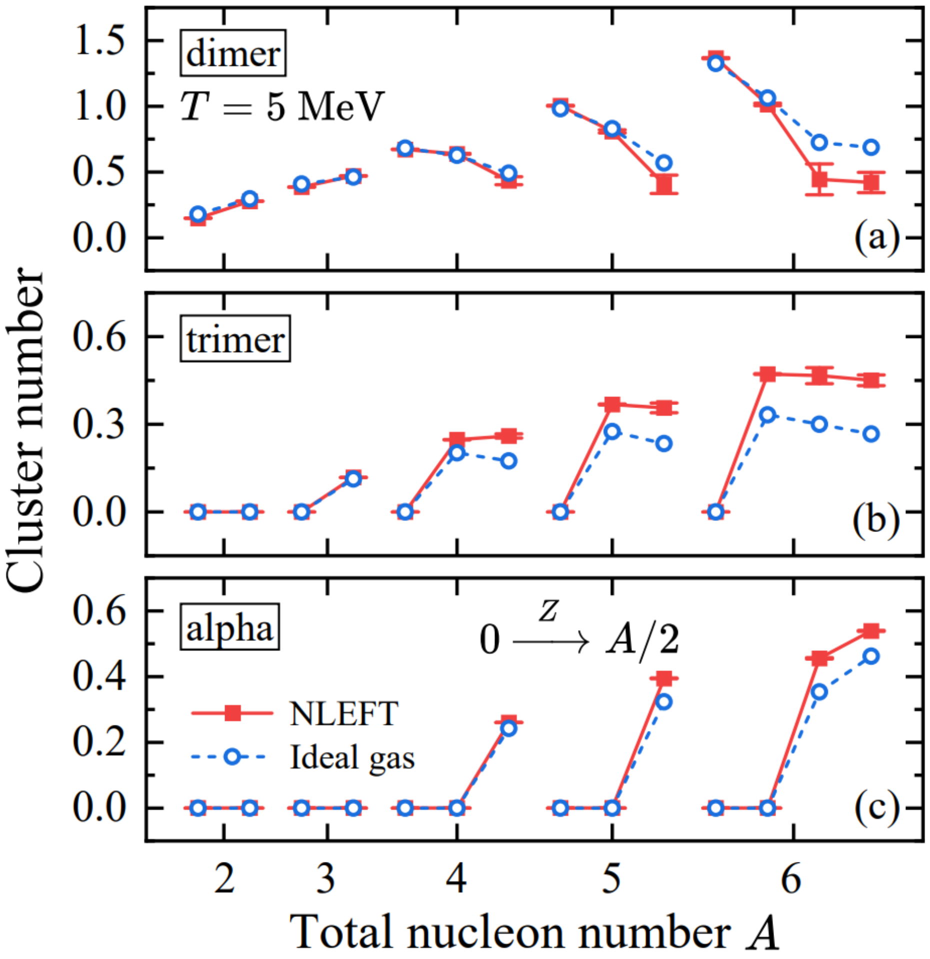
Clustering in ground states and hot nuclear matter

Many clustering indicator can be built from wave functions.
Works for ground states / excited states / finite temperature.

Monitoring how clustering evolves with N/Z, ρ, T, ⋯







$$G_{11}(n) = L^3 \sum_{\{\sigma_i \tau_i\}} : \rho_{\sigma_1\tau_1}(0)\rho_{\sigma_2\tau_2}(\boldsymbol{n}) :,$$

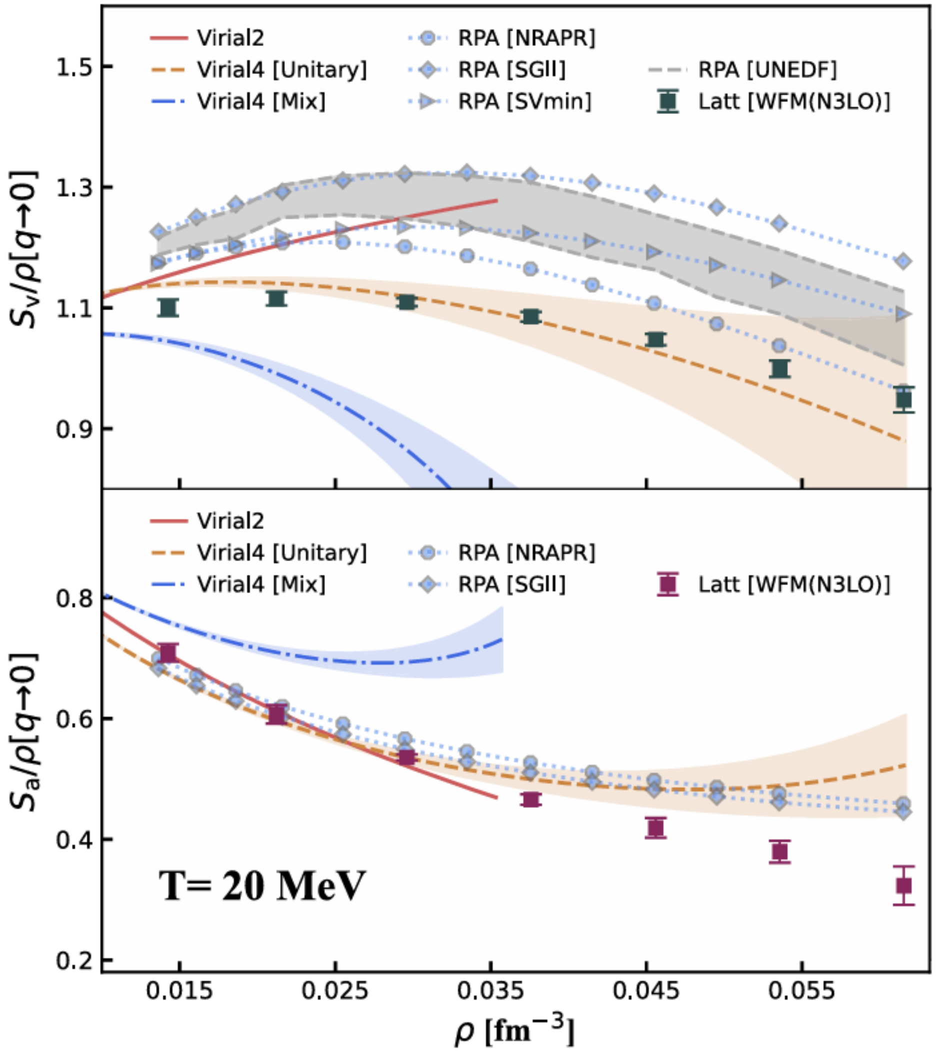
$$G_{21}(n) = L^3 \sum_{\{\sigma_i \tau_i\}} : \rho_{\sigma_1\tau_1}(0)\rho_{\sigma_2\tau_2}(0)\rho_{\sigma_3\tau_3}(\boldsymbol{n}) :,$$

$$G_{31}(n) = L^3 \sum_{\{\sigma_i \tau_i\}} : \rho_{\sigma_1\tau_1}(0)\rho_{\sigma_2\tau_2}(0)\rho_{\sigma_3\tau_3}(0)\rho_{\sigma_4\tau_4}(\boldsymbol{n}) :,$$

$$G_{22}(n) = L^3 \sum_{\{\sigma_i \tau_i\}} : \rho_{\sigma_1\tau_1}(0)\rho_{\sigma_2\tau_2}(0)\rho_{\sigma_3\tau_3}(\boldsymbol{n})\rho_{\sigma_4\tau_4}(\boldsymbol{n}) : .$$

Ren et al., PLB 850, 138463 (2024)

Correlations in hot nuclear matter



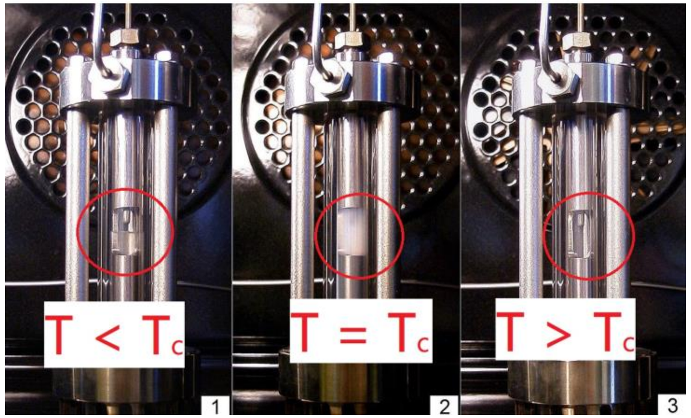
- Structure factors are Fourier transforms of correlation functions

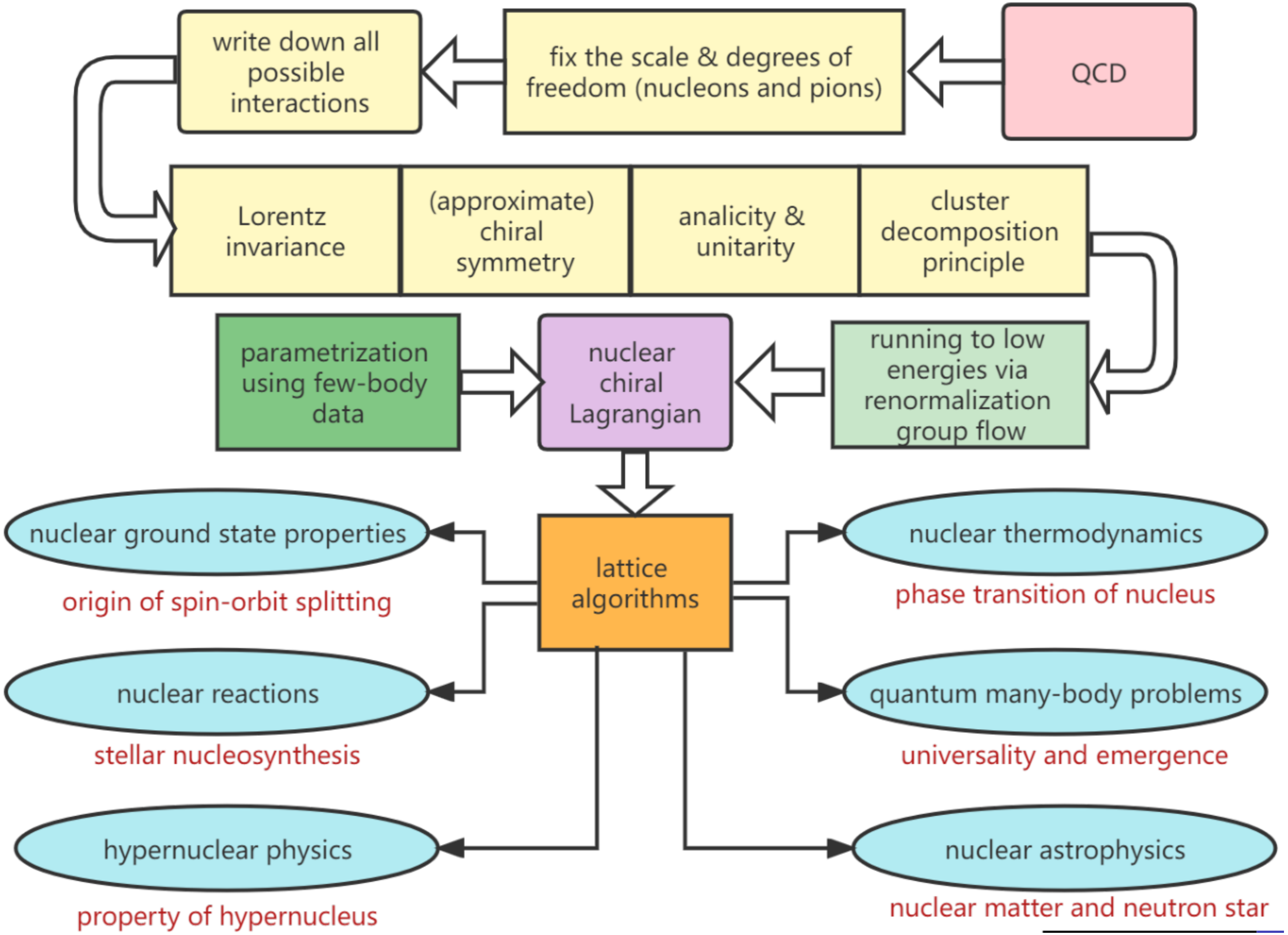
$$S_V(\boldsymbol{q}) = \int d^3r[\langle\hat{\rho}(\boldsymbol{r}+\boldsymbol{r}')\hat{\rho}(\boldsymbol{r}')\rangle - \rho_0^2]e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$$

$$S_a(\boldsymbol{q}) = \int d^3r[\hat{\rho}_z(\boldsymbol{r}+\boldsymbol{r}')\hat{\rho}_z(\boldsymbol{r}')\rangle - \rho_{z0}^2]e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$$

- Key for modeling Core-collapse supernovae explosions via neutrino-nucleon scattering
- Lattice EFT provides ab initio calculation with a N^{3}LO chiral interaction based on a rank-one operator method

  Ma et al., PRL 132, 232502 (2024)





write down all possible interactions
fix the scale & degrees of freedom (nucleons and pions)
QCD
Lorentz invariance
(approximate) chiral symmetry
analicity & unitarity
cluster decomposition principle
parametrization using few-body data
nuclear chiral Lagrangian
running to low energies via renormalization group flow
lattice algorithms
nuclear ground state properties
origin of spin-orbit splitting
nuclear reactions
stellar nucleosynthesis
hypernuclear physics
property of hypernucleus
nuclear thermodynamics
phase transition of nucleus
quantum many-body problems
universality and emergence
nuclear astrophysics
nuclear matter and neutron star

Summary and perspective

- Nuclear Lattice EFT is a quantum many-body problem solver designed for low-energy nuclear physics.
- The **wave functions for ground state / excited states** and **density matrices for finite temperature systems** can be solved exactly. **Collective correlations** can be extracted accordingly.
- Future with NLEFT:
  - More precision nuclear forces
  - Advanced quantum many-body algorithms
  - Ab initio nuclear spectroscopy
  - Ab initio nuclear thermodynamics
  - Ab initio calculation of electroweak observables
  - …

Summary and perspective

- Lattice EFT is a quantum many-body problem solver designed for low-energy nuclear physics.
- The **wave functions for ground state / excited states** and **density matrices for finite temperature systems** can be solved exactly. **Collective correlations** can be extracted accordingly.
- Future with NLEFT:
  - More precision nuclear forces
  - Advanced quantum many-body algorithms
  - Renormalization of effective field theory
  - Ab initio nuclear spectroscopy
  - Ab initio nuclear thermodynamics
  - Ab initio calculation of electroweak observables
  - …

Thank you for you attention!