Multi-Higgs Production at LHC and future Hadron Colliders

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Overview

Based on

C. Y. Chen, Q. S. Yan, X. Zhao, Y. M. Zhong and **ZZ**, [arXiv:1510.04013 [hep-ph]]
W. Kilian, S. Sun, Q. S. Yan, X. Zhao and **ZZ**, [arXiv:1702.03554 [hep-ph]]
W. Kilian, S. Sun, Q. S. Yan, X. Zhao and **ZZ**, [arXiv:1808.05534 [hep-ph]]
W. Kilian, S. Sun, Q. S. Yan, X. Zhao and **ZZ**, [arXiv:2011.xxxxx [hep-ph]]

Motivation

Higgs Lagrangian in SM

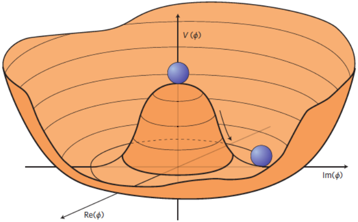
$$\mathcal{L}_H = (D^\mu H)^\dagger (D_\mu H) - V(H^\dagger H)$$

Higgs potential

$$V(H^\dagger H) = -\mu^2 (H^\dagger H) + \frac{\lambda}{4}(H^\dagger H)^2$$

Higgs self-interaction

$V_{\text{self}} = \frac{\lambda}{4} v h^3 + \frac{\lambda}{16} h^4$, where $\lambda = \frac{2m_h^2}{v^2}$



Motivation

After the discovery of Higgs boson, further researches are needed to answer following questions

- Are the properties of Higgs boson agreed with the SM prediction?
- Is it composite or fundamental?
- What is the shape of Higgs potential?
- How the EW phase transitions occur at the early universe?
- How to answer the baryogensis and CP violation problem?
- How New Physics contributes to the Higgs sector?
- ...

The measurements of multi-Higgs final state are important to answer these questions.

Why future collider?

We compare the cross sections of mutli-Higgs processes at 14 TeV LHC and a 100 TeV collider:

Process	$\sigma(14\ \text{TeV})$ (fb)	err.[th]	err.[exp]
$gg \to h$	4.968×10^4	$^{+7.5\%}_{-9.0\%}$	$\pm 1\%$
$gg \to hh$	45.05	$^{+7.3\%}_{-8.4\%}$	< 120 fb
$gg \to hhh$	0.0892	$^{+8.0\%}_{-6.8\%}$	$-$
	$\sigma(100\ \text{TeV})$ (fb)	err.[th]	err. [exp]
$gg \to h$	8.02×10^5	$^{+7.5\%}_{-9.0\%}$	$\pm 0.1\%$
$gg \to hh$	1749	$^{+5.7\%}_{-6.6\%}$	$\pm 5\%$
$gg \to hhh$	4.82	$^{+4.1\%}_{-3.7\%}$	< 30 fb

If we can reduce the background and the integrated luminosity is high enough, it is possible to observe $gg \to hhh$ at a 100 TeV machine.

Why quartic coupling?

Consider a simple model:

$$V(\phi_0, S) = \lambda\left(\phi_0^2 - \frac{v^2}{2}\right)^2 + \frac{a_1}{2}\left(\phi_0^2 - \frac{v^2}{2}\right) S + \frac{a_2}{2}\left(\phi_0^2 - \frac{v^2}{2}\right) S^2 + \frac{1}{4}\left(2b_2 + a_2 v^2\right) S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4,$$

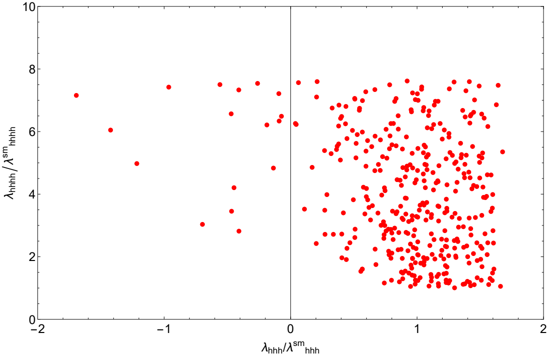
	B1	B2	B3
m_H (GeV)	460	500	490
θ	0.354	0.354	0.354
a_2	3.29	3.48	3.43
b_3 (GeV)	-706	-612	-637
b_4	8.38	8.38	8.38

	B1	B2	B3
$\Gamma_{\rm tot}(H)$ (GeV)	5.6	7.5	7.0
$BR(H \to W^+W^-)$	0.57	0.56	0.57
$BR(H \to ZZ)$	0.27	0.27	0.27
$BR(H \to t\bar{t})$	0.15	0.16	0.16
$BR(H \to b\bar{b})$	3.4×10^{-4}	2.8×10^{-4}	2.9×10^{-4}
$BR(H \to hh)$	5.3×10^{-7}	8.8×10^{-7}	1.5×10^{-7}
$BR(H \to hhh)$	1.0×10^{-3}	1.4×10^{-3}	1.3×10^{-3}
$\sigma(gg \to h)$ @ 14 TeV (fb)	3.2×10^2	2.3×10^2	2.5×10^2
$\sigma(gg \to hhh)$ @ 14 TeV (fb)	0.70	0.69	0.71
$\sigma(gg \to h)$ @ 100 TeV (fb)	1.4×10^4	1.1×10^4	1.2×10^4
$\sigma(gg \to hhh)$ @ 100 TeV (fb)	37	38	39

[C. Y. Chen *et al.*, 2015]

Why quartic coupling?

Another example is the strongly first order phase transition at Minimal Left-Right Symmetry Model:



In preparation

The Effective Field Theory

$$
\begin{aligned}
\mathcal{L}_{EFT} &= \mathcal{L}_{\overline{SM}} + \mathcal{L}_t + \mathcal{L}_{ggh} + \mathcal{L}_{Vh} + \mathcal{L}_{VVh} + \mathcal{L}_h \\
\mathcal{L}_t &= -a_1 \frac{m_t}{v} \bar{t}t\, h - a_2 \frac{m_t}{2v^2} \bar{t}t\, h^2 - a_3 \frac{m_t}{6v^3} \bar{t}t\, h^3 \\
\mathcal{L}_{ggh} &= \frac{g_s^2}{48\pi^2} \left(c_1 \frac{h}{v} + c_2 \frac{h^2}{2v^2} \right) G^a_{\mu\nu} G^{a\,\mu\nu} \\
\mathcal{L}_{Vh} &= g_{W,a1} \frac{2m_W^2}{v} h W^{+,\mu} W^-_\mu + g_{W,a2} \frac{m_W^2}{v^2} h^2 W^\mu W_\mu + g_{W,a3} \frac{m_W^2}{3v^3} h^3 W^\mu W_\mu \\
&\quad + g_{Z,a1} \frac{m_Z^2}{v} h Z^\mu Z_\mu + g_{Z,a2} \frac{m_Z^2}{2v^2} h^2 Z^\mu Z_\mu + g_{Z,a3} \frac{m_Z^2}{6v^3} h^3 Z^\mu Z_\mu + \cdots \\
\mathcal{L}_{VVh} &= -\left(g_{W,b1} \frac{h}{v} + g_{W,b2} \frac{h^2}{2v^2} + g_{W,b3} \frac{h^3}{6v^3} + \cdots \right) W^+_{\mu\nu} W^{-\,\mu\nu} \\
&\quad -\left(g_{Z,b1} \frac{h}{2v} + g_{Z,b2} \frac{h^2}{4v^2} + g_{Z,b3} \frac{h^3}{12v^2} + \cdots \right) Z_{\mu\nu} Z^{\mu\nu} \\
\mathcal{L}_h &= -\lambda_3 \frac{m_h^2}{2v} h^3 - \frac{\kappa_5}{2v} h \partial^\mu h \partial_\mu h - \lambda_4 \frac{m_h^2}{8v^2} h^4 - \frac{\kappa_6}{4v^2} h^2 \partial^\mu h \partial_\mu h
\end{aligned}
$$

In SM, we have

$$
\begin{aligned}
&a_1 = g_{V,a1} = g_{V,a2} = \lambda_3 = \lambda_4 = 1 \\
&a_2 = a_3 = c_1 = c_2 = g_{V,a3} = g_{V,b1} = g_{V,b2} = g_{V,b3} = \kappa_5 = \kappa_6 = 0
\end{aligned}
$$

The Effective Field Theory

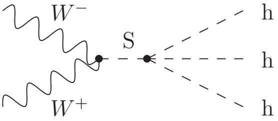
We implement a full Monte-Carlo simulation to study these operators

- **Generator:** WHIZARD/MadGraph5
- **PDF:** CTEQ6L1
- **Parton Shower:** PYTHIA8
- **Jet Reconstruction:** Fastjet
- **Detector simulation:** DELPHES

The Effective Field Theory

For the 5 points vertices, we introduced an auxiliary field S in WHIZARD with a Lagrangian

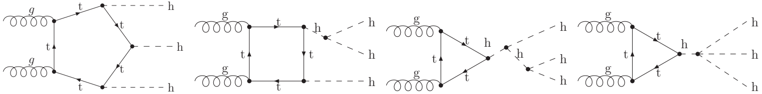
$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - g_{Shhh}(\partial^2 S)h^3 + g_{w,a3}\frac{2m_W^2}{v^3}SW^\mu W_\mu - \frac{g_{w,b3}}{v^3}SW^{\mu\nu}W_{\mu\nu}.$$



- Results are agreed with MadGraph5+UFO.
- In a new version of WHIZARD, this can be done by the UFO interface.

Multi-Higgs production via gluon-gluon fusion mode

- At a hadron collider, the dominant SM process of multi-Higgs production is gluon-gluon fusion (ggF) via a heavy top quark loop.



- This production mode involves many anomalous couplings.

	$gg \to h$	$gg \to hh$	$gg \to hhh$
Parameters involved	a_1, c_1	a_1, c_1	a_1, c_1
	-	$a_2, c_2, \lambda_3, \kappa_5$	$a_2, c_2, \lambda_3, \kappa_5$
	-	-	a_3, λ_4, κ_6

Multi-Higgs production via gluon-gluon fusion mode

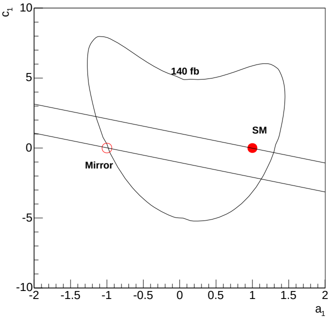
Decay channel	Branching ratio
$hhh \to b\bar{b}b\bar{b}W^+W^-$	22.34%
$hhh \to b\bar{b}b\bar{b}b\bar{b}$	20.30%
$hhh \to b\bar{b}W^+W^-W^+W^-$	8.20%
$hhh \to b\bar{b}b\bar{b}\tau^+\tau^-$	7.16%
$hhh \to b\bar{b}b\bar{b}gg$	6.54%
$hhh \to b\bar{b}b\bar{b}ZZ$	2.69%
$hhh \to W^+W^-W^+W^-W^+W^-$	1.00%
$hhh \to W^+W^-W^+W^-\tau^+\tau^-$	0.96%
$hhh \to W^+W^-W^+W^-gg$	0.88%
$hhh \to W^+W^-W^+W^-ZZ$	0.36%
$hhh \to b\bar{b}b\bar{b}\gamma\gamma$	0.29%

Background for $4b2\gamma$:
$b\bar{b}jj\gamma\gamma$, $ht\bar{t}$

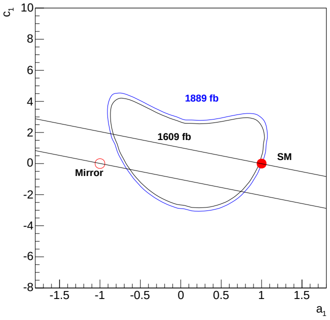
Background for $2b4W \to 2b4j2\ell^{\pm}$:
$ht\bar{t}$, $t\bar{t}W^+W^-$

The bounds on a_1 and c_1

The processes $gg \to h$ and $gg \to hh$ give a strong constraint on a_1 and c_1 (assuming 30 ab^{-1} luminosity for 100 TeV)

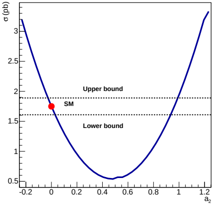


14 TeV

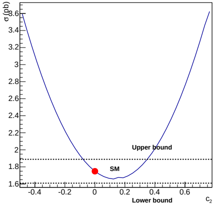


100 TeV

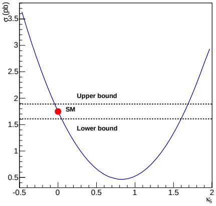
Bounds on a_2, λ_3 and κ_5 from $gg \to hh$ at a 100 TeV with $\mathcal{L} = 30$ ab^{-1}.



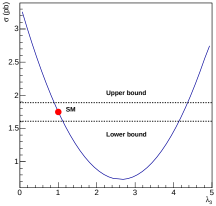
$a_2 \sim 10\%$



$c_2 \in [-0.1, 0.4]$



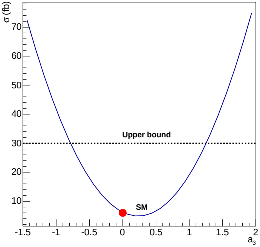
$\kappa_5 \sim 10\%$



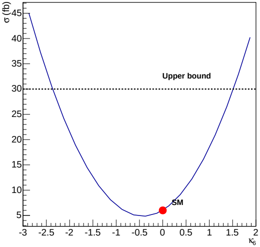
$\lambda_3 \sim 10\%$

The bounds from $gg \to hhh$

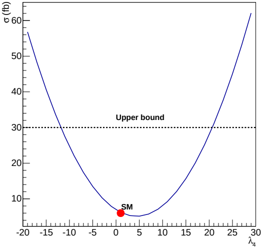
Bounds on a_3, λ_4 and κ_6 from $gg \to hhh$ at a 100 TeV with $\mathcal{L} = 30$ ab^{-1}.



$a_3 \in [-0.8, 1.3]$



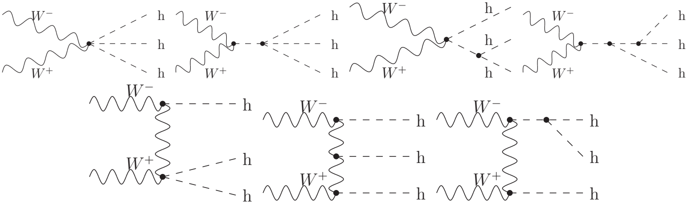
$\kappa_6 \in [-2.3, 1.5]$



$\lambda_4 \in [-13, 20]$

Multi-Higgs production via vector boson fusion mode

- The subdominant process of multi-Higgs production in hadron collisions is so-called vector-boson-fusion (VBF).



- This production mode involves different set of anomalous couplings.

	$VV \to h$	$VV \to hh$	$VV \to hhh$
Parameters involved	$g_{V,a1}$, $g_{V,b1}$	$g_{V,a1}$, $g_{V,b1}$	$g_{V,a1}$, $g_{V,b1}$
	-	$g_{V,a2}$, $g_{V,b2}$, λ_3, κ_5	$g_{V,a2}$, $g_{V,b2}$, λ_3, κ_5
	-	-	$g_{V,a3}$, $g_{V,b3}$, λ_4, κ_6

Constraints on parameters from the unitarity of S matrix

The scattering of observable particles is described by a $\mathcal{S}$ operator, which satisfies $\mathcal{S}^\dagger \mathcal{S} = 1$. Its nontrivial part is defined by $\mathcal{S} = 1 + i\mathcal{T}$, where the $\mathcal{T}$ satisfies the universal relation

$$-i(\mathcal{T} - \mathcal{T}^\dagger) = \mathcal{T}^\dagger \mathcal{T}$$

The matrix elements of the scattering amplitude operator $\mathcal{M}$ between the initial state $|\alpha, \Phi_a\rangle$ and the final state $|\beta, \Phi_b\rangle$ can be written as

$$\langle \beta, \Phi_b | \mathcal{T} | \alpha, \Phi_a \rangle = (2\pi)^4 \delta^{(4)}(p_a - p_b) \langle \beta, \Phi_b | \mathcal{M} | \alpha, \Phi_a \rangle.$$

We introduce a bijective mapping $\{x_a \in \mathbb{R}^{d_a};\ 0 < (x_a)_i < 1\}$ and a Jacobian $J_a(x_a) = \mathrm{d}\Phi_a / \mathrm{d}x_a$ and we have

$$M_{\beta\alpha}(x_b, x_a) = J_b^{1/2}(x_b) \langle \beta, \Phi_b(x_b) | \mathcal{M} | \alpha, \Phi_a(x_a) \rangle J_a^{1/2}(x_a)$$

Constraints on parameters from the unitarity of S matrix

The amplitudes can be expanded as

$$M^{\beta\alpha}(x_b, x_a) \quad = 2 \sum_{AB} a^{\alpha\beta}_{AB} H^{\alpha}_{A}(x_a)\, H^{\beta *}_{B}(x_b).$$

and the coefficients are

$$a^{\alpha\beta}_{AB} = \frac{1}{2} \int \mathrm{d}x_a \, \mathrm{d}x_b \, H^{\alpha *}_{A}(x_a)\, H^{\beta}_{B}(x_b)\, M^{\beta\alpha}(x_b, x_a),$$

where $\{H^{\alpha}_{A}(x_a)\}$ is an orthonormal basis on each α phase space.
The unitarity condition gives following bounds

$$|\mathrm{Re}\, a^{\alpha\alpha}_{AA}|^2 \leq \frac{1}{4}, \ |\mathrm{Im}\, a^{\alpha\alpha}_{AA} - \frac{1}{2}|^2 \leq \frac{1}{4}$$

$$\sum_{C \neq A} |a^{\alpha\alpha}_{AC}|^2 \leq \frac{1}{4}, \ \sum_{\gamma \neq \alpha} \sum_{C} |a^{\alpha\gamma}_{AC}|^2 \leq \frac{1}{4}$$

Unitarity Constraints from $VV \to hh$

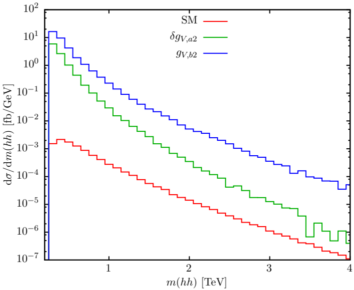
We define

$$b_A^{\alpha\gamma} = \sum_C |a_{AC}^{\alpha\gamma}|^2.$$

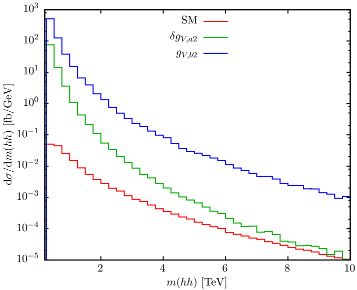
Finally, we obtain the strongest bounds on the EFT parameters:

$$\begin{aligned}
b_0(00) &= \frac{s^2}{2^9\pi^2 v^4}|g_{W,a2} - g_{W,a1}^2 + \frac{1}{2}\kappa_5 g_{W,a1}|^2 \leq \frac{1}{4} \\
b_0(++) &= \frac{s^2}{2^9\pi^2 v^4}|g_{W,b2} + 2g_{W,b1}^2 + \frac{1}{2}\kappa_5 g_{W,b1}|^2 \leq \frac{1}{4} \\
b_2(+-) &= \frac{s^2}{3\times 2^{10}\pi^2 v^4} g_{W,b1}^4 \leq \frac{1}{4}
\end{aligned}$$

Unitarity Constraints from $VV \to hh$



14 TeV



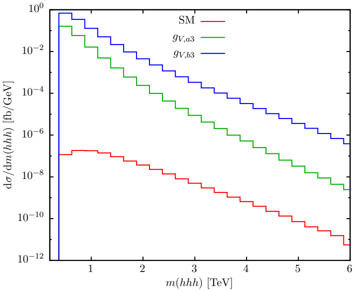
100 TeV

Unitarity Constraints from $VV \to hhh$

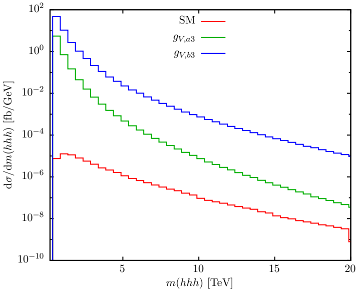
Similarly, we obtain the strongest bounds on the EFT parameters:

$$
\begin{aligned}
b_0(00) &= \frac{s^3}{3 \times 2^{14}\pi^4 v^6}|g_{W,a3} + \frac{1}{2}g_{W,a1}\kappa_6 + \frac{3}{2}g_{W,a2}\kappa_5 + g_{W,a1}\kappa_5^2 \\
&\quad -4g_{W,a1}g_{W,a2} + 4g_{W,a1}^3 - 2g_{W,a1}^2\kappa_5|^2 \leq \frac{1}{4} \\
b_0(++) &= \frac{s^3}{3 \times 2^{14}\pi^4 v^6}(|g_{W,b3} + \frac{1}{2}g_{W,b1}\kappa_6 + \frac{3}{2}g_{W,b2}\kappa_5 + g_{W,b1}\kappa_5^2 \\
&\quad +6g_{W,b1}g_{W,b2} + f_1 g_{W,b1}^3 - 3g_{W,b1}^2\kappa_5|^2 + f_2 g_{W,b1}^6) \leq \frac{1}{4} \\
b_2(+-) &= \frac{s^3}{3 \times 2^{14}\sqrt{6}\pi^4 v^6}\left|g_{W,b1}g_{W,b2} + 2g_{W,b1}^3 + \frac{1}{2}g_{W,b1}^2\kappa_5\right|^2 \leq \frac{1}{4}
\end{aligned}
$$

Unitarity Constraints from $VV \to hhh$



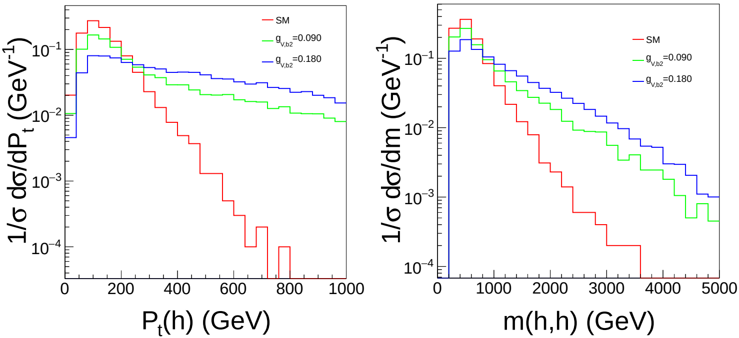
14 TeV



100 TeV

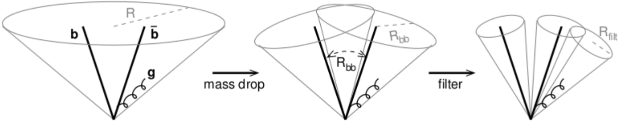
Highly boosted double Higgs production

Consider the VBF process $pp \to hhjj$@14 TeV



$$\frac{g_{W,b2}}{2v^2} h^2 W^+_{\mu\nu} W^{-\,\mu\nu},$$
$$\frac{g_{Z,b2}}{4v^2} h^2 Z_{\mu\nu} Z^{\mu\nu}$$

Mass-drop (MD) tagger [J. M. Butterworth *et al.*, 2008]

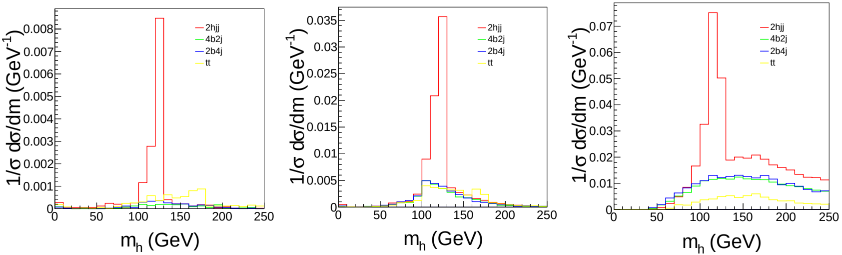


- Obtain hard jet j by some jet algorithm.
- Undo the last step of the splitting of jet j to get two subjets j_1 and j_2, and $m_{j1} > m_{j2}$;
- If they satisfy

$$m_{j1} < \mu m_j, \ \frac{\min(P_t(j_1), P_t(j_2))}{m_j^2} \Delta R^2_{j1,j2} > y_{cut}$$

  we can say that j is a heavy particle with substructure.
- Otherwise redefine j to be equal to j_1 and repeat.

We reconstruct the Higgs bosons in SM, and find



2-boosted 1-boosted 0-boosted

Analysis Results of 14 TeV LHC

Assuming $\mathcal{L} = 3000$ fb^{-1}, b-tagging efficiency $\epsilon_b = 0.7$, and miss tagging rate $\epsilon_{miss} = 0.001$

Process	$\sigma \times \mathcal{L}$	$n_b = 4$	VBF
SM signal	993	238	171
$pp \to 4b2j$	2.28×10^8	5.47×10^7	1.86×10^7
$pp \to 2b4j$	2.38×10^{10}	1.14×10^4	3.85×10^4
$pp \to t\bar{t} \to 2b4j$	7.89×10^8	387	58

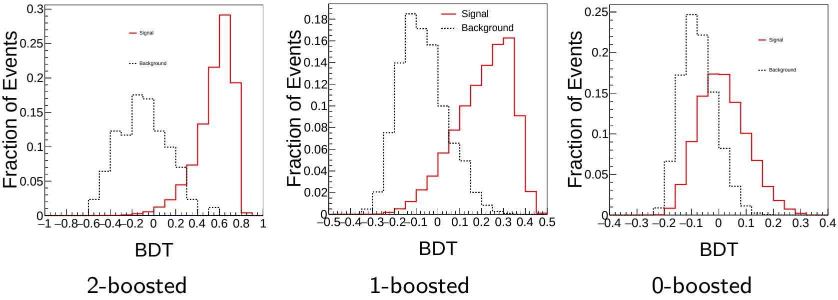
	2-boosted	1-boosted	0-boosted
SM Signal	4	21	146
$pp \to 4b2j$	1.17×10^5	1.56×10^6	1.69×10^7
$pp \to 2b4j$	28	349	3.81×10^4
$pp \to t\bar{t} \to 2b4j$	3	13	42

Analysis Results of 14 TeV LHC

Process	$\sigma \times \mathcal{L}$	$n_b = 4$	VBF
$g_{V,b2} = 0.18$ signal	5088	1222	694
$pp \to 4b2j$	2.28×10^8	5.47×10^7	1.86×10^7
$pp \to 2b4j$	2.38×10^{10}	1.14×10^4	3.85×10^4
$pp \to t\bar{t} \to 2b4j$	7.89×10^8	387	58

	2-boosted	1-boosted	0-boosted
$g_{V,b2} = 0.18$ Signal	184	235	275
$pp \to 4b2j$	1.17×10^5	1.56×10^6	1.69×10^7
$pp \to 2b4j$	28	349	3.81×10^4
$pp \to t\bar{t} \to 2b4j$	3	13	42

Analysis Results of 14 TeV LHC

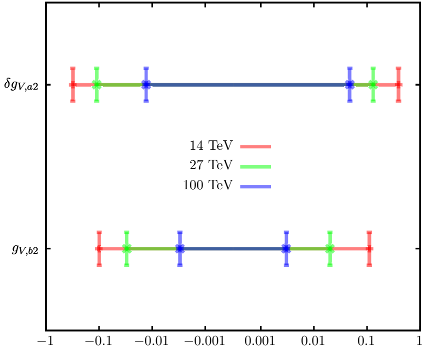


	2-boosted	1-boosted	0-boosted
$g_{V,b2} = 0.18$ Signal with BDT cut	90	49	31
Background with BDT cut	~ 0	~ 0	3.84×10^4
S/B	$-$	$-$	2.51×10^{-3}
$S/\sqrt{S+B}$	9.48	7.00	0.156

Bounds on coefficients

Repeating the analysis for coefficients, we can obtain the 5σ bounds on them:

	14 TeV (3 ab^{-1})	27 TeV (3 ab^{-1})	100 TeV (30 ab^{-1})
$\delta g_{V,a2}$	$(-0.31, 0.39)$	$(-0.11, 0.13)$	$(-0.013, 0.047)$
$g_{V,b2}$	$(-0.10, 0.11)$	$(-0.03, 0.02)$	$(-0.003, 0.003)$



Summary

- After the discovery of Higgs boson, many questions are still remained. Measurements of Higgs self-couplings are important to answer them.
- We use the EFT method to study triple and double Higgs productions at LHC and future hadron colliders.
- Full MC simulations are performed to obtain the bounds on these EFT parameters.
- The unitarity bound for VBF process is obtained.
- Tagging highly boosted Higgs boson is helpful to measure the Higgs self-couplings and search for new physics.

Thank You

Backup slides