12th International Workshop
on Charm Physics
12–16 MAY 2025
TSUNG-DAO LEE INSTITUTE, SHANG HAI

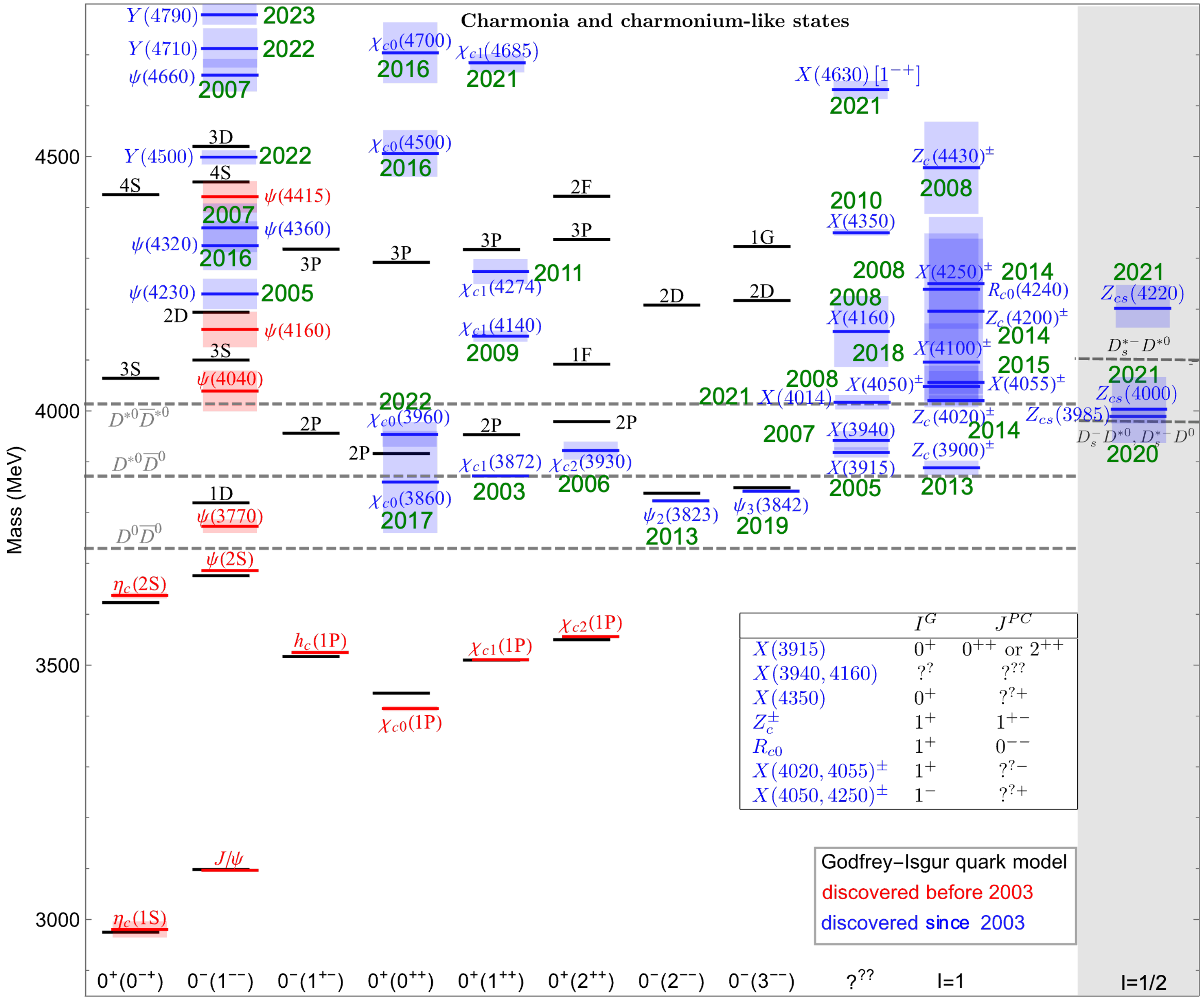
Precise determination of the properties of X(3872) and of its isovector partner

Feng-Kun Guo

Institute of Theoretical Physics, Chinese Academy of Sciences

Zhen-Hua Zhang, T. Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, A. Rusetsky, Predicting isovector charmonium-like states from X(3872) properties, JHEP 08 (2024) 130;
Jorgivan Dias, Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, Y. Zhang, Z.-H. Zhang, Dispersive analysis of the isospin breaking in the X(3872) → $J/\psi\pi^+\pi^-$ and X(3872) → $J/\psi\pi^+\pi^0\pi^-$ decays, Phys. Rev. D 111 (2025) 014031;
Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, Precise determination of the properties of X(3872) and of its isovector partner W_{c1}, arXiv:2502.04458

Charmonia and charmonium-like (XYZ) states



X(3872) and possible isospin-1 partners

- X(3872) has been discovered by Belle for more than 20 years, debates are still ongoing!
- Excellent observable for distinguishing models: Isospin-1 partners!
  - No, in charmonium model

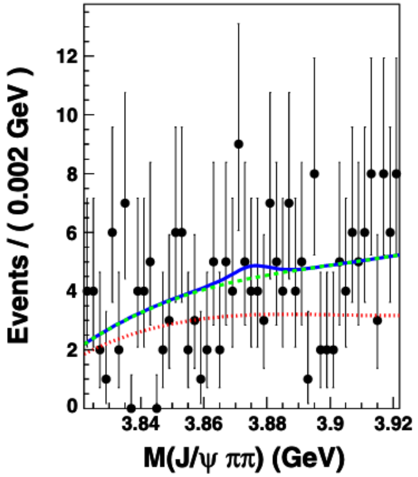
    L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2005) 014028
  - Quark bound states, in compact tetraquark model
    - With isospin-independent quark interactions, isoscalar and isovector tetraquarks must coexist

$$I = 1 \text{ multiplet: } [cu][\bar{c}\bar{d}],\ \frac{1}{\sqrt{2}}([cu][\bar{c}\bar{u}] - [cu][\bar{c}\bar{d}]),\ [cd][\bar{c}\bar{u}]$$

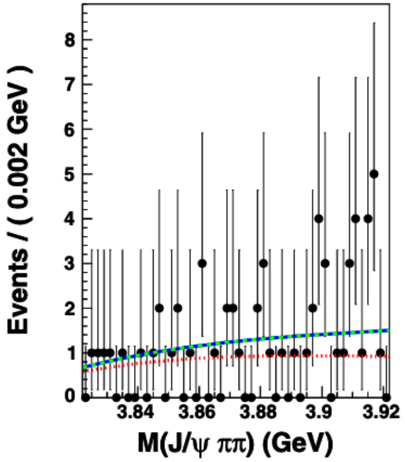
  - How about hadronic molecular picture?
    - Thought to be non-existing, but never carefully investigated
    - Will be shown to exist as virtual states in this talk

So far negative signal

- No signal in the charged channel so far



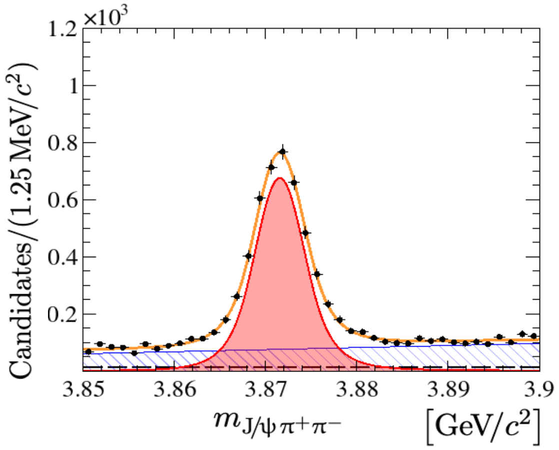
$\bar{B}^0 \to K^- \pi^+ \pi^0 J/\psi$



$B^+ \to K^0 \pi^+ \pi^0 J/\psi$

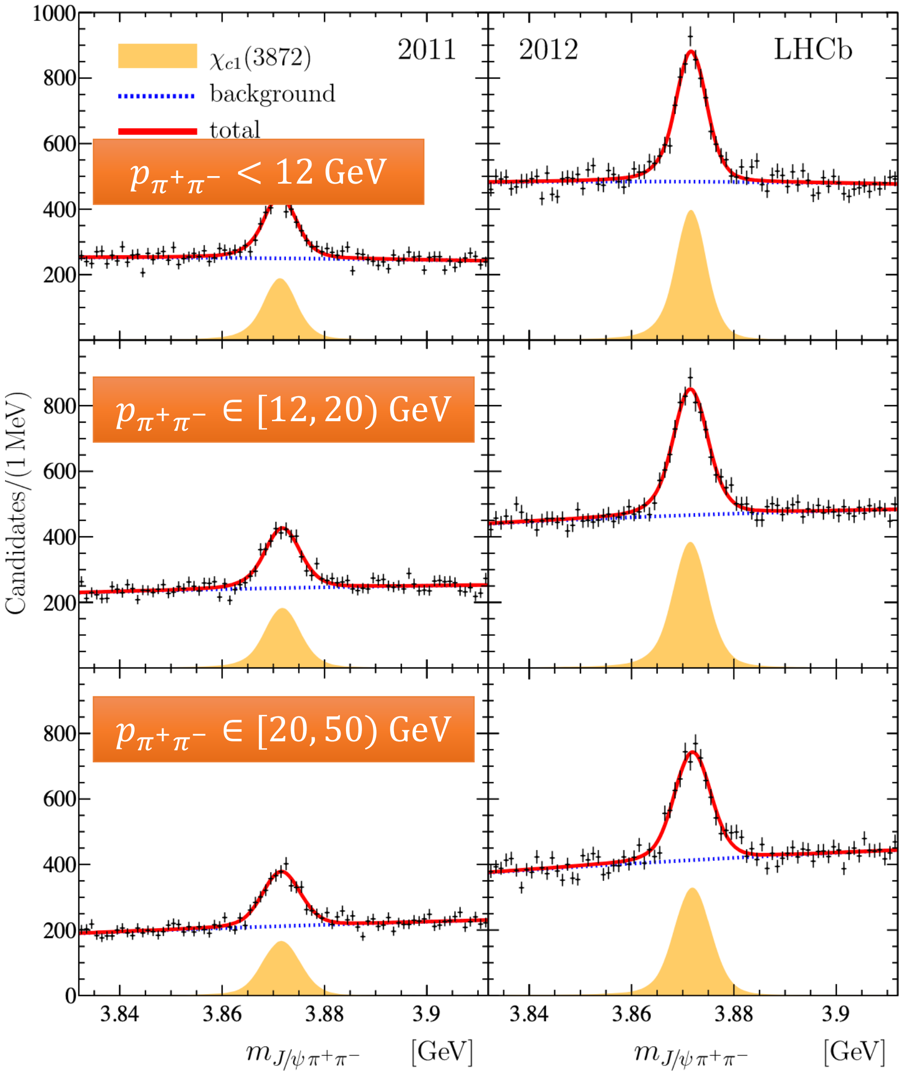
Belle, PRD 84 (2011) 052004

- No signal around the $D^+ D^{*-}$ threshold



$B^+ \to K^+ \pi^+ \pi^- J/\psi$

LHCb, JHEP 08 (2020) 123



$\pi^+ \pi^- J/\psi$ from b-hadrons

LHCb, PRD 102 (2020) 092005

$J^{PC} = 1^{++}$ sector

- Hadronic molecules: consider S-wave interactions between charm and anti-charm mesons
- For each isospin, only two low-energy constants (LECs) at LO in nonrelativistic expansion for S-wave interactions of 6 meson pairs
- For the $J^{PC} = 1^{++}$ sector, also two LECs at LO:
  - $I = 0$: C_{0X}; $I = 1$: C_{1X}
- Two inputs from $X(3872)$ properties :

> Mass
>
> $$M_X = 3871.69^{+0.00+0.05}_{-0.04-0.13}\text{MeV}$$ LHCb, PRD 102 (2020) 092005
>
> $$M_{D^0} + M_{D^{*0}} = 3871.69(7)\ \text{MeV}$$ PDG 2024
>
> Isospin breaking in decays LHCb, PRD 108 (2023) L011103
>
> $$R_X = \left|\frac{\mathcal{M}_{X(3872)\to J/\psi\rho^0}}{\mathcal{M}_{X(3872)\to J/\psi\omega}}\right| = 0.29 \pm 0.04$$

- Neutral systems X and W_{c1}^0: coupled channels
  - ✓ $(D\bar{D}^*)_0 \equiv (D^0\bar{D}^{*0} - \bar{D}^0 D^{*0})/\sqrt{2}$
  - ✓ $(D\bar{D}^*)_\pm \equiv (D^+ D^{*-} - D^- D^{*+})/\sqrt{2}$
- Charged systems $W_{c1}^\pm$: single channel

Extracted using BW for resonances;
Updated using Omnes representation for $\pi\pi$ P-wave
J. Dias et al., PRD 111 (2025) 014031

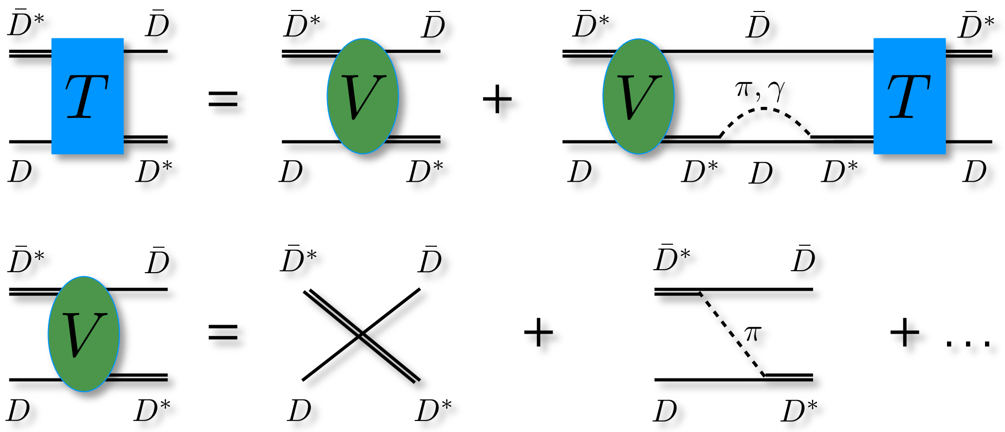
$$\mathcal{M}_{X\to J/\psi\pi\pi} = \mathcal{N}\varepsilon_{ijk}\varepsilon_\psi^i\varepsilon_X^j q_\pi^k \underbrace{P(s)\Omega(s)}_{\text{Omnes}}\left[1 + \underbrace{\kappa_X G_\omega(s)}_{\rho\omega\ \text{mixing}}\right]$$

Lippmann-Schwinger equation (LSE)

Z.-H. Zhang et al., JHEP 08 (2024) 130

- Coupled channels: $D^0\bar{D}^{*0}, D^+D^{*-}$ with $C=+$
- T matrix is given by the LSE:

$$T(E;p',p) = V(E;p',p) + \int \frac{l^2 dl}{2\pi^2} V(E;p',l)G(E;l)T(E;l;p)$$



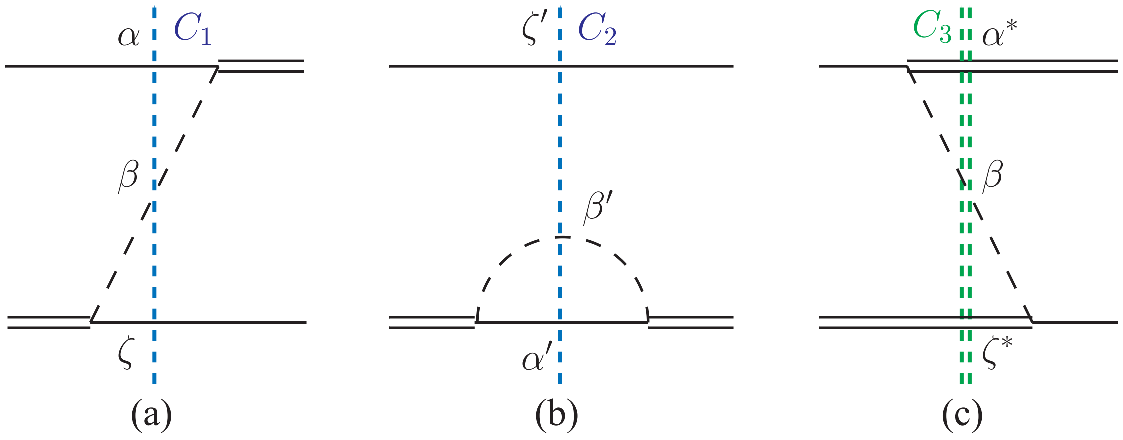
Potential: contact term (C_{0X}, C_{1X}) + one-pion exchange (OPE)

$$V_{\mathrm{ct}} = \frac{1}{2}\begin{pmatrix} C_{0X}+C_{1X} & C_{0X}-C_{1X} \\ C_{0X}-C_{1X} & C_{0X}+C_{1X} \end{pmatrix}$$

➢ 3-body effects: OPE, D^* selfenergy

$$V_\pi(E;p',p) = \frac{g^2}{6F_\pi^2}\begin{pmatrix} \frac{1}{2}V^{SS}_{D^0\pi^0\bar{D}^0} & V^{SS}_{D^0\pi^+D^-} \\ V^{SS}_{D^+\pi^-\bar{D}^0} & \frac{1}{2}V^{SS}_{D^+\pi^0D^-} \end{pmatrix}$$

$$V^{SS}_{\alpha\beta\zeta} = \frac{1}{2}\int_{-1}^{1} dz \frac{q^2}{2E_\beta(q)}\left(\frac{1}{D^R_{\alpha\beta\zeta}} + \frac{1}{D^A_{\alpha^*\beta\zeta^*}}\right)$$

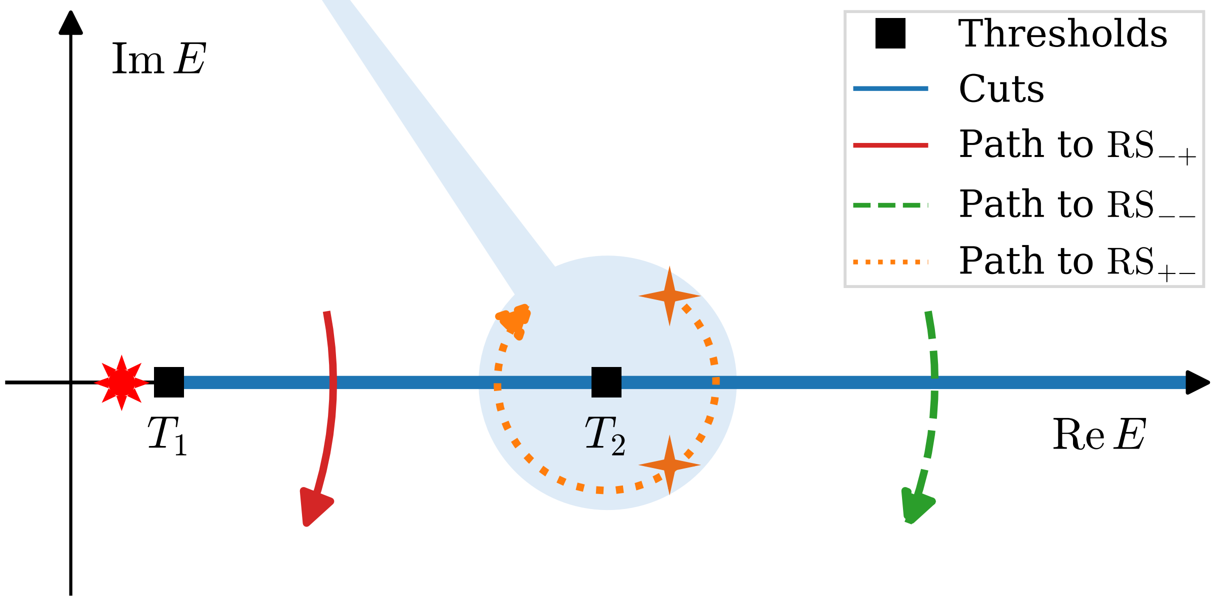


retarded OPE — D^* selfenergy — advanced OPE

Lippmann-Schwinger equation (LSE)

Z.-H. Zhang et al., JHEP 08 (2024) 130

- Two poles of the T-matrix for the $(D\bar{D}^*)_0 - (D\bar{D}^*)_\pm$ scattering amplitudes (4 Riemann sheets)
  - $X(3872)$ pole on the 1st RS (RS_{++})
  - $W_{c1}(3880)$ pole on the 4th RS (RS_{+-})
    - Shaded by D^+D^{*-} threshold
    - Cusp at the D^+D^{*-} threshold!!!



Prediction of an isospin vector partner of $X(3872)$

- There must be near-threshold isovector W_{c1} states
  - Virtual state pole in the stable D^* limit
    - W_{c1}^{+} in $D^+\bar{D}^{*0}$ single-channel scattering amplitude: pole on the 2nd Riemann sheet (RS), 8^{+8}_{-5} MeV below $D^0 D^{*-}$ threshold

$$W_{c1}^{\pm}: \quad 3866.9^{+4.6}_{-7.7} - i(0.07 \pm 0.01)\ \mathrm{MeV}$$

    - W_{c1}^{0} in $(D\bar{D}^*)_0 - (D\bar{D}^*)_\pm$ scattering amplitudes: pole on the 4th RS (RS_{+-}),

$$W_{c1}^{0}: \quad 3881.2^{+0.8}_{-0.0} + i1.6^{+0.7}_{-0.9}\ \mathrm{MeV}$$

  - Must appear as threshold cusps!!!
  - Compact tetraquarks (Maiani et al. (2005)) cannot be virtual states as they do not feel the thresholds

Cutoff insensitivity checked: poles relative to thresholds varied within 5% for $\Lambda \in [0.5, 1.0]$ GeV

- Virtual state W_{c1} was confirmed in lattice QCD calculation with $M_\pi = 280$ MeV

M. Sadl et al., PRD 111 (2025) 054513

J^{PC}	Interpolators	$1/a_0\,[\mathrm{fm}^{-1}]$	r_0 [fm]	χ^2/N_{dof}	Δm_{V} [MeV]
1^{+-}	All	$0.46^{+1.16}_{-0.45}$	$0.96^{+0.43}_{-0.73}$	0.13	$-3.0^{+3.0}_{-31.1}$
	$\eta_c\rho$ excl	$0.54^{+1.07}_{-0.44}$	$2.23^{+0.95}_{-1.08}$	0.24	$-2.8^{+2.6}_{-17.1}$
1^{++}	All	$0.62^{+1.30}_{-0.51}$	$1.78^{+0.25}_{-2.44}$	0.18	$-3.8^{+3.6\mathrm{a}}$
	$J/\psi\rho, \eta_c a_0$ excl	$0.96^{+1.42}_{-0.91}$	$2.19^{+0.36}_{-1.00}$	0.15	$-6.7^{+6.7}_{-19.5}$

[a]Uncertainty is so large that it is unbounded from below.

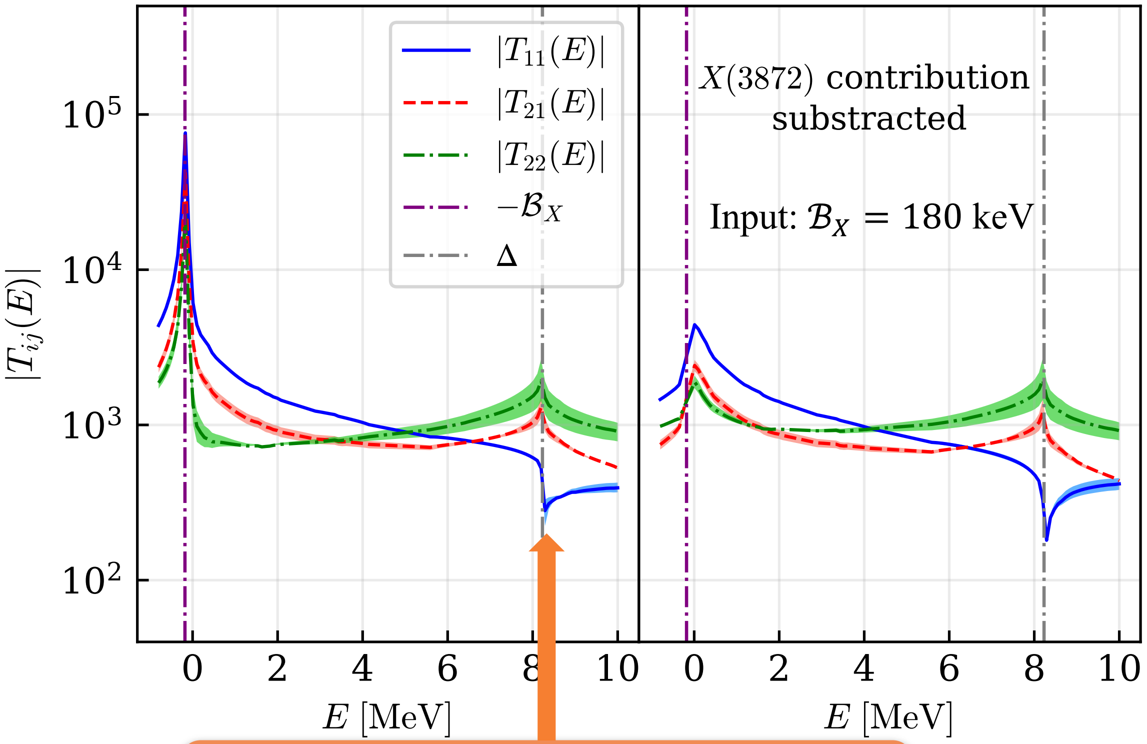
sign convention different from ours

Also obtained in one-boson exchange model in

X.-X. Chen, Z.-M. Ding, J. He, arXiv:2504.15534

Why have they not been observed?

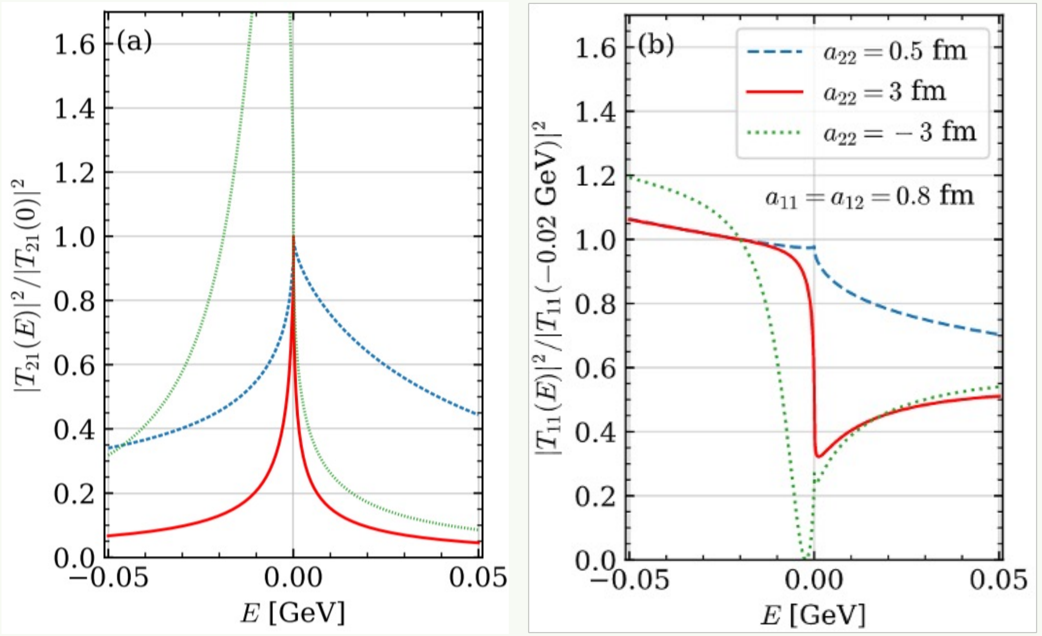
- W_{c1}^0 lives in the same amplitudes as the $X(3872)$, effects shielded by X
  - ➢ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



Threshold cusp!
peak or dip depends on processes

➢ Universality of dip for large scattering length

✓ For strongly interacting channel-2 (large a_{22}), there must be a dip around threshold

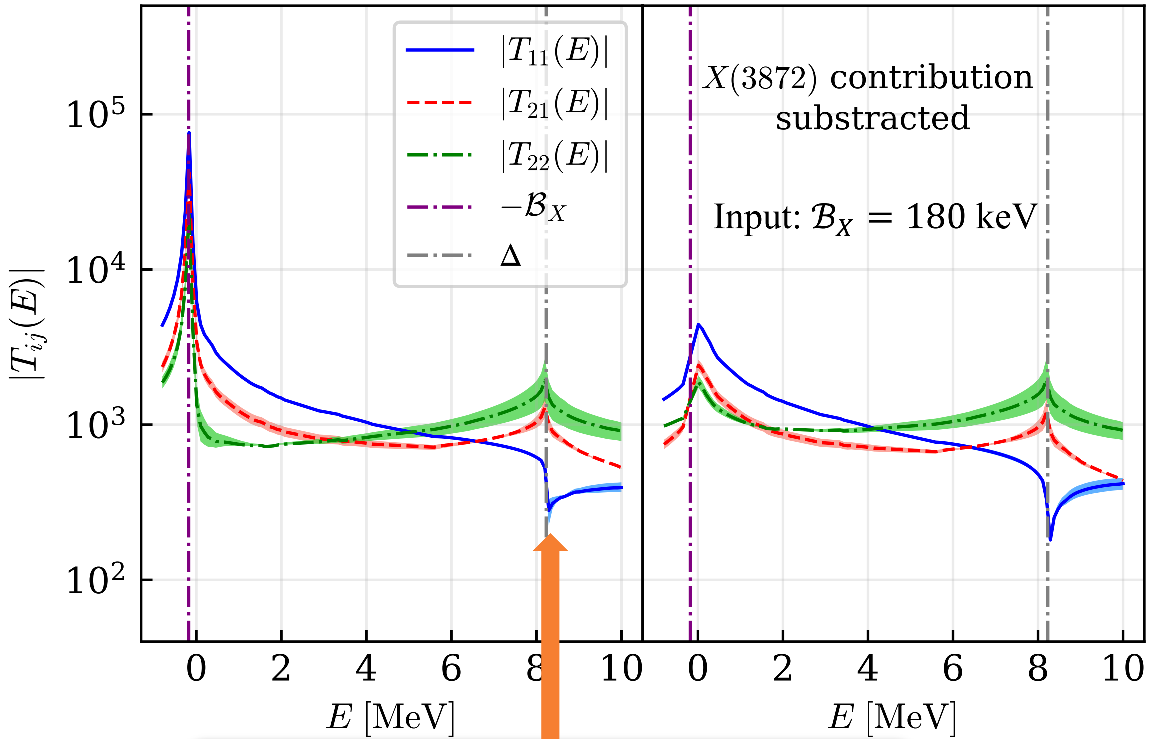


$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - ik_1\right)\left[\frac{1}{a_{22,\text{ eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

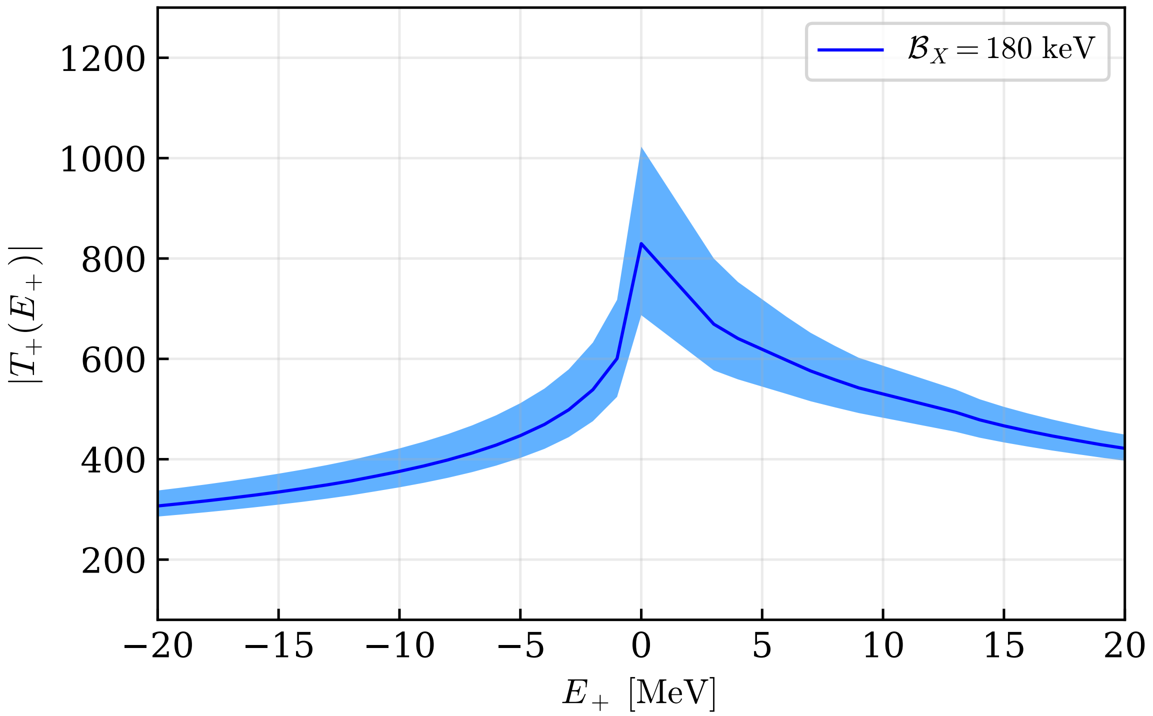
Why have they not been observed?

- W_{c1}^0 lives in the same amplitudes as the $X(3872)$, effects shaded by X
  - ➢ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes
  - ➢ Charged W_{c1}^+ in $D^+\bar{D}^{*0}$ scattering amplitude: height much lower than the X peak



Threshold cusp!
peak or dip depends on processes

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001



- ➢ should be searched for in high-statistic $J/\psi\pi^{\pm}\pi^0$ data

- The observed $X(3872)$ signals should contain the W_{c1}^0 contribution as well ⇒ combined analysis !!

Combined analysis of BESIII and LHCb data for $X(3872)$

Teng Ji et al., arXiv:2502.04458

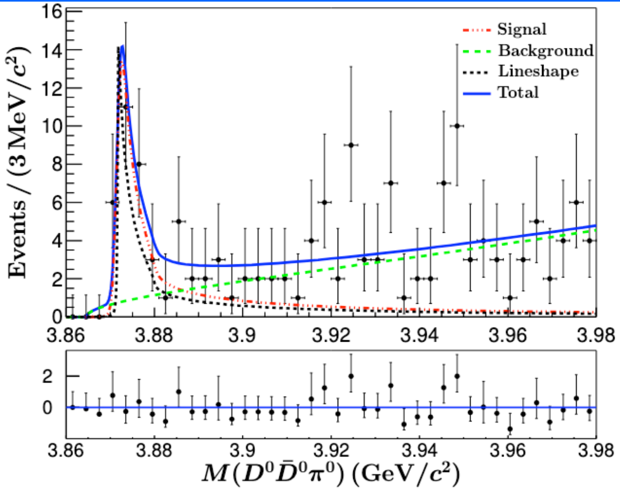
- $X(3872)$ line shapes $\Rightarrow X(3872)$ + possible $W_{c1}(3880)^0$
- $\pi^+\pi^-$ invariant mass distribution $\Rightarrow$ isospin breaking, information on $I=1$

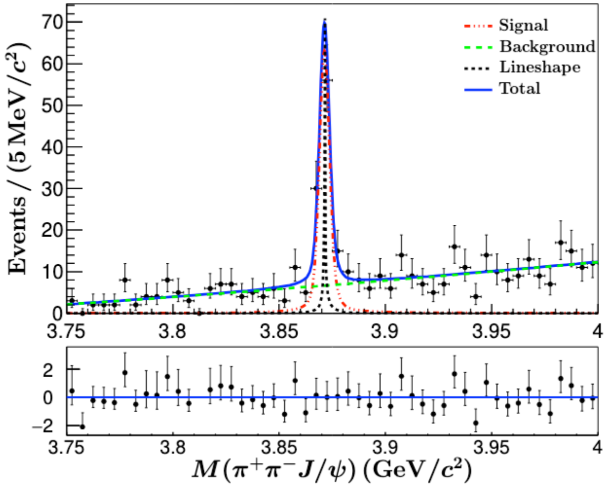
BESIII:

$e^+e^- \to \gamma[D^0\bar{D}^0\pi^0]$

$e^+e^- \to \gamma[J/\psi\pi^+\pi^-]$

BESIII, PRL 132 (2024) 151903

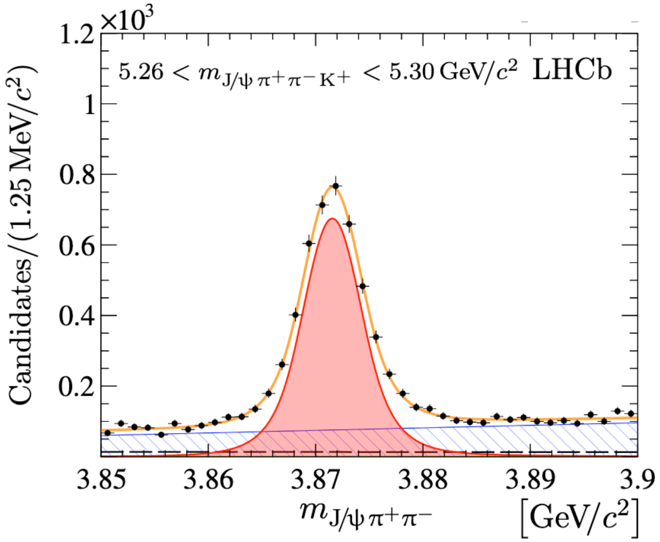


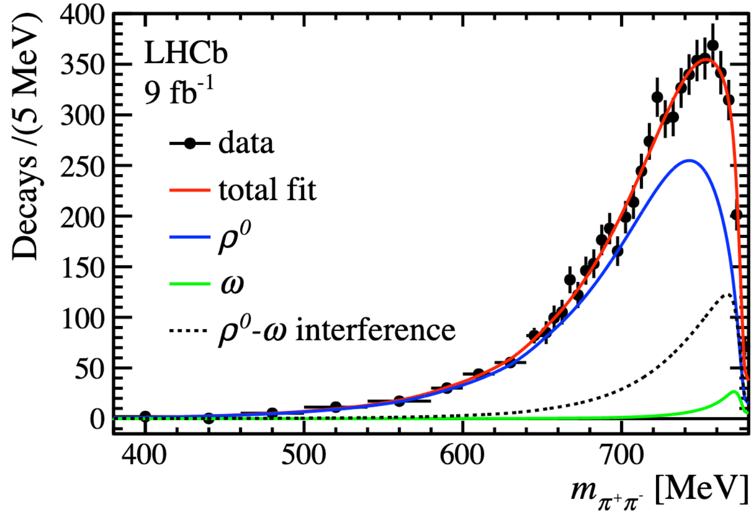


LHCb:

$B^+ \to K^+[J/\psi\pi^+\pi^-]$

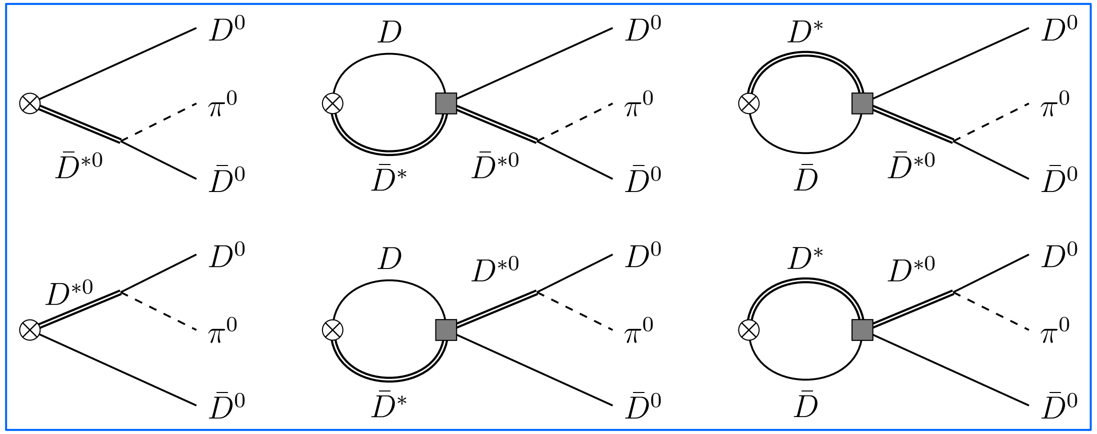
LHCb, JHEP 08 (2020) 123;
PRD 108 (2023) L011103

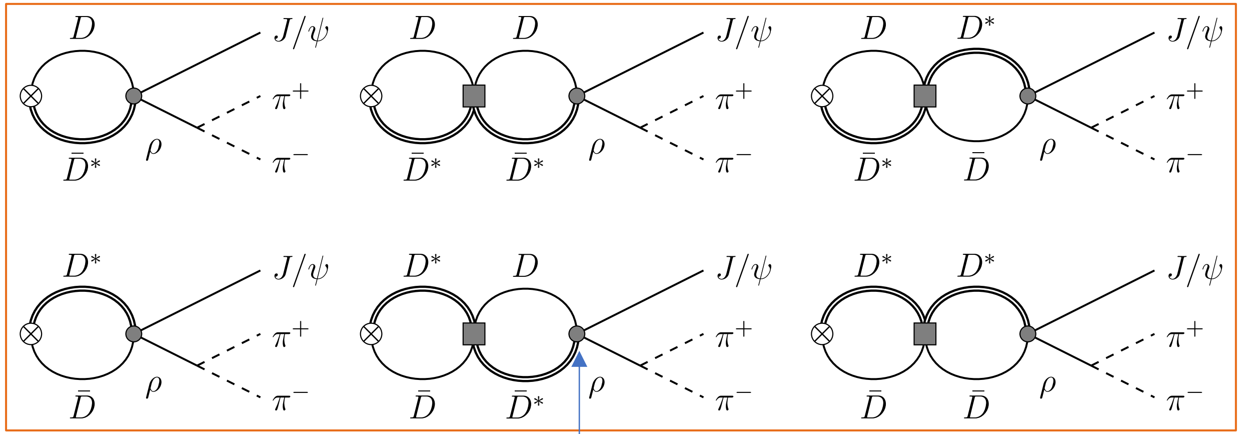




Combined analysis of BESIII and LHCb data for $X(3872)$

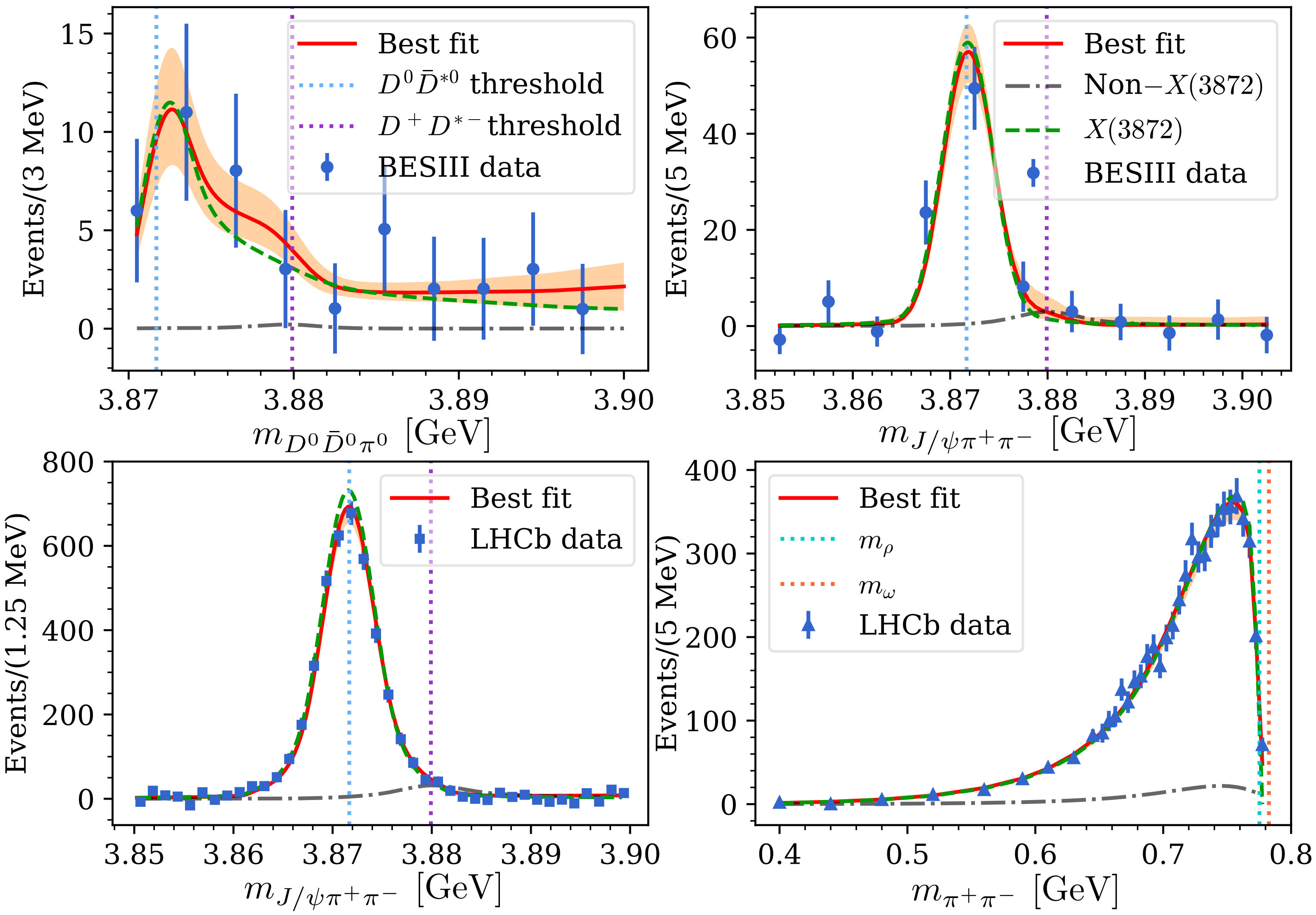
- Coupled channels
  - $(D\bar{D}^*)_0$, $(D\bar{D}^*)_\pm$: contact terms + OPE, $D\bar{D}\pi$ three-body effects considered
  - Inelastic channels:
    - $J/\psi\rho, J/\psi\omega$: ρ included using the Omnes dispersive approach, ρ-ω mixing considered
    - $J/\psi\gamma, \psi'\gamma, \chi_{cJ}(1P)\pi^0$: neglected in the baseline fit, included in uncertainty analysis





ρ-ω mixing included

Combined analysis of BESIII and LHCb data for $X(3872)$



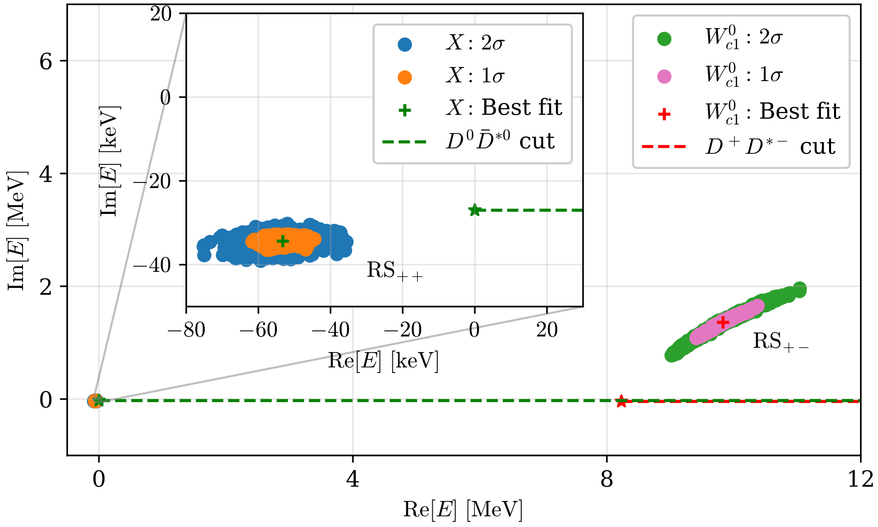
Best fit: $\chi^2/\text{dof} = 67.5/(88-9) = 0.78$

- Poles
  - ☐ $X(3872)$ as a bound state below $D^0\bar{D}^{*0}$ threshold (5σ)

    $$E_X = \left(-53^{+10}_{-25} - i34^{+2}_{-12}\right)\,\text{keV}$$

  - ☐ $W_{c1}(3880)^0$ pole on RS_{+-}, relative to the D^+D^{*-} threshold:

    $$E_W = \left(1.6^{+0.7}_{-0.9} + i1.4^{+0.3}_{-0.6}\right)\,\text{MeV}$$



Other properties of X(3872)

- Width (twice of the imaginary part of the pole): $69^{+24}_{-\ 4}$ keV
  - $D^0\bar{D}^0\pi^0 + D\bar{D}\gamma$: 54 keV ($\approx \Gamma(D^{*0})$)
  - $\Gamma(J/\psi\pi^+\pi^-) = 4.4^{+1.7}_{-0.9}$ keV, $\Gamma(J/\psi\pi^+\pi^-\pi^0) = 10.5^{+4.9}_{-2.4}$ keV

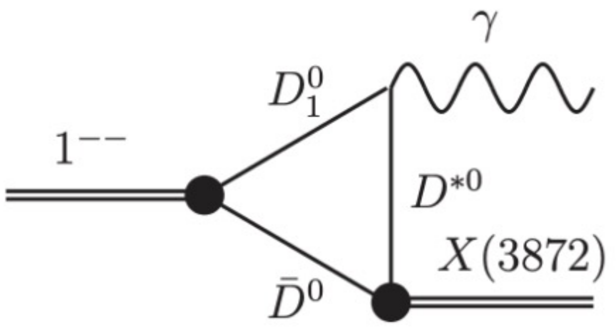
$$\frac{\mathrm{Br}(X(3872)\to J/\psi 3\pi)}{\mathrm{Br}(X(3872)\to J/\psi 2\pi)} = 2.4(6)$$

- Isospin breaking ratio $R_X \equiv \left|\frac{g_{XJ/\psi\rho}}{g_{XJ/\psi\omega}}\right| = 0.28(4)$
- Compositeness (including range corrections) Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

$$X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE\,\frac{\mathrm{Re}\,\delta(E)}{E - \mathrm{Re}\,E_X}\right) = 0.99(1)$$

Signal of $W_{c1}(3880)$?

- Signal of $W_{c1}(3880)^0$ almost invisible in the current data, reasons:
  - Virtual state, threshold cusp
  - $(D\bar{D}^*)_0$ easier produced than $(D\bar{D}^*)_\pm$ for both $e^+e^- \to \gamma D\bar{D}^*$ @ $\sqrt{s} \approx 4.23$ GeV and $B^+ \to K^+ D\bar{D}^*$



FKG et al., PLB 725 (2013) 127

$\Gamma(D_1^0 \to \gamma D^{*0}) \gg \Gamma(D_1^+ \to \gamma D^{*+})$

J.G. Korner et al., PRD 47 (1993) 3955;
Fayyazuddin et al., PRD 50 (1994) 2329

➢ For B^+ decays, fit parameters (ratio of production vertices): $P_\pm/P_0 = 0.45 \pm 0.05$

Data:

PDG 2024

$$\frac{\mathrm{Br}\left[B^+ \to K^+ \left(D^+ D^{*-} + D^- D^{*+}\right)\right]}{\mathrm{Br}\left[B^+ \to K^+ \left(D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0}\right)\right]} = 0.14 \pm 0.02$$

➢ Switching $u \leftrightarrow d$, situation should be different for B^0 decays

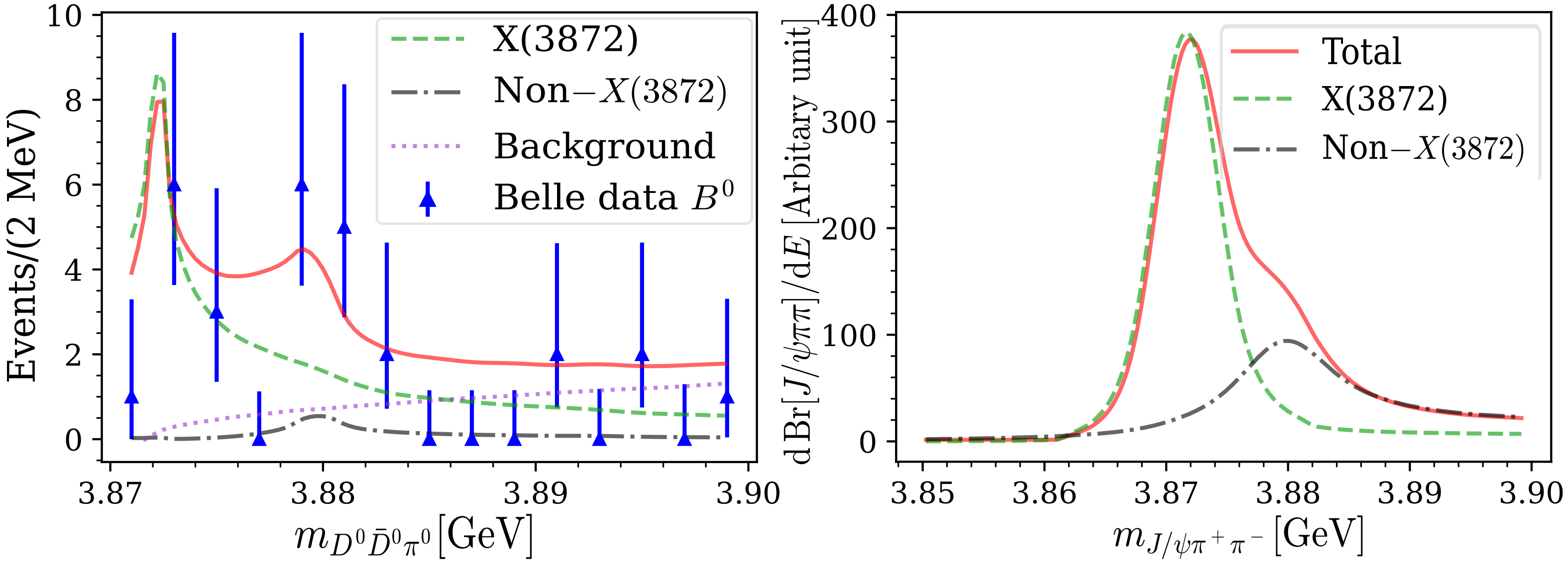
Data:

PDG 2024

$$\frac{\mathrm{Br}\left[B^0 \to K^0 \left(D^+ D^{*-} + D^- D^{*+}\right)\right]}{\mathrm{Br}\left[B^0 \to K^0 \left(D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0}\right)\right]} = 5.8 \pm 2.7$$

Implications of the existence of $W_{c1}(3880)$

- $W_{c1}(3880)^0$ signal should be stronger in $B^0 \to K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$ decays, to be checked @ LHCb, Belle II



- Cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^\pm\pi^0$

- Signal of $X(3872)$ and $W_{c1}(3880)$ in lepto-/photo-production?
  - Should depend on the dominant production mechanism!

Other implications to be explored

- $\mathrm{Br}(X \to \psi'\gamma)/\mathrm{Br}(X \to J/\psi\gamma)$: could be different at different experiments

$\Gamma(\chi_{c1}(3872) \to \gamma\psi(2S))/\Gamma(\chi_{c1}(3872) \to \gamma J/\psi)$

PDG 2024

VALUE	CL%	EVTS	DOCUMENT ID		TECN	COMMENT
			• • We do not use the following data for averages, fits, limits, etc. • •			
< 0.59	90		ABLIKIM	2020W	BES3	$e^+ e^- \to \gamma\chi_{c1}(3872)$
$2.46 \pm 0.64 \pm 0.29$		36 ± 9	[1] AAIJ	2014AH	LHCB	$B^+ \to \gamma\psi(2S)K^+$
< 2.1	90		BHARDWAJ	2011	BELL	$B^+ \to \gamma\psi(2S)K^+$
3.4 ± 1.4			AUBERT	2009B	BABR	$B^+ \to \gamma c\bar{c}K'$

[1] From 36.4 ± 9.0 events of $\chi_{c1}(3872) \to J/\psi\gamma$ decays with a statistical significance of 4.4σ.

- $\Gamma(X \to \chi_{cJ}\pi^0)$ with iso-vector final states: could receive sizable contribution from $W_{c1}(3880)^0$

$\pi^0\chi_{c2}$	$< 5\%$
$\pi^0\chi_{c1}$	$(3.8^{+1.9}_{-1.7})\%$
$\pi^0\chi_{c0}$	$< 16\%$

PDG 2024

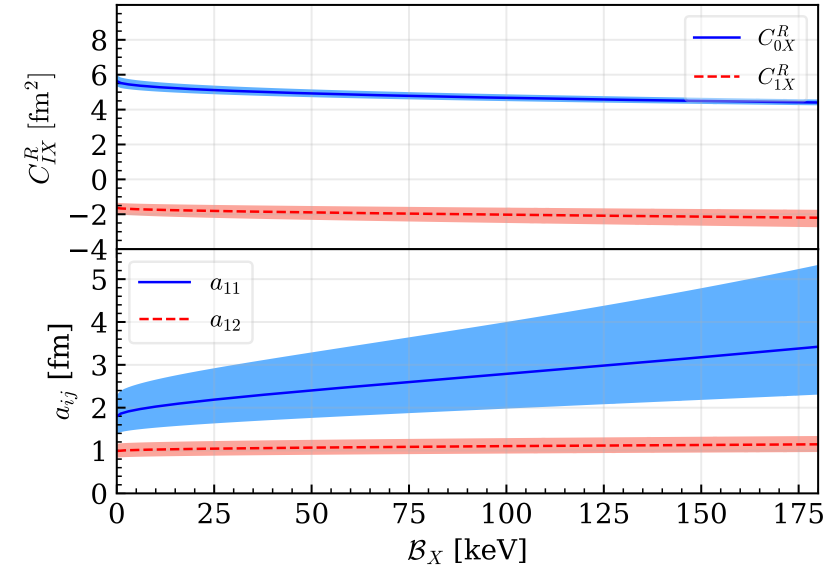
Summary and outlook

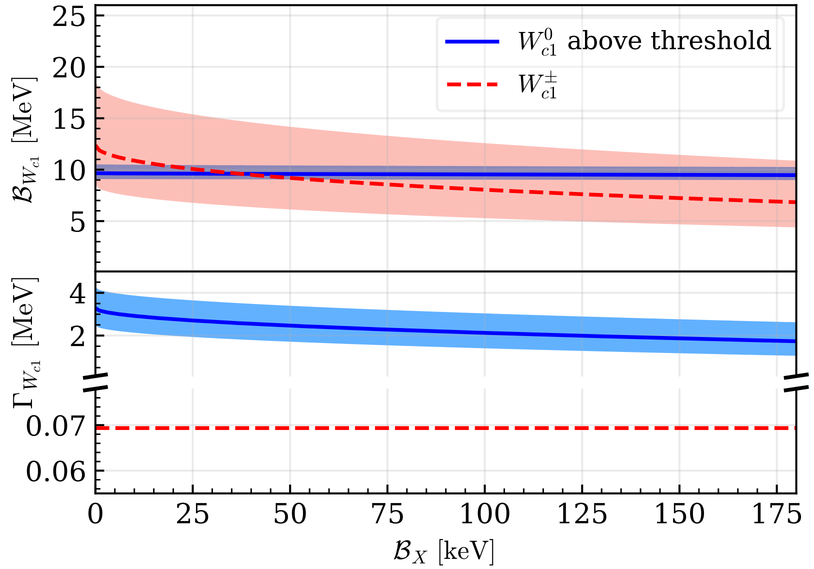
- Pole position of the $X(3872)$ precisely determined to be -53^{+10}_{-24} keV below the $D^0\bar{D}^{*0}$ threshold
- Existence of an isovector $W_{c1}(3880)$
  - Signal of $W_{c1}(3880)^0$ predicted to be more visible in $B^0 \to K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$
  - Signal of $W_{c1}(3880)^{\pm}$: threshold cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^{\pm}\pi^0$
- Some X(3872) decays need to be reanalyzed

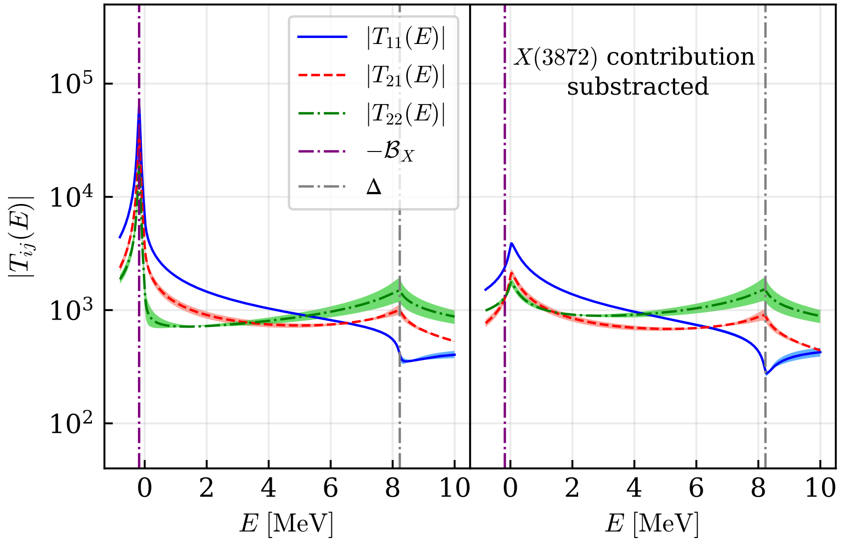
Thank you for your attention!

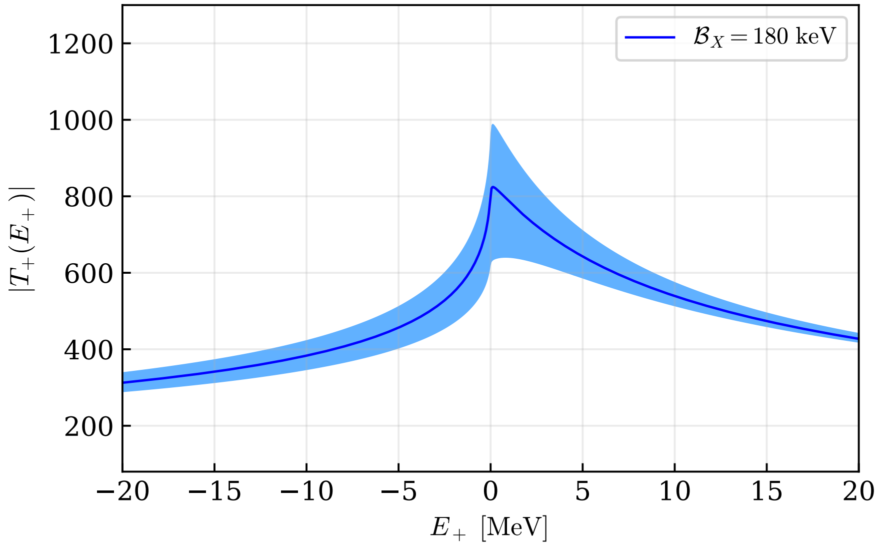
Results in the pionless theory

- All the qualitative features in the pion-full theory persist in the much simpler pionless theory









X(3872) line shapes

- Line shapes of a near-threshold resonance depend on reaction mechanism! X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

  - Peak for $|T_{21}|$ (1: lower inelastic channel; 2: elastic channel)

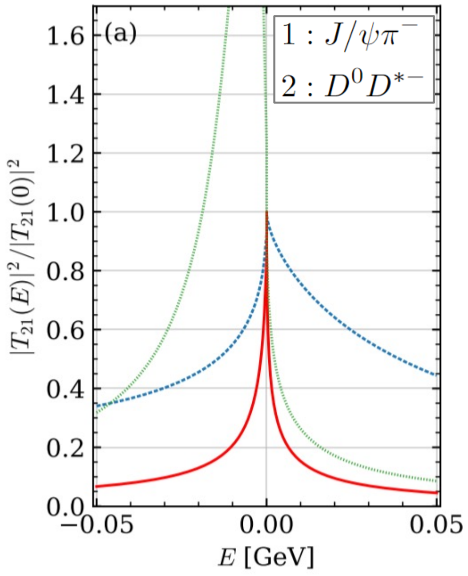
$$T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - ik_1)}\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}.$$

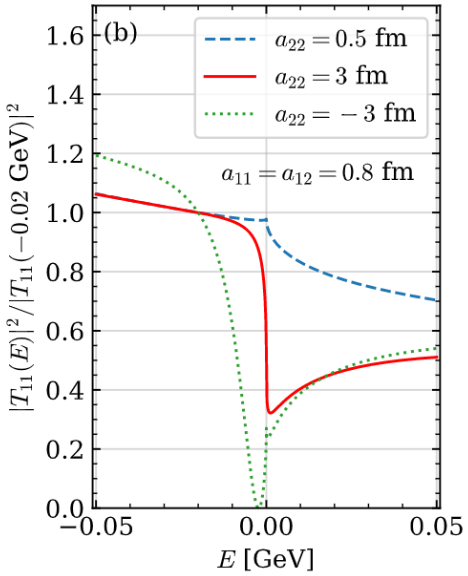
  - Dip for $|T_{11}|$ if scattering length for channel-2 is large

$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$

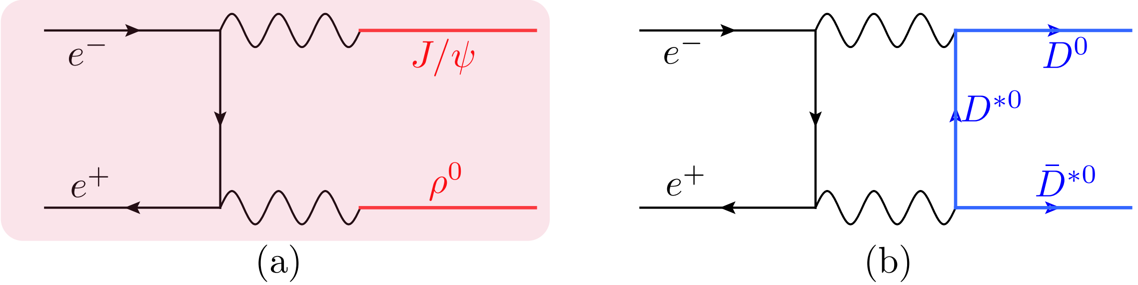
$$= -8\pi E_2^{\mathrm{thr}}\left(\underbrace{\frac{1}{a_{11}^{-1} - ik_1}}_{\text{background}} + \underbrace{\frac{a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-2}}{a_{22,\mathrm{eff}}^{-1} - ik_2}}_{\text{pole term}}\right)$$

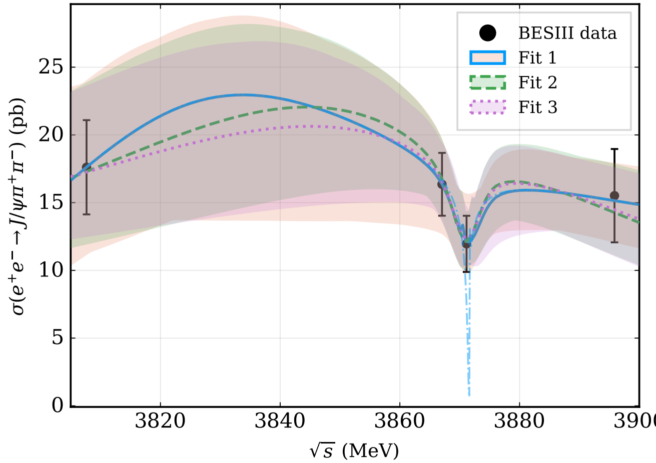
The interfering phase is fixed by unitarity!





- $X(3872)$ shows up as a dip in $e^+e^- \to X \to J/\psi\pi\pi$ direct production





V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109 (2024) L111501