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Spin correlation effects in heavy flavor physics

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@Tsung-Dao Lee Institute **2024.12.26**

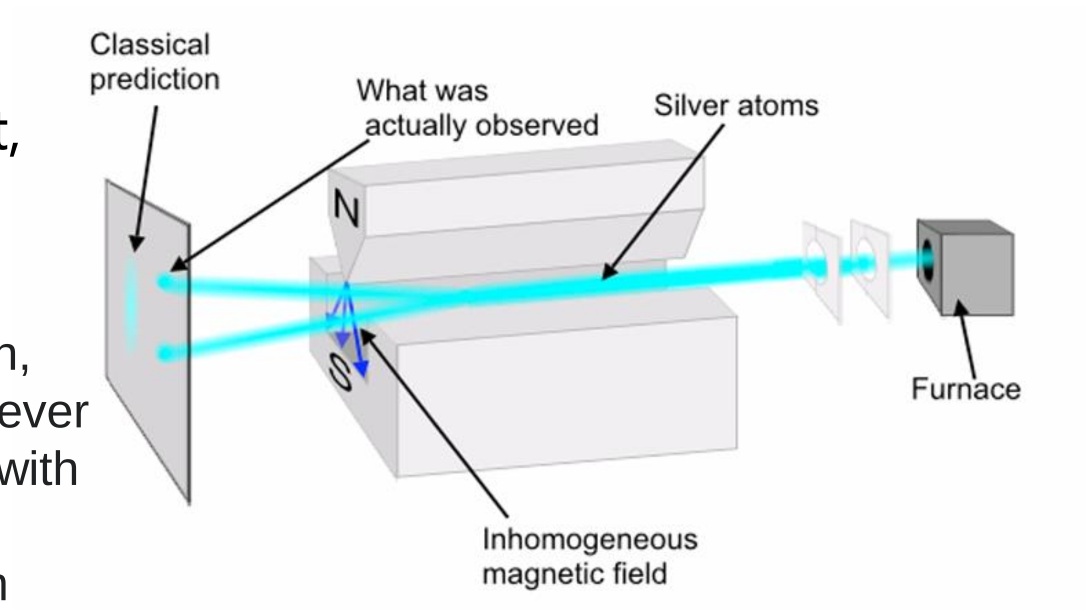
Early history of spin

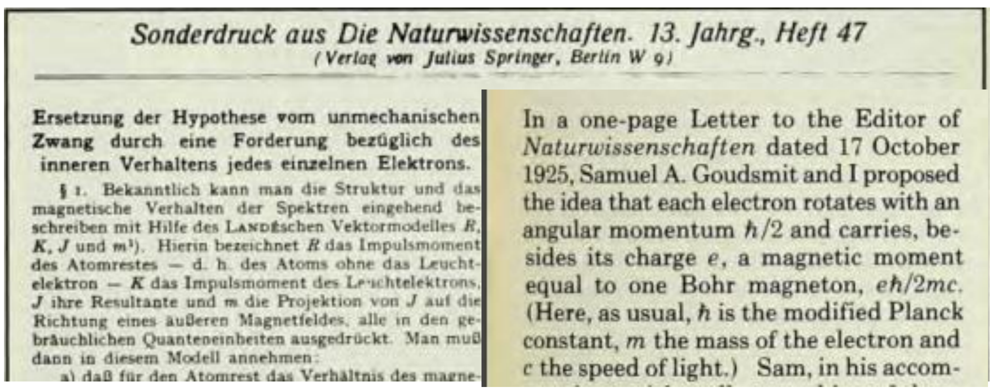
- **100 anniversaries in 2025**

Back to 1922,
Stern-Gerlach experiment,
to detect OAM

In January 1925,
Kronig first had an idea of spin,
Pauli saying "it is indeed very clever but of course has nothing to do with reality".
Pauli proposed four quantum numbers
(n,l,m,s)

In September1925,
Uhlenbeck and Goudsmit
proposed electron
intrinsic "Spin" concept





Sonderdruck aus Die Naturwissenschaften. 13. Jahrg., Heft 47
(Verlag von Julius Springer, Berlin W 9)

Ersetzung der Hypothese vom unmechanischen Zwang durch eine Forderung bezüglich des inneren Verhaltens jedes einzelnen Elektrons.

§ 1. Bekanntlich kann man die Struktur und das magnetische Verhalten der Spektren eingehend beschreiben mit Hilfe des Landéschen Vektormodelles R, K, J und m[1]). Hierin bezeichnet R das Impulsmoment des Atomrestes — d. h. des Atoms ohne das Leuchtelektron — K das Impulsmoment des Leuchtelektrons, J ihre Resultante und m die Projektion von J auf die Richtung eines äußeren Magnetfeldes, alle in den gebräuchlichen Quanteneinheiten ausgedrückt. Man muß dann in diesem Modell annehmen:

a) daß für den Atomrest das Verhältnis des magne-

In a one-page Letter to the Editor of *Naturwissenschaften* dated 17 October 1925, Samuel A. Goudsmit and I proposed the idea that each electron rotates with an angular momentum $\hbar/2$ and carries, besides its charge e, a magnetic moment equal to one Bohr magneton, $e\hbar/2mc$. (Here, as usual, $\hbar$ is the modified Planck constant, m the mass of the electron and c the speed of light.) Sam, in his accom-

Electron complete description

- **Complete set of quantum numbers for electron in the atom**

$$\psi_{nlmsm_s}(\mathbf{r}) = R_{nl}(r) Y_{lm}(\theta,\phi) \chi_{s,m_s}$$

$$\chi_{1/2,1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{1/2,-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **In 1928, Dirac Equation; During 1946-1949, QED by Tomonaga-Schwinger-Feynman； Lorentz(Poincare) group**

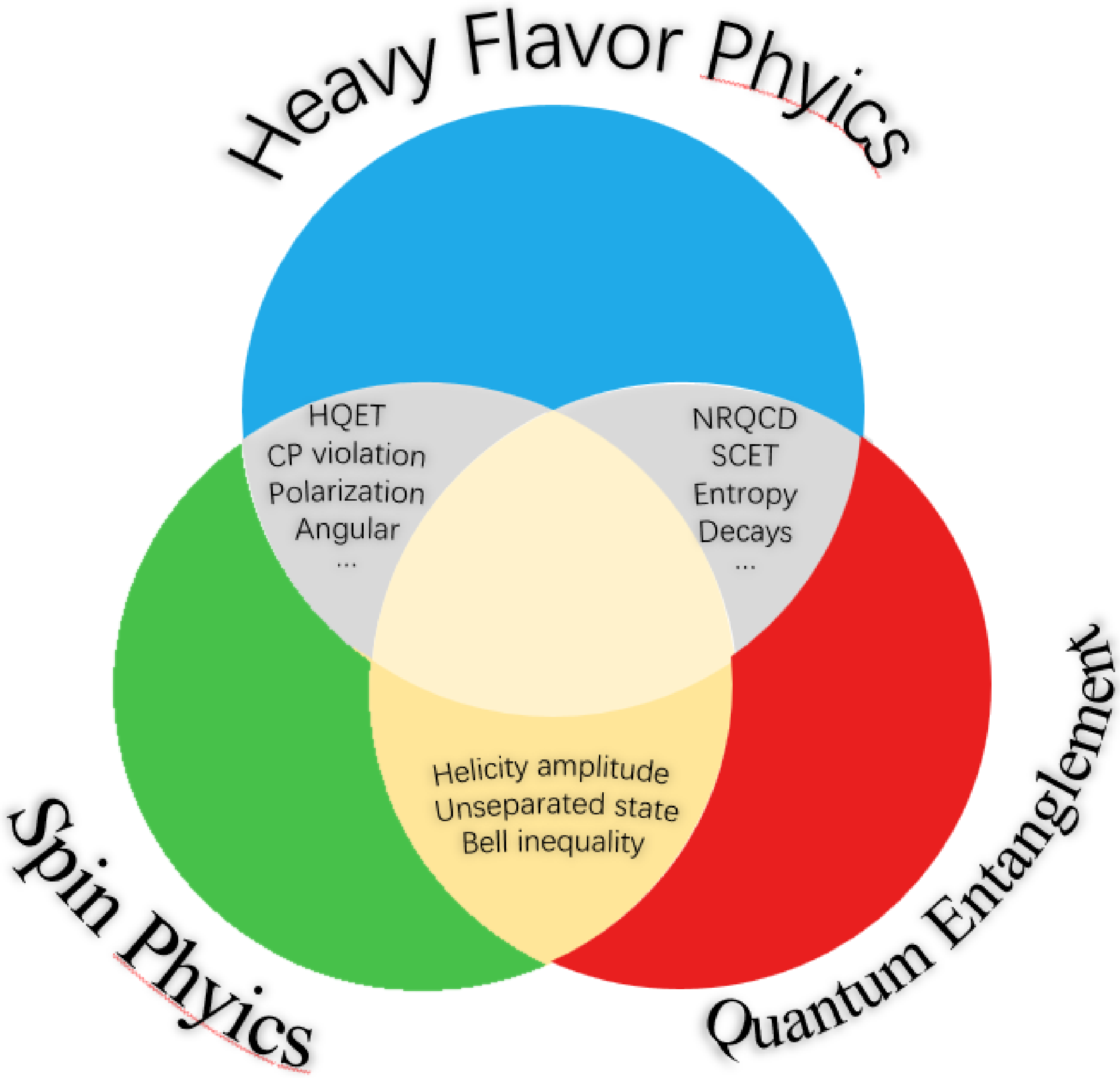
$$(i\gamma^\mu \partial_\mu - m)\psi = -e\gamma^\mu \psi A_\mu$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - e j^\mu A_\mu$$

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + e\phi - \frac{e\hbar}{2mc}\boldsymbol{\sigma} \cdot \mathbf{B}.$$

AM generator in Lorentz group

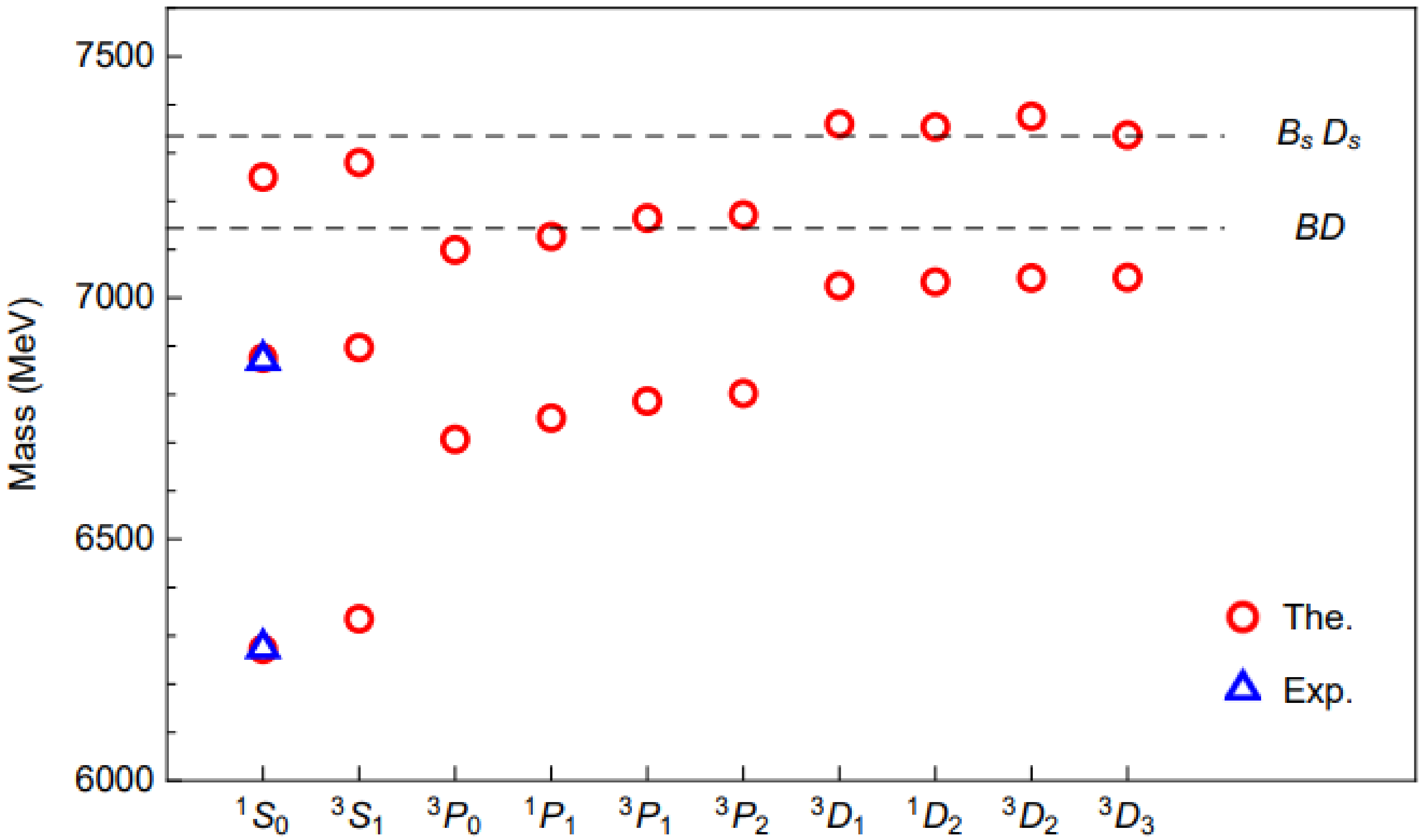
Spin correlation in heavy flavor physics



Outline

- Spin correlation in beauty-charm meson family decays
- Spin correlation in fully charmed tetraquark states decays
- Summary and Outlook

Beauty-charm meson family spectrum

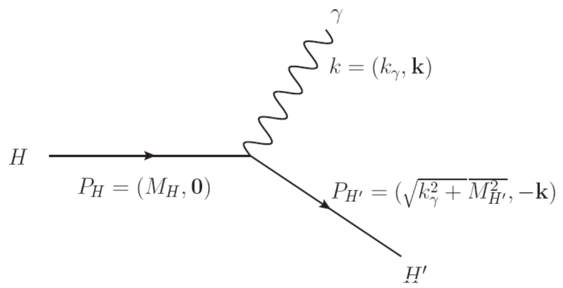


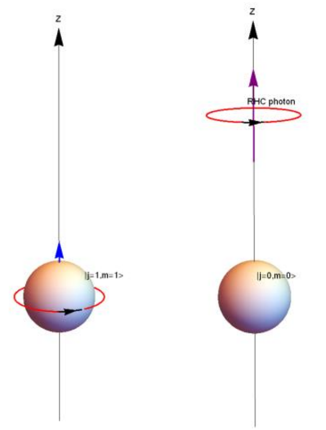
nonrelativistic potential model + Coupled channel analysis

Hao-Zhu, 2402.18898

Detect Bc* meson by EM decays

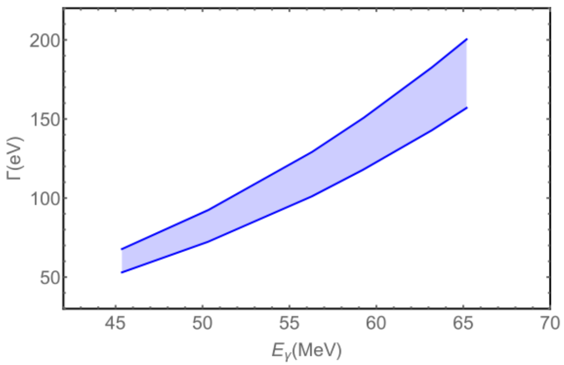
- **Bc* (1S) major (99.99%) electromagnetic decays to Bc(1S): M1 transition**





$$
\begin{aligned}
\mathcal{L}_{\gamma\mathrm{pNRQCD}} = \int d^3r\,\mathrm{Tr}\Big[& e\frac{e_Q - e'_Q}{2} V_A^{\mathrm{em}} \mathrm{S}^\dagger \mathbf{r}\cdot\mathbf{E}^{\mathrm{em}}\mathrm{S} \\
& + e\Big(\frac{e_Q m'_Q - e'_Q m_Q}{4 m_Q m'_Q}\Big)\Big[V_S^{\frac{\sigma\cdot B}{m}} \Big\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \Big\}\mathrm{S} \\
& + \frac{1}{8} V_S^{(r\cdot\nabla)^2 \frac{\sigma\cdot B}{m}} \Big\{ \mathrm{S}^\dagger, \mathbf{r}^i\mathbf{r}^j \left(\boldsymbol{\nabla}^i\nabla^j\boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}}\right) \Big\}\mathrm{S} \\
& + V_O^{\frac{\sigma\cdot B}{m}} \left\{ \mathrm{O}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \right\}\mathrm{O}\Big] \\
& + e\Big(\frac{e_Q m_{Q'}^2 - e'_Q m_Q^2}{32 m_Q^2 m_{Q'}^2}\Big)\Big[4\frac{V_S^{\frac{\sigma\cdot B}{m^2}}}{r} \Big\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \Big\}\mathrm{S} \\
& + 4\frac{V_S^{\frac{\sigma\cdot(r\times r\times B)}{m^2}}}{r} \Big\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{\mathrm{em}})] \Big\}\mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla\times E}{m^2}} \Big[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}\times, \mathbf{E}^{\mathrm{em}}] \Big]\mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla_r\times r\cdot\nabla E}{m^2}} \Big[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i\left(\boldsymbol{\nabla}^i\mathbf{E}^{\mathrm{em}}\right)\right] \Big]\mathrm{S}\Big] \\
& + e\Big(\frac{e_Q m_{Q'}^3 - e'_Q m_Q^3}{8 m_Q^3 m_{Q'}^3}\Big)\Big[V_S^{\frac{\nabla_r^2\sigma\cdot B}{m^3}} \Big\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}} \Big\}\nabla_r^2\,\mathrm{S} \\
& + V_S^{\frac{(\nabla r\cdot\sigma)(\nabla r\cdot B)}{m^3}} \Big\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}^i\mathbf{B}^{\mathrm{em}\,j} \Big\}\boldsymbol{\nabla}_r^i\nabla_r^j\,\mathrm{S}\Big]\Big],
\end{aligned}
$$

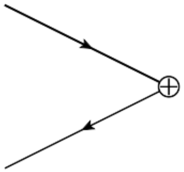
- **We generalize the pNRQCD to unequal mass case and obtain the effective Lagrangian**



Detect Bc* meson by weak decays

- **Bc* decay constants in QCD**

$$\left\langle 0\left|\bar{b}\gamma^{\mu}c\right|B_c^*(P,\varepsilon)\right\rangle = f_{B_c^*}^{v} m_{B_c^*}\varepsilon^{\mu},$$



(a) LO

- **Bc* decay constants in NRQCD**

mathing coefficients — NRQCD LDMEs

$$f_{B_c^*}^{v} = \sqrt{\frac{2}{m_{B_c^*}}}\,\underline{C_v(m_b, m_c, \mu_f)}\,\underline{\left\langle 0\left|\chi_b^{\dagger}\sigma\cdot\varepsilon\psi_c\right|B_c^*(\mathbf{P})\right\rangle}(\mu_f) + O(v^2)$$

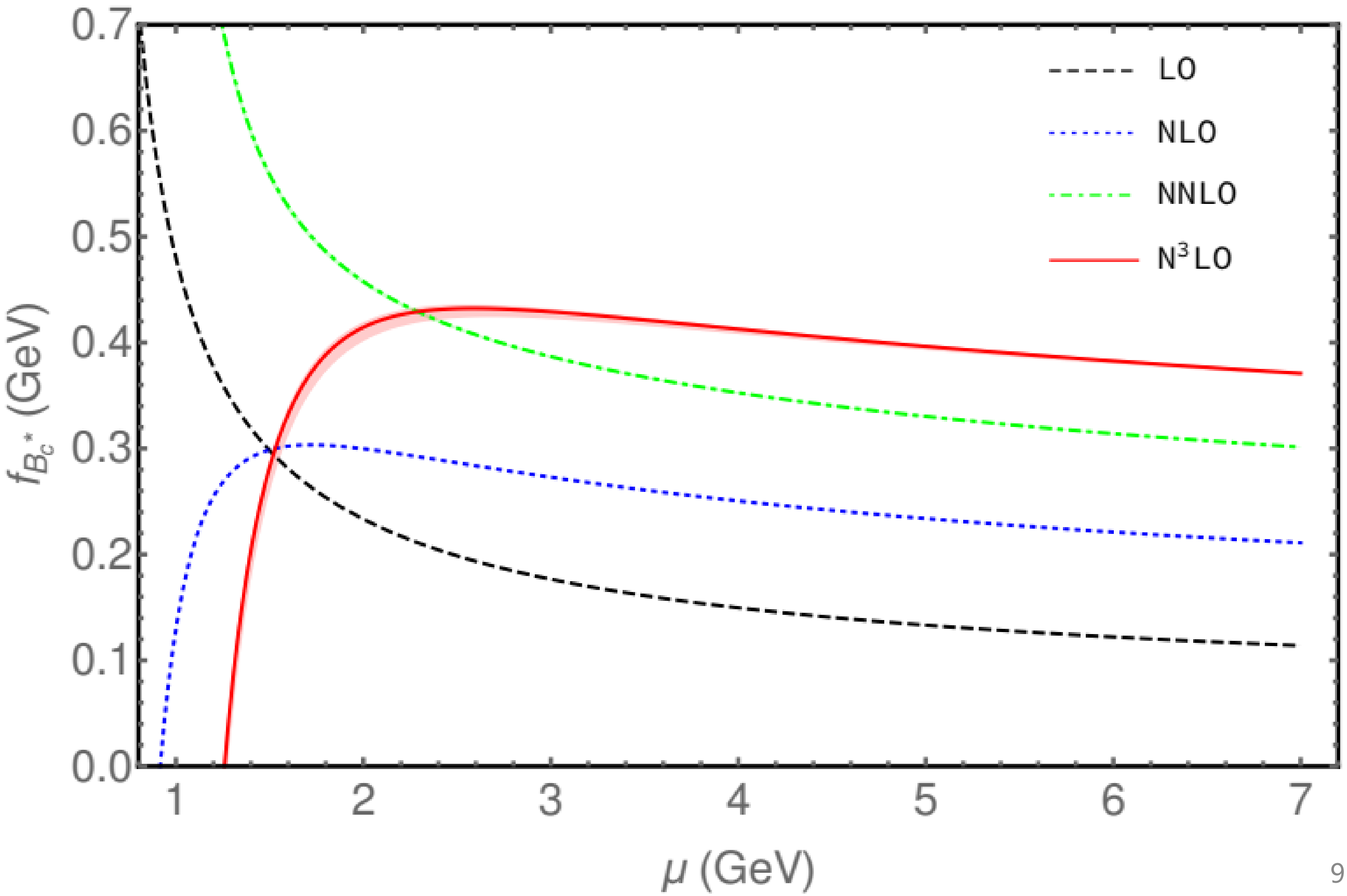
- **Matching Formulae**

Braaten-Fleming,PRD52,181(1995);
Lee-Sang-Kim,JHEP01,113(2011)

$$Z_J Z_{2,b}^{\frac{1}{2}} Z_{2,c}^{\frac{1}{2}} \Gamma_J = C_J \tilde{Z}_J^{-1} \tilde{Z}_{2,b}^{\frac{1}{2}} \tilde{Z}_{2,c}^{\frac{1}{2}} \tilde{\Gamma}_J$$

$\tilde{Z}_J$:NRQCD $\overline{\text{MS}}$ current renormalization constants

Vector Bc* meson decay constant



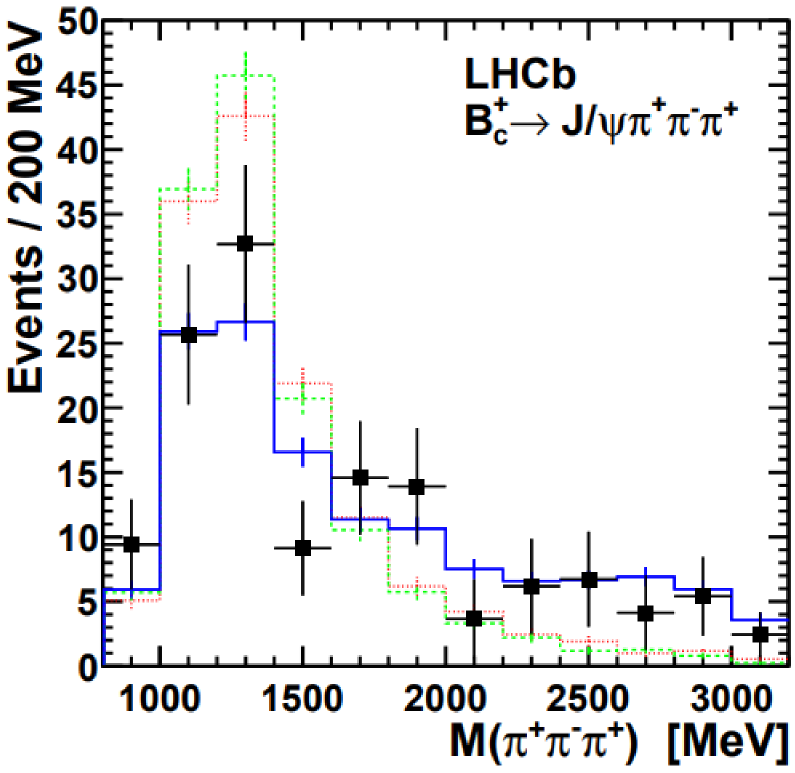
Leptonic decay branching ratios

Branching ratios	N^{3}LO
$\mathcal{B}(B_c^{*+} \to e^+ \nu_e)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \mu^+ \nu_\mu)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \tau^+ \nu_\tau)$	$(3.40^{+0.25-0.06-1.19}_{-0.41+0.03+0.33}) \times 10^{-6}$
$\mathcal{B}(B_c^{+} \to e^+ \nu_e)$	$(1.91^{+0.15-0.19-0.70}_{-0.23+0.12+0.22}) \times 10^{-9}$
$\mathcal{B}(B_c^{+} \to \mu^+ \nu_\mu)$	$(8.18^{+0.63-0.83-2.99}_{-1.00+0.52+0.94}) \times 10^{-5}$
$\mathcal{B}(B_c^{+} \to \tau^+ \nu_\tau)$	$(1.96^{+0.15-0.20-0.72}_{-0.24+0.12+0.23}) \times 10^{-2}$

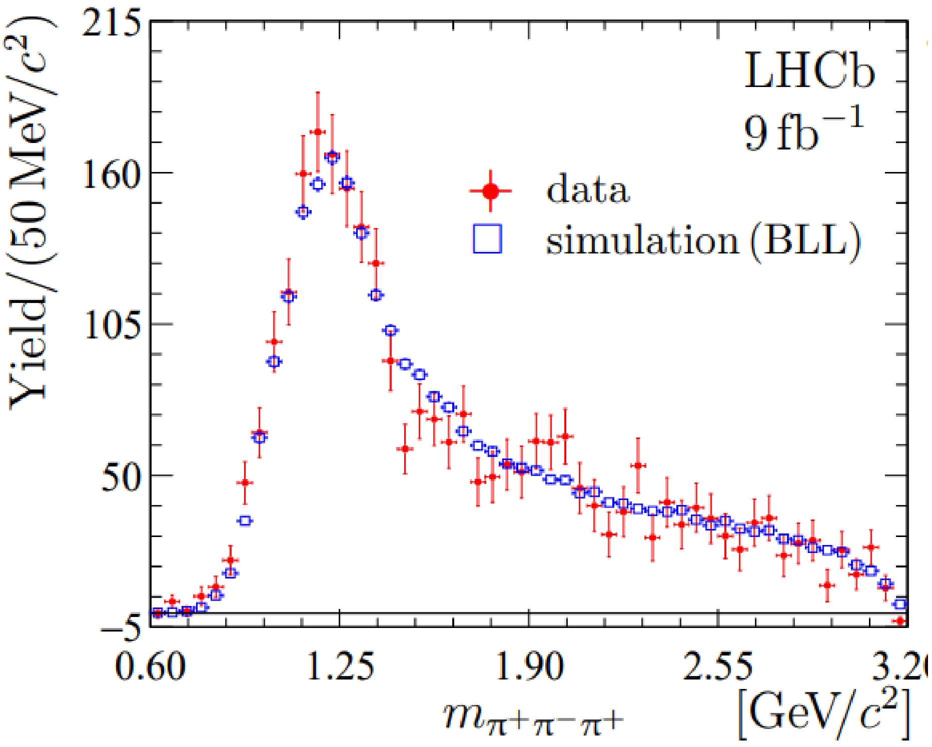
$$\Gamma(B_c^*(\lambda = \pm 1) \to \ell\nu_\ell) = \frac{|V_{cb}|^2}{12\pi} G_F^2 f_{B_c^*}^2 \left(1 - \frac{m_\ell^2}{m_{B_c^*}^2}\right)^2 \times m_{B_c^*}^3,$$

$$\Gamma(B_c^*(\lambda = 0) \to \ell\nu_\ell) = \frac{m_\ell^2 \Gamma(B_c^{*+}(\lambda = \pm 1) \to \ell\nu_\ell)}{2m_{B_c^*}^2},$$

Bc and Bc* decays along with 3 pions



LHCb, arXiv:1204.0079



LHCb, arXiv:2111.03001

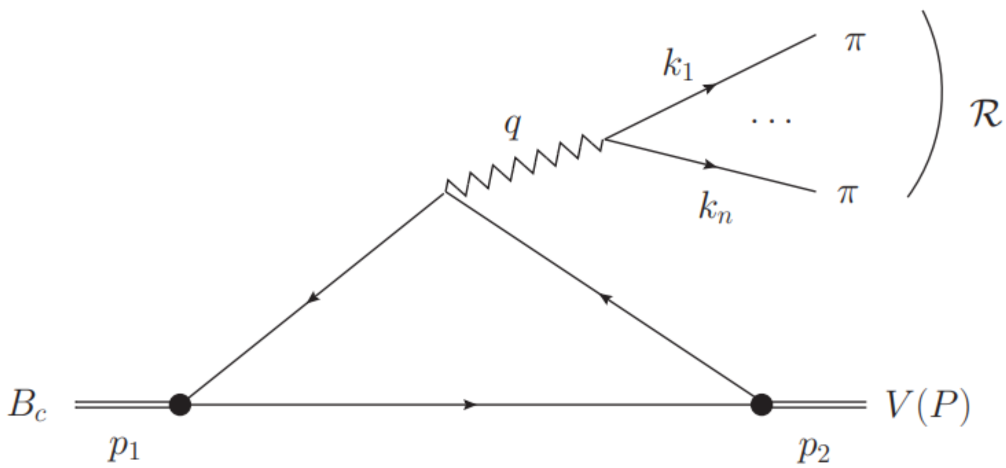
Around 10^5 Bc to Jpsi+X events

Invariant mass distribution in Bc* decays to J/psi +n h

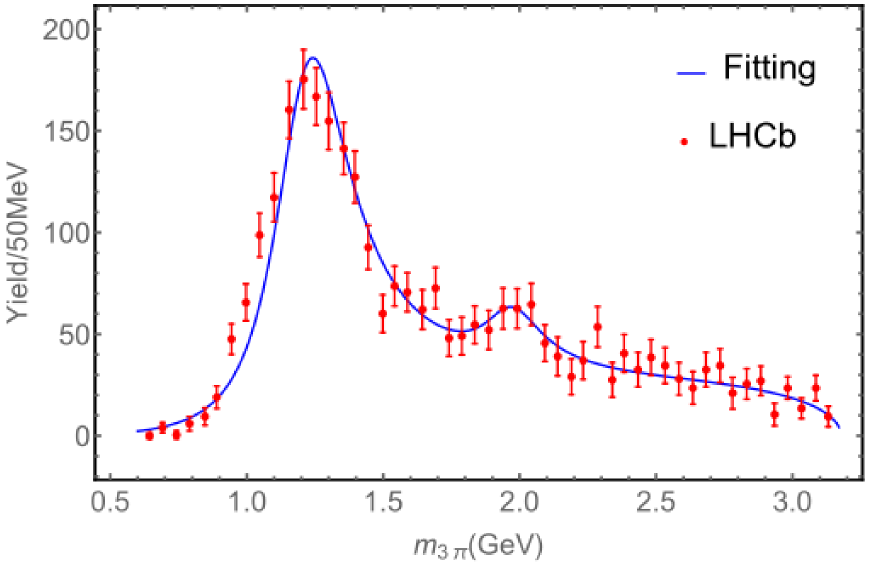
➢ Helicity decomposition of weak decay width

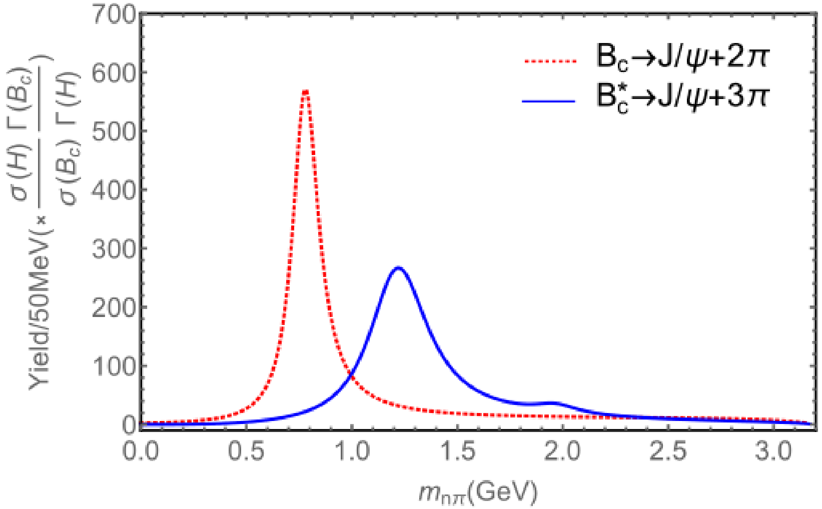
$$\frac{d\Gamma(B_c^{(*)} \to J/\psi + nh)}{dq^2} = \sum_{\lambda_i} \frac{|V_{cb}|^2 G_F^2 a_1^2 |\mathbf{p}'|}{32\pi M^2} \Gamma_{J_1\lambda_1 J_2\lambda_2\lambda_{nh}},$$

$$\Gamma_{11110} = 2\left[V_1^2\left((M-M')^2-q^2\right)\left((M'+M)^2-q^2\right) + \left(A_1\left(M^2-M'^2\right)+A_2 q^2\right)^2\right]\rho_T^{nh}(q^2),$$



Results of invariant mass distribution

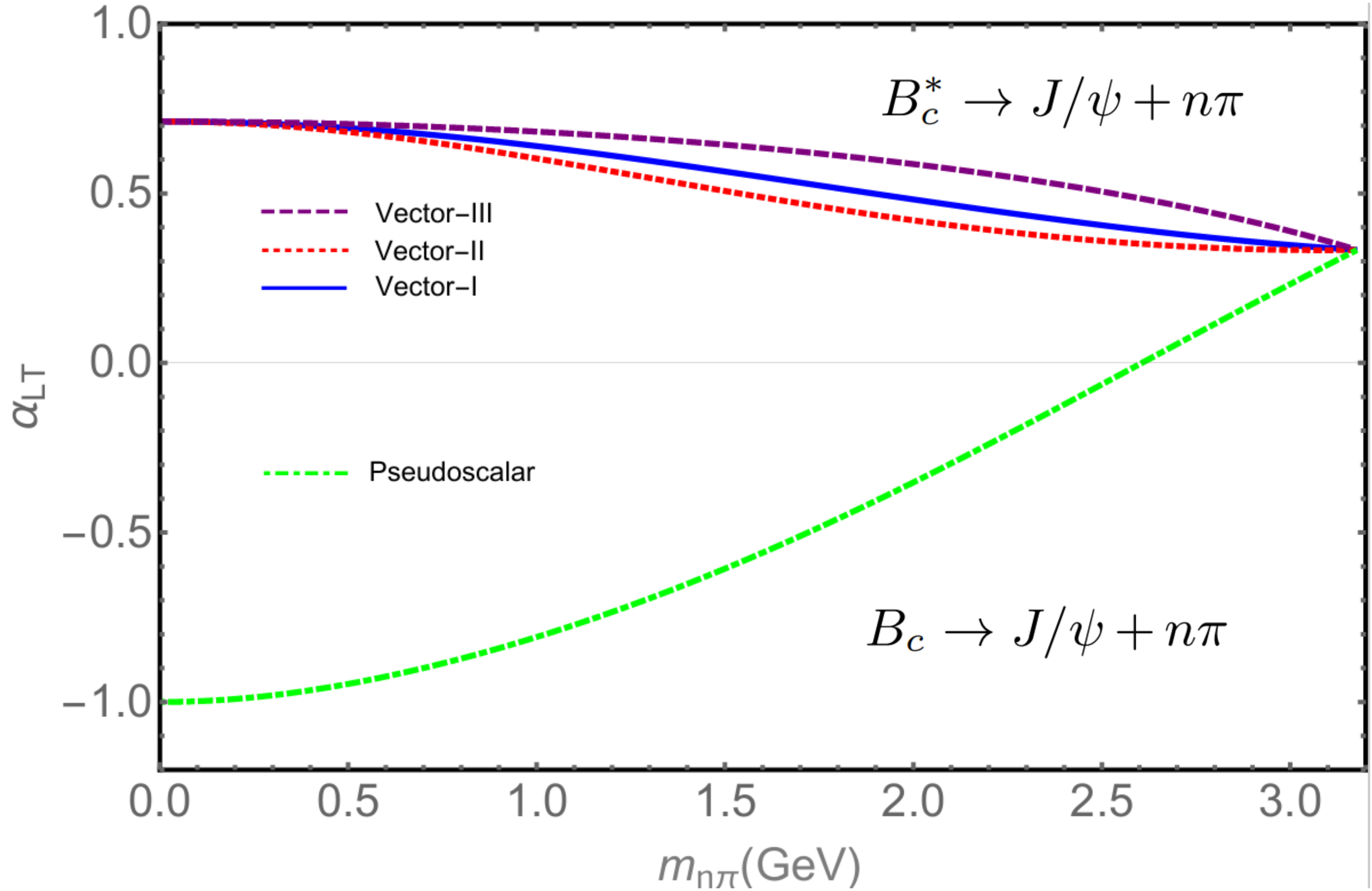




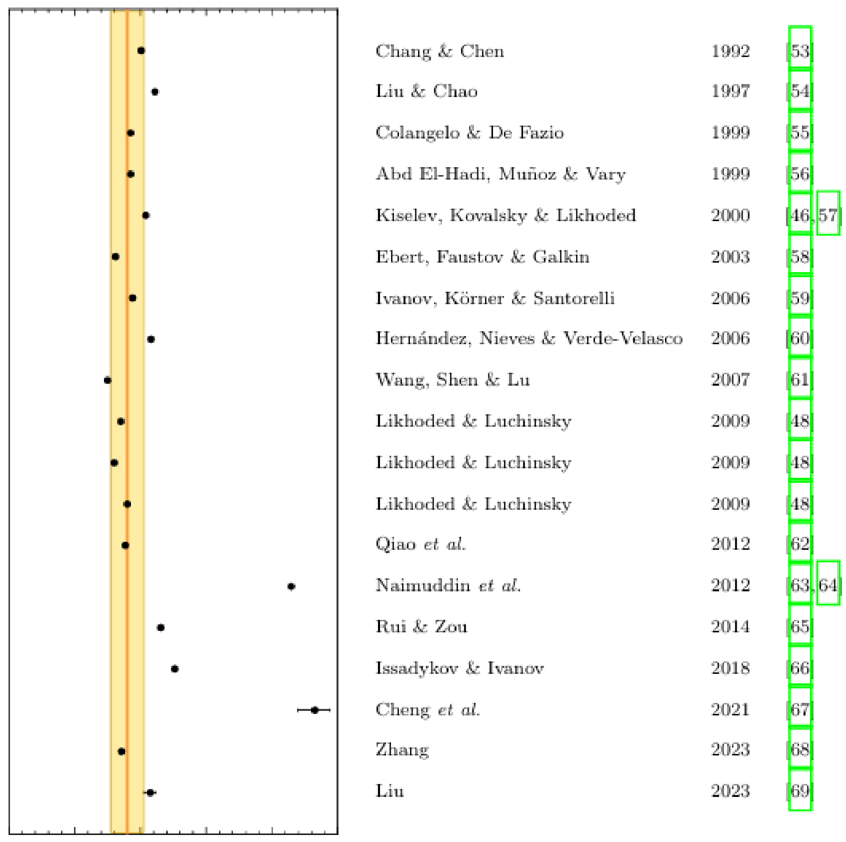
LHCb, arXiv:2111.03001

Polarization Asymmetry(A general law in V(P) to V transitions)

$$\alpha_{LT} = \sum_{\lambda_1,\lambda_{nh}} \frac{\Gamma_{J_1\lambda_1 11\lambda_{nh}} - \Gamma_{J_1\lambda_1 10\lambda_{nh}}}{\Gamma_{J_1\lambda_1 11\lambda_{nh}} + \Gamma_{J_1\lambda_1 10\lambda_{nh}}},$$

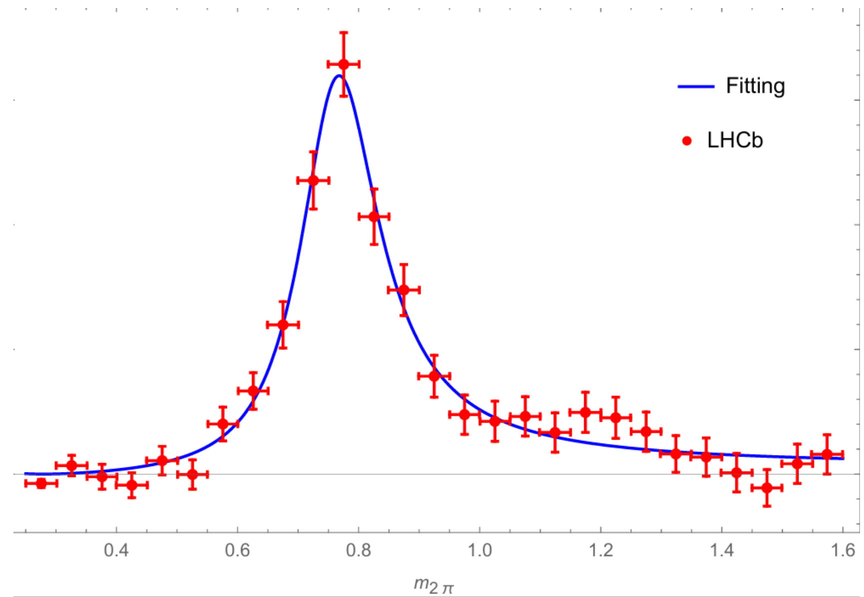


Bc decays along with 2 pions



$$\frac{\mathcal{B}_{B_c^+\to J/\psi\pi^+\pi^0}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}}$$

$$\mathcal{R} = \frac{\mathcal{B}_{B_c^+\to J/\psi\pi^+\pi^0}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}} = 2.80 \pm 0.15 \pm 0.11 \pm 0.16\,,$$



[62]Qiao-Sun-Yang-Zhu, 1209.5859

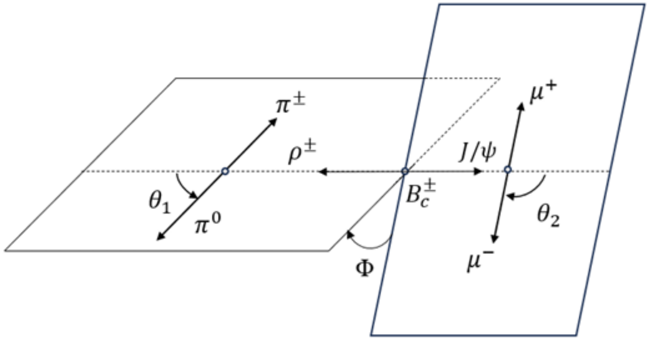
LHCb, arXiv: 2402.05523

2404.06221, 2310.03425

Helicity amplitudes and angular distributions

➢ weak decay amplitude

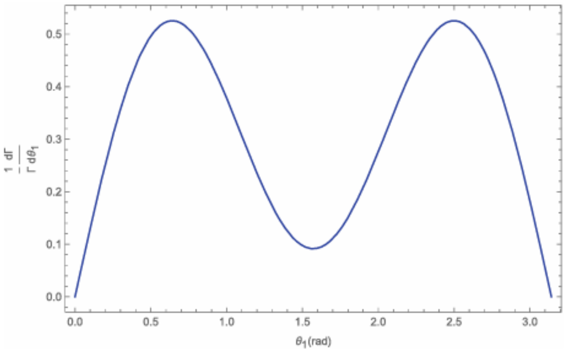
$$h_\lambda \equiv \langle \rho(p_1,\lambda) J/\psi(p_2,\lambda)|\mathcal{H}_{eff}|B_c(p)\rangle$$
$$= \epsilon_{1\mu}(\lambda)^* \epsilon_{2\nu}(\lambda)^* \left(a g^{\mu\nu} + \frac{b p^\mu p^\nu}{m_1 m_2} + \frac{i c \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_\beta}{m_1 m_2} \right),$$



➢ Angular distribution

$$\frac{d^3\Gamma\left(B_c \to J/\psi(\mu^+\mu^-) + \rho(\pi\pi)\right)}{d\cos\theta_1 d\cos\theta_2 d\phi} =$$

$$\frac{9p_m}{128\pi^2 M^2}\Big\{ \cos^2\theta_1 \sin^2\theta_2 H_{00} + \frac{1}{4}\sin^2\theta_1\left(1+\cos^2\theta_2\right)\left(H_{11}+H_{-1-1}\right)$$
$$-\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\left[\cos 2\phi\,\mathrm{Re}\left(H_{1-1}\right) - \sin 2\phi\,\mathrm{Im}\left(H_{1-1}\right)\right]$$
$$-\frac{1}{4}\sin 2\theta_1 \sin 2\theta_2\left[\cos\phi\,\mathrm{Re}\left(H_{10}+H_{-10}\right) - \sin\phi\,\mathrm{Im}\left(H_{10}-H_{-10}\right)\right]\Big\},$$



$$f_L = \frac{|h_0|^2}{|h_{+1}|^2 + |h_{-1}|^2 + |h_0|^2}. \qquad \alpha_{LT} = 1 - 2f_L. \qquad f_L(J/\psi) = f_L(\rho) \simeq 0.877,$$

Quantum spin entanglement

➢ Quantum spin entanglement state

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}}\left[h_{+1}|J/\psi(+1)\rho(+1)\rangle + h_0|J/\psi(0)\rho(0)\rangle + h_{-1}|J/\psi(-1)\rho(-1)\rangle\right],$$

➢ Von Neumann entropy

$$\varrho = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{+1}h_{+1}^* & 0 & h_{+1}h_0^* & 0 & h_{+1}h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_{+1}^* & 0 & h_0h_0^* & 0 & h_0h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{-1}h_{+1}^* & 0 & h_{-}h_0^* & 0 & h_{-1}h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon = -Tr[\varrho_A \ln \varrho_A] = -Tr[\varrho_B \ln \varrho_B].$$

$$\varrho_{J/\psi} = \varrho_\rho = \frac{1}{|H|^2}\begin{pmatrix} h_{+1}h_{+1}^* & 0 & 0 \\ 0 & h_0h_0^* & 0 \\ 0 & 0 & h_{-1}h_{-1}^* \end{pmatrix}.$$

$$\varepsilon = 0.405.$$

Test of Bell inequality

- **Bell inequality** $|s\rangle = \frac{1}{\sqrt{2}}(\ |\uparrow\downarrow> - |\downarrow\uparrow> \ ).$

$$P(\vec{a},\vec{b}) = \int d\lambda \rho(\lambda) A(a,\lambda) B(b,\lambda).$$

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \le \int d\lambda \rho(\lambda) \left(1 - A(b,\lambda) A(c,\lambda)\right)$$
$$= \int d\lambda \rho(\lambda) - \int d\lambda \rho(\lambda) A(b,\lambda) A(c,\lambda) = 1 + P(\vec{b},\vec{c}).$$

- **Collins-Gisin-Linden-Massar-Popescu qutrits inequality**

$$A_1, A_2, B_1, B_2 = 0, \ldots, d-1.$$

$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
$$- P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \le 2.$$

$$I_3 = tr(\rho B) = tr(|\psi\rangle\langle\psi| B) = \langle\psi|B|\psi\rangle.$$

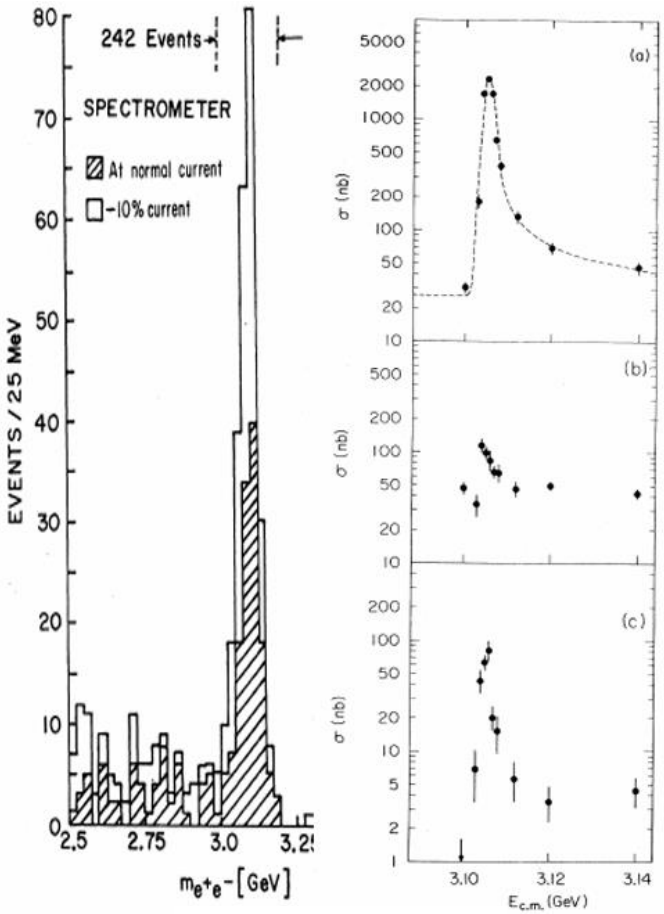
$$I_3 = |-2.91| = 2.91 > 2$$

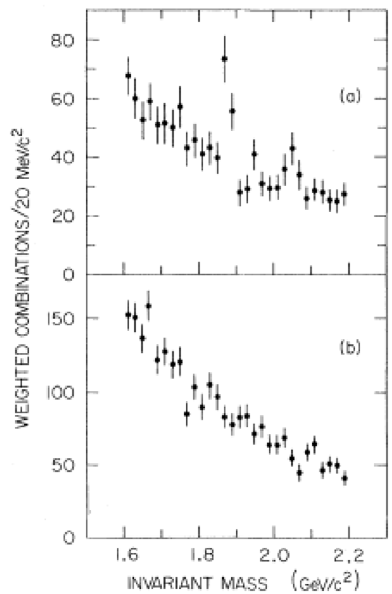
CGLMP inequality breaks down.

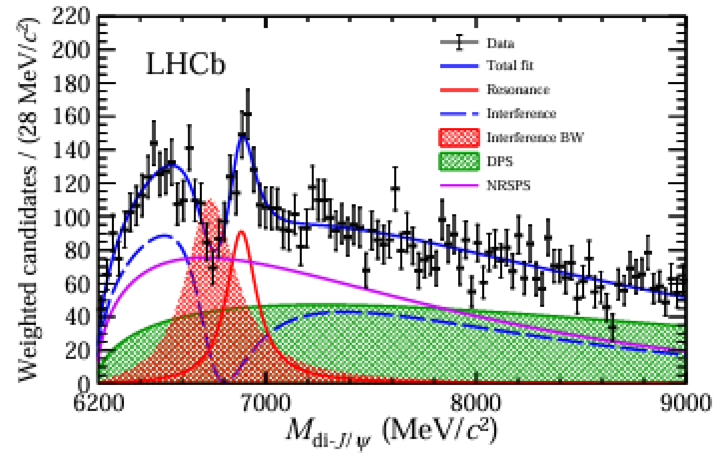
Charm family

$\Omega_{ccc} \quad (CCC)???$

$T_{3c} \quad (CC\bar{C}\bar{q})???$







$J/\psi \quad (C\bar{C})$

$D \quad (C\bar{q})$

$T_{4c} \quad (CC\bar{C}\bar{C})?$

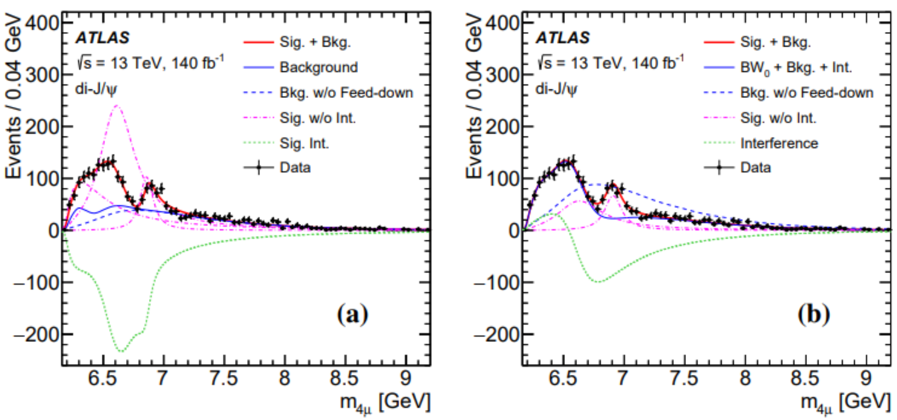
1974 by Ting and Richter

1976 at SLAC

LHCb, 2006.16957

Latest data on fully charmed tetraquarks

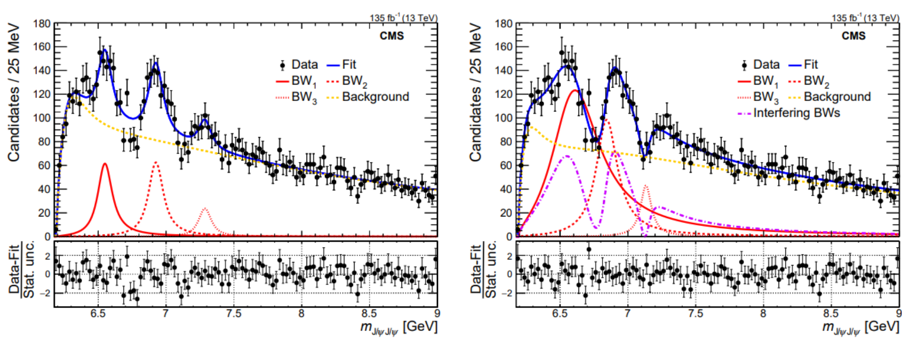
Exp.	Fit	M_{BW_1}	Γ_{BW_1}	$M_{X(6900)}$	$\Gamma_{X(6900)}$	M_{BW_3}	Γ_{BW_3}
LHCb	No interf.	–	–	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	–	–
CMS	No interf.	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 184$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$
LHCb	Interf.	6741 ± 6	288 ± 16	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	–	–
CMS	Interf.	6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$	6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}
ATLAS	Fit-A	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$
ATLAS	Fit-B	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	–	–



ATLAS, 2304.08962; 140fb-1 data;

P_t(mu1,2,3,4)>4,4,3,3GeV; |eta(mu1,2,3,4)|<2.5;

2.94(3.56)GeV<M(dimuon)<3.25(3.80)GeV

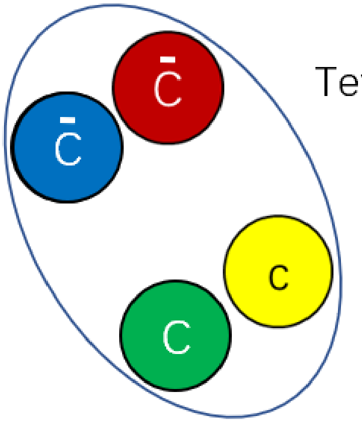


CMS, 2306.07164; 135fb-1 data;

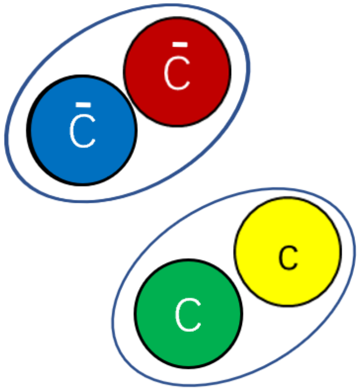
P_t(muon)>2GeV;|eta(muon)|<2.4;

P_t(di muon)>3.5GeV;

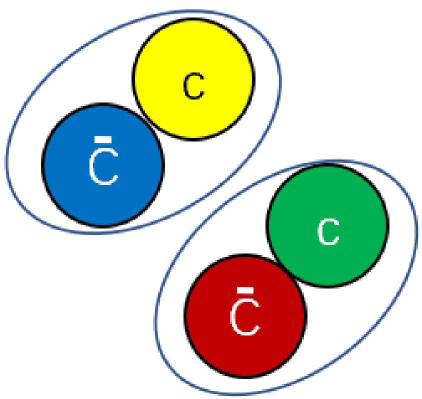
How to explain these exotic states?



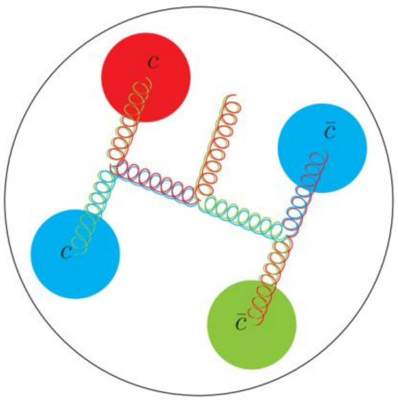
Tetraquark



Diquark-antidiquark



Charmonia Molecule



Gluonic Tetracharm Hybrid

diquark-antidiquark model?

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Nuclear Physics B 966 (2021) 115393
NUCLEAR PHYSICS B
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Fully-heavy tetraquark spectra and production at hadron colliders

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Received 27 October 2020; received in revised form 21 March 2021; accepted 1 April 2021
Available online 7 April 2021
Editor: Hong-Jian He

Abstract

Motivated by the observation of exotic structure around 6900 MeV in the J/ψ-pair mass spectrum using proton-proton collision data by the LHCb collaboration, we study the spectra of fully-heavy tetraquarks within Bethe-Salpeter equation and Regge trajectory relation. The $X(6900)$ may be explained as a radially excited state with quark content $cc\bar{c}\bar{c}$ and spin-parity $0^{++}(3S)$ or $2^{++}(3S)$ or an orbitally excited $2P$ state. New $cc\bar{c}\bar{c}$ structures around 6.0 GeV, 6.5 GeV, and 7.1 GeV are predicted together. Other $bb\bar{b}\bar{b}$ and $bc\bar{b}\bar{c}$ structures which may be experimentally prominent are discussed. On the other hand, the fully-heavy S-wave tetraquark production at hadron colliders is investigated and their cross sections are obtained.

PHYSICAL REVIEW LETTERS **132**, 111901 (2024)

Editors' Suggestion

New Structures in the $J/\psi J/\psi$ Mass Spectrum in Proton-Proton Collisions at $\sqrt{s} = 13$ TeV

A. Hayrapetyan *et al.**
(CMS Collaboration)

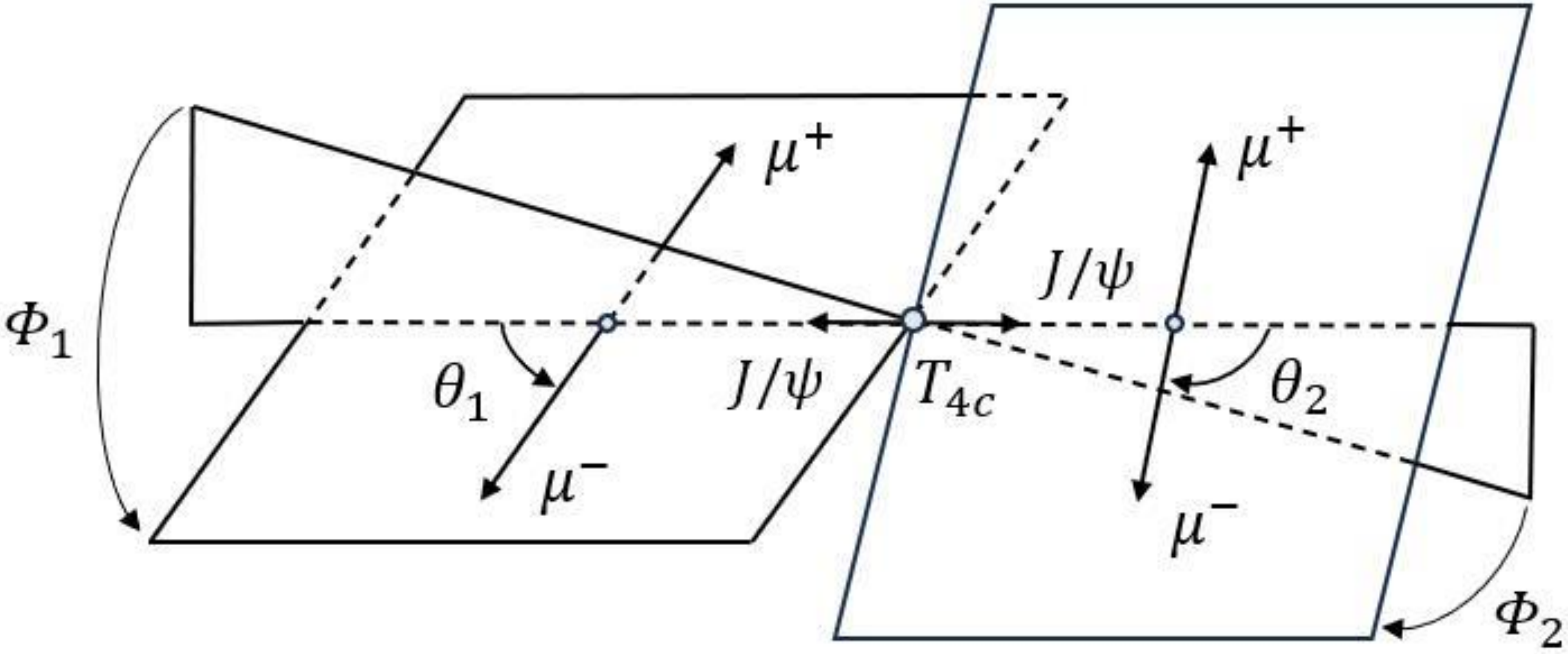
Our measured masses appear compatible with recent calculations of the $cc\bar{c}\,\bar{c}$ spectrum [21,71], which would indicate that these three structures may be a family of radial excitations of the same J^{PC}. This is the case for both no-interference and interference masses, albeit for different theoretical models.

[21] R. Zhu, Fully-heavy tetraquark spectra and production at hadron colliders, Nucl. Phys. **B966**, 115393 (2021).

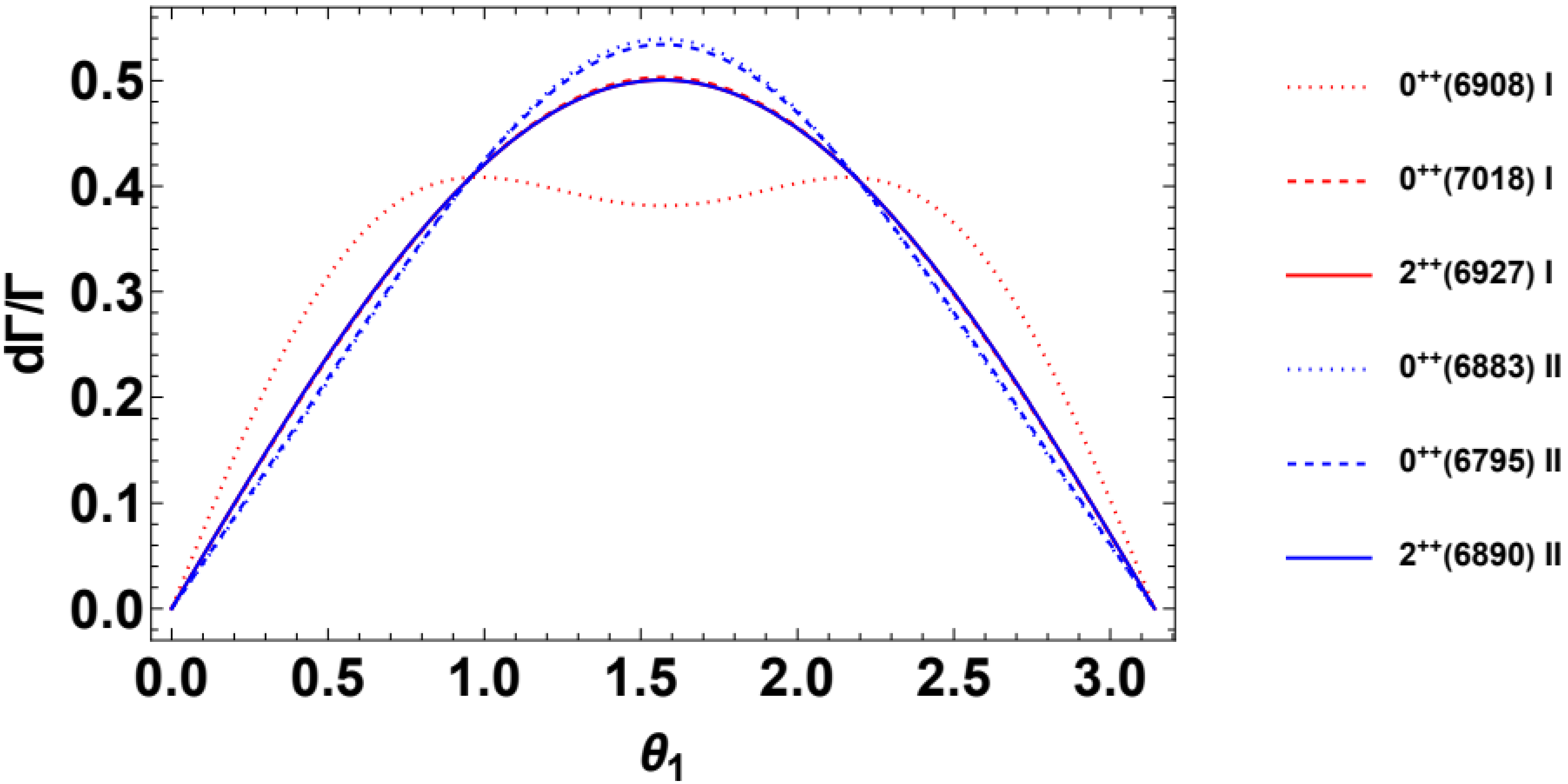
These masses are predicted correctly and confirmed by CMS.

Next, are their spin-parities correct?

spin-correlated amplitudes will tell the truth

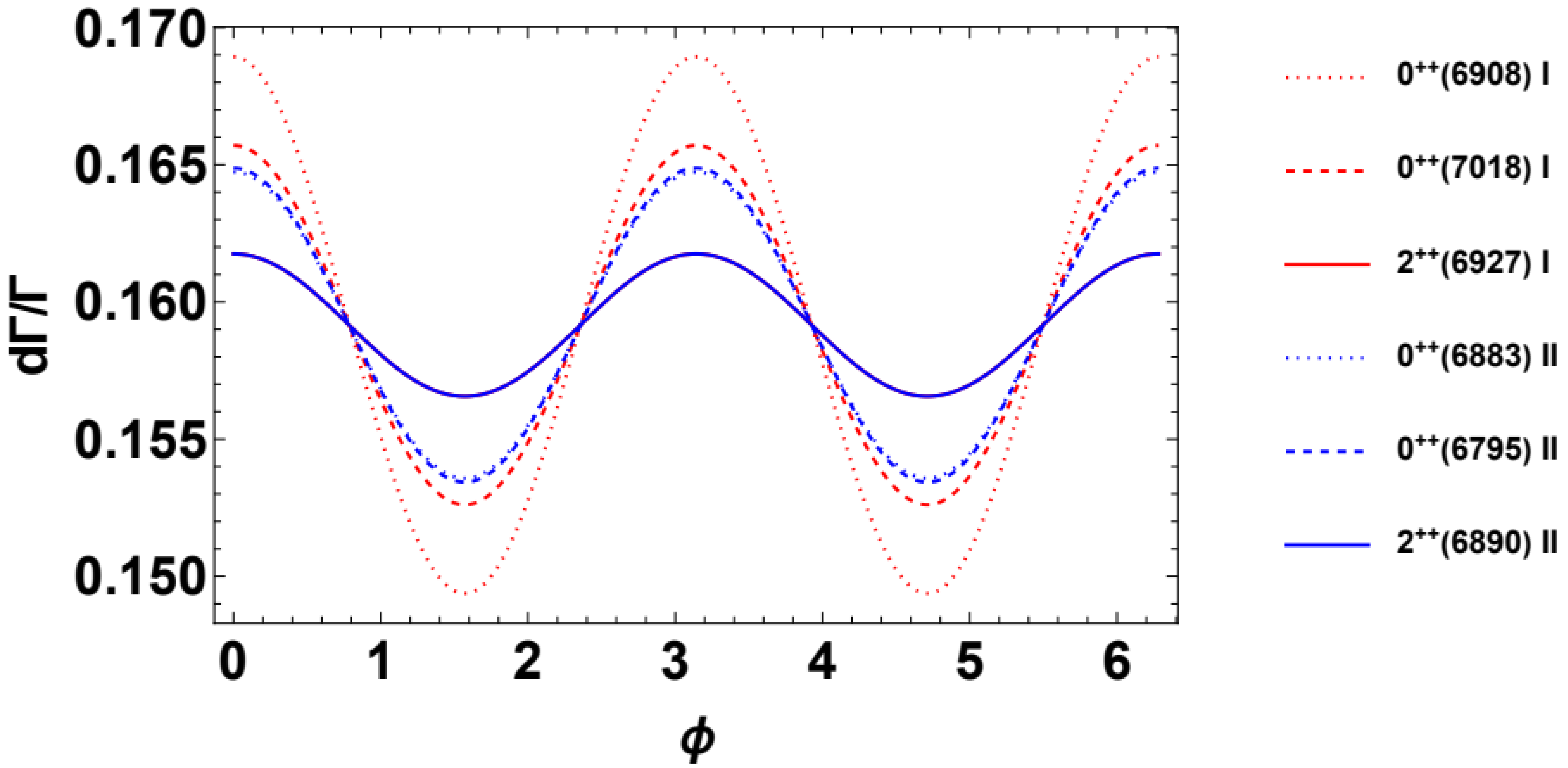


polar angular distribution

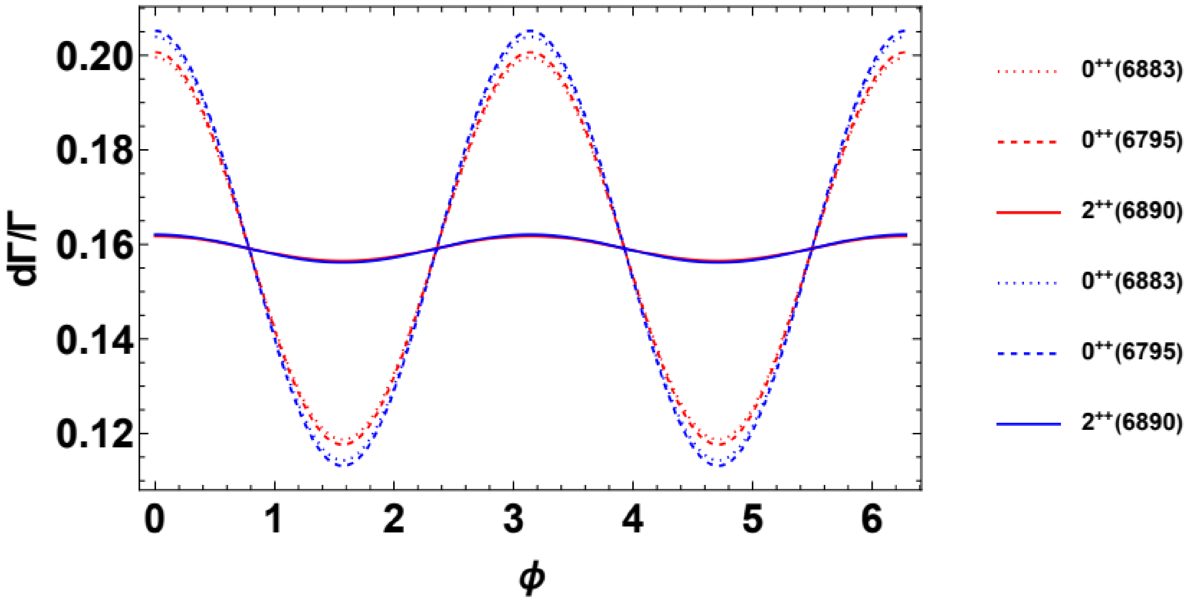


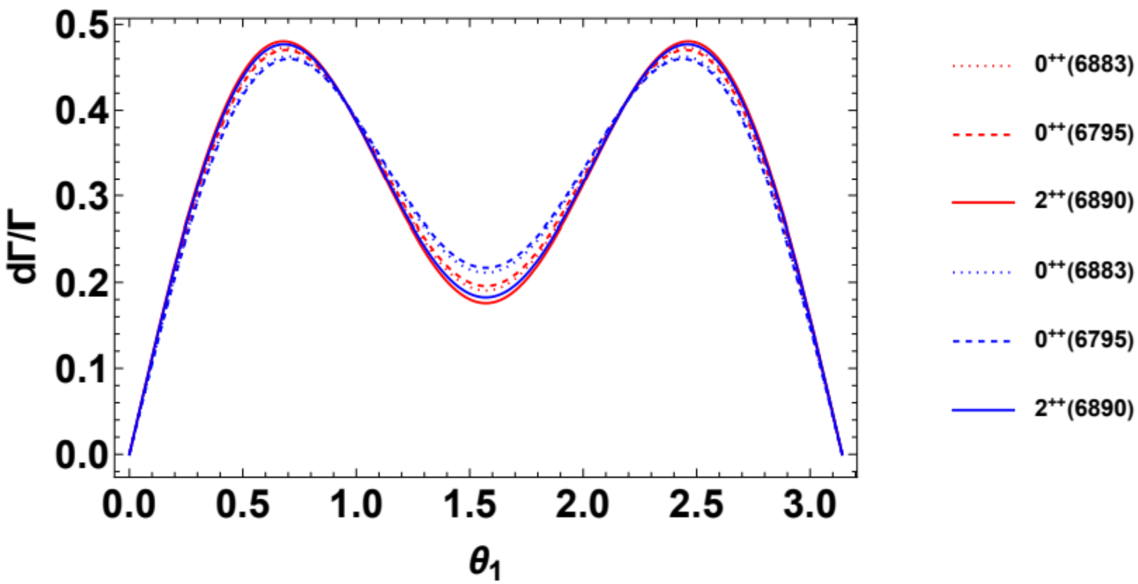
Model I: quark model (QM);
Model II: heavy quark effective theory(HQET)

plane angular distribution



fully charmed tetraquark decays to charmed meson pair





The θ_1 and Φ distributions for various tetraquarks near 6.9 GeV into $D^*(\to D\pi)$ and $\bar{D}^*(\to \bar{D}\pi)$ using Model II (HQET).

Summary and outlook

- ✓ Spin-correlation effects are involved in heavy flavor physics
- ✓ Polarization analysis are helpful to detect the vector beauty-charm meson and to determine the spin-parity of fully charmed tetraquarks

Outlook: Spin-correlation effects leads to quantum entanglement；What can quantum entanglement tell us in QCD/QFT/SM?

Thank you a lot!

Backup

Matching coefficients up to three loops

- **for vector current**

$$\mathcal{C} = 1 - 2.29\left(\frac{\alpha_s^{(n_l)}}{\pi}\right) - 35.44\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 - 1686.27\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4),$$

for $n_l = 3, n_c = 1, n_b = 0,$

Sang-Zhang-Zhou, arXiv:2210.02979

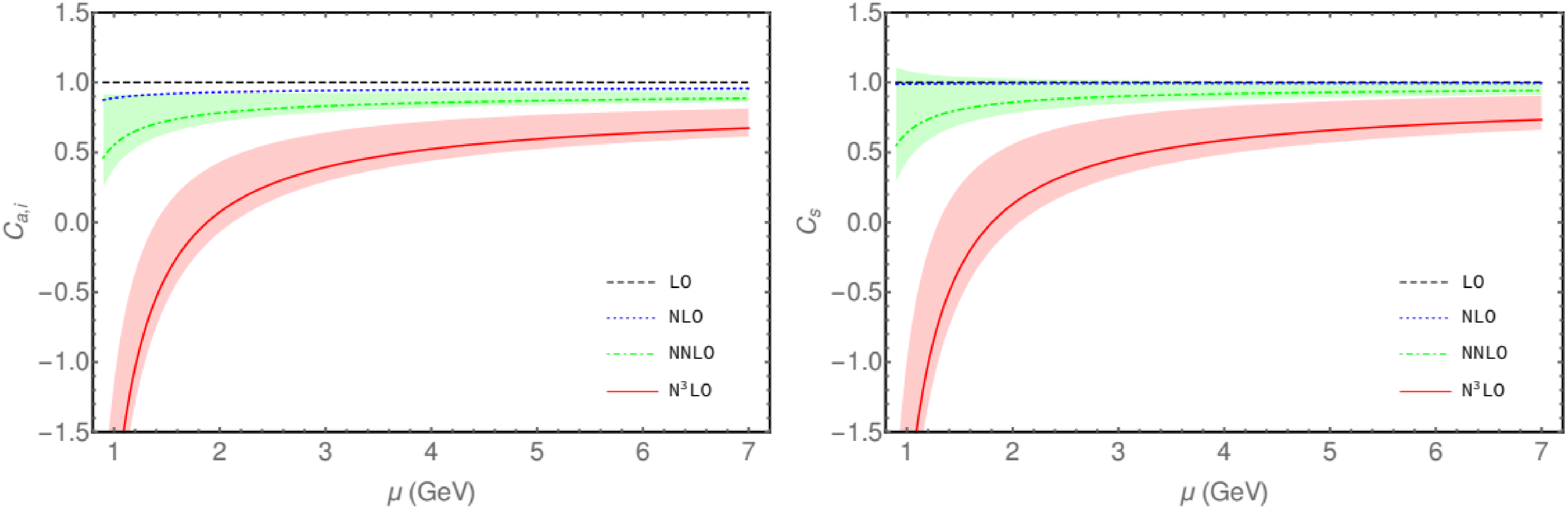
- **for pseudoscalar current**

$$\mathcal{C}(x_{\rm phys}) = 1 - 1.62623\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right) - 6.51043\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^2 - 1520.59\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

Feng-Jia-Mo-Pan-Sang-Zhang, arXiv:2208.04302

for axial-vector and scalar currents

- **Matching coefficients for axial-vector and scalar up to three loops**



Nonconvergence behaviors also in other two currents

Multi-loop integral calculation performed by AMFlow (Ma et al)

Sub-leading Contribution

➢ Relativistic corrections

$$\langle 0|\overline{Q_1}\gamma^5 Q_2\,|Q_2\overline{Q_1}\rangle_{\text{QCD}}$$

$$= \sqrt{2M_H}\left[C_0^P\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\text{NRQCD}} + C_2^P\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\cdot\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\text{NRQCD}} + \cdots\right]$$

Employing EOM: $\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\,\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle = -2m_r E\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle.$

$$f_{B_c^*} = 2\sqrt{\frac{N_c}{m_{B_c^*}}}\left[\mathcal{C}_v + \frac{d_v E_{B_c^*}}{12}\left(\frac{8}{M} - \frac{3}{m_r}\right)\right]|\Psi_{B_c^*}(0)|,$$

$$f_{B_c} = 2\sqrt{\frac{N_c}{m_{B_c}}}\left[\mathcal{C}_p - \frac{d_p E_{B_c}}{4m_r}\right]|\Psi_{B_c}(0)|,$$

Wave function scale dependence

- Wave function at origin

For Power-law potential $V(r) = Ar^a + C$

Exact solution $|\psi_\mu^n(0)|^2 = f(n,a)(A\mu)^{3/(2+a)}$

Scale relation $|\Psi_{B_c^*}(0)| = |\Psi_{J/\psi}(0)|^{1-y}\ |\Psi_\Upsilon(0)|^y,$

$$y = y_c = \ln((1 + m_c/m_b)/2)/\ln(m_c/m_b)$$

Collins-Imbo-King-Martell, PLB 393 (1997) 155–160

$$|\psi_1(0)|^2 = |\psi_1^{(0)}(0)|^2\left(1 + \sum_{k=1}^{n} f_k a_s^k\right).$$

$$\left|\psi_1^{(0)}(0)\right|^2 = \frac{(m_b C_F \alpha_s)^3}{8\pi},$$

$$E_1^{(0)} = -\frac{1}{4} m_b (C_F\alpha_s)^2,$$

Beneke et al., PRL. 112, 151801 (2014)

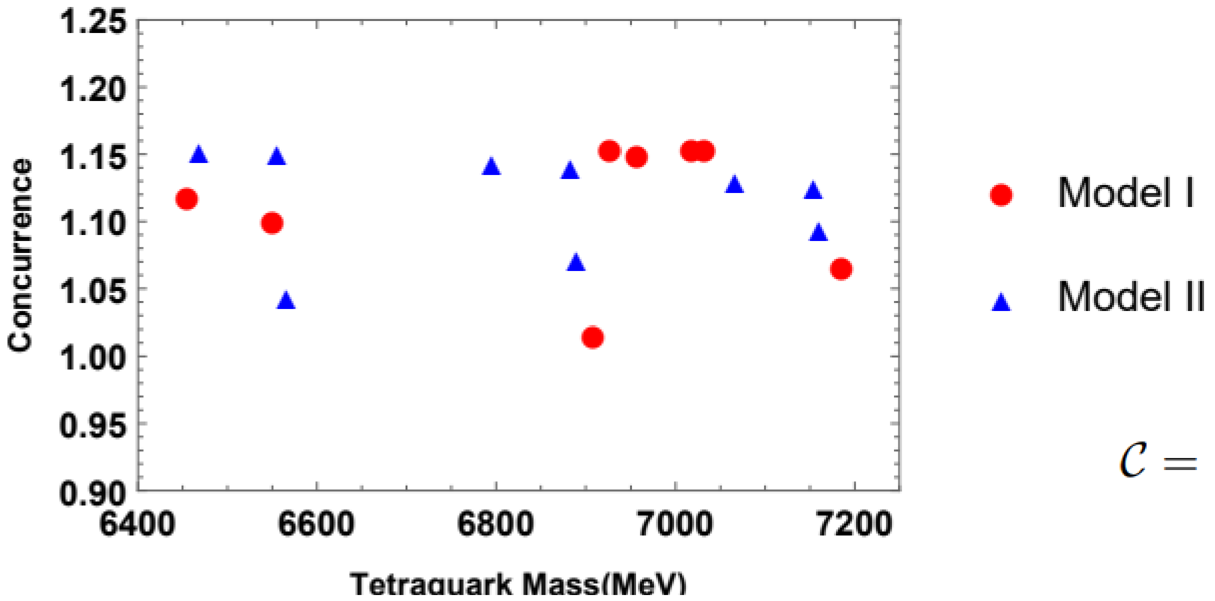
fully charmed tetraquark decay widths

Exp.	Fit method	M_{BW_1}	Γ_{BW_1}	M_{BW_2}	Γ_{BW_2}	M_{BW_3}	Γ_{BW_3}
LHCb [3]	No interf.	-	-	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	-	-
LHCb [3]	Interf.	6741 ± 6	288 ± 16	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	-	-
ATLAS [4]	Fit-A	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$
ATLAS [4]	Fit-B	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	-	-
CMS [5]	No interf.	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 184$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$
CMS [5]	Interf.	6638^{+43+16}_{-38-31}	$440^{+230}_{-200-240}$	6847^{+44}_{-28-20}	191^{+66+25}_{-49-17}	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}
Theo.	$n, {}^{2S+1}L_J, J^{PC}$	M_T	Γ_T	M_T	Γ_T	M_T	Γ_T
QM	$1, {}^1S_0, 0^{++}$	6455	20.5	-	-	-	-
	$1, {}^1S_0, 0^{++}$	6550	7.5	-	-	-	-
	$1, {}^5S_2, 2^{++}$	6524	31.0	-	-	-	-
	$2, {}^1S_0, 0^{++}$	-	-	6908	38.7	-	-
	$2, {}^1S_0, 0^{++}$	-	-	6957	83.3	-	-
	$2, {}^1S_0, 0^{++}$	-	-	-	-	7018	34.6
	$2, {}^1S_0, 0^{++}$	-	-	-	-	7185	36.9
	$2, {}^5S_2, 2^{++}$	-	-	6927	27.1	-	-
	$2, {}^5S_2, 2^{++}$	-	-	-	-	7032	670.9
HQET	$2, {}^1S_0, 0^{++}$	6555^{+36}_{-37}	15.5	-	-	-	-
	$3, {}^1S_0, 0^{++}$	-	-	6883^{+27}_{-27}	16.2	-	-
	$4, {}^1S_0, 0^{++}$	-	-	-	-	7154^{+22}_{-22}	15.9
	$2, {}^1S_0, 0^{++}$	6468^{+35}_{-35}	23.6	-	-	-	-
	$3, {}^1S_0, 0^{++}$	-	-	6795^{+26}_{-26}	21.3	-	-
	$4, {}^1S_0, 0^{++}$	-	-	-	-	7066^{+21}_{-22}	27.1
	$2, {}^5S_2, 2^{++}$	6566^{+34}_{-35}	39.4	-	-	-	-
	$3, {}^5S_2, 2^{++}$	-	-	6890^{+27}_{-26}	36.0	-	-
	$4, {}^5S_2, 2^{++}$	-	-	-	-	7160^{+21}_{-22}	29.0

fully charmed tetraquark major decay modes

Theo.	$n,^{2S+1}L_J,J^{PC},M_T$	$J/\psi J/\psi$	$H_c H_c'$	$D^{(*)}\bar{D}^{(*)}$	$D_s^{(*)}\bar{D}_s^{(*)}$	gg	$\gamma\gamma(\times 10^{-3})$
QPM[a]	$1,^1S_0,0^{++},6455$	0.7	1.45	$9.6(\frac{\xi_D}{0.1})^2$	$6.9(\frac{\xi_{D_s}}{0.1})^2$	$1.9(\frac{f_0^{\prime 1}}{100})^2$	$1.3(\frac{f_0^{\prime 1}}{100})^2$
	$1,^1S_0,0^{++},6550$	1.78	0.12	$3.0(\frac{\xi_D}{0.1})^2$	$2.1(\frac{\xi_{D_s}}{0.1})^2$	$0.5(\frac{f_0^{1}}{100})^2$	$0.3(\frac{f_0^{1}}{100})^2$
	$1,^5S_2,2^{++},6524$	-	-	$15(\frac{\xi_D}{0.1})^2$	$14.2(\frac{\xi_{D_s}}{0.1})^2$	$1.8(\frac{f_2^{2}}{100})^2$	$1.3(\frac{f_2^{2}}{100})^2$
	$2,^1S_0,0^{++},6908$	0.12	23.75	$7.6(\frac{\xi_D}{0.1})^2$	$5.5(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0^{\prime 2}}{100})^2$	$1.3(\frac{f_0^{\prime 2}}{100})^2$
	$2,^1S_0,0^{++},6957$	4.66	74.03	$2.4(\frac{\xi_D}{0.1})^2$	$1.8(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^{2}}{100})^2$	$0.3(\frac{f_0^{2}}{100})^2$
	$2,^1S_0,0^{++},7018$	1.87	18.01	$7.8(\frac{\xi_D}{0.1})^2$	$5.2(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0^{\prime 2}}{100})^2$	$1.2(\frac{f_0^{\prime 2}}{100})^2$
	$2,^1S_0,0^{++},7185$	0.48	32.25	$2.2(\frac{\xi_D}{0.1})^2$	$1.6(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^{2}}{100})^2$	$0.3(\frac{f_0^{2}}{100})^2$
	$2,^5S_2,2^{++},6927$	0.36	1.45	$12(\frac{\xi_D}{0.1})^2$	$11.6(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_2^{2}}{100})^2$	$1.3(\frac{f_2^{2}}{100})^2$
	$2,^5S_2,2^{++},7032$	7.12	640.06	$11(\frac{\xi_D}{0.1})^2$	$11(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_2^{2}}{100})^2$	$1.2(\frac{f_2^{2}}{100})^2$
HQET	$1,^1S_0,0^{++},6055$	-	-	$4.0(\frac{\xi_D}{0.1})^2$	$2.8(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_0^{\prime 1}}{100})^2$	$1.4(\frac{f_0^{\prime 1}}{100})^2$
	$1,^1S_0,0^{++},5984$	-	-	$16.4(\frac{\xi_D}{0.1})^2$	$8.7(\frac{\xi_{D_s}}{0.1})^2$	$0.5(\frac{f_0^{1}}{100})^2$	$0.4(\frac{f_0^{1}}{100})^2$
	$1,^5S_2,2^{++},6090$	-	-	$19.3(\frac{\xi_D}{0.1})^2$	$17.6(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^{1}}{100})^2$	$1.4(\frac{f_2^{1}}{100})^2$
	$2,^1S_0,0^{++},6555$	$2.6(\frac{\xi_\psi}{0.1})^2$	$6.0(\frac{\xi_{\eta_c}}{0.1})^2$	$3.0(\frac{\xi_D}{0.1})^2$	$2.1(\frac{\xi_{D_s}}{0.1})^2$	$1.8(\frac{f_0^{\prime 2}}{100})^2$	$1.3(\frac{f_0^{\prime 2}}{100})^2$
	$2,^1S_0,0^{++},6468$	$5.5(\frac{\xi_\psi}{0.1})^2$	$1.2(\frac{\xi_{\eta_c}}{0.1})^2$	$9.6(\frac{\xi_D}{0.1})^2$	$6.8(\frac{\xi_{D_s}}{0.1})^2$	$0.5(\frac{f_0^{2}}{100})^2$	$0.3(\frac{f_0^{2}}{100})^2$
	$2,^5S_2,2^{++},6566$	$8.5(\frac{\xi_\psi}{0.1})^2$	$0.01(\frac{\xi_{\eta_c}}{0.1})^2$	$14.9(\frac{\xi_D}{0.1})^2$	$14(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^{2}}{100})^2$	$1.4(\frac{f_2^{2}}{100})^2$
	$3,^1S_0,0^{++},6883$	$3.6(\frac{\xi_\psi}{0.1})^2$	$5.7(\frac{\xi_{\eta_c}}{0.1})^2+0.8(\frac{\xi_{\chi_{c0}}}{0.1})^2$	$2.6(\frac{\xi_D}{0.1})^2$	$1.8(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0^{\prime 3}}{100})^2$	$1.3(\frac{f_0^{\prime 3}}{100})^2$
	$3,^1S_0,0^{++},6795$	$6.3(\frac{\xi_\psi}{0.1})^2$	$0.7(\frac{\xi_{\eta_c}}{0.1})^2$	$8(\frac{\xi_D}{0.1})^2$	$5.9(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^{3}}{100})^2$	$0.3(\frac{f_0^{3}}{100})^2$
	$3,^5S_2,2^{++},6890$	$9.6(\frac{\xi_\psi}{0.1})^2$	$0.03(\frac{\xi_{\eta_c}}{0.1})^2$	$12.5(\frac{\xi_D}{0.1})^2$	$11.9(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^{3}}{100})^2$	$1.4(\frac{f_2^{3}}{100})^2$
	$4,^1S_0,0^{++},7154$	$4.1(\frac{\xi_\psi}{0.1})^2$	$5.3(\frac{\xi_{\eta_c}}{0.1})^2+1.8(\frac{\xi_{\chi_{c0}}}{0.1})^2+0.4(\frac{\xi_{h_c}}{0.1})^2$	$2.2(\frac{\xi_D}{0.1})^2$	$0.8(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0^{\prime 4}}{100})^2$	$1.2(\frac{f_0^{\prime 4}}{100})^2$
	$4,^1S_0,0^{++},7066$	$6.2(\frac{\xi_\psi}{0.1})^2$	$0.4(\frac{\xi_{\eta_c}}{0.1})^2+4(\frac{\xi_{\chi_{c0}}}{0.1})^2+4(\frac{\xi_{h_c}}{0.1})^2$	$7.0(\frac{\xi_D}{0.1})^2$	$5.1(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^{4}}{100})^2$	$0.3(\frac{f_0^{4}}{100})^2$
	$4,^5S_2,2^{++},7160$	$9.8(\frac{\xi_\psi}{0.1})^2$	$0.04(\frac{\xi_{\eta_c}}{0.1})^2+1.4(\frac{\xi_{h_c}}{0.1})^2$	$10.9(\frac{\xi_D}{0.1})^2$	$10.3(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^{4}}{100})^2$	$1.4(\frac{f_2^{4}}{100})^2$

Concurrence



$$\mathcal{C} = \sqrt{2(1 - \mathrm{Tr}\rho_A^2)},$$

$$max\left(0, \sqrt{\frac{1}{3}\left[\frac{h_{00}^{00} + 2h_{01}^{01} + 4h_{11}^{00} + 2h_{11}^{11}}{N} - 1\right]}\right) \leq \sqrt{2(1 - \frac{\left(h_{00}^{00}\right)^2 + 2\left(h_{11}^{11}\right)^2}{N^2})} \leq \frac{2}{\sqrt{3}}.$$

$$max\,(0, LB_2) \leq \sqrt{2(1 - \left(\frac{h_{00}^{00} + 2h_{10}^{10}}{N}\right)^2 - \left(\frac{h_{10}^{10} + h_{11}^{11} + h_{1-1}^{1-1}}{N}\right)^2 - \left(\frac{h_{10}^{10} + h_{1-1}^{1-1} + h_{11}^{11}}{N}\right)^2)} \leq \frac{2}{\sqrt{3}}.$$

If assume quantum entanglement as a basic principle, then a constraint formula for helicity amplitudes.

That may make sense because we have no strict solution in QCD.