# Reinvigorating high-$T$ 3d EFT approach for the Electroweak phase transition


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In collaboration with:
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based on [arXiv:2009.10080].

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# Intro

- Thermal history of the EW symmetry breaking is interesting!
- Pipeline from collider phenomenology to baryogenesis and gravitational wave production.
- EWPT: playground for BSM physics near EW scale, with relatively light fields and strong enough couplings to Higgs $\rightarrow$ collider targets!

# Primordial gravitational waves from 1st order transition

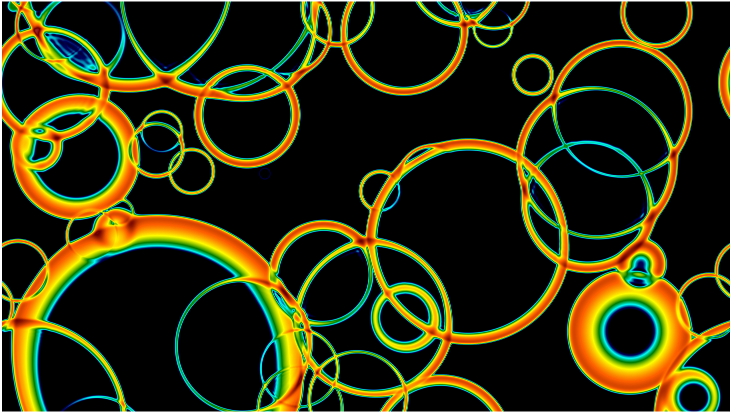
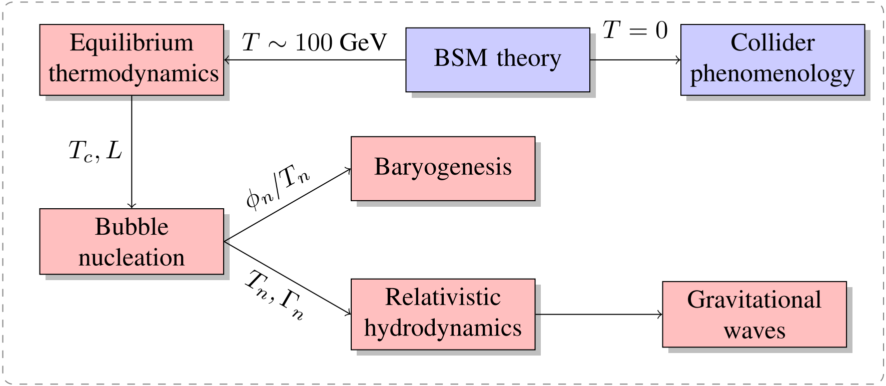


Figure by David Weir.

- Relativistic hydrodynamic simulations: how do we relate these to BSM models?

# Pipeline: EWPT in BSM theories.



# Outline

- State-of-art thermodynamics: dimensionally reduced 3-d EFT's and combination of perturbative and non-perturbative methods.
- This talk: IR resummation and 2-loop effective potential in perturbation theory.
- Beyond leading order daisy resummation, construction of high-$T$ EFTs and resummed effective potential.

Thermodynamic parameters from perturbation theory: **thermal effective potential** (free energy) and 3d action (nucleation rate, semi-classical approximation).

At high temperatures, effective expansion parameter for light bosons is

$$\frac{g^2}{e^{E/T}-1} \approx \frac{g^2 T}{E} \geq \frac{g^2 T}{m}.$$

So for $m \lesssim g^2 T$, the perturbative expansion breaks down:
Light bosons are nonperturbative at finite temperature! (Linde's IR problem.)

# Bubble integral ($D = d + 1 = 4 - 2\epsilon$ )

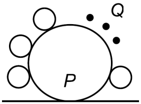
$$
\begin{aligned}
&\sum\!\!\!\!\!\!\int_P \frac{1}{P^2 + m^2} \\
&= \underbrace{\left(\frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \int \frac{\mathrm{d}^D p}{(2\pi)^D} \frac{1}{p^2 + m^2}}_{T=0,\ \text{UV-div.}} + \underbrace{\int_p \frac{n_B(E_p, T)}{E_p}}_{T\neq 0,\ \text{UV-finite, IR-sensitive}} \qquad (1) \\
&= \underbrace{\int_p \frac{T}{p^2 + m^2}}_{\text{UV-finite, IR-sensitive}} + \underbrace{\sum\!\!\!\!\!\!\int_P{}' \frac{1}{P^2 + m^2}}_{\text{UV-div., IR-safe}}\,, \qquad (2)
\end{aligned}
$$

$$
\int_p \equiv \left(\frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \int \frac{\mathrm{d}^d p}{(2\pi)^d}, \qquad (3)
$$

$$
n_B(E_p, T) = 1/(e^{E_p/T} - 1) \qquad (4)
$$

# Daisy diagrams mixing soft/hard modes

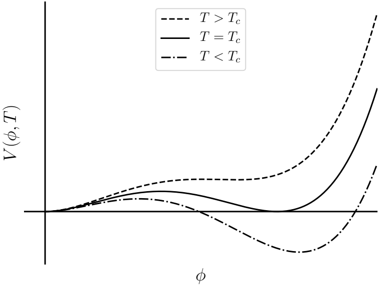
When the inner loop is a zero-mode, i.e. has a soft momenta $P = (0, \mathbf{p})$ and all $N$ outer loops or petals have hard momenta $Q$ with non-vanishing Matsubara frequencies:




$$\propto g^{2N} \left[ \int_p \frac{T}{(p^2 + m_T^2)^N} \right] \left[ \sum\!\!\!\!\!\!\!\int_Q{}' \frac{1}{Q^2} \right]^N \propto m_T^3 T \left( \frac{gT}{m_T} \right)^{2N} , \tag{5}$$

This contribution is of order $\mathcal{O}(g^3)$ for any $N$. Furthermore, it is IR-divergent for $N \geq 2$ in the limit of vanishing mass.

# Effective potential




$$V_{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \# T^2)\phi^2 + \frac{1}{4}\lambda^2 + \#\phi^3 + \dots$$

# Computation of effective potential

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}} + V_{\text{1-loop}} \,, \tag{6}$$

$$V_{\text{1-loop}} = \underbrace{\frac{1}{2}\Big(\frac{\bar{\mu}^2 e^{\gamma_{\text{E}}}}{4\pi}\Big)^{\epsilon} \int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln(p^2 + m^2)}_{\equiv V_{\text{CW}}(m)} \underbrace{- \int_p T \ln\Big(1 \mp n_{\text{B/F}}(E_p, T)\Big)}_{V_{T,b/f}\left(\frac{m^2}{T^2}\right)} \,, \tag{7}$$

$$V_{\text{1-loop}} = \underbrace{\frac{T}{2}\int_p \ln(p^2 + m^2)}_{\equiv V_{\text{soft}}(m)} + \underbrace{\frac{1}{2} \sum\!\!\!\!\!\!\!\int_{P/\{P\}}' \ln(P^2 + m^2)}_{\equiv V_{\text{hard}}(m)} \,. \tag{8}$$

Re-organisation of perturbation theory (to cancel daisy diagrams mixing soft/hard modes order-by-order):

$$\mathcal{L} = (\mathcal{L}_{\text{free}} + \Pi_T \phi_0^\dagger \phi_0) + (\mathcal{L}_{\text{interaction}} - \Pi_T \phi_0^\dagger \phi_0), \tag{9}$$

Resummation of soft parts:

$$T J_{\text{soft}}(m) = -\frac{T}{12\pi}(m^2)^{\frac{3}{2}} \to T J_{\text{soft}}^{\text{resummed}}(m) = -\frac{T}{12\pi}(\underbrace{m^2 + \Pi_T}_{m_T^2})^{\frac{3}{2}} , \tag{10}$$

$$J_{\text{daisy}}(m) \equiv J_{\text{soft}}^{\text{resummed}}(m) - J_{\text{soft}}(m) = -\frac{T}{12\pi}\Big((m^2 + \Pi_T)^{\frac{3}{2}} - (m^2)^{\frac{3}{2}}\Big) , \tag{11}$$

$$V_{\text{eff}}^{\text{A-E res.}}(\phi, T, \bar{\mu}) = V_{\text{tree}} + V_{\text{CW}} + V_T + V_{\text{daisy}} . \tag{12}$$

$$V_{\text{eff}}^{\text{A-E res.}}(\phi, T, \bar{\mu}) = V_{\text{tree}} + V_{\text{CW}} + V_T + V_{\text{daisy}} \ . \tag{13}$$

$$V_{\text{eff}}^{\text{resummed}}(\phi, T, \bar{\mu}) = V_{\text{tree}} + V_{\text{soft}}^{\text{resummed}} + V_{\text{hard}} \ , \tag{14}$$

These expressions are equal, if only masses are resummed! (A-E resummation resums at leading order.)

However, resummation (thermal screening) could be included to couplings and fields as well (hints of dimensional reduction to EFT of soft zero modes only! )

$$V_{\text{eff}}^{\text{resummed}}(\phi, T, \bar{\mu}) = \underbrace{V_{\text{tree}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{soft}}^{\text{resummed}}}_{\mathcal{O}(g^3)} + \underbrace{V_{\text{hard}}}_{\mathcal{O}(\phi^2 g^2)+\mathcal{O}(\phi^4 g^4)} , \tag{15}$$

Power counting $\mu^2 \sim g^2 T^2$, $\lambda \sim g^2$.

$$V_{\text{eff}} \simeq \underbrace{\frac{1}{2}( \underbrace{-\mu^2}_{\text{tree-level}} + \underbrace{\# T^2}_{\text{1-loop}})}_{\mathcal{O}(g^2)} \phi^2 + \underbrace{(\text{2-loop})}_{\mathcal{O}(g^4)} \phi^2 \dots$$

Full $\mathcal{O}(g^4)$ requires 2-loop thermal masses! (also one-loop field renormalisation contributes at $\mathcal{O}(g^4)$)

Caution: to cancel running of $\# T^2 \phi^2$ one-loop pieces, log-terms in 2-loop piece are crucial!

# Resummation beyond leading order?

How to compute full $\mathcal{O}(g^4)$ accurate result?

We need: two-loop thermal masses, one-loop field renormalisation corrections, consistent resummation of soft pieces...

# Answer: make use of the thermal mass hierarchy and 3d EFT approach

- Matsubara decomposition:

$$\phi(\tau, \mathbf{x}) = T \sum_n \tilde{\phi}(\mathbf{p}) e^{i\omega_n \tau}, \ \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

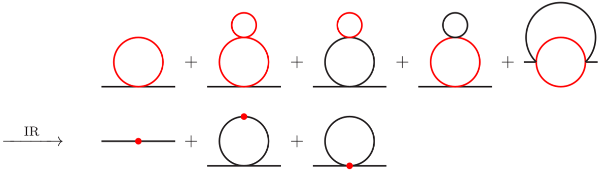
- Propagators: $\dfrac{1}{\mathbf{p}^2 + m^2 + \omega_n^2}$

- Modes with $\omega_n \neq 0$ are heavy and decouple at distances $\gg 1/T \rightarrow$ can be integrated out! (dimensional reduction)

1/T 

# Dimensional reduction

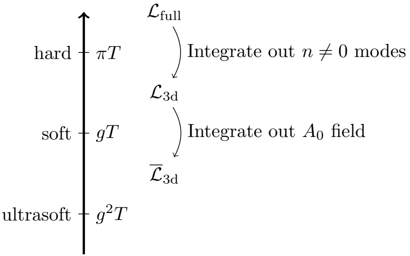
- Construct 3-d effective theory for zero modes.
- Zero modes are screened by the heavy excitations in the plasma.
- Construction of EFT is perturbative but IR safe! Incorporates thermal resummations!



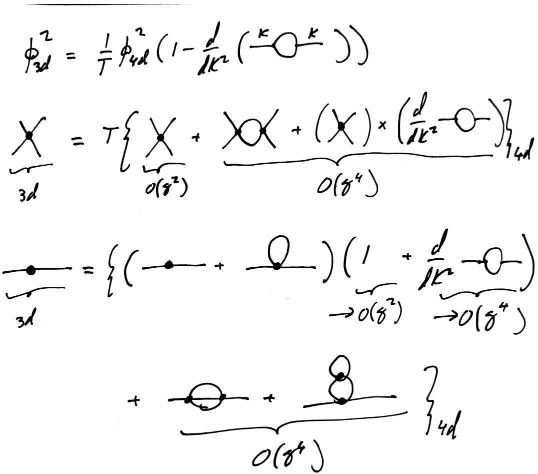
# Dimensional reduction

- Gauge field temporal component $A_0$ gets screened and can be integrated out as well: $m_D \sim gT$.

- Final EFT of light bosonic modes (gauge fields + scalar zero modes **only**) is non-perturbative $\rightarrow$ requires lattice Monte Carlo simulations!

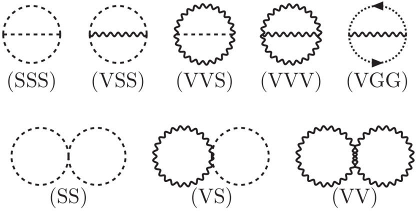


# Dimensional reduction: matching correlators



# Two-loop effective potential in 3d EFT [1]

Easy in the EFT: just 3d integrals instead of sumintegrals!




General electroweak 3d EFT literature: [hep-ph/9508379], [hep-ph/9501375], [hep-ph/9707415] etc. (A lot of hot QCD DR literature too!)

[1] [hep-ph/9404201], [hep-ph/9406268], [arXiv:2005.11332].

# Compare effective potentials in perturbation theory

$$T\ V^{\rm 3d}_{\rm eff} \simeq V^{\rm 4d}_{\rm eff}\ , \tag{16}$$

$$T\ \Big(V^{\rm 3d}_{\rm tree} + V^{\rm 3d}_{\rm loops}\Big) \simeq V^{\rm 4d}_{\rm tree} + V^{\rm 4d}_{\rm hard} + V^{\rm 4d}_{\rm soft,\ resummed}\ . \tag{17}$$

$$T\ V^{\rm 3d}_{\rm tree} \simeq V^{\rm 4d}_{\rm tree} + V^{\rm 4d}_{\rm hard} \tag{18}$$

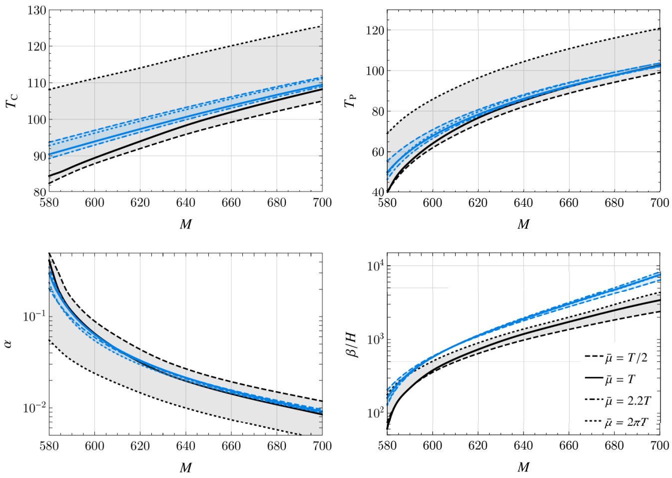
$$T\ V^{\rm 3d}_{\rm loops} \simeq V^{\rm 4d}_{\rm soft,\ resummed}\ . \tag{19}$$

$$T\ V^{\rm 3d}_{\rm tree} \approx T^4(\#g^2 + \#g^4 + \dots) \tag{20}$$

$$T\ V^{\rm 3d}_{\rm loops} \approx T^4(\#g^3 + \#g^4 + \dots) \tag{21}$$

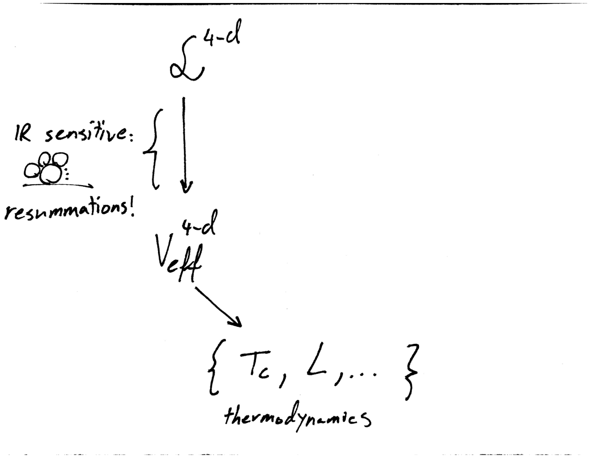
# Is $\mathcal{O}(g^4)$ accuracy important? ... Yes!

Dependence on $\bar{\mu}$ in the 3d approach and the 4d approach



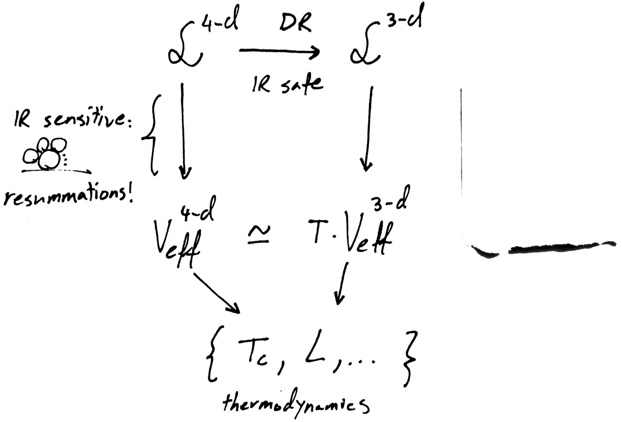

Toy(ish) model: $SM + \frac{1}{M^2}(\phi^\dagger\phi)^3$ ("SMEFT").

# How do I resum thee? Let me count the ways.



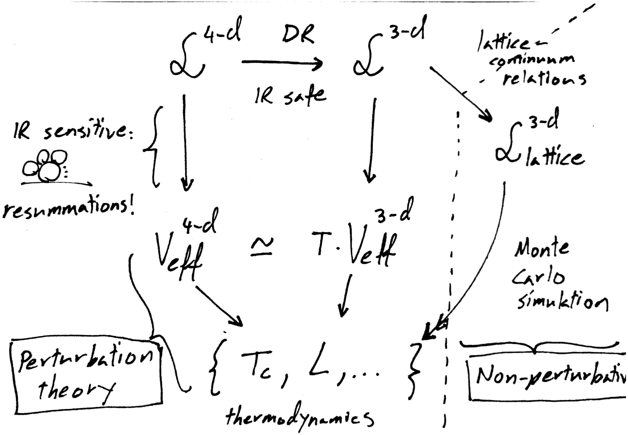

- Resummations in 4-d $V_{\text{eff}}$ are only included at leading order.

# How do I resum thee? Let me count the ways.




- DR takes care of resummations in systematic manner, but pertubation theory still cannot be expected to work near $T_c$.

# How do I resum thee? Let me count the ways.




- Lattice Monte Carlo provides appropriate non-perturbative treatment, but DR step is still perturbative. Caution: higher dimensional operators in EFT, if large portal couplings!

# Reduction to 3-d EFT is very economical

- High-$T$ resummations are implemented automatically in DR.
- Universality: many different 4-d theories map into same 3-d EFT.
- 4-d simulations (with fermions) are not available, while DR + 3-d simulations have many strenghts: relations to 4-d physics perturbative (in DR), exact lattice-continuum relations due to super-renormalisability, fewer length scales in 3-d.
- In 3-d perturbation theory: consistent nucleation, gauge invariance... See [arXiv:2009.10080]!

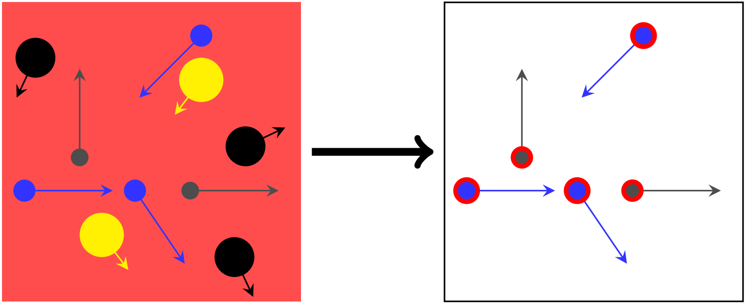
# Summary

- Dimensional reduction and use of 3d EFT is arguably the best way to organise thermal resummations and attack the IR problem:

  $\rightarrow$ Only known fully consistent way to perform simulations!

  $\rightarrow$ Furthermore: should be preferred even for purely perturbative calculations! (Simulations are demanding and take ages so perturbation theory has to be used as first approximation!)

- Theoretically sound methods are required to make pipeline between collider phenomenology and thermal history of the Universe quantitatively reliable.



Thanks!