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Gravitational waves from strongly supercooled phase transitions

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Based on work done in collaboration with J. Ellis, M. Lewicki, C. Marzo, L. Marzola, J. M. No and A. Racioppi.

Content

M. Lewicki and VV, 2007.04967:

1. Gravitational wave spectrum from scalar field gradients in a first order phase transition

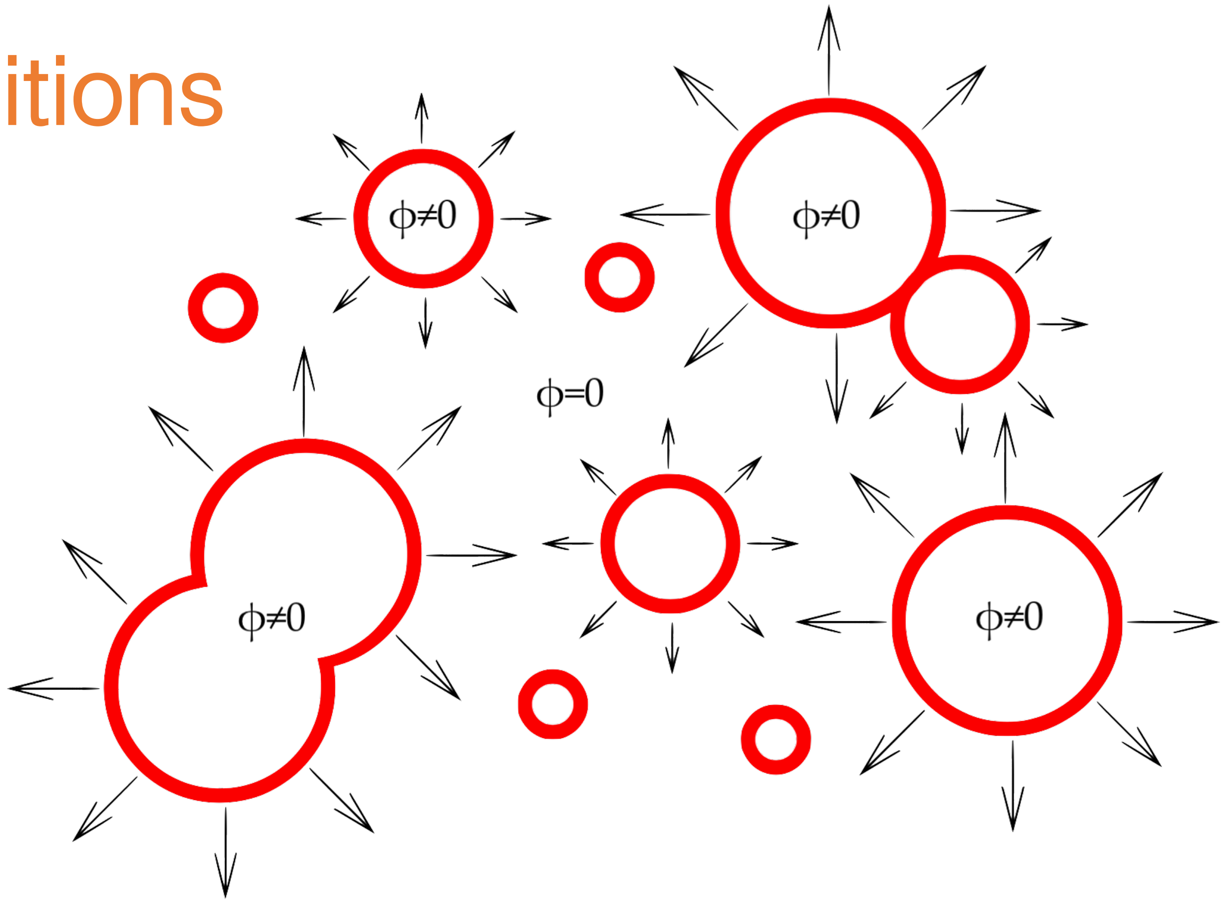
J. Ellis, M. Lewicki and VV, 2007.15586:

2. Energy budget of strongly supercooled phase transitions and detectability of the produced GW background

Gravitational wave spectrum from scalar field gradients in a first order phase transition

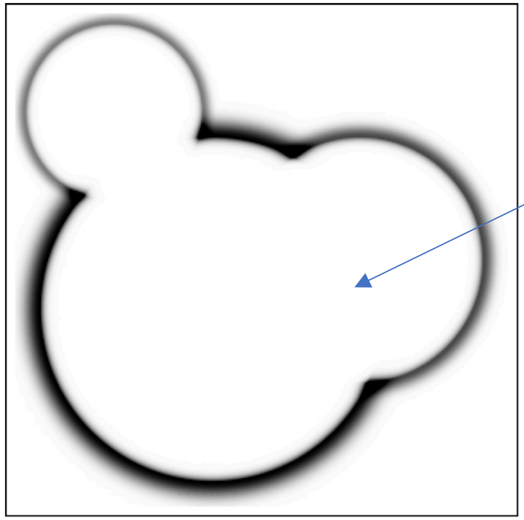
First order phase transitions

- Predicted by many extensions of the SM.
- Proceed via nucleation and expansion of bubbles of the new phase.

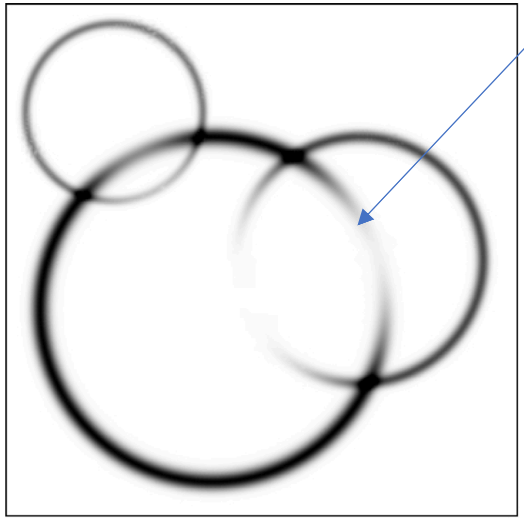


Gravitational waves are sourced by
1. plasma motions and
2. <u>scalar field gradients.</u>

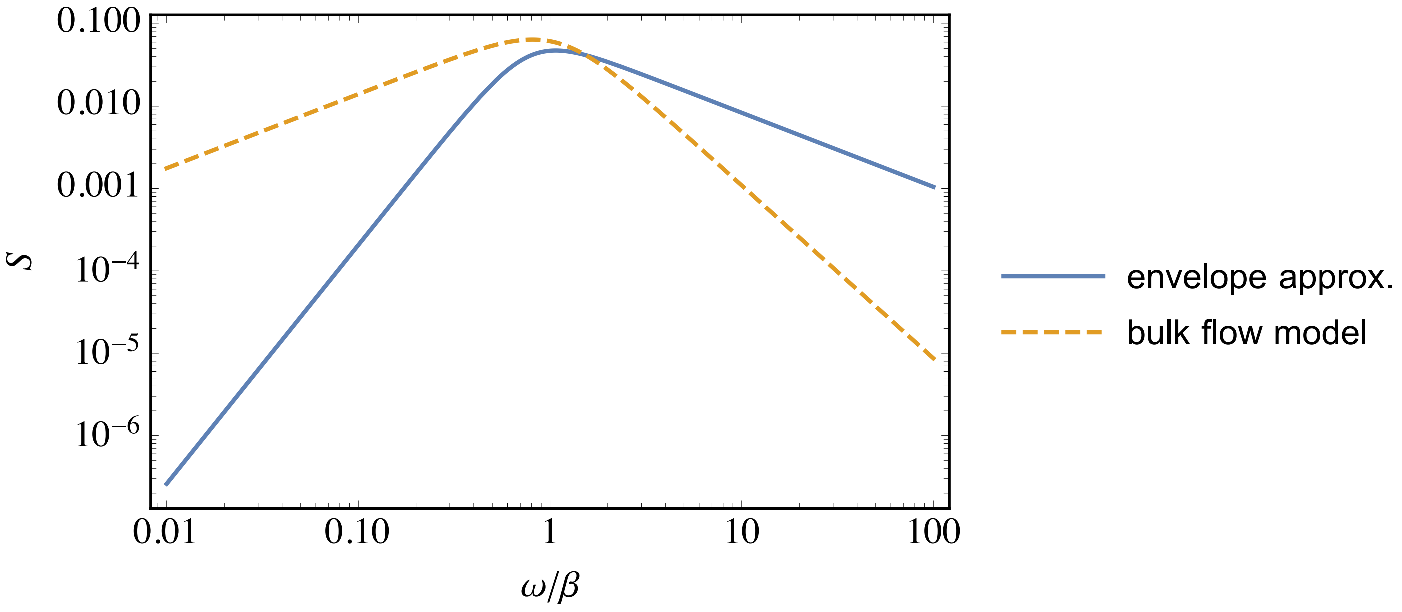
Review of previous studies



A. Kosowsky and M. S. Turner, astro-ph/9211004, (1993): envelope approximation for the GW signal from scalar field gradients. Neglects collided parts.



T. Konstandin, 1712.06869, (2018): bulk flow model to account for collided fluid shells. Assumes $\propto R^{-2}$ scaling of the surface energy density after collision.



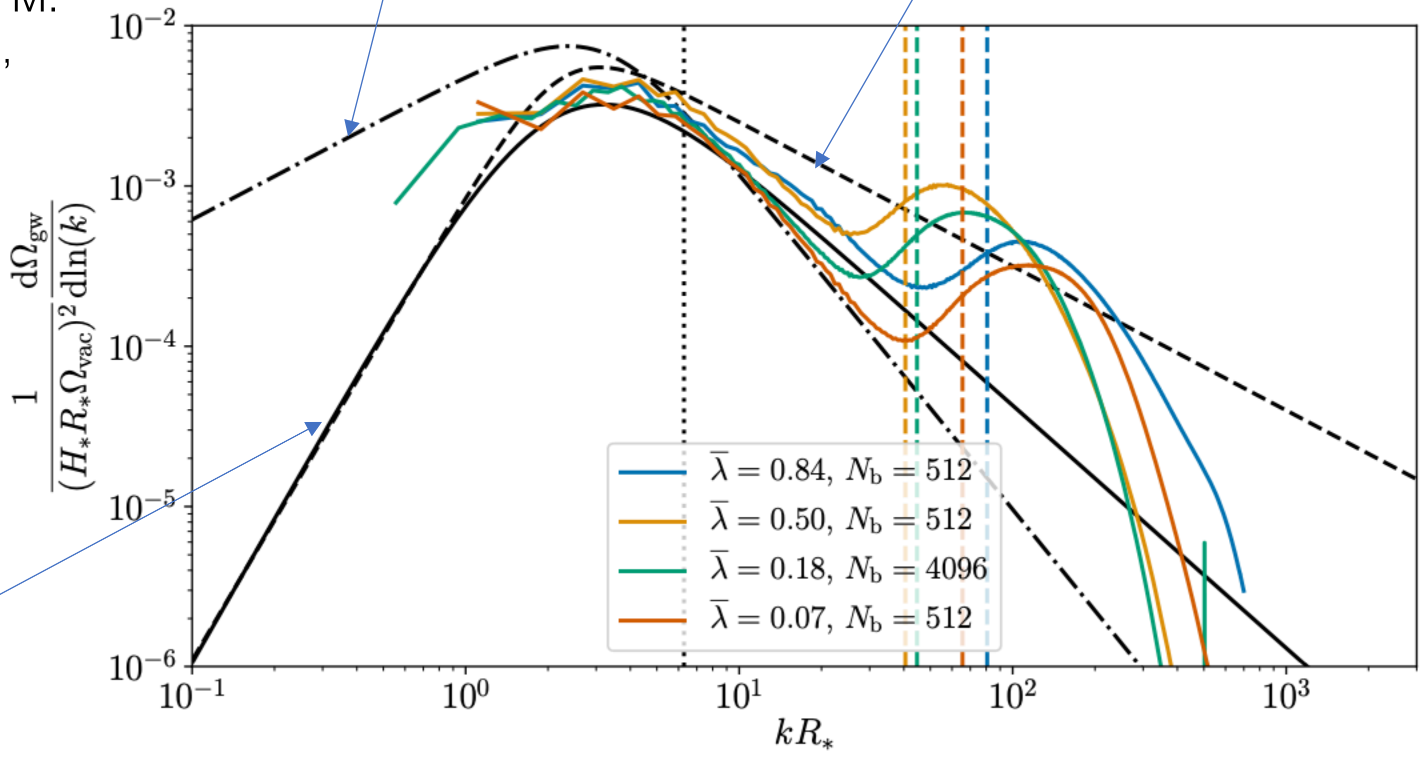
Review of previous studies

3D lattice simulations solving the scalar field equations of motion: computationally very expensive as bubbles are large compared to their wall thickness.

D. Cutting, E. G. Escartin, M. Hindmarsh and D. J. Weir, 2005.13537:

bulk flow model

envelope approx.



Fit assuming $\propto \omega^3$ low-frequency tail

GWs from scalar field gradients

M. Lewicki and V. Vaskonen, 2007.04967:

Plan: improve the envelope approximation by taking the collided parts into account.

1. Study scaling of the gradient energy after collision in two-bubble lattice simulations.
2. Calculate the GW spectrum in thin-wall approximation by performing many-bubble simulation.

GWs from scalar field gradients

The abundance of GWs is

$$\Omega_{\rm GW}(\omega) = \left(\frac{H}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 S(\omega)\,, \qquad S(\omega) = \left(\frac{\omega}{\beta}\right)^3 \frac{3\beta^5}{8\pi V_s} \underbrace{\int {\rm d}\Omega_k \left(|C_+|^2 + |C_\times|^2\right)}_{\propto V_s/\beta^5}$$

$\Gamma \propto e^{\beta t}$

$\alpha = \dfrac{\Delta V}{\rho_{\rm tot} - \Delta V}$

where in the thin wall limit

$$C_{+,\times}(\hat{\boldsymbol{k}},\omega) \approx \frac{1}{6\pi}\sum_{n=1}^{N}\int_{t_n} {\rm d}t\,{\rm d}\Omega_x\,\sin^2\theta_x\, g_{+,\times}(\phi_x) f(t,t_{n,c}) R_n^3\, e^{i\omega(t-z_n-R_n\cos\theta_x)}$$

$R_n \approx t - t_n$

sum over all bubbles

nucleation time

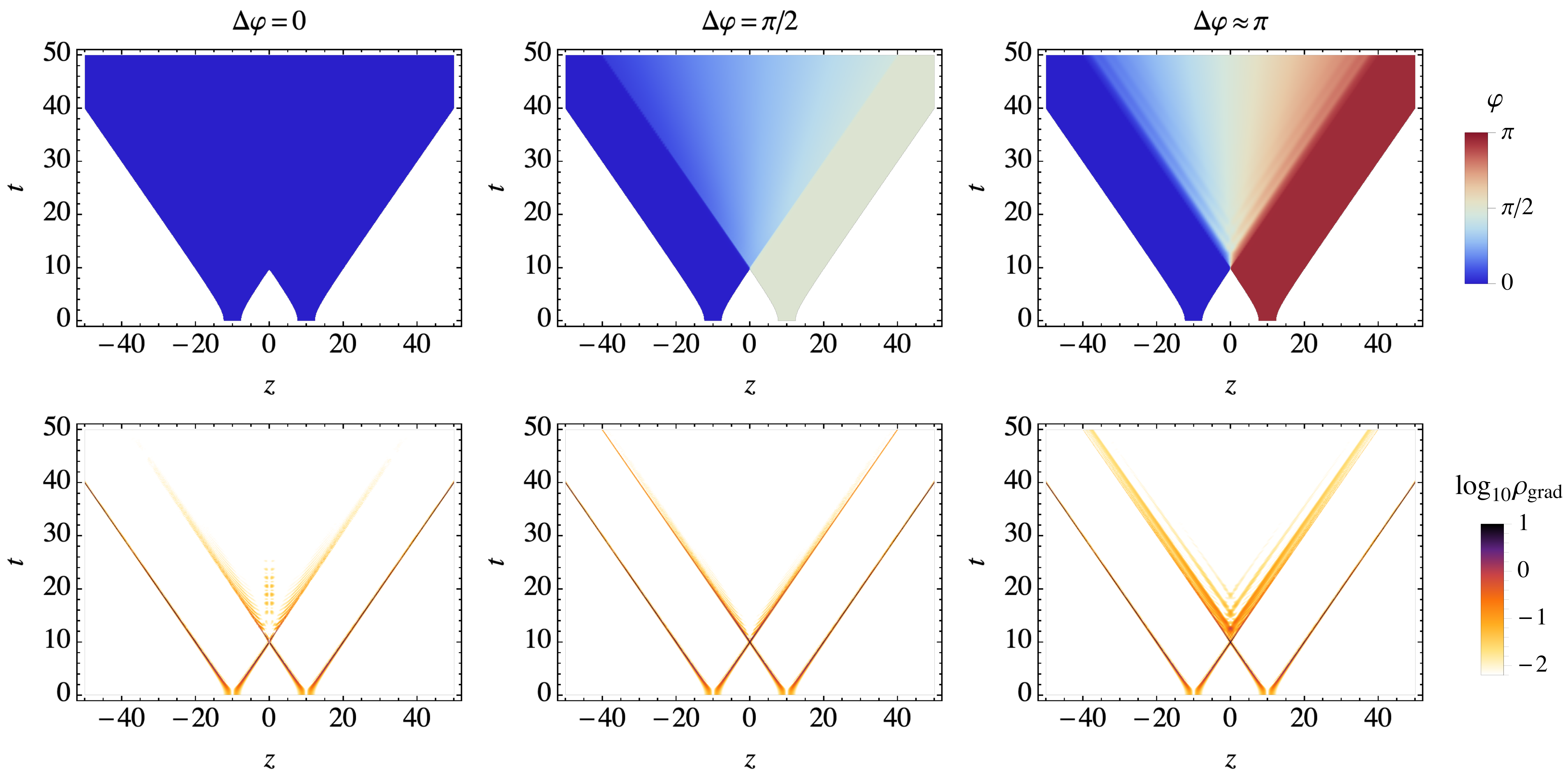
Time when the wall element collides with another bubble wall

Envelope approx.: $f(t,t_{n,c}) = \theta(t_{n,c}-t)$

Two-bubble collisions

Complex scalar field with U(1) symmetric potential:

$$\Box\phi + \frac{\mathrm{d}V}{\mathrm{d}\phi^*} = 0$$



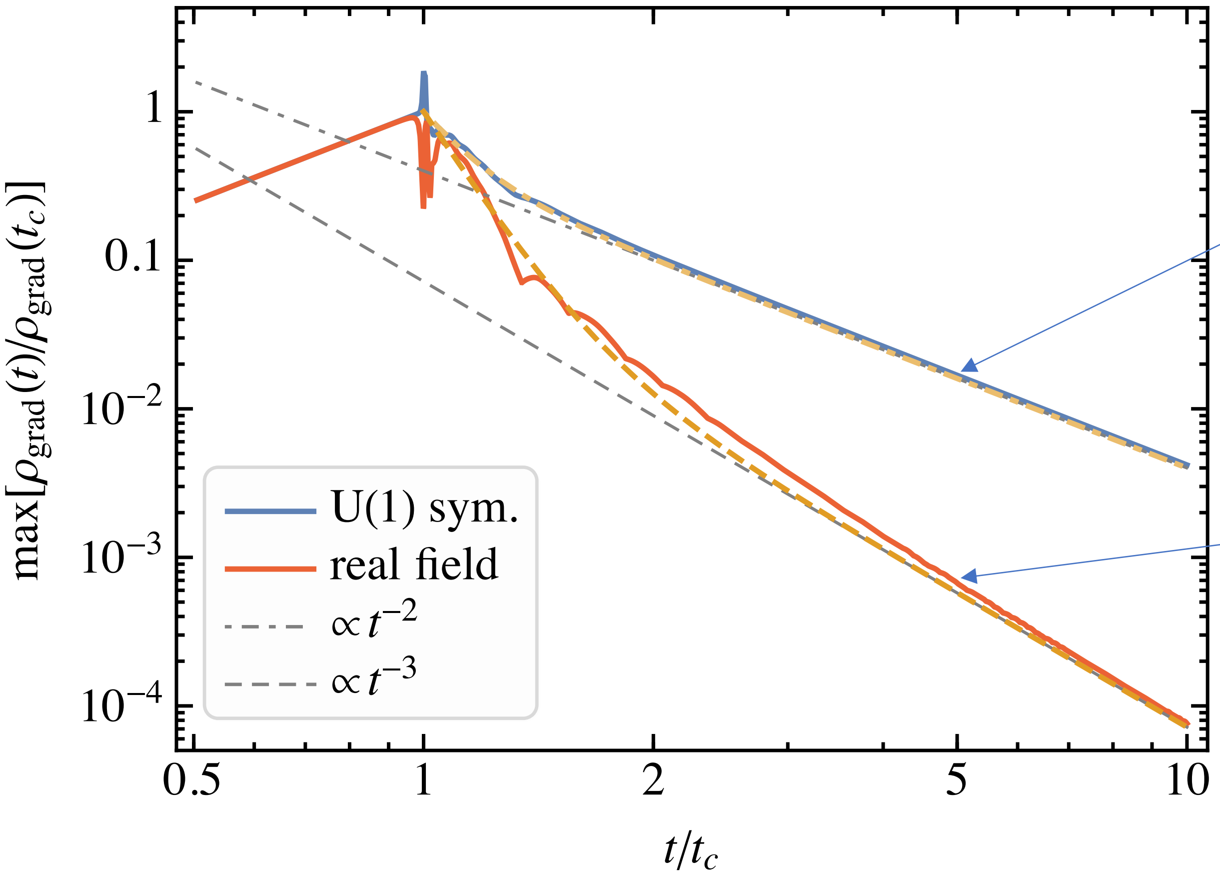
Complex phase of the scalar field

$$\phi \propto e^{i\varphi}$$

Gradient energy density

$$\rho_{\mathrm{grad}} = \frac{1}{2}|\partial_z\phi|^2$$

Evolution of the maximum of the gradient energy density:

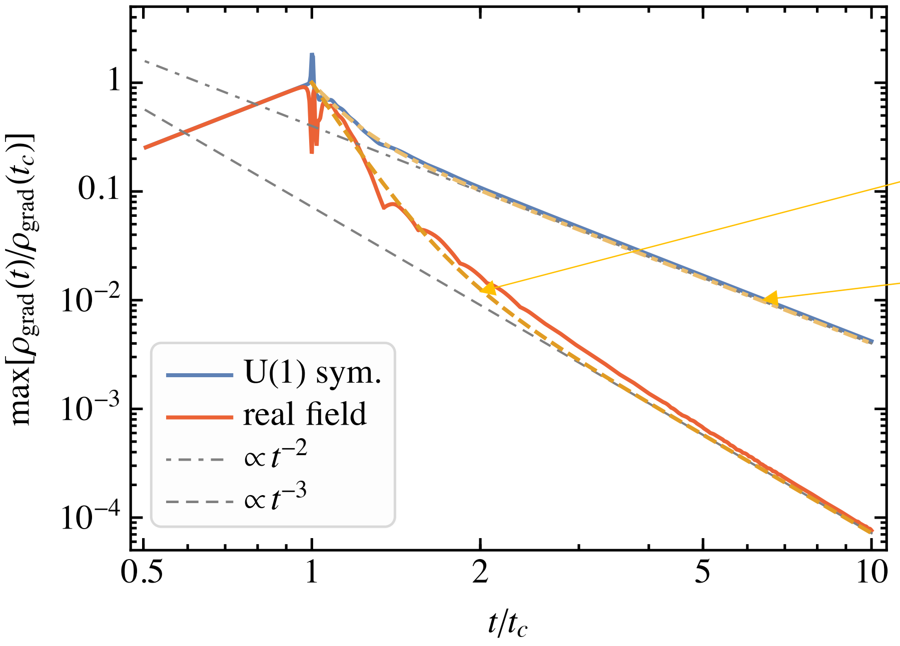


Average over different initial complex phase differences.

$\Delta\varphi = 0$

wall thickness

$$f(t, t_c) = \frac{L}{R} \frac{\max[\rho_{\rm grad}(t)]}{\max[\rho_{\rm grad}(t_c)]}$$



broken power-law fit:

from $\xi = 8$ to $\xi = 3$

from $\xi = 3$ to $\xi = 2$

Before collision:

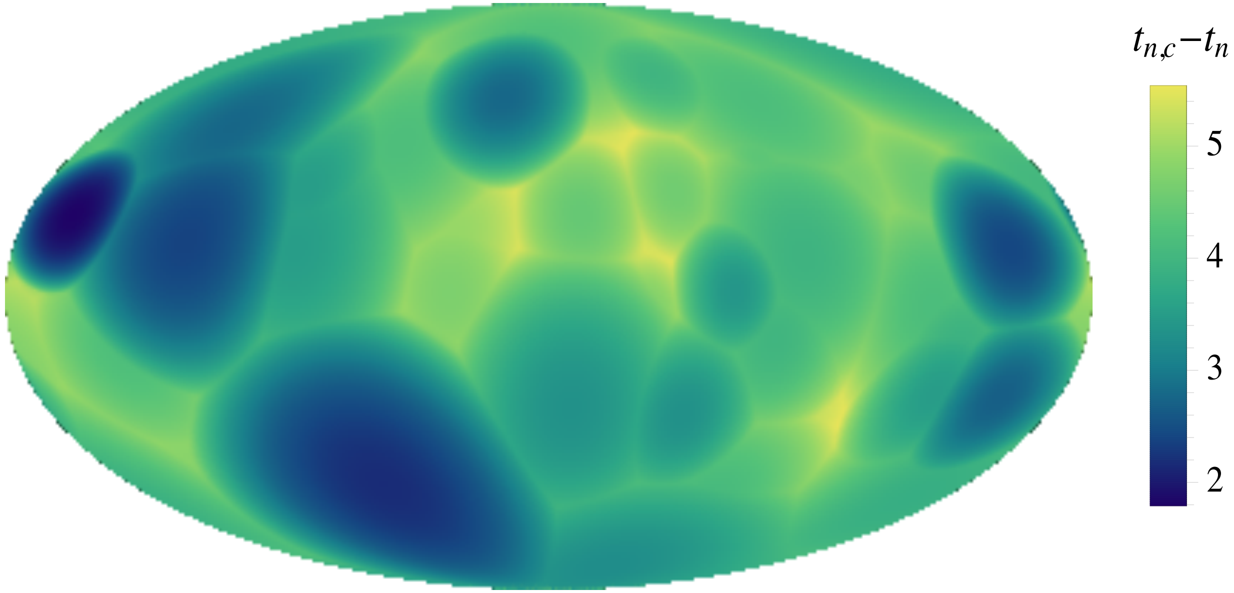
$$L \propto 1/R, \quad \max[\rho_{\rm grad}] \propto R^2 \quad \Longrightarrow \quad f(t < t_c) = 1.$$

After collision:

$$L = \text{const.}, \quad \max[\rho_{\rm grad}] \propto R^{-\xi} \quad \Longrightarrow \quad f(t > t_c) \propto R^{-(\xi+1)}$$

Bulk flow model: $\quad \xi = 2$

Many-bubble simulation



1. Nucleate bubbles according to the rate

$$\Gamma \propto e^{\beta t}$$

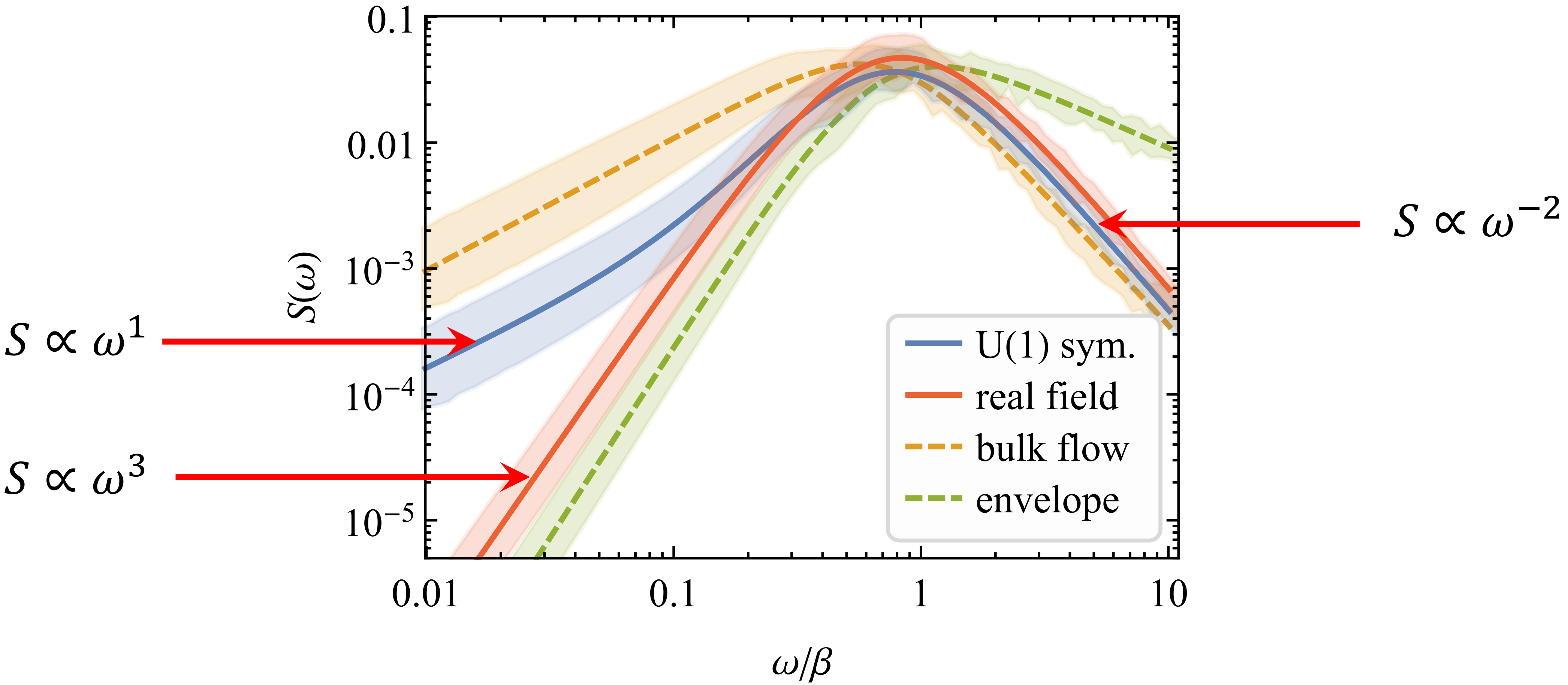
2. Discretize the surface of each bubble and find the moment when a given wall element collides with another bubble wall.

3. Evaluate the functions (time integral can be done analytically for a (broken) power-law f)

$$C_{+,\times}(\hat{\boldsymbol{k}}, \omega) \approx \frac{1}{6\pi} \sum_{n=1}^{N} \int_{t_n} \mathrm{d}t \, \mathrm{d}\Omega_x \, \sin^2\theta_x \, g_{+,\times}(\phi_x) f(t, t_{n,c}) R_n^3 \, e^{i\omega(t - z_n - R_n \cos\theta_x)}$$

GW spectrum

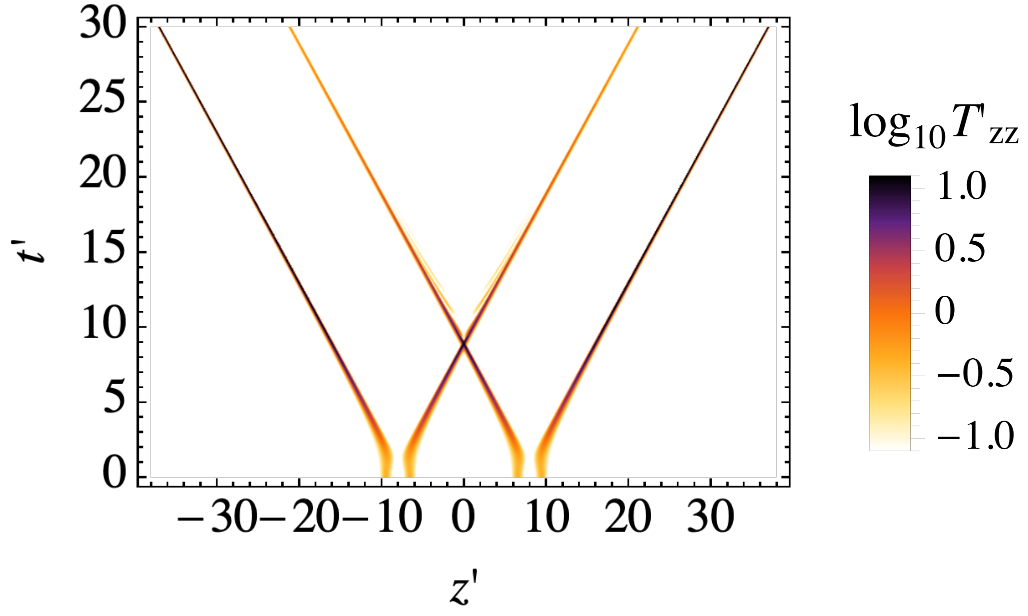
$$\Omega_{\rm GW}(\omega) = \left(\frac{H}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 S(\omega)\,, \quad S(\omega) = \left(\frac{\omega}{\beta}\right)^3 \frac{3\beta^5}{8\pi V_s} \int \mathrm{d}\Omega_k \left(|C_+|^2 + |C_\times|^2\right)$$



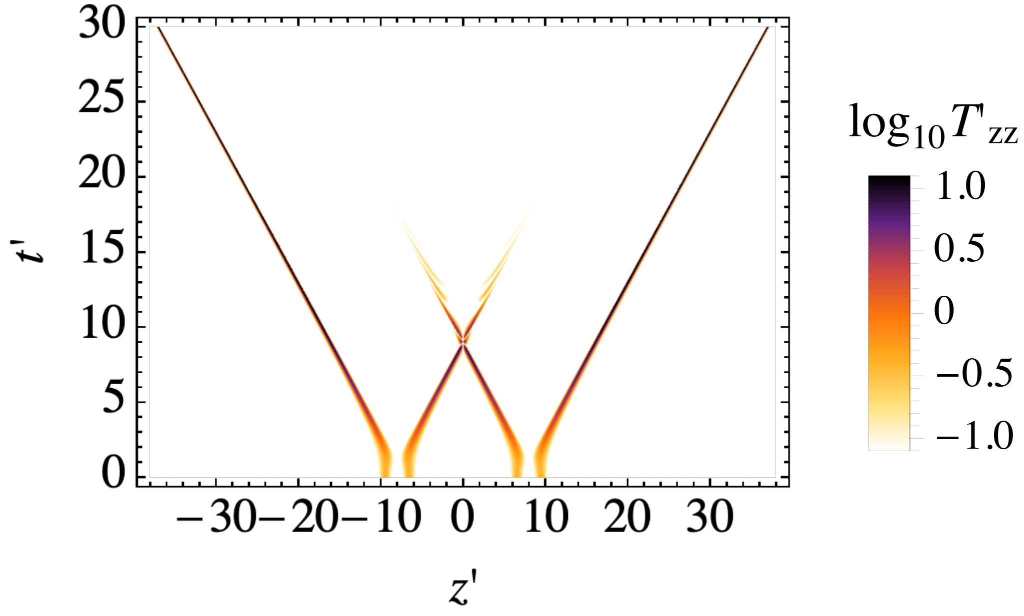
Future work

Effects of the gauge field related to the U(1) symmetry:

$$gv^2/\sqrt{\Delta V} \ll 1$$



$$gv^2/\sqrt{\Delta V} \gg 1$$



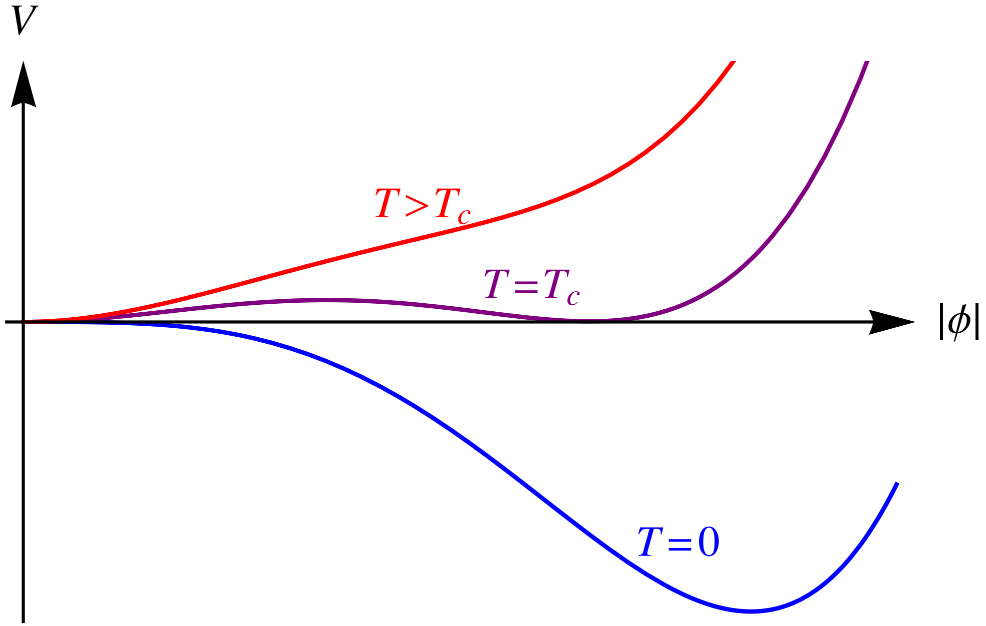
Energy budget of strongly supercooled phase transtions and detectability of the produced GW background

Benchmark model: classically conformal $U(1)_{B-L}$ extension of the SM

$$V \approx \underbrace{\frac{3g^4}{4\pi^2}|\phi|^4\left[\log\frac{|\phi|^2}{w^2}-\frac{1}{2}\right]}_{\text{CW potential}} + \underbrace{g^2T^2|\phi|^2}_{\text{thermal correction}}$$

B-L gauge coupling

VEV of ϕ



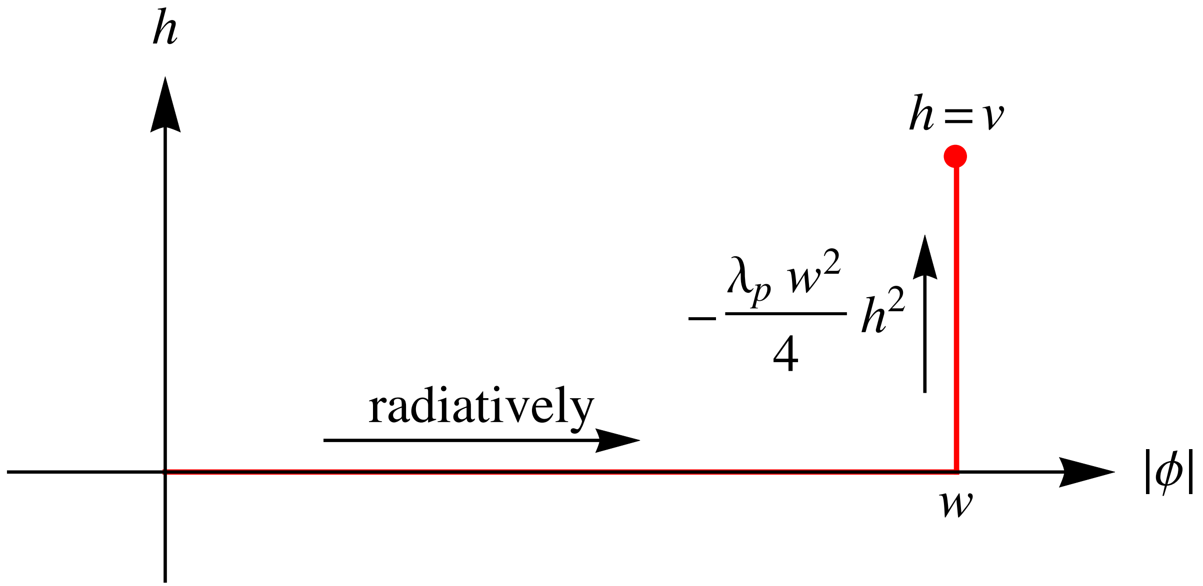
The potential energy barrier vanishes only at $T = 0$,

$\Longrightarrow$ first order phase transition.

Higgs portal

$$V = \lambda_H (H^\dagger H)^2 + \lambda_\phi (\phi^\dagger \phi)^2 - \lambda_p (H^\dagger H)(\phi^\dagger \phi)$$

Quantum effects induce VEV for ϕ that triggers EW symmetry breaking:



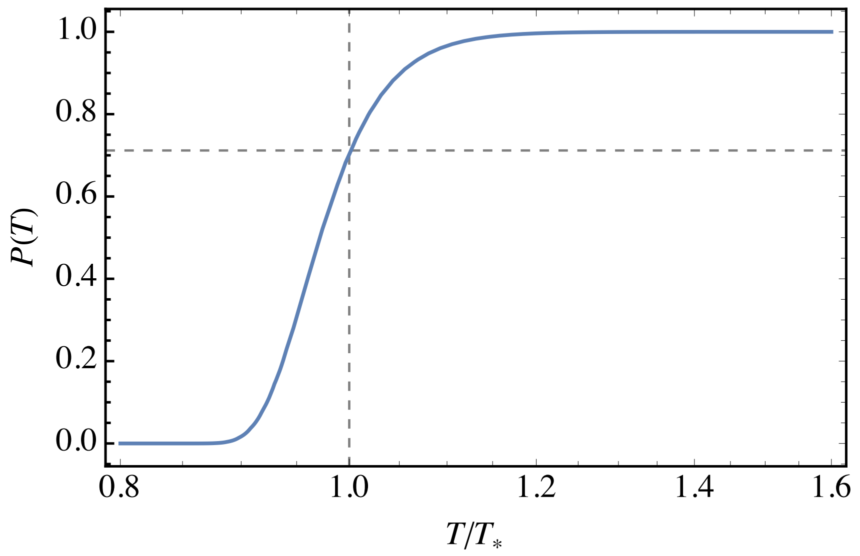
Phase transition

Bubble nucleation rate:

$$\Gamma(T) \simeq T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} e^{-S_3/T} \qquad \text{where} \qquad S_3 = 4\pi \int r^2 \mathrm{d}r \left[\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)^2 + V(\phi, T)\right]$$

Probability that a given point remains in the unstable vacuum:

$$P(T) = \exp\left[-\frac{4\pi}{3}\int_T^{T_c} \frac{\mathrm{d}T'\,\Gamma(T')}{T'^4 H(T')}\left(\int_T^{T'} \frac{\mathrm{d}\tilde{T}}{H(\tilde{T})}\right)^3\right]$$



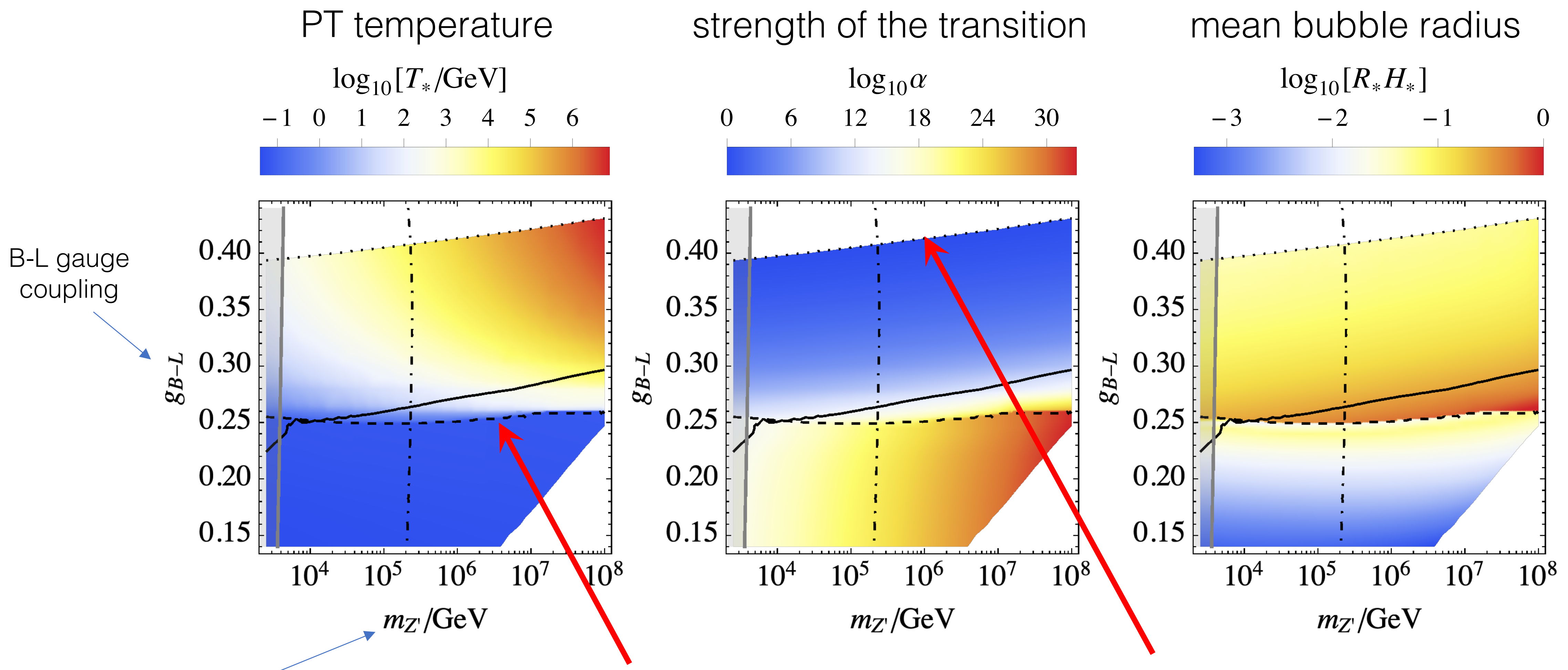
Phase transition temperature T_*: $\quad P(T_*) \approx 0.7$

Strength of the transition: $$\alpha \approx \frac{\Delta V}{\rho_R(T_*)}$$

radiation

Mean bubble radius: $$R_* = \left[T_* \int_{T_*}^{T_c} \frac{\mathrm{d}T'}{T'^2} \frac{\Gamma(T')}{H(T')} e^{-I(T')}\right]^{-\frac{1}{3}}$$

Related to simulations with exponential nucleation rate $\Gamma \propto e^{\beta t}$ through $\beta = (8\pi)^{\frac{1}{3}}/R_*$



$$T_* = T_{\rm QCD}$$

QCD scale induces a linear term for the Higgs field

$$\alpha = 1$$

vacuum dominance before the transition

Bubble wall Lorentz factor

Interactions of the bubble wall with plasma can stop its acceleration:

$$\Delta P = \Delta V - P_{1\to 1} - P_{1\to N}$$

$$P_{1\to 1} \approx 0.04 \Delta m^2 T_*^2$$

D. Bodecker and G. D. Moore, 0903.4099.

$$P_{1\to N} \approx 0.005 g^2 \gamma^2 T_*^4$$

S. Höche et.al., 2007.10343.

If $\Delta V \gg P_{1\to 1}$ then the bubble wall accelerates until $\Delta P = 0$:

$$\gamma_{\rm eq} \approx \frac{\sqrt{\Delta V}}{0.07 g T_*^2}$$

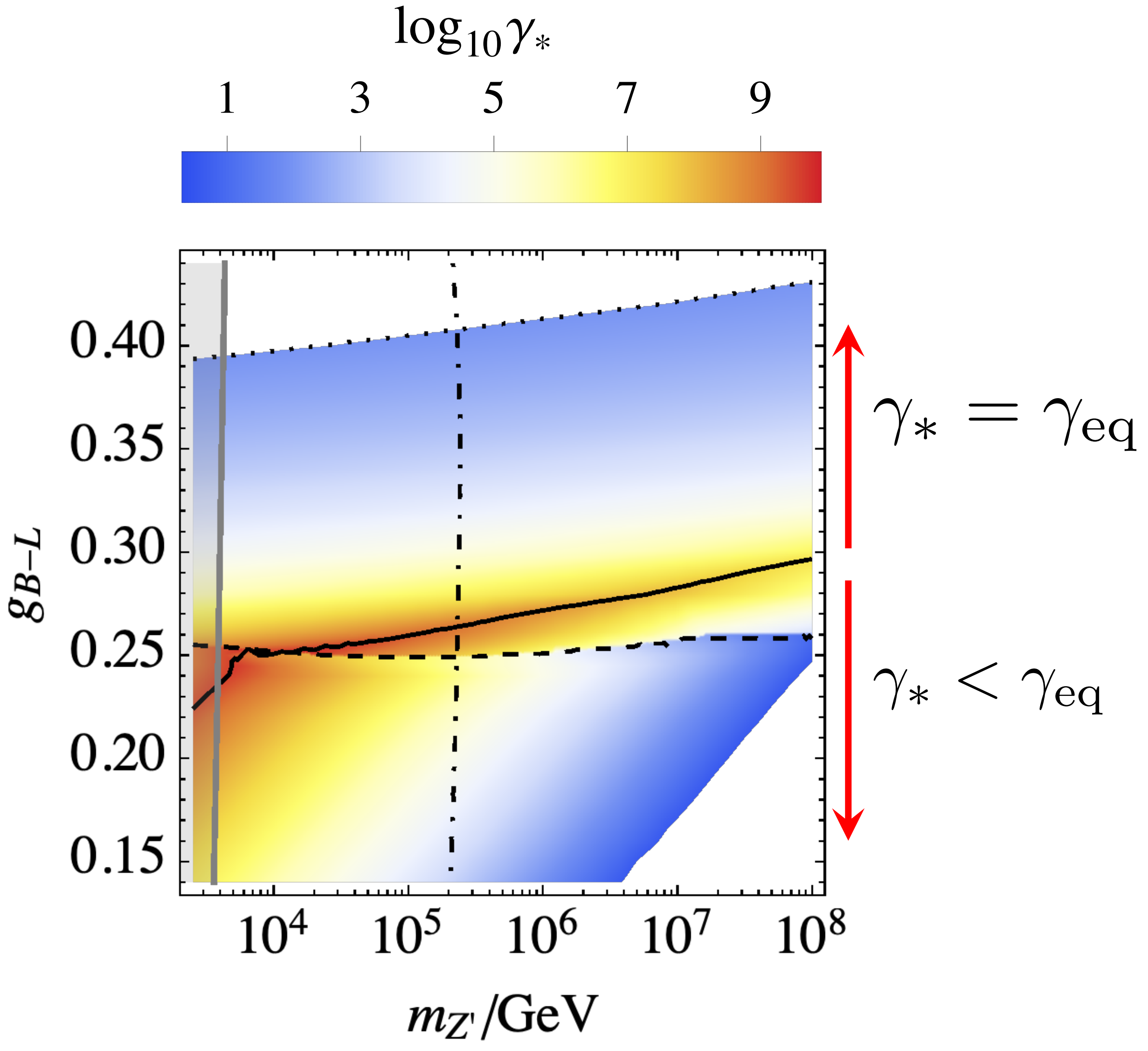
Bubble wall Lorentz factor:

$$\gamma_* = \min[\tilde{\gamma}_*, \gamma_{\rm eq}]$$

where

$$\tilde{\gamma}_* \propto \frac{R_*}{R_0}$$

Lorentz factor of the wall if the friction terms are neglected.



Efficiency factors

Fraction of the released vacuum energy that is in the scalar field gradients:

$$E_{\mathrm{wall}} = 4\pi R^2 \int_0^R \frac{\mathrm{d}R'}{3} \left[\Delta V - P_{1\to 1} - P_{1\to N}(R')\right]$$

$$\kappa_{\mathrm{col}} = \frac{E_{\mathrm{wall}}}{E_V} \approx \begin{cases} 1 - \frac{1}{3}\left(\frac{\tilde{\gamma}_*}{\gamma_{\mathrm{eq}}}\right)^2 , & \tilde{\gamma}_* < \gamma_{\mathrm{eq}} \\ \frac{2}{3}\frac{\gamma_{\mathrm{eq}}}{\tilde{\gamma}_*} , & \tilde{\gamma}_* > \gamma_{\mathrm{eq}} \end{cases}$$

collisions before terminal velocity is reached

collisions after terminal velocity is reached

$$E_V = \frac{4\pi}{3} R^3 \Delta V$$

GW sources related to plasma motions are separated into two distinct periods:

1. sound waves and
2. turbulence.

The sound wave period lasts for

M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, 1704.05871 and 1504.03291.

$$\tau_{\rm sw} = \frac{R_*}{U_f}$$

where

$$U_f = \sqrt{\frac{3}{4}\frac{\alpha_{\rm eff}}{1+\alpha_{\rm eff}}\kappa_{\rm sw}}$$

is the RMS fluid velocity.

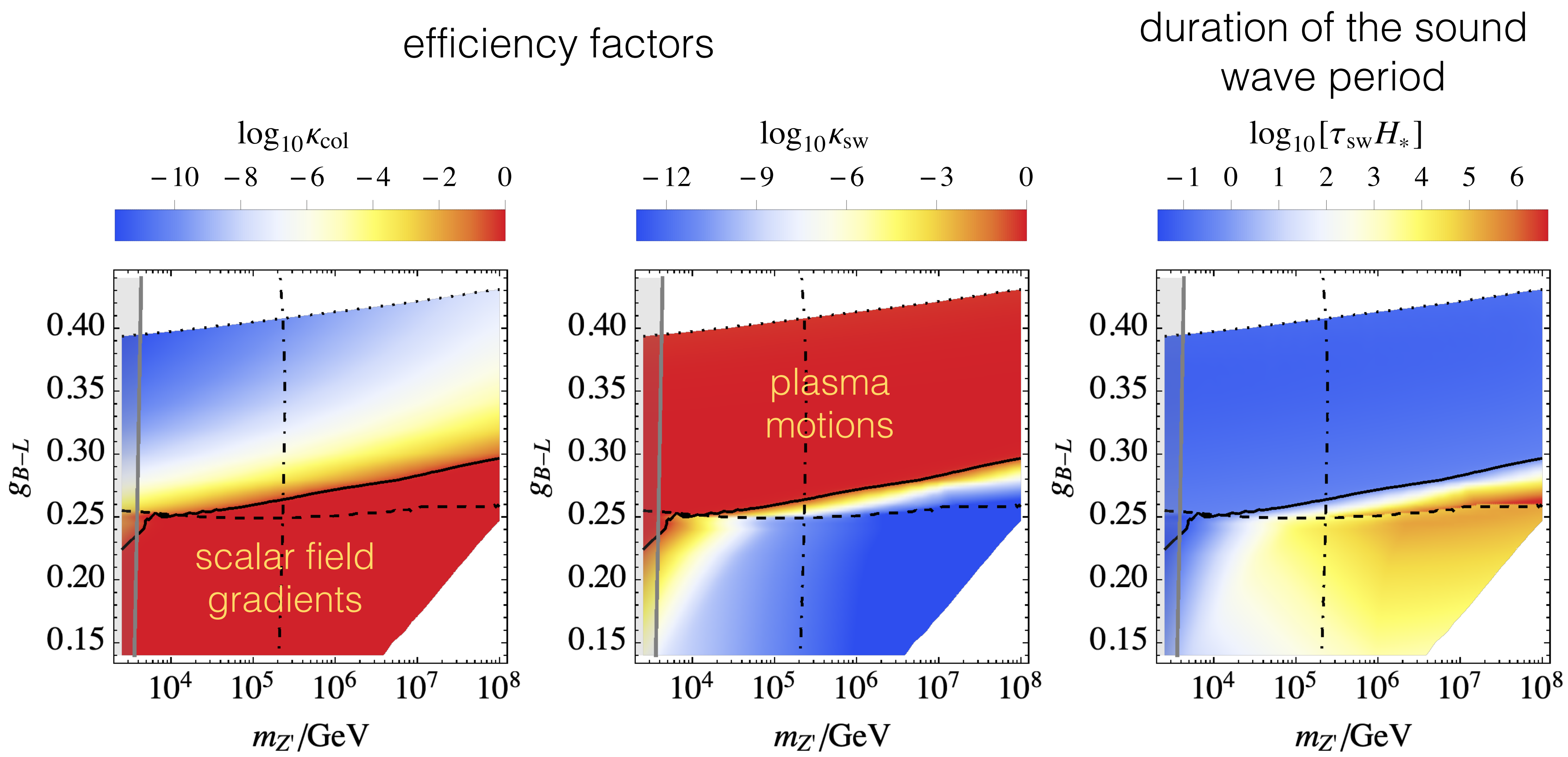
The related efficiency factor is given by

J. R. Espinosa. T. Konstandin, J.M. No and G. Servant, 1004.4187.

$$\kappa_{\rm sw} = \frac{\alpha_{\rm eff}}{\alpha}\frac{\alpha_{\rm eff}}{0.73+0.083\sqrt{\alpha_{\rm eff}}+\alpha_{\rm eff}}$$

where

$$\alpha_{\rm eff} = \alpha(1-\kappa_{\rm col})$$



efficiency factors
duration of the sound wave period
$\log_{10}\kappa_{\rm col}$
-10 -8 -6 -4 -2 0
$\log_{10}\kappa_{\rm sw}$
-12 -9 -6 -3 0
$\log_{10}[\tau_{\rm sw}H_*]$
-1 0 1 2 3 4 5 6
g_{B-L}
0.40
0.35
0.30
0.25
0.20
0.15
scalar field gradients
plasma motions
10^4 10^5 10^6 10^7 10^8
$m_{Z'}/\mathrm{GeV}$

GW signal

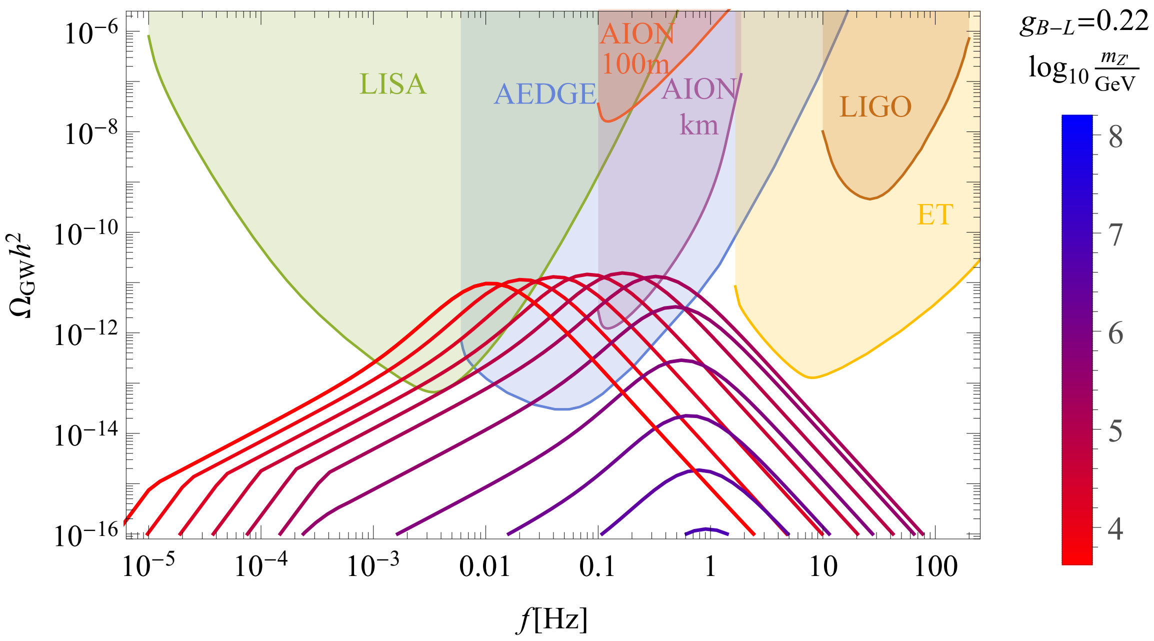
Temperature after all released energy has decayed to radiation

$$f_{\text{peak}} \propto \frac{T_{\text{reh}}}{R_* H_*} \left(\frac{\Gamma}{H_*} \right)^{-1/3}$$

Decay rate of the oscillating scalar field

GWs from scalar field gradients:

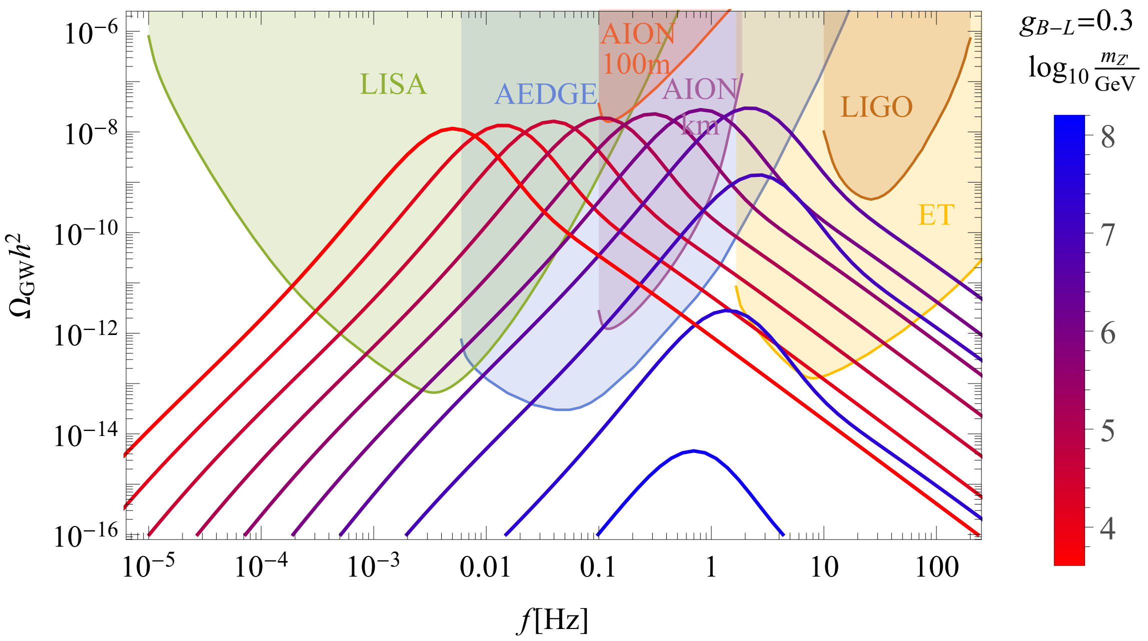
$$\Omega_{\text{col}} \propto (R_* H_*)^2 \left(\kappa_{\text{col}} \frac{\alpha}{1+\alpha} \right)^2 \left(\frac{\Gamma}{H_*} \right)^{2/3}$$



GWs from sound waves and turbulence:

$$\Omega_{\text{sw}} \propto (R_* H_*)(H_* \tau_{\text{sw}}) \left(\kappa_{\text{sw}} \frac{\alpha}{1+\alpha} \right)^2 \left(\frac{\Gamma}{H_*} \right)^{2/3}$$

$$\Omega_{\text{turb}} \propto (R_* H_*)(1 - H_* \tau_{\text{sw}}) \left(\kappa_{\text{sw}} \frac{\alpha}{1+\alpha} \right)^{3/2} \left(\frac{\Gamma}{H_*} \right)^{2/3}$$



Detectability

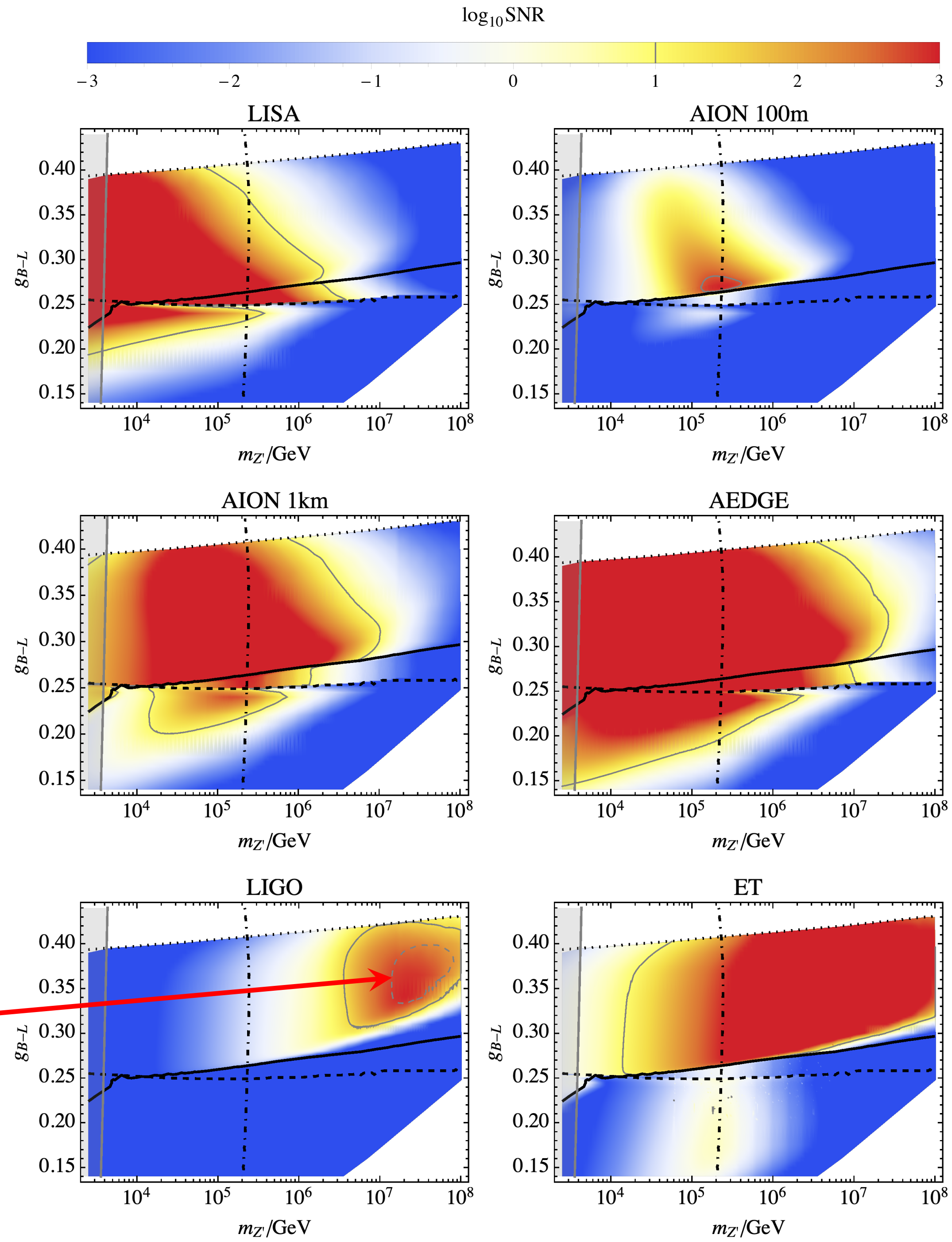
Prospects for observing the GW signals is determined by the signal-to-noise ratio:

$$\mathrm{SNR} = \sqrt{\mathcal{T} \int \mathrm{d}f \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{noise}}(f)}\right]^2}$$

observation period

sensitivity

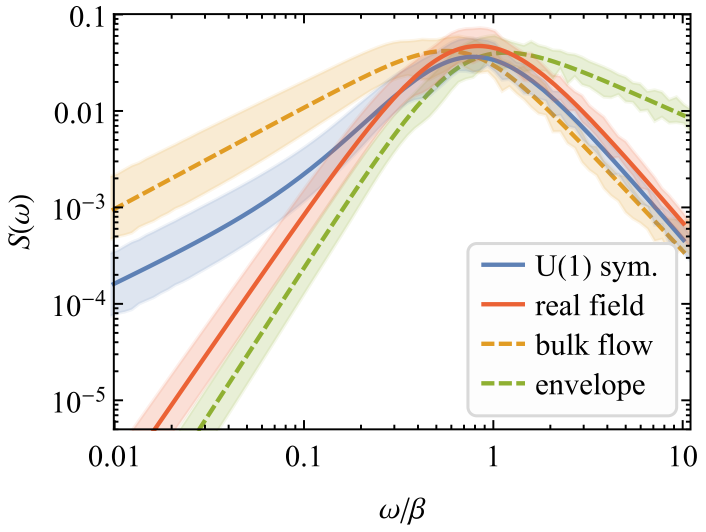
SNR = 1 for the current LIGO sensitivity



Summary

- New calculation of the GW spectrum from scalar field gradients: low-frequency tail depends on the scalar potential.
- For strongly supercooled phase transitions the GW signal can be sourced dominantly by scalar field gradients.
- This is possible in classically scale invariant models.
- Future GW experiments can probe large parts of the parameter space of these models.

GW spectrum from scalar field gradients:



Efficiency factors: