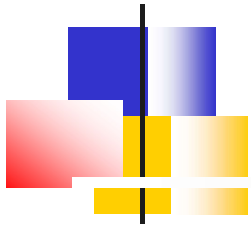


Theoretical and Observational Investigation of Modified Gravity through Gravitational Waves



Emmanuel N. Saridakis

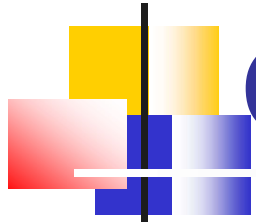
Physics Department, National and Technical University of Athens, Greece

Physics Department, Baylor University, Texas, USA



National
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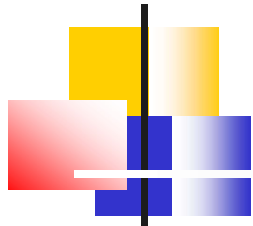
Goal

- We investigate **cosmological scenarios** arising from **modified gravity** that can describe the **observed Universe** as a whole
- **Astrophysical cosmology** has become a **precision science** with a huge amount of data. The advancing **gravitational wave multi-messenger astronomy** opens a **new era**



Talk Plan

- 1) Observational Cosmology: the **Standard Model of Cosmology**.
- 2) **Standard Model of Cosmology**. Do we need **new physics**?
- 3) We can **modify** the **Universe content**, or/and the **gravitational theory**.
- 4) Use of various **observational data** (SnIa, CMB, BAO, $H(z)$, LSS etc) in order to **constrain** the proposed **theories**.
- 5) **Torsional modified gravity**: A good candidate.
- 6) **GWs**: basic **properties** and **evolution**.
- 7) **Gravitational wave astronomy**, and **multi-messenger astronomy**: a **novel tool** to test General Relativity and cosmological scenarios in **great accuracy**.

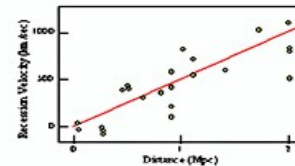


Observations

- **Cosmological Principle** “axiom” (indirect result): the Universe is **homogeneous and isotropic**

- **Hubble** (1929): The **Universe expands**

Hubble's Data (1929)



$$v = H r \quad H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

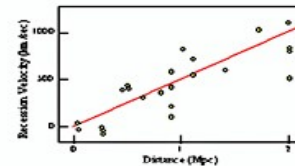
- **Alpher, Bethe, Gamow** (1948): The Universe **begun to expand** from a very **high-density and high-temperature** state towards less dense and hot states. Hoyle named the theory “**The Big Bang Theory**”.

Observations

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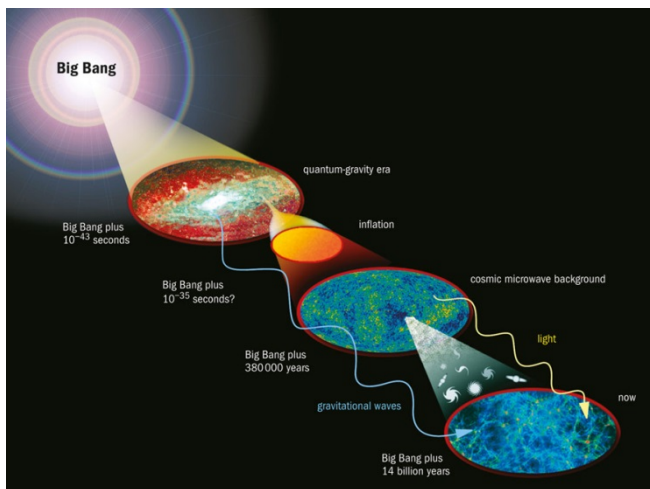
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- **Theoretical Problems:**

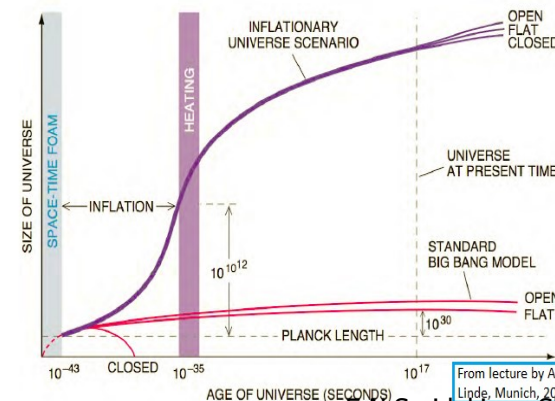
- I) **Horizon problem:** Why points at opposite directions have the same properties
- II) **Flatness problem:** Why the universe is today almost spatially flat
 $\Omega_k \sim 0.001$. It must have started with $\sim 10^{-50}$!
- **Monopole problem:** They are not observed.

Inflation

- Kazanas, Guth, Linde (1982): The Universe 10^{-36} sec after the Big Bang, through some mechanism went into an exponential expansion up to 10^{-32} sec increasing in size $\sim 10^{30}$ times: Inflation.
- I) The observable Universe is a tiny part of the total one, and originates from a small, causally connected region.
- II) Due to the huge expansion, the spatial curvature became almost zero.
- III) Due to the huge expansion the monopoles spread in all regions, and thus our own, observable universe, has at most one.

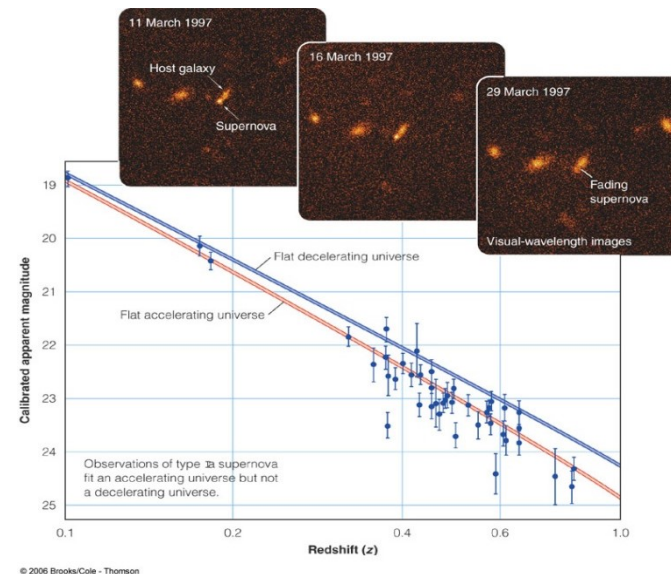


Inflationary Universe

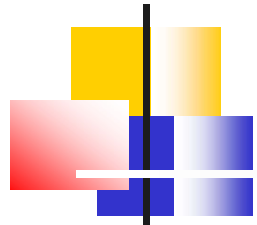


Dark Energy

- The **accelerated expansion** is verified by independent observations, **Supernovae type Ia (SNIa)**, **Cosmic Microwave Background (CMB)**, **Baryon Acoustic Oscillations (BAO)**, **Large Scale Structure (LSS)**, etc

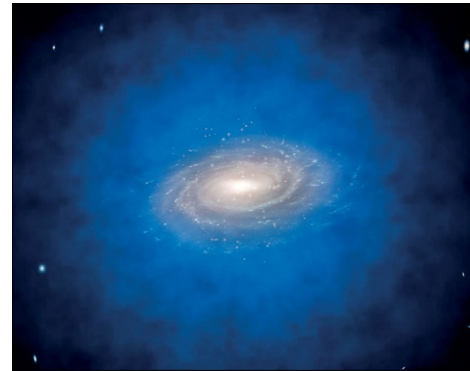
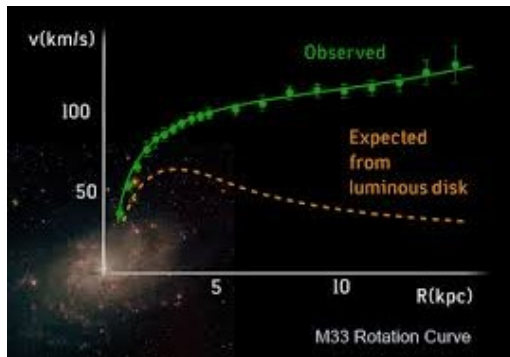


- Around **70%** of the **total energy density** of the Universe is this unknown **dark energy** (it does not interact electromagnetically).
- Possible explanation: **The cosmological constant Λ** (**Einstein's "greatest blunder"**). A term that produces the extra **"repulsion"**.

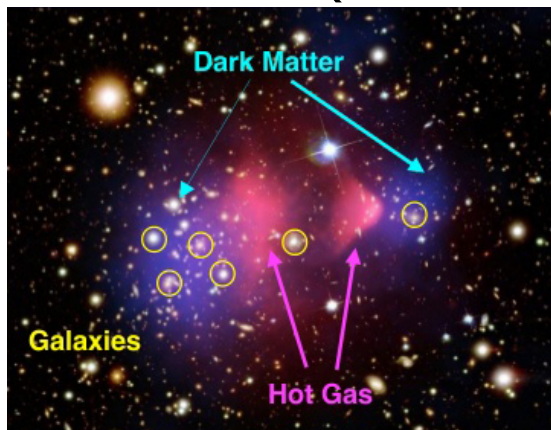


Dark Matter

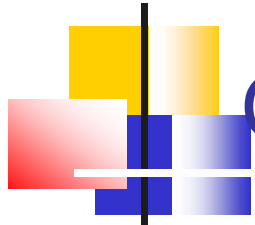
- Galaxy rotation curves:



- Bullet cluster (collision of two galaxy clusters)

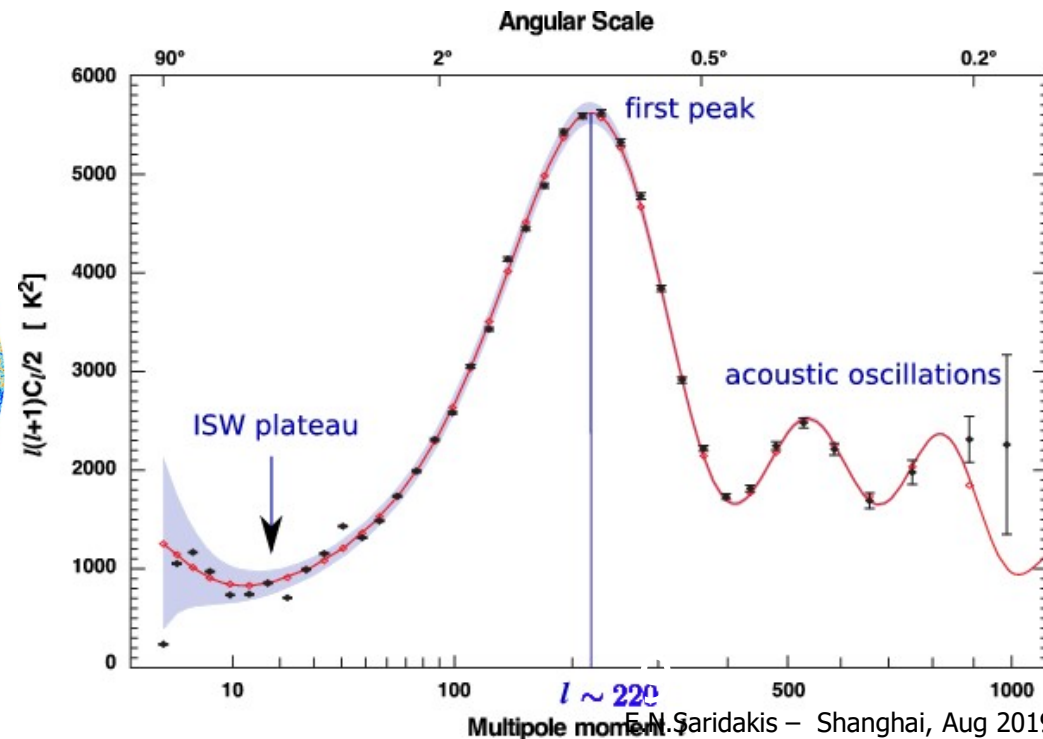
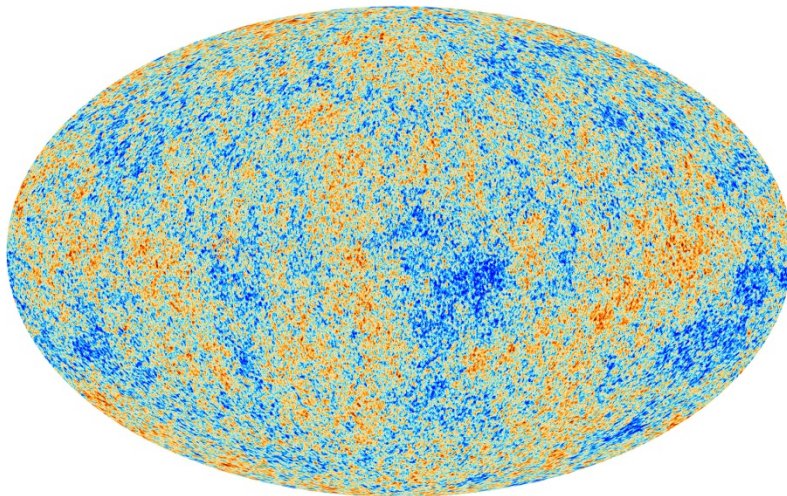


- 80% of matter is an “unknown” dark matter (it does not interact electromagnetically)!



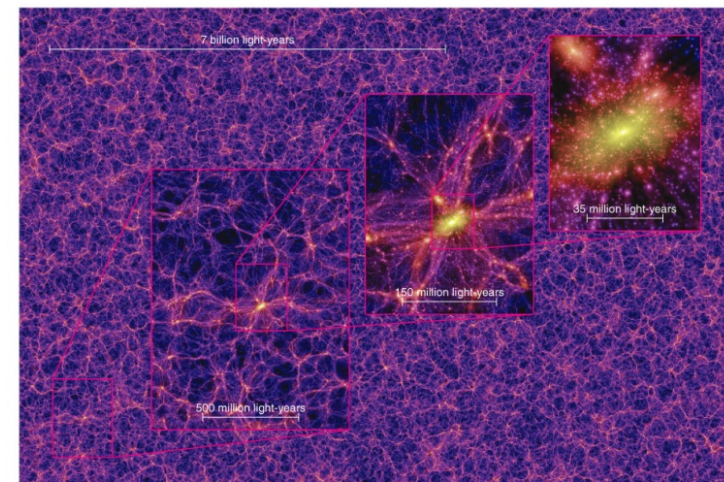
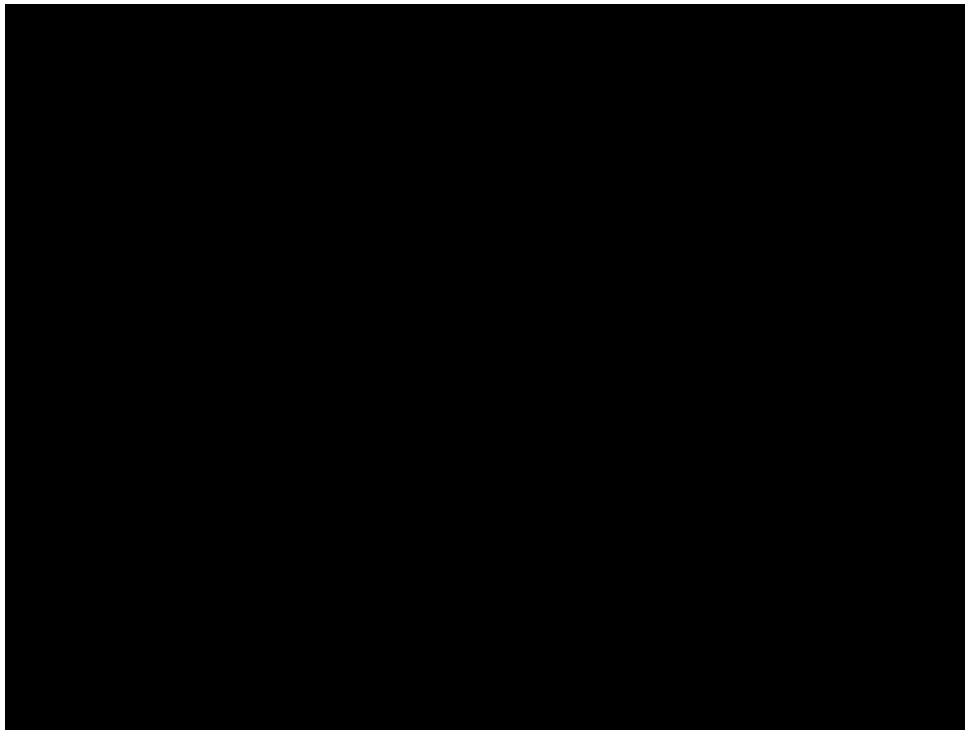
Cosmic Microwave Background radiation

- From the **fluctuation spectrum** we extract information: The **first peak** provides the spatial **curvature** (it results to flat universe), the **second peak** the **baryon energy density parameter**, the **third peak** the **dark matter energy density parameter**, etc.

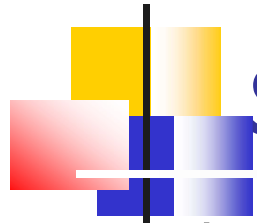


Inflation can also explain CMB and seeds of LSS

- Additional success: **Inflation** provides the necessary **primordial fluctuations**, which later gave the **Large Scale Structure** of matter:

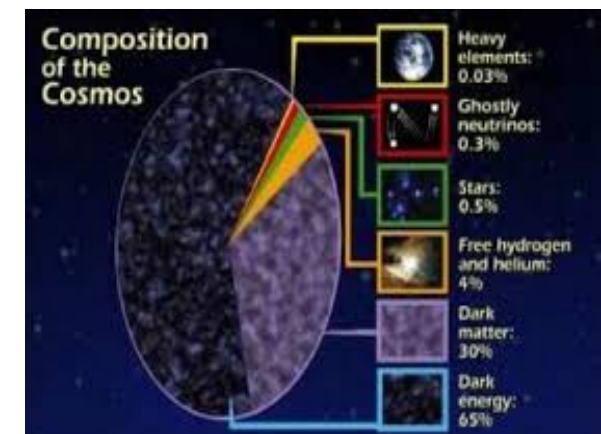
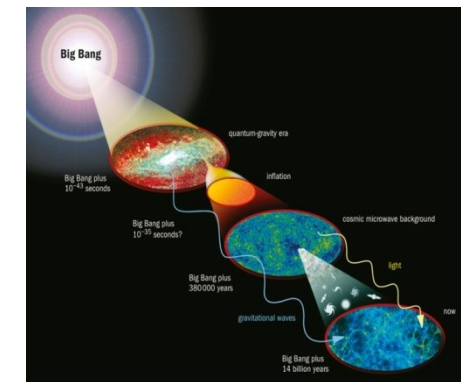
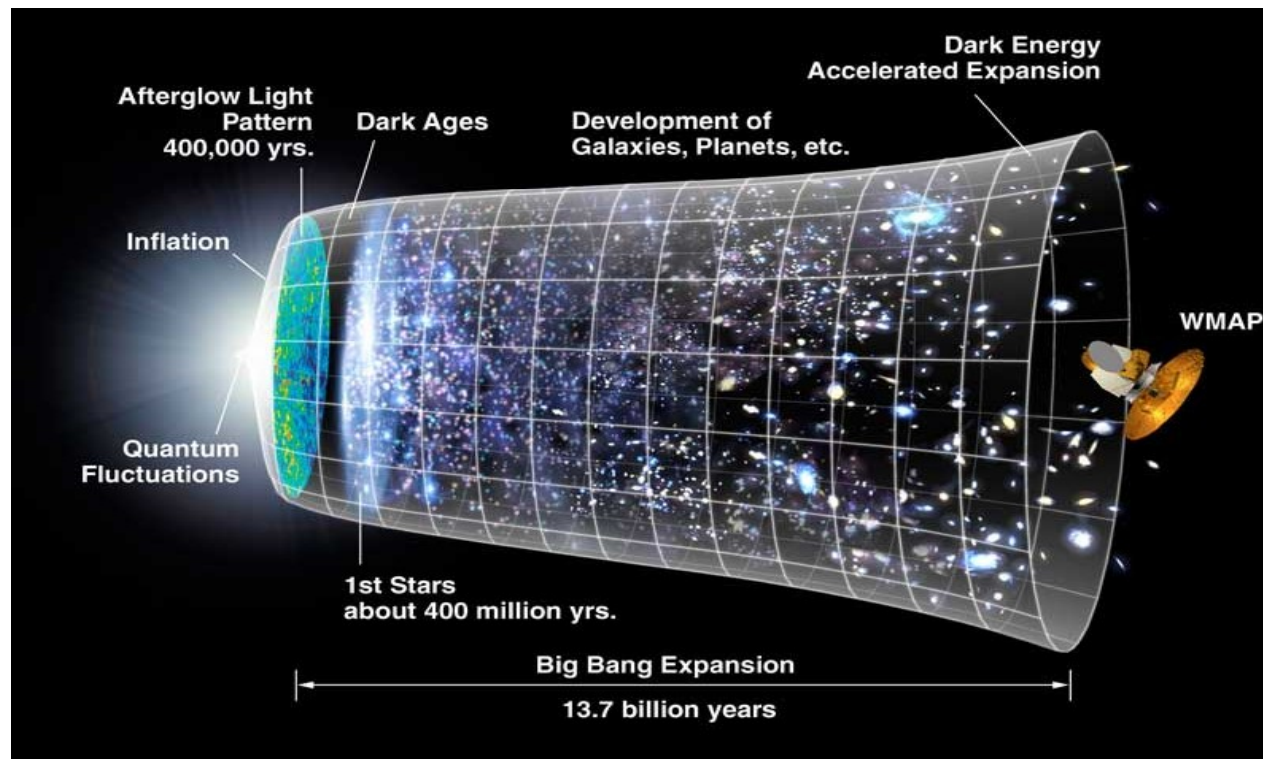


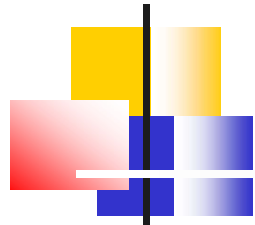
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Summary of Observations

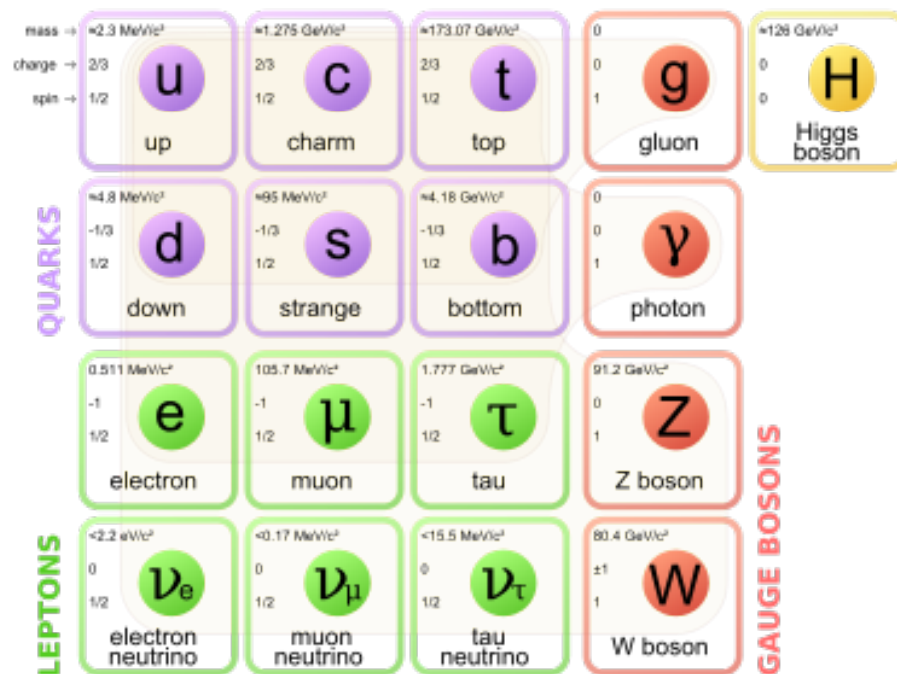
The **Universe history:**

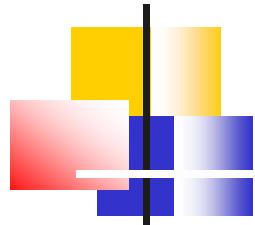




Knowledge of Physics

Knowledge of Physics: Standard Model

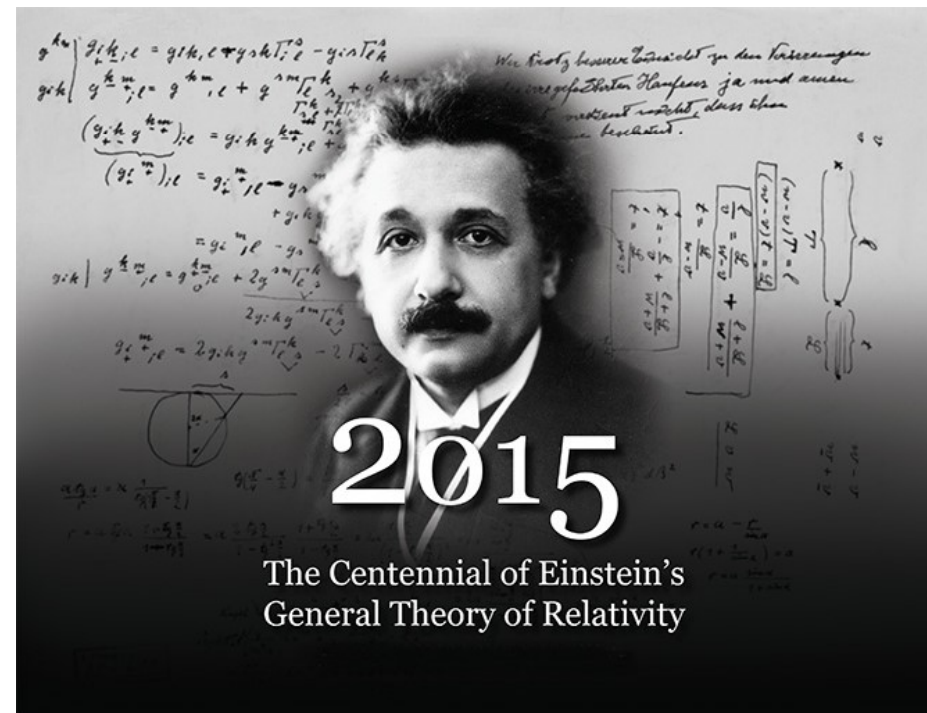


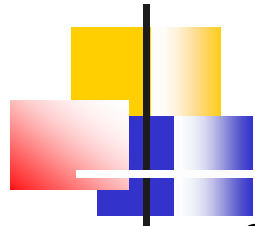


Knowledge of Physics

Knowledge of Physics: **Standard Model** + **General Relativity**

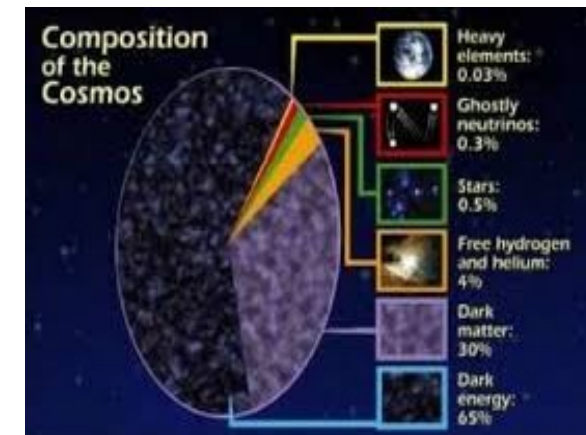
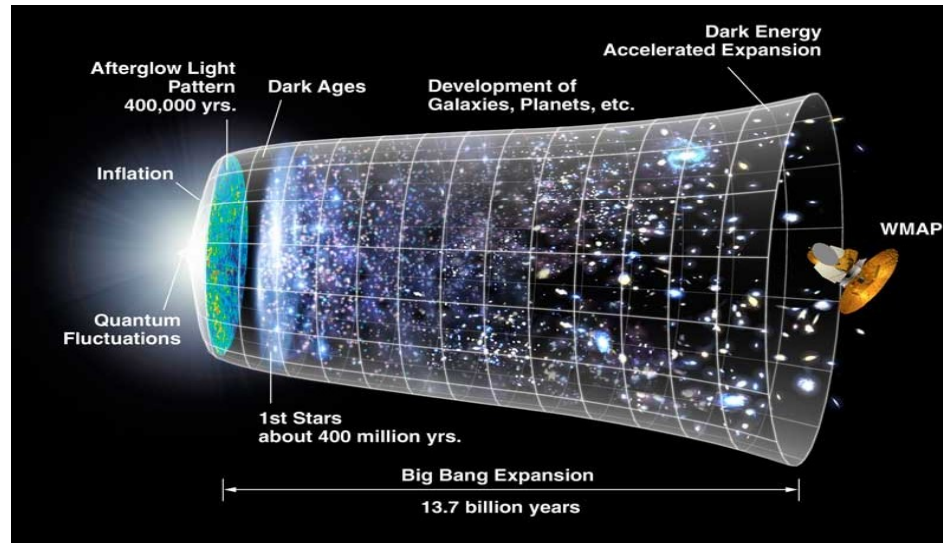
mass →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	≈4.8 MeV/c ²	≈95 MeV/c ²	≈4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	γ photon	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
					GAUGE BOSONS

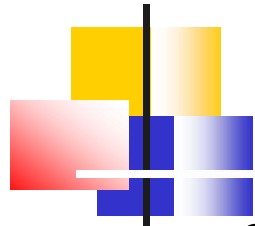




Modified/new knowledge of physics

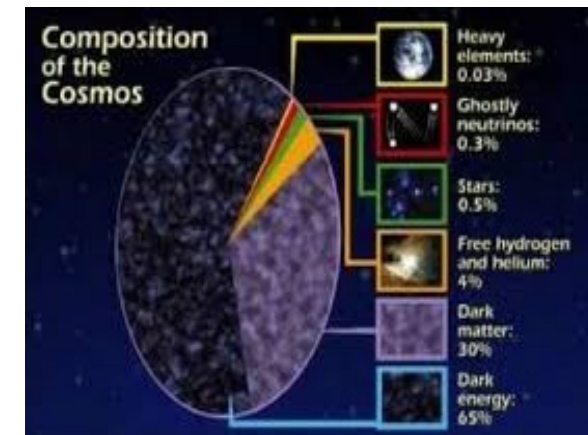
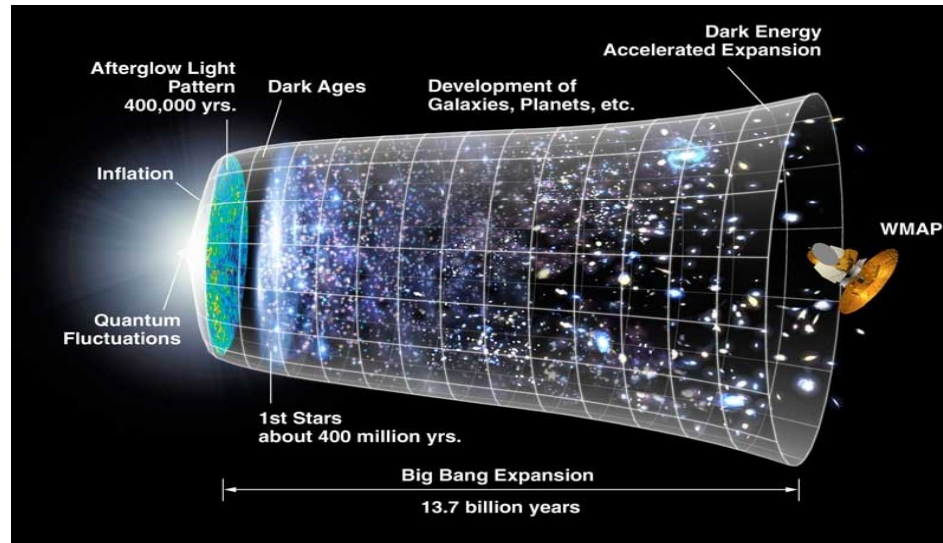
So can our **knowledge of Physics** describes all these?





Modified/new knowledge of physics

So can our **knowledge of Physics** describes all these?



Most probably, no!

We definitely need **new physics** for **Inflation** and **Dark matter**. Maybe for **dark energy**.



Cosmology

- A **successful** cosmological model must:
 - 1) Describe the **evolution** of the universe at the **background level**
 - 2) Describe the **evolution** of the universe at the **perturbation level**

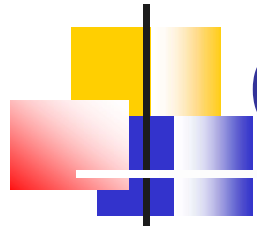


Cosmology

- A **successful cosmological model** must:
 - 1) Describe the **evolution** of the universe at the **background level**
 - 2) Describe the **evolution** of the universe at the **perturbation level**

- **Λ CDM paradigm** seems to succeed in **both**, at **post-inflationary** eras

- **Open issues:**
 - 1) The **cosmological-constant problem**. Calculation of Λ gives a number **120 orders of magnitude larger** than observed.
~~Worst error in the history of physics, history of science, history~~
 - 2) How to describe **primordial universe** (inflation)
 - 3) **Tensions** with some data sets, e.g. **H0 and σ_8** data



Cosmology-background

- Homogeneity and isotropy: $ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$
- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE})$$

$$\dot{H} = -4\pi G (\rho_m + p_m + \rho_{DE} + p_{DE}),$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a $H(z)$ and $\Omega_m(z)$ and $w_{DE}(z)$ in agreement with observations (SNIa, BAO, CMB shift parameter, $H(z)$ etc)



Cosmology-perturbations

- **Perturbation evolution:** $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta \approx 0$ where $\delta \equiv \delta\rho/\rho$
 where $G_{\text{eff}}(z, k)$ is the **effective Newton's constant**, given by

$$\nabla^2 \phi \approx 4\pi G_{\text{eff}} \rho \delta,$$

under the scalar **metric perturbation** $ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\vec{x}^2$

- Hence: $\delta'' + \left(\frac{(H^2)'}{2H^2} - \frac{1}{1+z} \right) \delta' \approx \frac{3}{2}(1+z) \frac{H_0^2}{H^2} \frac{G_{\text{eff}}(z, k)}{G_N} \Omega_{0m} \delta$

with $f(a) = \frac{d \ln \delta}{d \ln a}$ the **growth rate**, with $f(a) = \Omega_m(a)^{\gamma(a)}$ and $\Omega_m(a) \equiv \frac{\Omega_{0m} a^{-3}}{H(a)^2/H_0^2}$

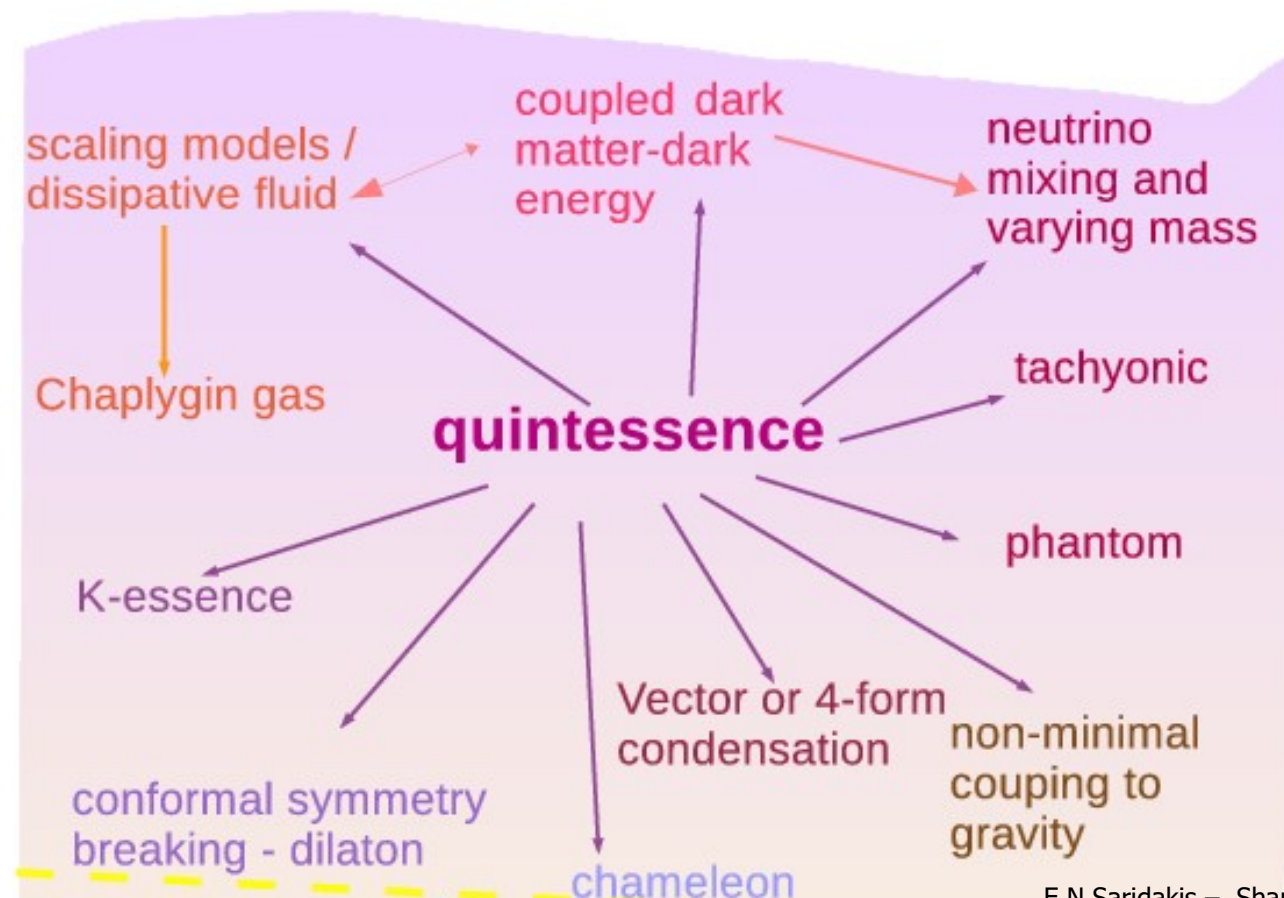
- One can define the **observable**: $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$

with $\sigma(a) = \sigma_8 \frac{\delta(a)}{\delta(1)}$ the z-dependent rms fluctuations of the linear density field within spheres of radius $R = 8h^{-1}\text{Mpc}$, and σ_8 its value today.

Dark Energy-Inflation

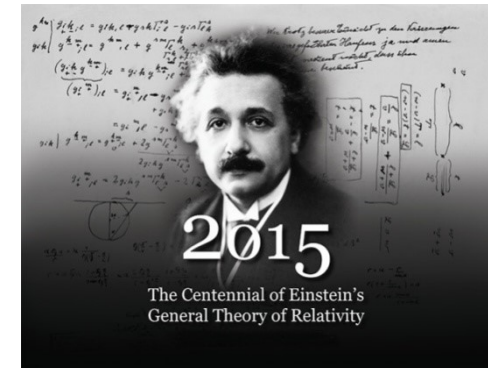
- Add a **scalar field ϕ** in the Universe content

mass = $4.2 \text{ MeV}/c^2$ charge = $2/3$ spin = $1/2$	mass = $1.275 \text{ GeV}/c^2$ charge = $2/3$ spin = $1/2$	mass = $1.732 \text{ GeV}/c^2$ charge = $2/3$ spin = $1/2$	mass = 0 charge = 0 spin = 1	mass = $125 \text{ GeV}/c^2$ charge = 0 spin = 0
u up	c charm	t top	g gluon	H Higgs boson
mass = $4.8 \text{ MeV}/c^2$ charge = $-1/3$ spin = $1/2$	mass = $95 \text{ MeV}/c^2$ charge = $-1/3$ spin = $1/2$	mass = $4.18 \text{ GeV}/c^2$ charge = $-1/3$ spin = $1/2$	mass = 0 charge = 0 spin = 1	
d down	s strange	b bottom	γ photon	
mass = $0.511 \text{ MeV}/c^2$ charge = -1 spin = $1/2$	mass = $105.7 \text{ MeV}/c^2$ charge = -1 spin = $1/2$	mass = $1.777 \text{ GeV}/c^2$ charge = -1 spin = $1/2$	mass = $91.2 \text{ GeV}/c^2$ charge = 0 spin = 1	
e electron	μ muon	τ tau	Z Z boson	
mass = $< 2.2 \text{ eV}/c^2$ charge = 0 spin = $1/2$	mass = $10.57 \text{ MeV}/c^2$ charge = 0 spin = $1/2$	mass = $1.777 \text{ GeV}/c^2$ charge = 0 spin = $1/2$	mass = $80.4 \text{ GeV}/c^2$ charge = ± 1 spin = 1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	



General Relativity

- Einstein 1915: **General Relativity**:



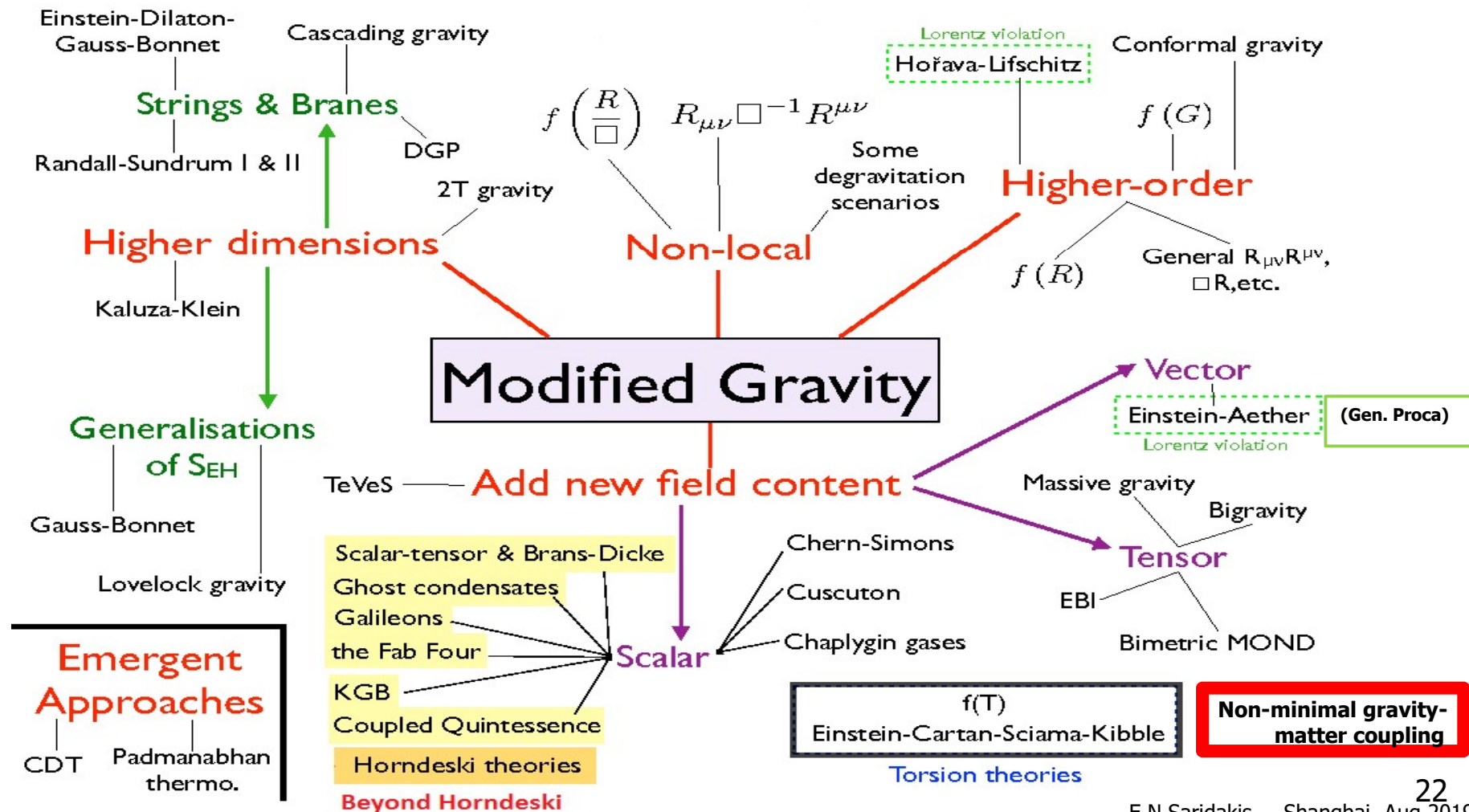
energy-momentum source of spacetime **Curvature**

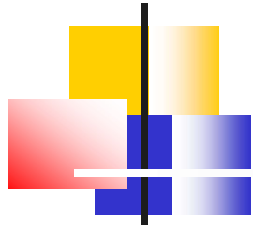
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x L_m(g_{\mu\nu}, \psi)$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

$$\text{with } T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta L_m}{\delta g_{\mu\nu}}$$

Modified Gravity





Scalar-Tensor Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$L_H = \sum_{i=2}^5 L_i$$

$$X = -\partial^\mu \phi \partial_\mu \phi / 2$$

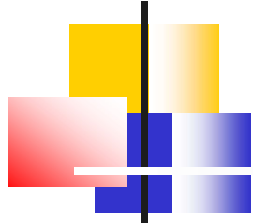
$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X) \diamond \phi$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X} [(\diamond \phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi)]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6}G_{5,X} [(\diamond \phi)^3 - 3(\diamond \phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi)]$$

[G. Horndeski, Int. J. Theor. Phys. 10]



Horndeski Theories

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$$L_H = \sum_{i=2}^5 L_i$$

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$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6}G_{5,X} [(\diamond \phi)^3 - 3(\diamond \phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi)]$$

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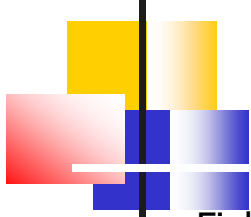
[G. Horndeski, Int. J. Theor. Phys. 10]



- Coincides with Generalized Galileon theories

$$\phi \rightarrow \phi + c, \quad \partial_\mu \phi \rightarrow \partial_\mu \phi + b_\mu$$

[Nicolis, Rattazzi, Trincherini, PRD 79]



Horndeski Cosmology (background)

- Field Equations: $L.H.S = R.H.S$

- In flat FRW:

- $$2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) - 12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m$$

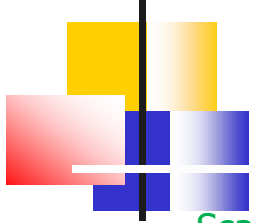
- $$K - 2X(G_{3,\phi} + \ddot{\phi}G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4,X} - 4H\dot{X}G_{4,X} - 8\dot{H}XG_{4,X} - 8HX\dot{X}G_{4,XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + 4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} - 2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} - 4H^2X^2\ddot{\phi}G_{5,XX} + 4HX(\dot{X} - HX)G_{5,\phi X} + 2[2(\dot{H}X + H\dot{X}) + 3H^2X]G_{5,\phi} + 4HX\dot{\phi}G_{5,\phi\phi} = -p_m$$

- $$\frac{1}{a^3} \frac{d}{dt}(a^3 J) = P_\phi$$

with $J = \dot{\phi}K_{,X} + 6HXG_{3,X} - 2\dot{\phi}G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) - 12HXG_{4,\phi X} + 2H^3X(3G_{5,X} + 2XG_{5,XX}) - 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X})$

$$P_\phi = K_{,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi}G_{3,\phi X}) + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} - 6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}G_{5,\phi X}$$

[De Felice, Tsujikawa JCAP 1202]



Horndeski Cosmology (perturbations)

- **Scalar perturbations:** $ds^2 = -(1 + 2\psi)dt^2 + a^2(1 - 2\phi)\delta_{ij}dx^i dx^j \Rightarrow L.H.S = R.H.S$
- **No-ghost condition:** $Q_s \equiv \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2} > 0$
- **No Laplacian instabilities condition:** $c_s^2 \equiv \frac{3(2w_1^2w_2H - 4w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2) - 6w_1^2(\rho_m + p_m)}{w_1(4w_1w_3 + 9w_2^2)} > 0$

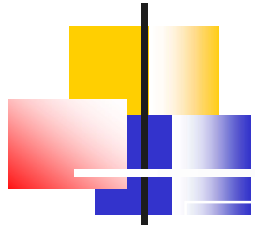
with $w_1 \equiv 2(G_4 - 2XG_{4,X}) - 2X(G_{5,X}\dot{\phi}H - G_{5,\phi})$

$$w_2 \equiv -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H + 4(\dot{\phi}G_{4,\phi X} - 4HG_{4,X})X + 2G_{4,\phi}\dot{\phi} \\ + 8X^2G_{5,\phi X}H + 2HX(6G_{5,\phi} - 5HG_{5,X}\dot{\phi}) - 4G_{5,XX}\dot{\phi}X^2H^2$$

$$w_3 \equiv 3X(K_{,X} + 2XK_{,XX}) + 6X(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6\dot{\phi}HG_{3,X}) \\ + 18H(4HX^3G_{4,XXX} - HG_4 - 5X\dot{\phi}G_{4,\phi X} - G_{4,\phi}\dot{\phi} + 7HG_{4,X}X + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,X\phi X}) \\ + 6H^2X(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} + 13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X} - 18G_{5,\phi})$$

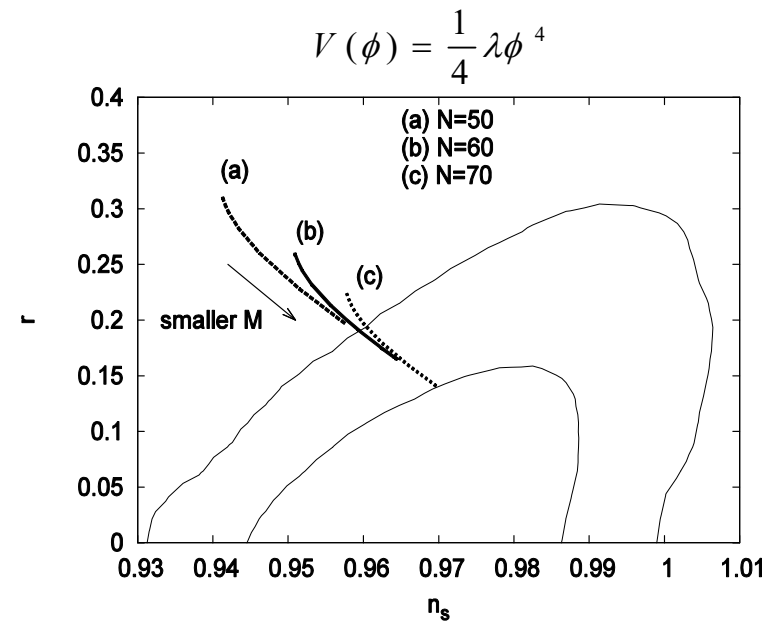
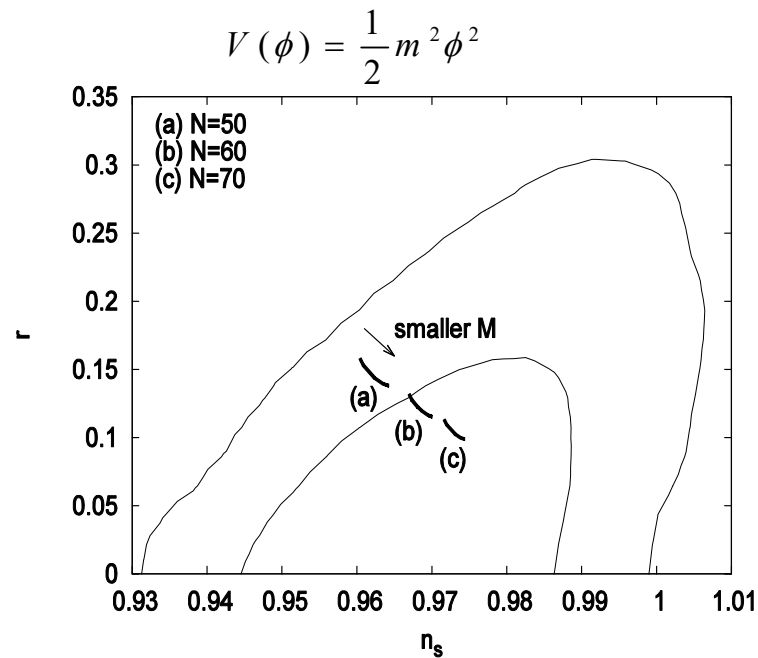
$$w_4 \equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

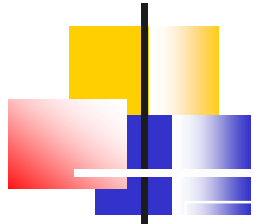
[De Felice, Tsujikawa JCAP 1202]



Inflation in Horndeski Theories

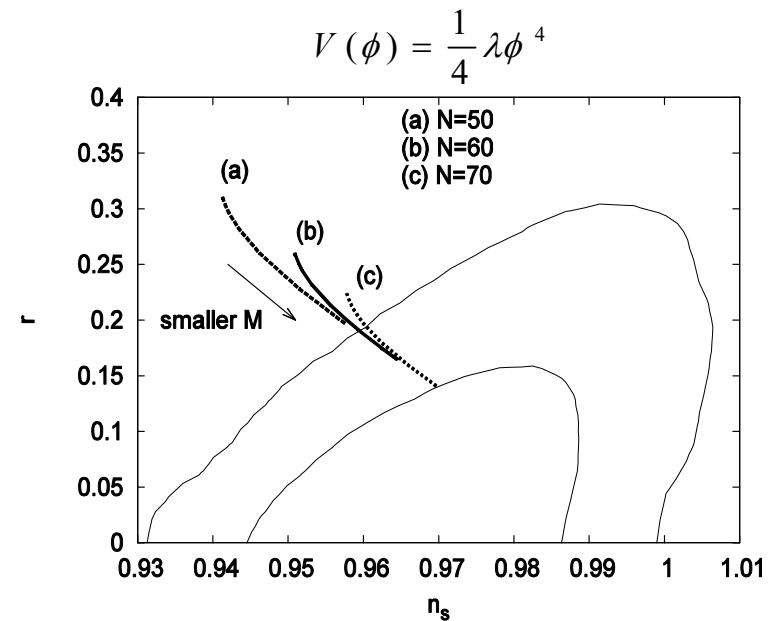
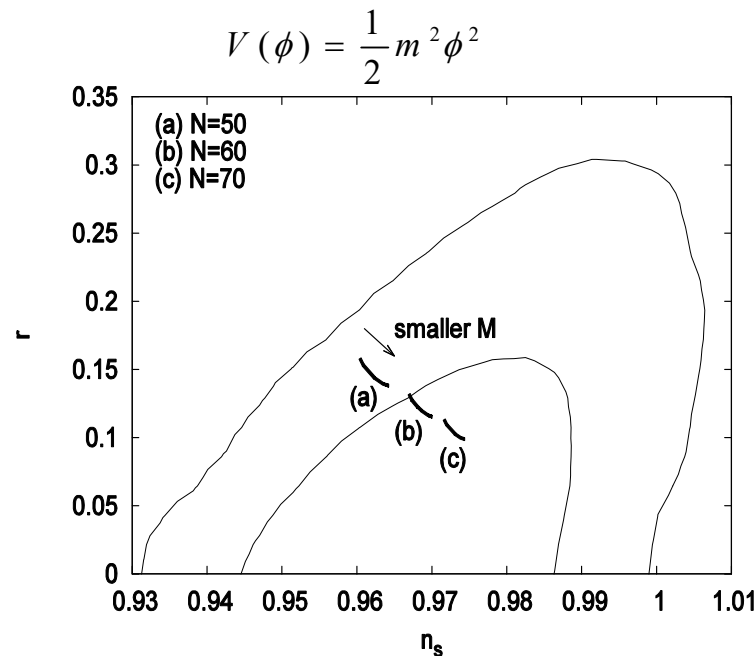
$K(\phi, X) = X - V(\phi), \quad G_3(\phi, X) = \frac{c_3}{M^3} X, \quad G_4 = G_5 = 0$
[Ohashi, Tsujikawa, JCAP 1210]





Inflation in Horndeski Theories

- $K(\phi, X) = X - V(\phi)$, $G_3(\phi, X) = \frac{c_3}{M^3} X$, $G_4 = G_5 = 0$ [Ohashi, Tsujikawa, JCAP 1210]

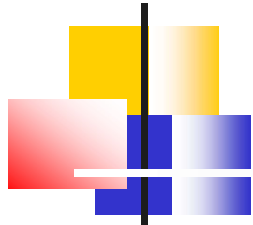


- G-Inflation (Shift-symmetric):** $K(\phi, X) = X + \frac{X^2}{2M^3\mu}$, $G_3(\phi, X) = \frac{1}{M^3} X$, $G_4 = G_5 = 0$

$$r \approx 0.17$$

[Kobayashi, Yamaguchi, Yokoyama PRL 105]

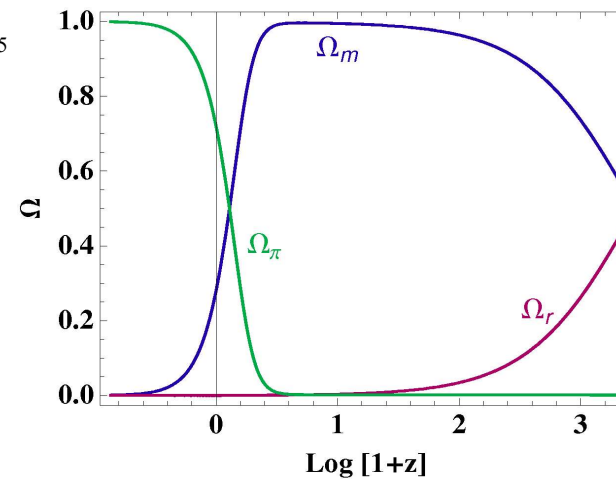
[Banerjee, Saridakis PRD 95]

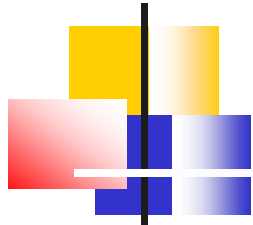


Dark Energy in Horndeski Theories

- $K(\phi, X) = c_2 X$, $G_3(\phi, X) = c_3$, $G_4 = 1$, $G_5 = c_5$
- Background evolution: Universe thermal history

[Ali, Gannouji, Sami PRD 82] [Leon, Saridakis JCAP 1303]



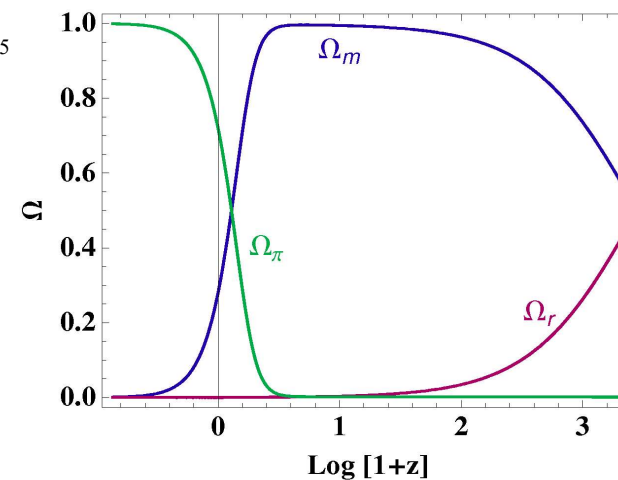


Dark Energy in Horndeski Theories

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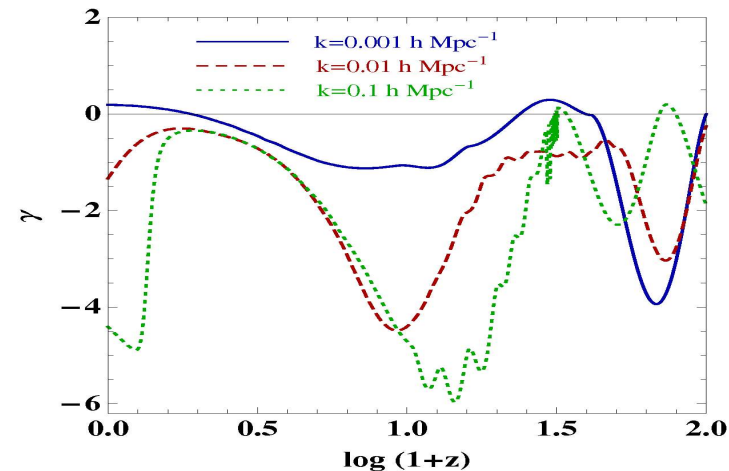
[Leon, Saridakis JCAP 1303]



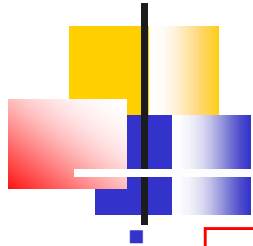
- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}} \rho_m \delta_m$
with $G_{\text{eff}} = G_{\text{eff}}(\phi, K, G_3, G_4, G_5)$

- Clustering growth rate: $\frac{d \ln \delta_m}{d \ln a} = \Omega_m^\gamma(a)$

$\gamma(z)$: Growth index.



[Ali, Gannouji, Sami PRD 82]



f(R) gravity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_m(g_{\mu\nu}, \psi)$$

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu} \diamond] f'(R) = 8\pi G T_{\mu\nu}$$

- **Field Equations** (metric formalism):

- **Conformal transformation:** $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = f'(R)g_{\mu\nu} \equiv \phi g_{\mu\nu}, \quad d\phi = \sqrt{\frac{2\omega_0 + 3}{16\pi G}} \frac{d\phi}{\phi}$

$$\Rightarrow_{\omega_0=0} S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{16\pi G} - \frac{1}{2} \partial^\alpha \phi \partial_\alpha \phi - U(\phi) \right] + S_m(e^{-\sqrt{16\pi G/3}} \tilde{g}_{\mu\nu}, \psi) \quad U(\phi) = \frac{Rf'(R) - f(R)}{16\pi G [f'(R)]^2}$$



f(R) cosmology - Inflation

Friedmann Equations (metric formalism):

$$3FH^2 = \frac{FR - f}{2} - 3H\dot{F} + 8\pi G \rho_m$$
$$-2F\dot{H} = \ddot{F} - H\dot{F} + 8\pi G(\rho_m + p_m)$$

$$F(R) \equiv f'(R)$$

$$R = 12H^2 + 6\dot{H}$$



f(R) cosmology - Inflation

■ **Friedmann Equations** (metric formalism):

$$3FH^2 = \frac{FR - f}{2} - 3H\dot{F} + 8\pi G \rho_m$$

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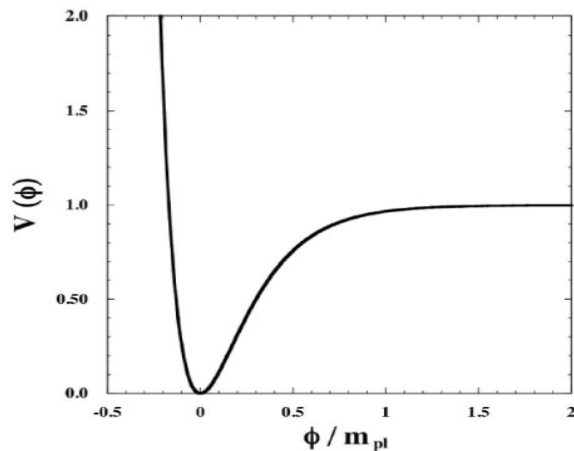
$$F(R) \equiv f'(R)$$

$$R = 12H^2 + 6\dot{H}$$

■ **Inflation:** e.g. Starobinsky inflation

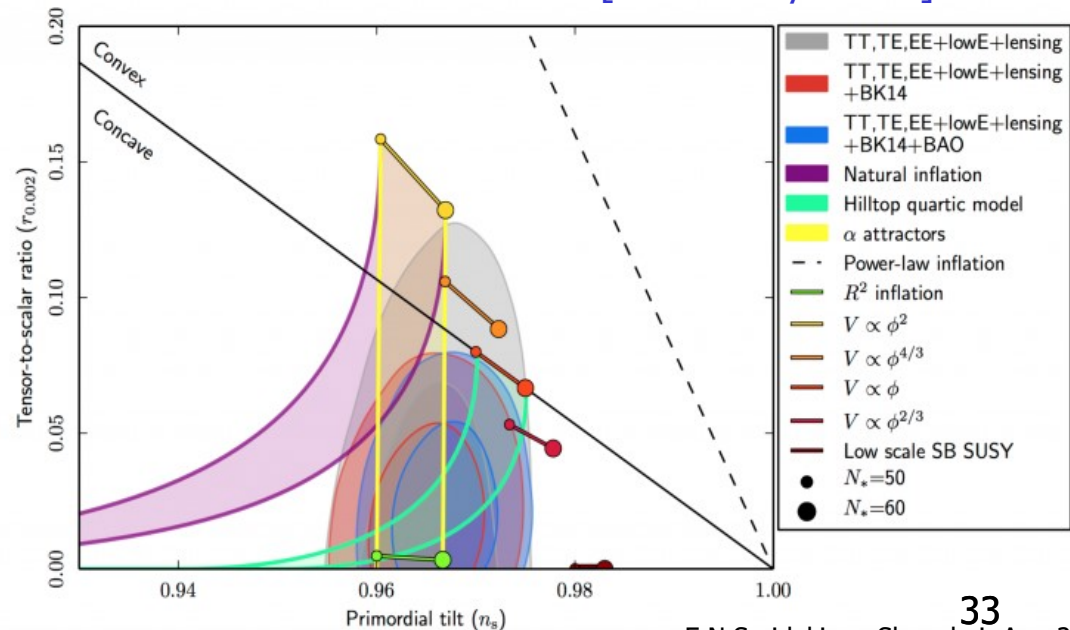
$$H \approx H_i - \frac{M^2}{6}(t - t_i)$$

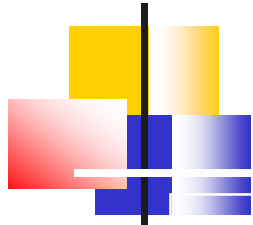
$$T_{reh} \leq 3 \times 10^{17} g_*^{1/4} \left(\frac{M}{m_*} \right)^{3/2} GeV \quad M \approx 3 \times 10^{13} GeV$$



$$f(R) = R + \frac{R^2}{6M^2} \Rightarrow V(\phi) = \frac{3M^2}{32\pi G} \left(1 - e^{-\sqrt{2/3}\pi G \phi} \right)$$

[Starobinsky PL 91]





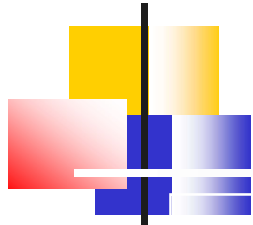
$f(R)$ cosmology – Dark energy

$$8\pi G \rho_{DE} = \frac{FR - f}{2} - 3H\dot{F} + 3H^2(1 - F)$$

for **viable**: $f_{,R} > 0$, $f_{,RR} > 0$, for $R \geq R_0 (> 0)$

$$8\pi G p_{DE} = \ddot{F} + 2H\dot{F} - \frac{FR - f}{2} - (3H^2 + 2\dot{H})(1 - F)$$

[Starobinsky PLB 91]



f(R) cosmology – Dark energy

$$8\pi G \rho_{DE} = \frac{FR - f}{2} - 3H\dot{F} + 3H^2(1 - F)$$

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[Starobinsky PLB 91]

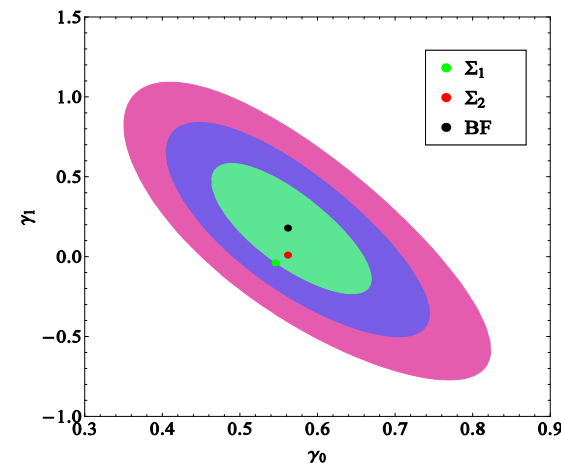
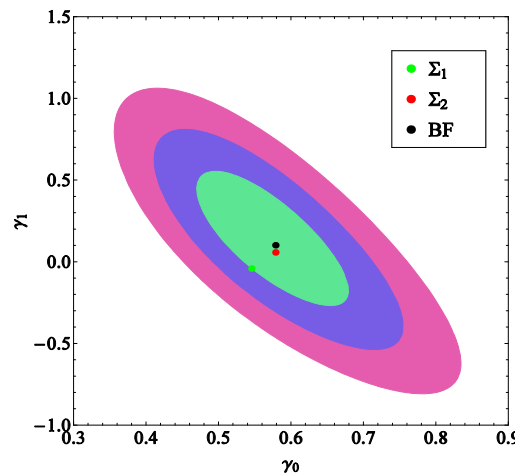
model	$f(R)$	Constant parameters
(i) Hu-Sawicki	$R - \frac{c_1 R_{HS} (R/R_{HS})^p}{c_2 (R/R_{HS})^{p+1} + 1}$	$c_1, c_2, p(> 0), R_{HS}(> 0)$
(ii) Starobinsky	$R + \lambda R_S \left[\left(1 + \frac{R^2}{R_S^2} \right)^{-n} - 1 \right]$	$\lambda(> 0), n(> 0), R_S$
(iii) Tsujikawa	$R - \mu R_T \tanh\left(\frac{R}{R_T}\right)$	$\mu(> 0), R_T(> 0)$
(iv) Exponential	$R - \beta R_E (1 - e^{-R/R_E})$	β, R_E

[Bamba,Geng,Lee JCAP 1011]

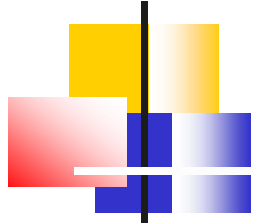
$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff} \rho_m \delta_m$$

$$G_{eff} = \frac{G}{f'} \frac{1 + 4 \frac{k^2}{a^2} \frac{f''}{f'}}{1 + 3 \frac{k^2}{a^2} \frac{f''}{f'}}$$

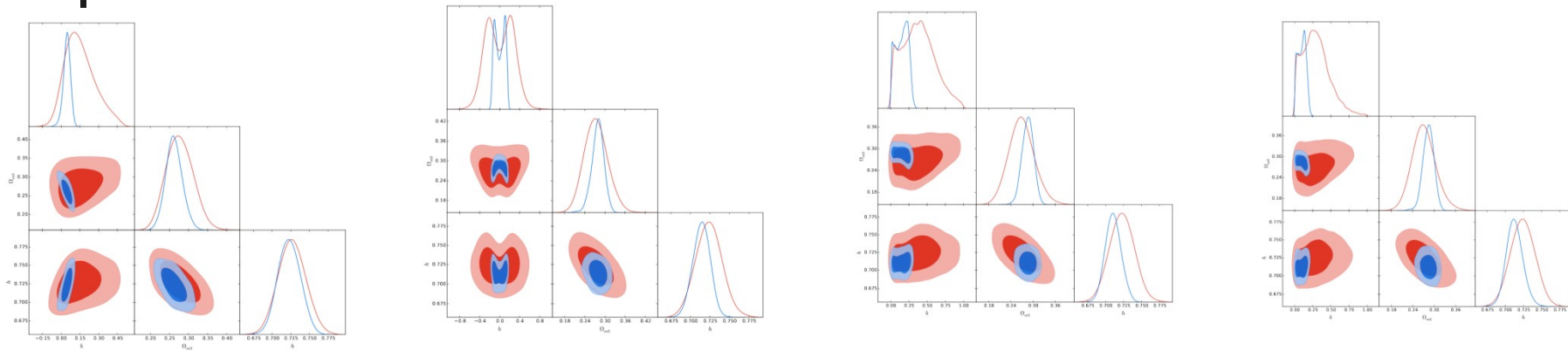
$$\frac{d \ln \delta_m}{d \ln a} = \Omega_m^\gamma(a)$$



[Basilakos,Nesseris,Perivolaropoulos PRD 87]

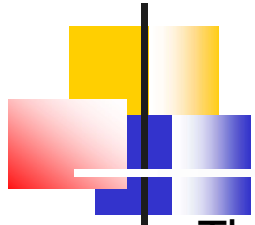


f(R) cosmology – Dark energy



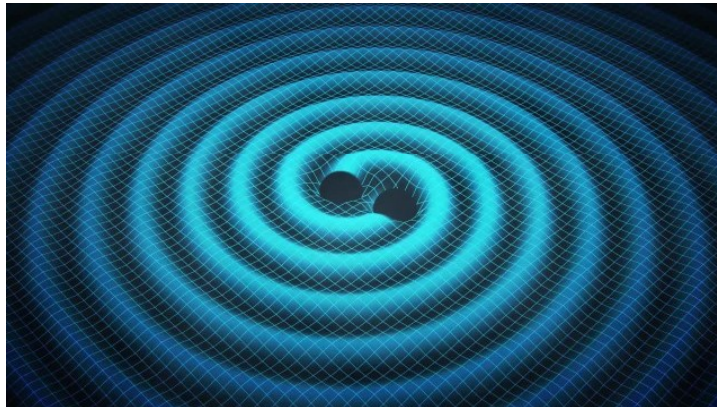
Models	CC+ H_0				JLA + BAO + CC + H_0			
	AIC	ΔAIC	BIC	ΔBIC	AIC	ΔAIC	BIC	ΔBIC
Λ CDM Model	28.205	0	36.809	0	721.084	0	749.017	0
Hu-Sawicki Model	28.744	0.539	38.782	1.973	720.840	-0.244	753.428	4.411
Starobinsky Model	29.096	0.891	39.134	2.325	721.726	0.642	754.314	5.297
Tsujikawa Model	29.407	1.202	39.445	2.636	722.966	1.882	755.554	6.537
Exponential Model	29.310	1.105	39.347	2.538	722.548	1.464	755.136	6.119

[Nunes, Pan, Saridakis, Abreu JCAP 1701]



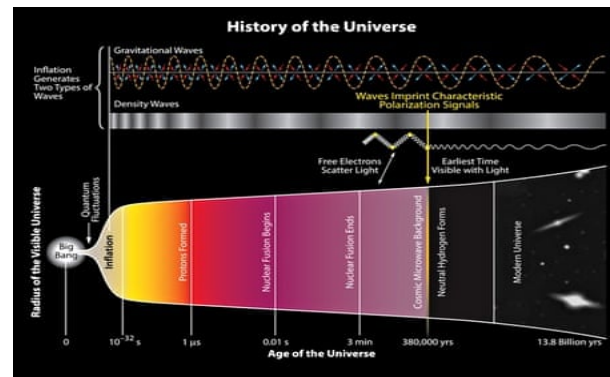
Gravitational waves

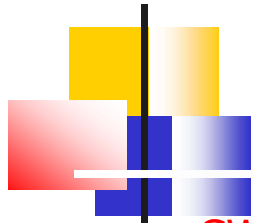
- The **GWs** are the **tensor perturbations** of the metric. Predicted in 1915, first observed in 2015. **First astronomical observation** ever, **not related to E/M**.
- **GWs from mergers:**



[Abbott et al, LIGO Virgo PRL 116]

- **Primordial GWs:**

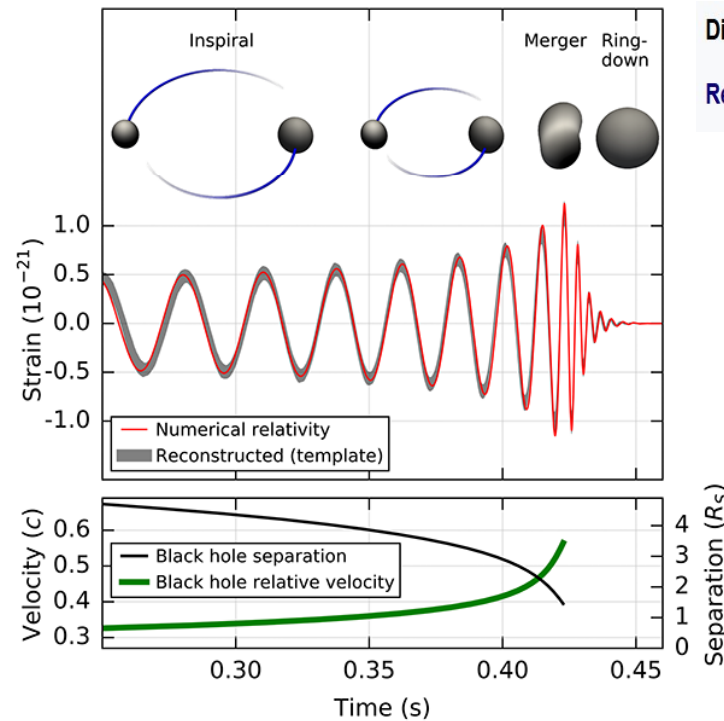
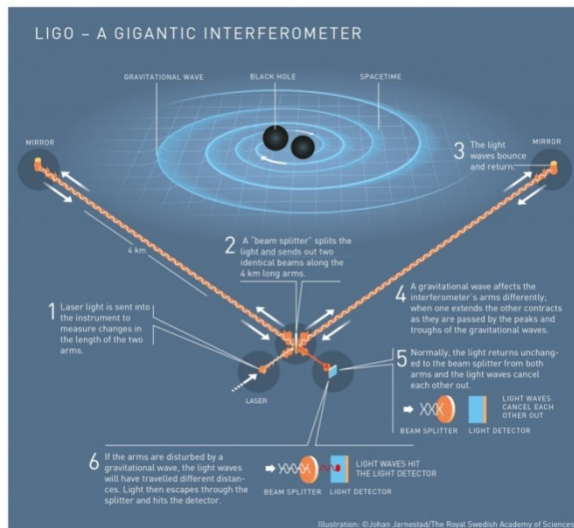




Gravitational waves

GW150914: Two **black holes** with $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, resulting in a $62^{+4}_{-4} M_{\odot}$ black hole

Louisiana.
Washington
4km
 $10^{-18}m$



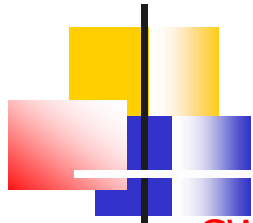
Distance	$410^{+160}_{-180} \text{ Mpc}$
Redshift	$0.093^{+0.030}_{-0.036}$

[Abbott et al, LIGO Virgo PRL 116]

2017 Nobel Prize in Physics

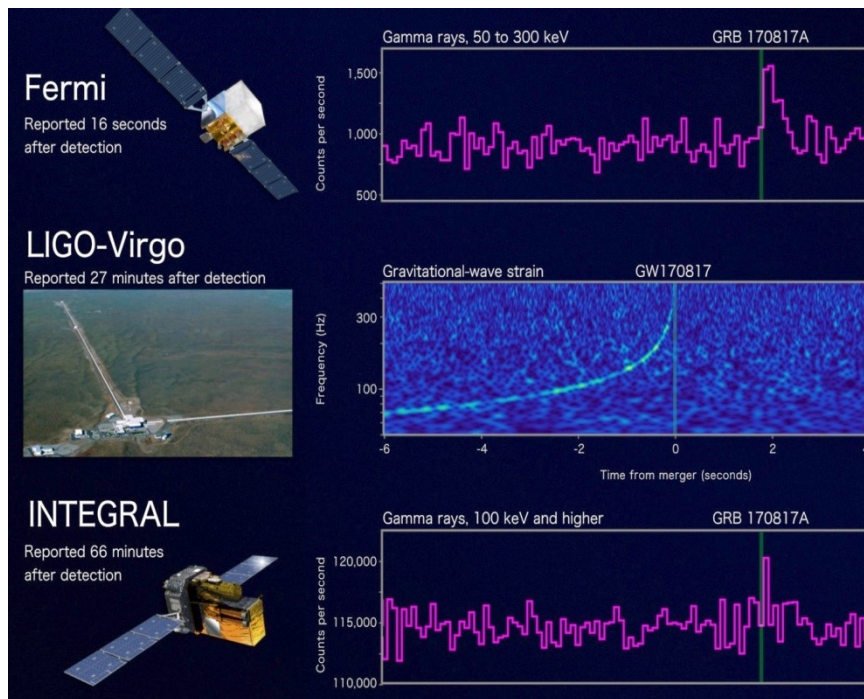
38

E.N.Saridakis – Shanghai, Aug 2019



Gravitational waves

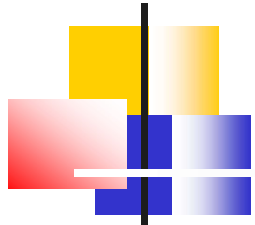
- **GW170817**: Two **neutron stars**, distance 40 Mpc, redshift 0.0099
- **GRB170817A**: The Electromagnetic counterpart.



[Goldstein et al, Fermi Gamma Ray Burst Monitor
Astrophys.J 848]

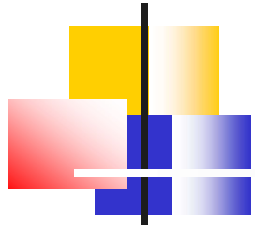
[Abbott et al, LIGO Virgo PRL 119]

- The **era** of **multi-messenger astronomy** begins!



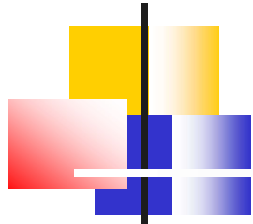
Gravitational waves

- In case of GWs from **black hole mergers** we know their **properties** at the **moment of detection**, and their direction (in case of three detectors).
Assuming GR and Λ CDM we can extract their speed, distance, and properties at the **moment of emission**.



Gravitational waves

- In case of GWs from **black hole mergers** we know their **properties** at the **moment of detection**, and their direction (in case of three detectors).
Assuming GR and Λ CDM we can extract their speed, distance, and properties at the **moment of emission**.
- In case of GWs from **neutron star mergers**, and their **E/M counterpart**, we know their **properties** at the **moment of detection** and their direction, but using the implied physics from the E/M information we can extract their speed, distance and **properties** at the **moment of emission**, **independently** of the **underlying gravitational theory and cosmological scenario**.
- **Great tool** for **testing General Relativity** and **cosmological scenarios**!



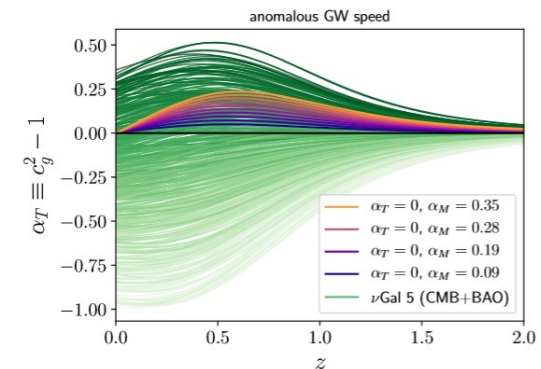
Gravitational waves

- An immediate result: **The speed of GWs is equal to the speed of light!**

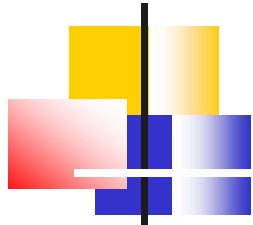
GW170817 time delay $1.74 \pm 0.05\text{s}$ constrains: $-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$

- Excludes** a large number of theories that were consistent with other data!

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/ $f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)\cdot\text{Gauss-Bonnet}$ [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817



[Ezquiaga, Zumalacarregui PRL 119]



Gravitational waves

- For **tensor perturbations**:

$$g_{00} = -1, \quad g_{0i} = 0,$$

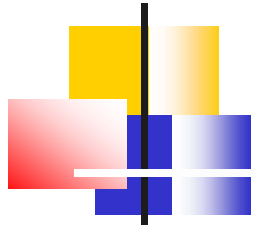
$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} \right)$$

$$\ddot{h}_{ij} + (3 + \alpha_M) \dot{h}_{ij} + (1 + \alpha_T) \frac{k^2}{a^2} h_{ij} = 0$$

$$\alpha_M = \frac{d \log(M_*^2)}{d \log a} \quad c_g^2 = (1 + \alpha_T)$$

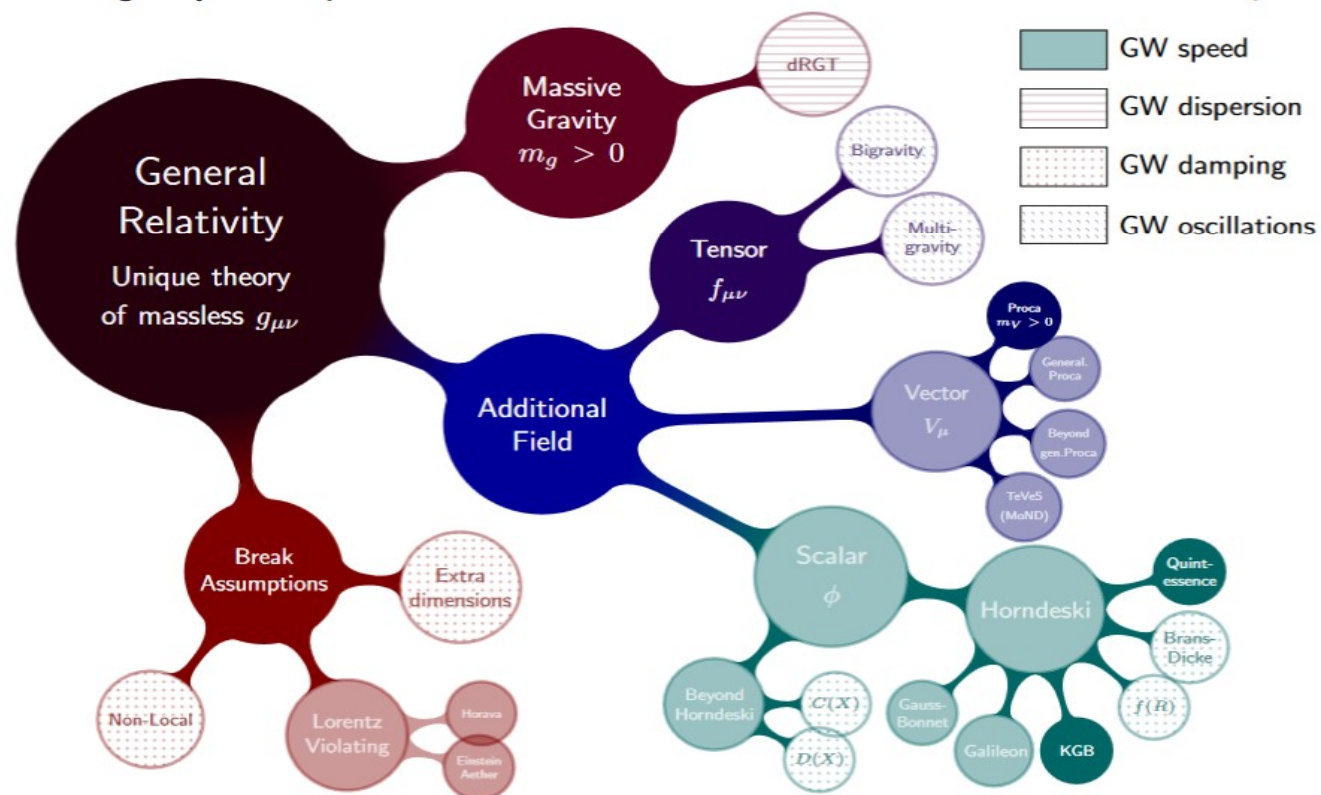
- $$h_{\text{GW}} \sim h_{\text{GR}} \underbrace{e^{-\frac{1}{2} \int \nu \mathcal{H} d\eta}}_{\text{Affects amplitude}} \underbrace{e^{ik \int (\alpha_T + a^2 m^2 / k^2)^{1/2} d\eta}}_{\text{Affects phase}}$$

[Ezquiaga, Zumalacarregui PRL 119]

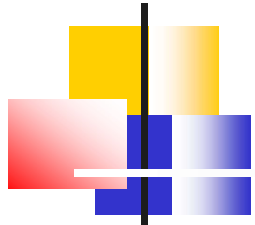


Gravitational waves

Modified gravity roadmap

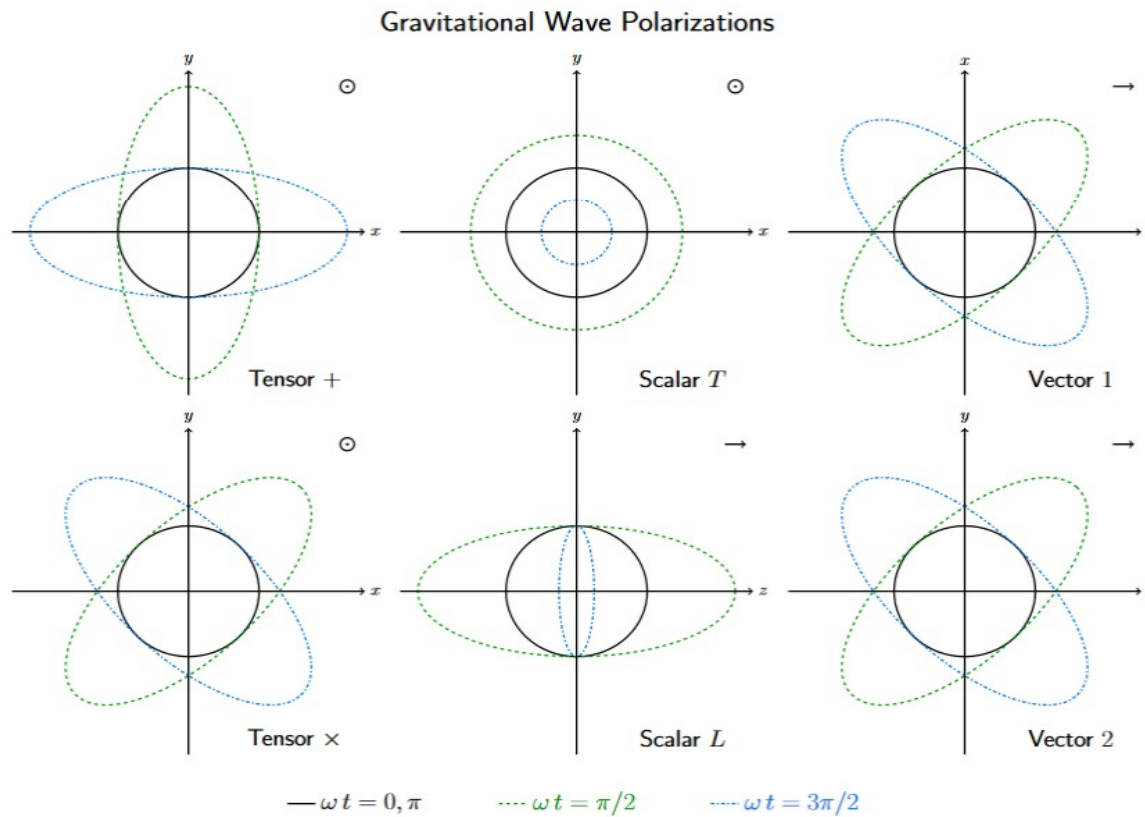


[Ezquiaga, Zumalacarregui PRL 119]

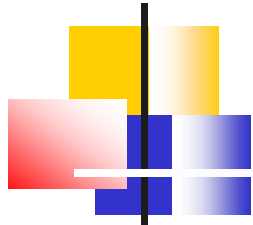


Gravitational waves

- Polarizations:

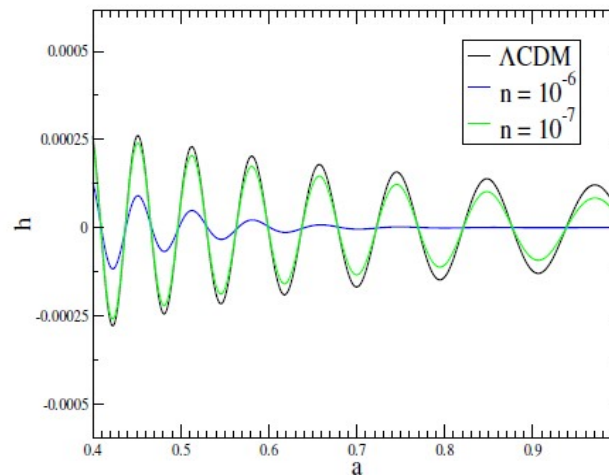
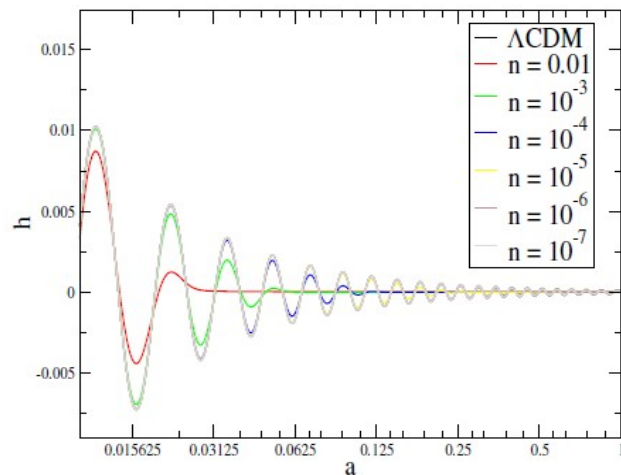


[Ezquiaga, Zumalacarregui PRL 119]



Gravitational waves

- Testing General Relativity, modified gravities, and various cosmological scenarios.
- The GWs properties at emission and detection are determined by them.
- Examples: $f(T)$, $f(R)$, $f(Q)$, etc



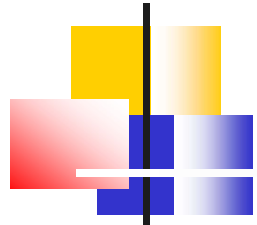
$$h_{\mu\nu}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\gamma_1^{(1)1} & B_1^2 \exp(ip_\mu x^\mu) & 0 \\ 0 & B_1^2 \exp(ip_\mu x^\mu) & -2\gamma_1^{(1)1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

[Cai, Li, Saridakis, Xue PRD97] [Li, Cai, Cai, Saridakis, JCAP 1810]

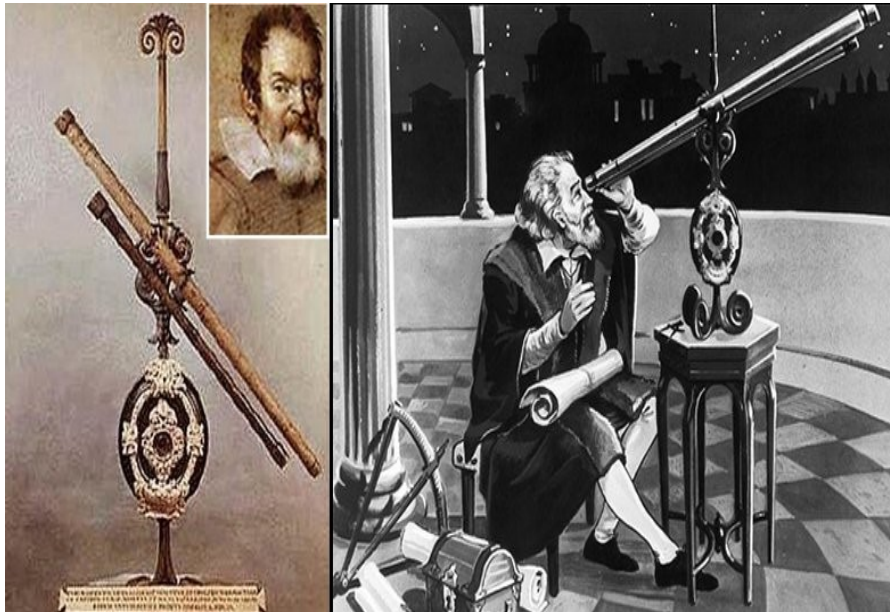
[Farrugia, Said, Gakis, Saridakis, PRD97]

[Soudi, Farrugia, Gakis, Said, Saridakis, 1810.08220 (to appear in PRD)]

[Nunes, Pan, Saridakis, PRD98]



Multi-messenger Astronomy Era!



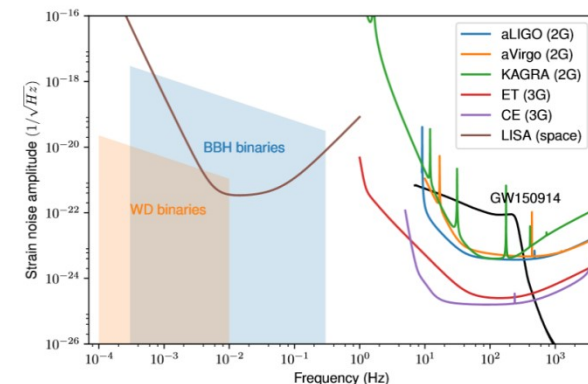
EM observations: 400 years



GW observations: 4 years

Conclusions

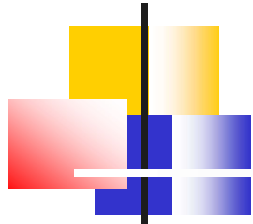
- i) The **Standard Model of Cosmology** may ask for **new physics**, definitely for **inflation** and **dark matter**, probably for **dark energy**.
- ii) We can **modify** the **Universe content**, or/and the **gravitational theory**. **Torsional gravity** is a good candidate.
- iii) We use various **observational data** (SnIa, CMB, BAO, H(z), LSS etc) in order to **constrain** the proposed **theories**.
- iv) The advancing **gravitational wave astronomy**, and especially **multi-messenger astronomy** offers a **novel tool** to test General Relativity and cosmological scenarios in **great accuracy**.
- v) **A new era has begun!**





Outlook

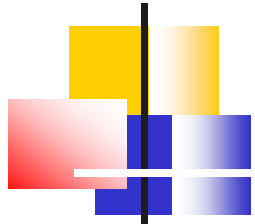
- A **huge project** is ahead for the community:
- i) Calculate the **exact form of GWs** created from mergers in various **gravitational theories** (needs numerical gravity).
- ii) Calculate the **propagation of these GWs** from emission to detection for various **cosmological scenarios**.
- iii) Use **multi-messenger data** to test General Relativity, break degeneracies and constrain or exclude the **various theories**.
- iv) Elaborate also the creation and possible detection of **primordial GWs**.
- v) For $f(T)$ gravity, $f(R,G)$, running vacuum, higher-order theories, $f(T,TG)$ gravity, $f(Q)$ gravity, etc, **currently under investigation**
[Saridakis, Capozziello, Cai, Marciano, Modesto Nunes, Erices, Said, Basilakos]
- vi) **Get prepared** for the **huge flow of data** that **will come!**



- “There are the ones that **invent occult fluids** to understand the Laws of Nature. They come to conclusions, but they now run out into **dreams** and **chimeras** neglecting the **true constitutions** of the things...
However there are those that from the **simplest observation of Nature**, they reproduce **New Forces**”...

From the Preface of PRINCIPIA (II edition) 1687
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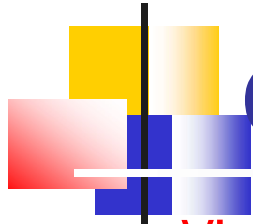


THANK YOU!









Curvature and Torsion

- **Vierbeins** e_A^μ : four linearly independent fields in the **tangent space**

$$g_{\mu\nu}(x) = \eta_{AB} e_\mu^A(x) e_\nu^B(x)$$

- **Connection**: ω_{ABC}

- **Curvature tensor**: $R_{B\mu\nu}^A = \omega_{B\nu,\mu}^A - \omega_{B\mu,\nu}^A + \omega_{C\mu}^A \omega_{B\nu}^C - \omega_{C\nu}^A \omega_{B\mu}^C$

- **Torsion tensor**: $T_{\mu\nu}^A = e_{\nu,\mu}^A - e_{\mu,\nu}^A + \omega_{B\mu}^A e_\nu^B - \omega_{B\nu}^A e_\mu^B$

- **Levi-Civita connection and Contorsion tensor**: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

$$K_{ABC} = \frac{1}{2}(T_{CAB} - T_{BCA} - T_{ABC}) = -K_{BAC}$$

- **Curvature and Torsion Scalars**: $R = \bar{R} + T - 2(T_\nu^{\nu\mu})_{;\mu}$

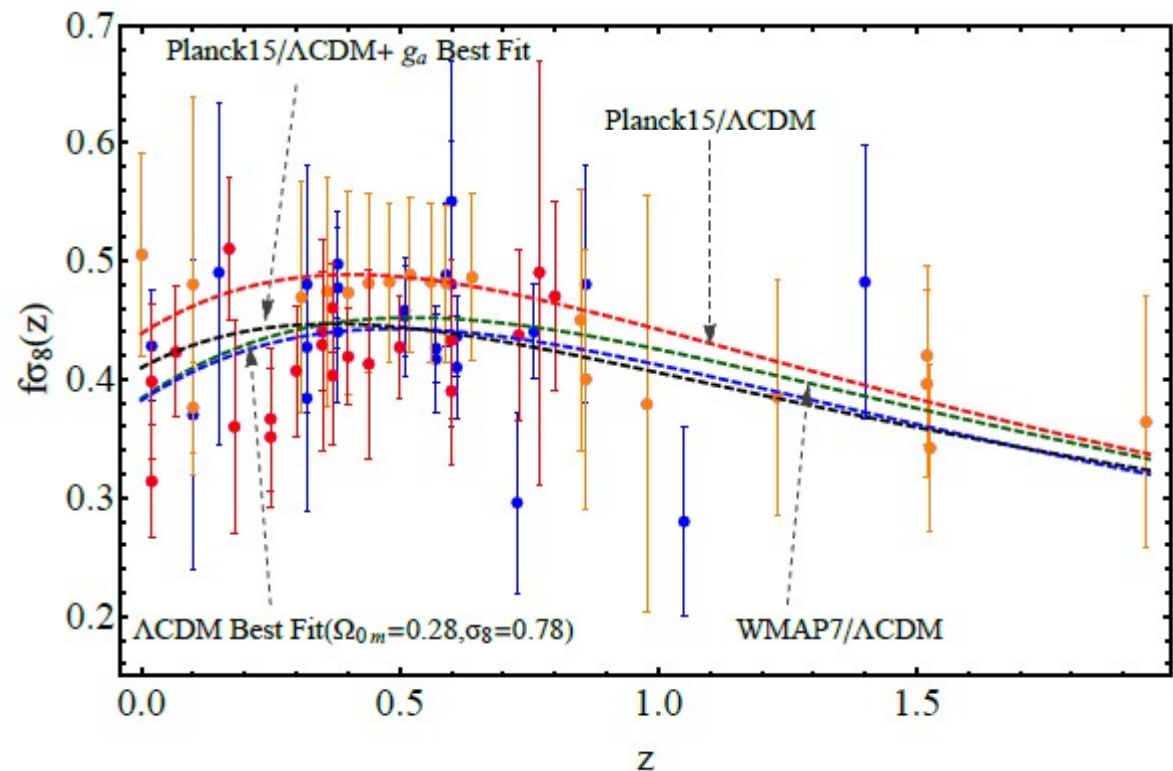
$$R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} R_{\mu\rho\nu}^\rho$$

$$T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^\rho T_\nu^{\nu\mu}$$

Tension1 – $f\sigma_8$

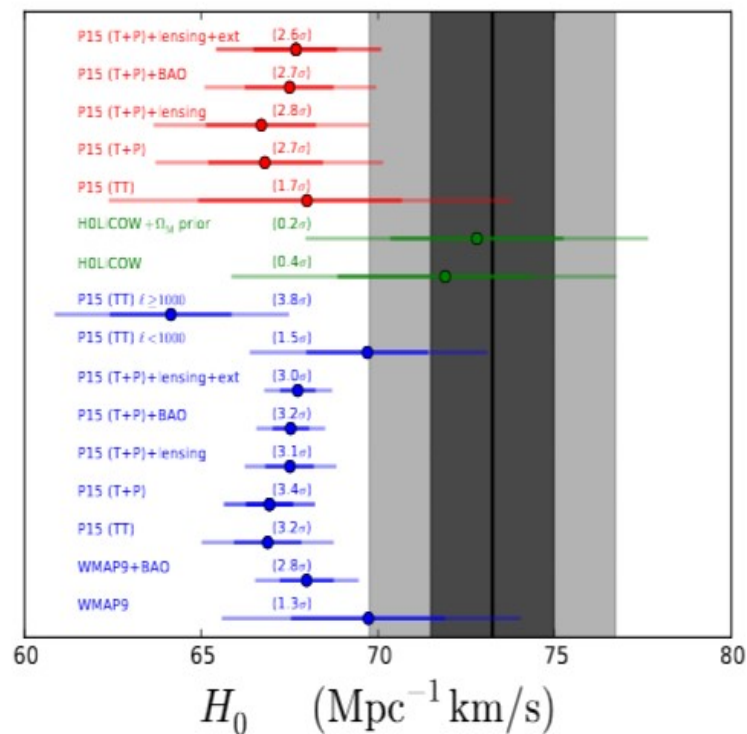
- **Tension** between the **data** and **Planck/ Λ CDM**. The data indicate a **lack of “gravitational power”** in structures on intermediate-small cosmological scales.

Parameter	Planck15/ Λ CDM [12]	WMAP7/ Λ CDM [45]
$\Omega_b h^2$	0.02225 ± 0.00016	0.02258 ± 0.00057
$\Omega_c h^2$	0.1198 ± 0.0015	0.1109 ± 0.0056
n_s	0.9645 ± 0.0049	0.963 ± 0.014
H_0	67.27 ± 0.66	71.0 ± 2.5
Ω_{0m}	0.3156 ± 0.0091	0.266 ± 0.025
w	-1	-1
σ_8	0.831 ± 0.013	0.801 ± 0.030

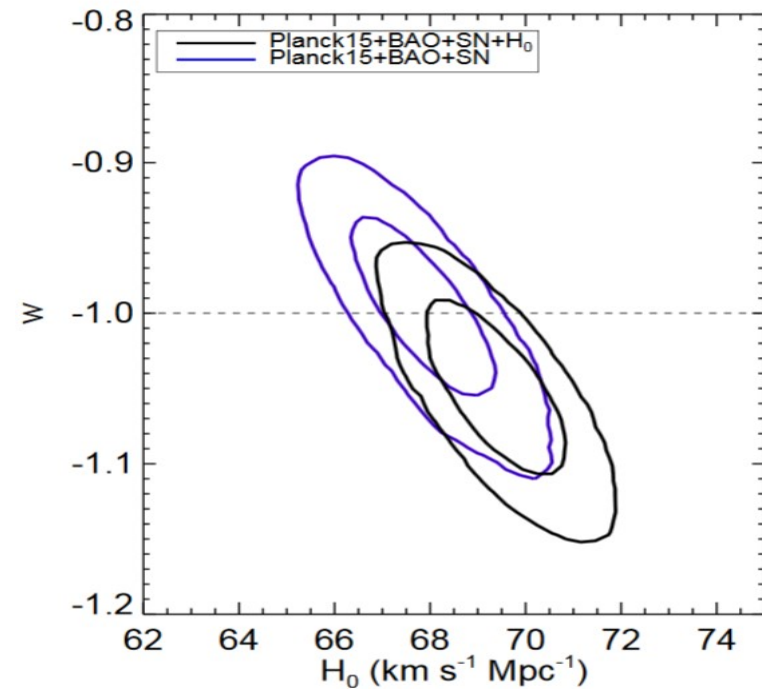


Tension2 – H0

- **Tension** between the **data** (direct measurements) and **Planck/ Λ CDM** (indirect measurements). The data indicate a **lack of “gravitational power”**.



[Bernal, Verde, Riess, JCAP1610]



[Riess et al, Astrophys.J 826]