Neutrino mass models based on symmetries

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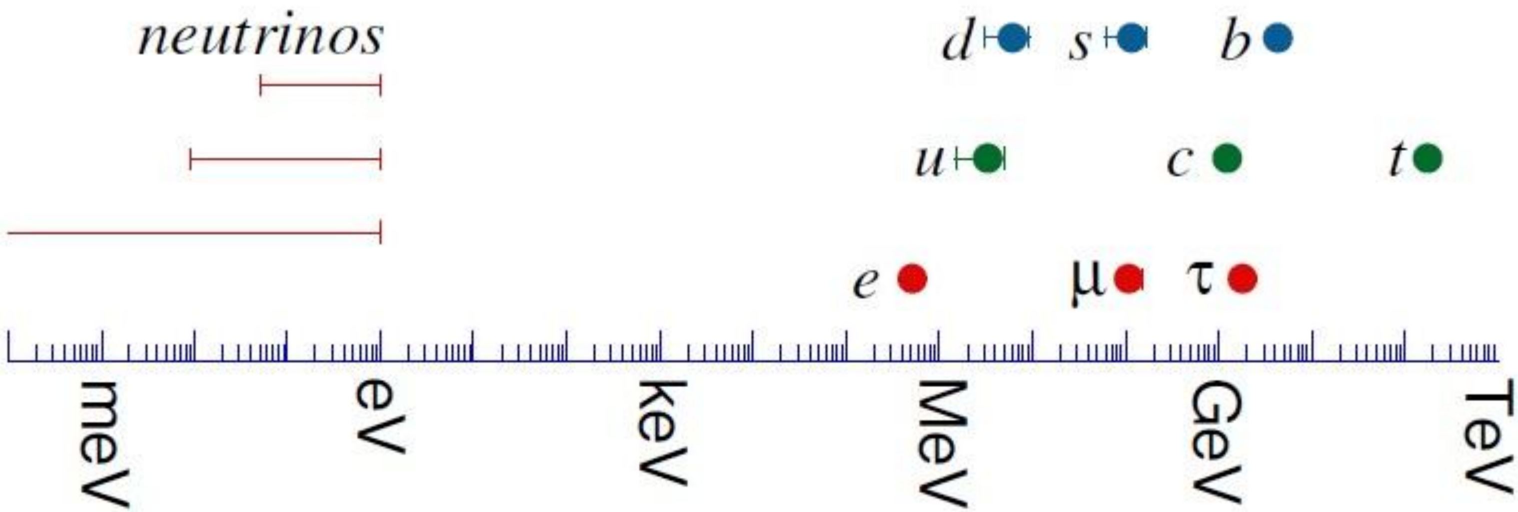
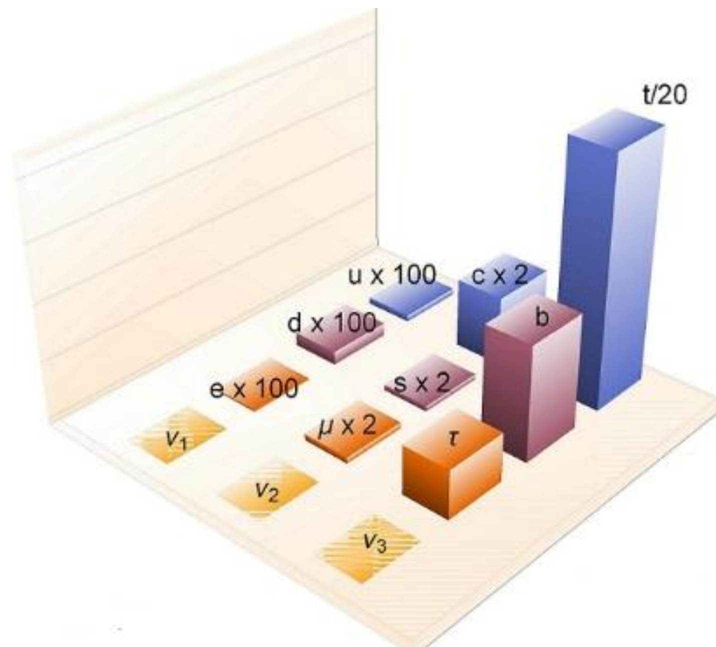
Fermion masses in SM



One big open question: why the neutrino masses are so tiny ?What is the origin of the observed neutrinos masses?



Seesaw mechanism

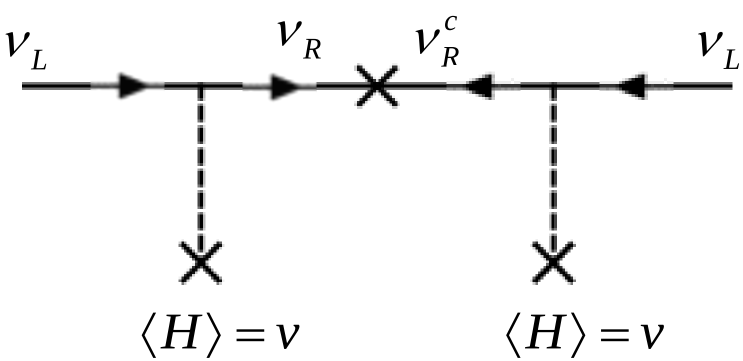
- Adding right-handed neutrinos ν_R which are SM singlets

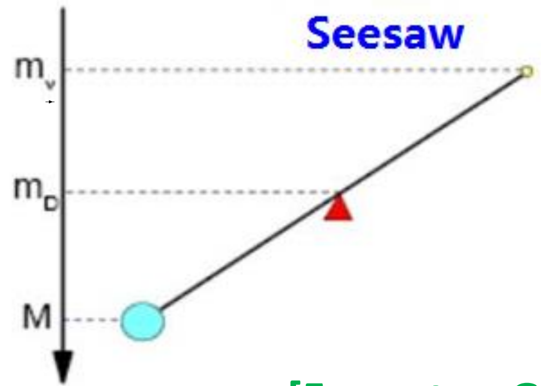
$$\mathcal{L}^{\nu}_{mass} = -\frac{1}{2}\left[\left(\overline{\nu}_L, \overline{\nu^c_R}\right)\begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}\begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix}\right] + h.c.$$

In the seesaw limit $M >> m_D$

$$m_\nu = -m_D M^{-1} m_D^T$$

[Minkowski(1977); Yanagida(1979); Glashow (1979); Gell-Mann, Ramond, Slansky(1979); Mohapatra, Senjanovic(1980)]





The formalism is less predictive in general

[Frampton,Glashow, Yanagida,hep-ph/0208157]

$$m_D = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

$$m_D = \begin{pmatrix} \times & \times \\ \times & \times \\ \times & \times \end{pmatrix}, \quad m_N = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

# of parameters: $2\times 9 - 3 + 3 = 18$

$2\times 6 - 3 + 2 = 11$

Littlest seesaw

one texture zero

[King,1512.07531;Luhn, King, 1607.05276]

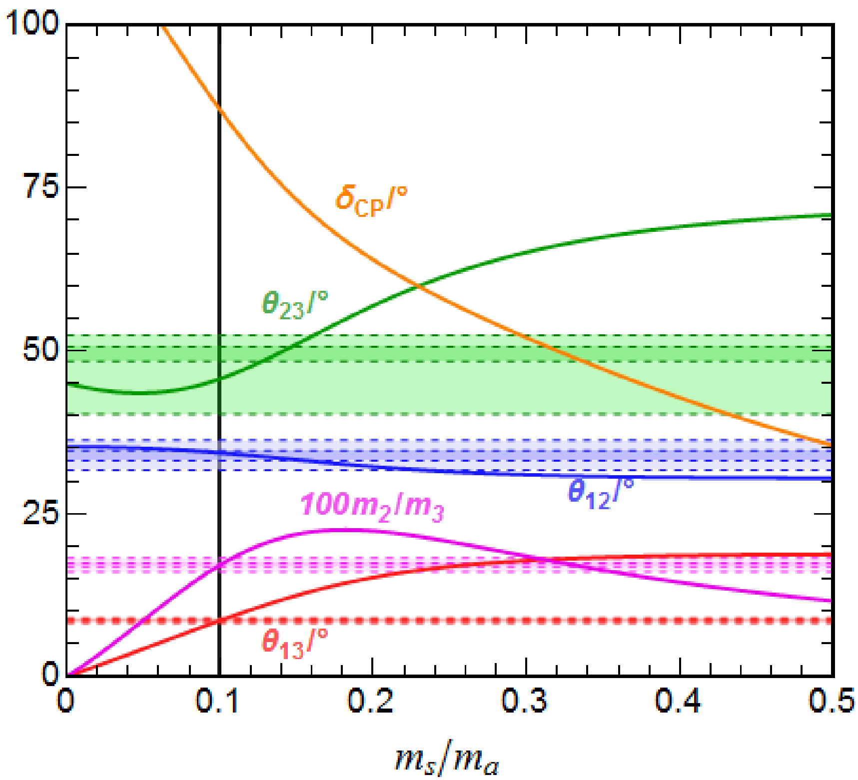
$$m_D = \begin{pmatrix} 0 & be^{i\pi/3} \\ a & 3be^{i\pi/3} \\ a & be^{i\pi/3} \end{pmatrix}, \quad m_N = \begin{pmatrix} M_{\text{atm}} & 0 \\ 0 & M_{\text{sol}} \end{pmatrix}$$

seesaw

$$\Rightarrow m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_s e^{2\pi i/3} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

m_a and m_s are fixed by the neutrino mass squared differences.

m_a (meV)	26.796
m_s (meV)	2.706
$\sin^2\theta_{13}$	0.0225
$\sin^2\theta_{12}$	0.318
$\sin^2\theta_{23}$	0.513
δ_{CP}	-0.482π
m_1 (meV)	0
m_2 (meV)	8.632
m_3 (meV)	50.210
m_{ee} (meV)	2.696

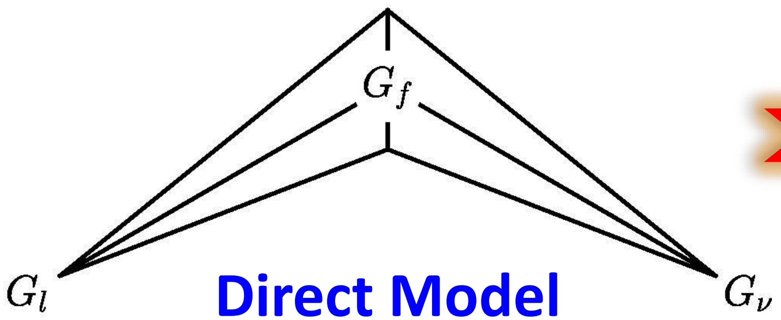


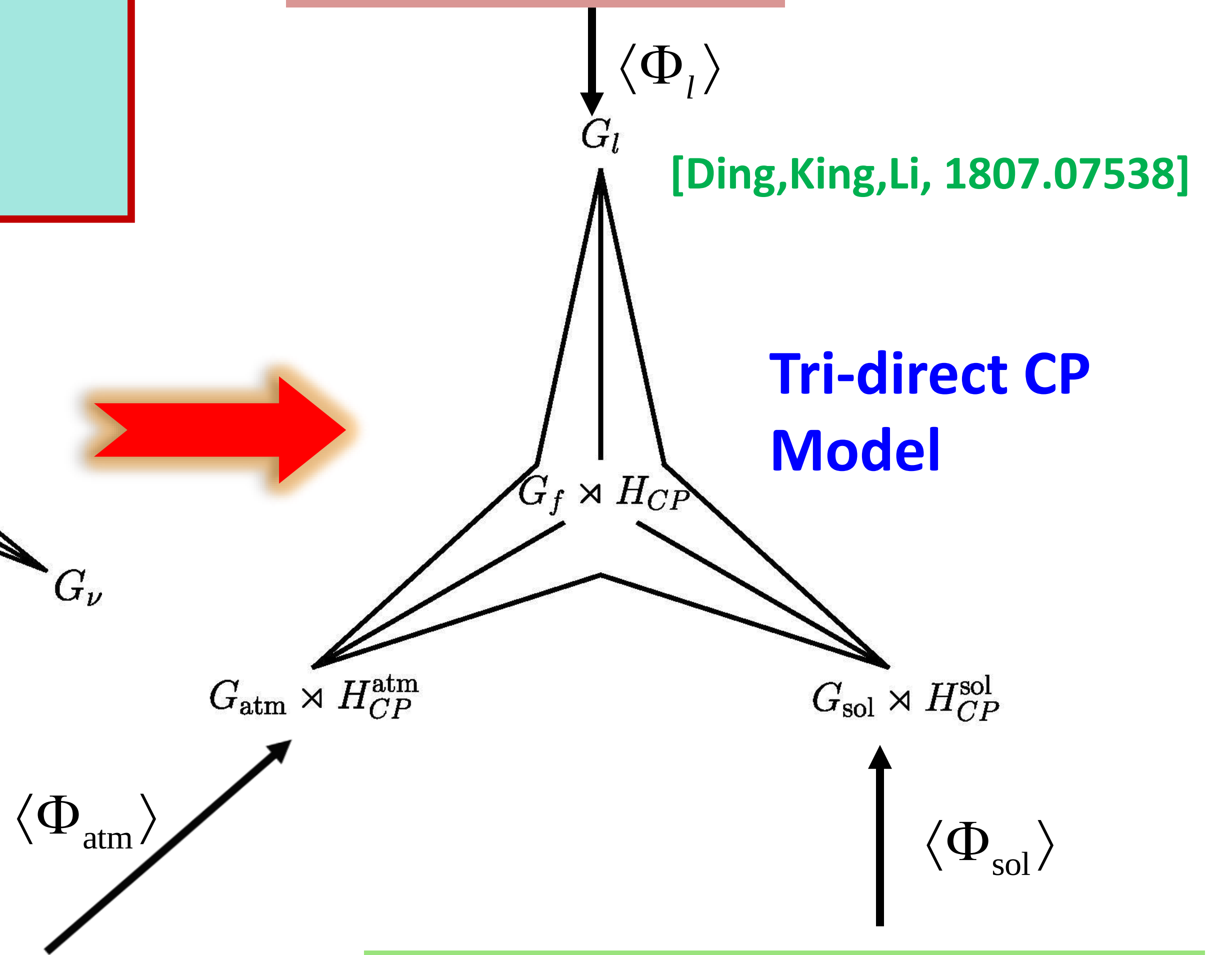
A model building paradigm: Tri-direct CP approach

$$m_D = \left(y_{\text{atm}} \langle \Phi_{\text{atm}} \rangle, \quad y_{\text{sol}} \langle \Phi_{\text{sol}} \rangle \right),$$

$$m_N = \begin{pmatrix} M_{\text{atm}} & 0 \\ 0 & M_{\text{sol}} \end{pmatrix}$$

$$\mathcal{L}_c = -y_l (L.\Phi_l) l^c$$





$$\mathcal{L}_{\text{atm}} = -\frac{1}{2} x_{\text{atm}} \xi_{\text{atm}} N^c_{\text{atm}} N^c_{\text{atm}} - y_{\text{atm}} (L.\Phi_{\text{atm}}) N^c_{\text{atm}}$$

$$\mathcal{L}_{\text{sol}} = -\frac{1}{2} x_{\text{sol}} \xi_{\text{sol}} N^c_{\text{sol}} N^c_{\text{sol}} - y_{\text{sol}} (L.\Phi_{\text{sol}}) N^c_{\text{sol}}$$

General results of Tri-direct CP approach

- Vacuum alignment constrained by residual symmetry

$$g_{\rm atm}\langle\Phi_{\rm atm}\rangle=\langle\Phi_{\rm atm}\rangle,\quad g_{\rm atm}\in G_{\rm atm}$$

$$X_{\rm atm}\langle\Phi_{\rm atm}\rangle^{*}=\langle\Phi_{\rm atm}\rangle,\quad X_{\rm atm}\in H^{CP}_{\rm atm}$$

[Ding,King,Li, 1807.07538]

Similar condition for solar neutrino sector.

- Light neutrino mass matrix

$$m_\nu=-\frac{y^2_{\rm atm}}{x_{\rm atm}}\frac{\langle\Phi_{\rm atm}\rangle\langle\Phi_{\rm atm}\rangle^T}{\langle\xi_{\rm atm}\rangle}-\frac{y^2_{\rm sol}}{x_{\rm sol}}\frac{\langle\Phi_{\rm sol}\rangle\langle\Phi_{\rm sol}\rangle^T}{\langle\xi_{\rm sol}\rangle}$$

- One column of the mixing matrix is fixed

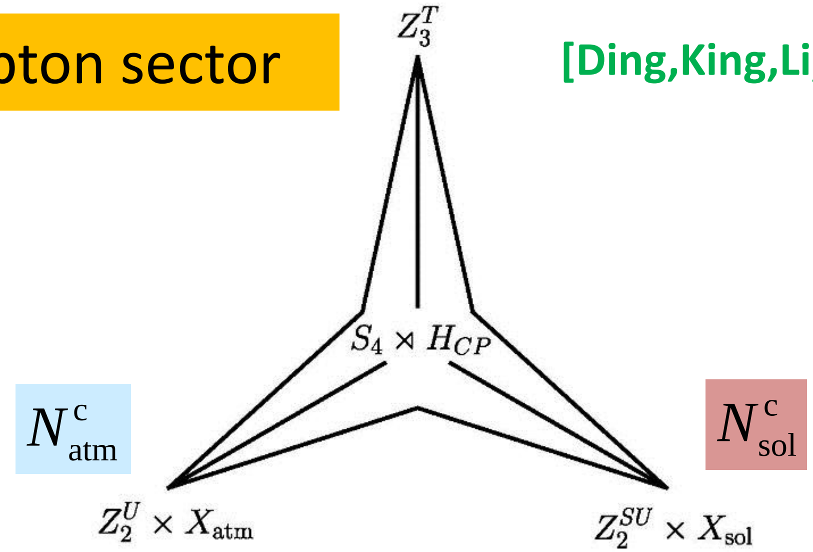
$$m_\nu\left[\langle\Phi_{\rm atm}\rangle\times\langle\Phi_{\rm sol}\rangle\right]=(0,0,0)^T$$

- 1st column $\langle\Phi_{\rm atm}\rangle\times\langle\Phi_{\rm sol}\rangle$ → $m_1=0$ → Normal ordering
- 3rd column $\langle\Phi_{\rm atm}\rangle\times\langle\Phi_{\rm sol}\rangle$ → $m_3=0$ → Inverted ordering

Littlest seesaw from Tri-direct CP with S_4

Charged lepton sector

[Ding,King,Li, 1807.07538, 1811.12340]



- Vacuum alignment

$$\langle \Phi_{\mathbf{atm}} \rangle = v_{\mathbf{atm}} \left(0, \quad 1, \quad -1\right)^T, \quad \langle \Phi_{\mathbf{sol}} \rangle = v_{\mathbf{sol}} \left(1, \quad x, \quad 2-x\right)^T$$

x real

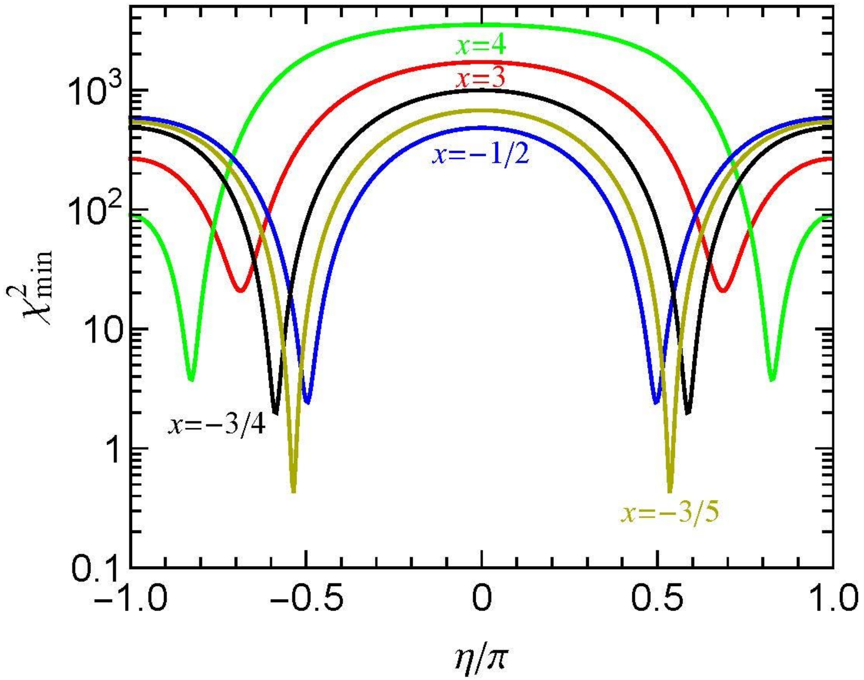
- Neutrino mass matrix

Only one phase

$$m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + m_s e^{i\eta} \begin{pmatrix} 1 & 2-x & x \\ 2-x & (2-x)^2 & x(2-x) \\ x & x(2-x) & x^2 \end{pmatrix}$$

TM1 mixing pattern; **NO** mass spectrum with $\mathbf{m_1=0}$

- Benchmark values



Original Littlest seesaw: $(x,\eta)=(3,\,2\pi/3),\,(-1,-2\pi/3)$

$$\longrightarrow \sin^2\theta_{23}\approx 0.5, \quad \delta_{CP}\approx -\pi/2$$

New Littlest seesaw: $(x,\eta)=(-1/2,-\pi/2)$ **More close to present data!**

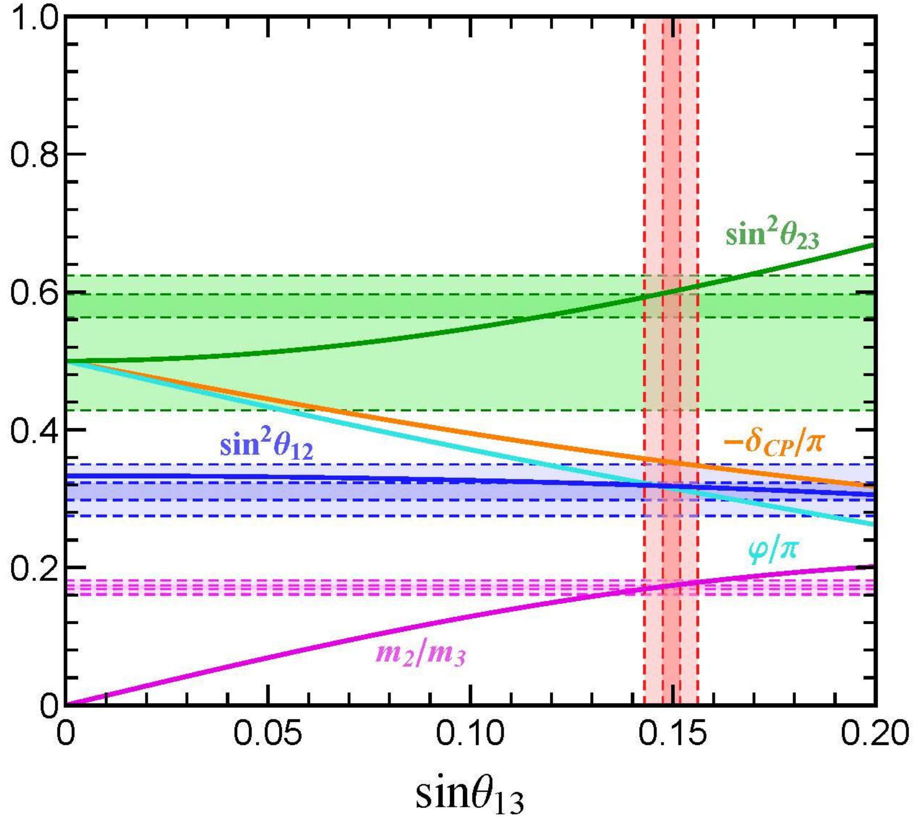
$$\longrightarrow 0.593\le\sin^2\theta_{23}\le 0.609, \quad -0.358\pi\le\delta_{CP}\le -0.348\pi$$

The new Littlest seesaw is **more easy** to be realized in a model.

[Chen, Ding,King,Li, 1906.11414]

➢ Phenomenological predictions of new Littlest seesaw

mixing parameters



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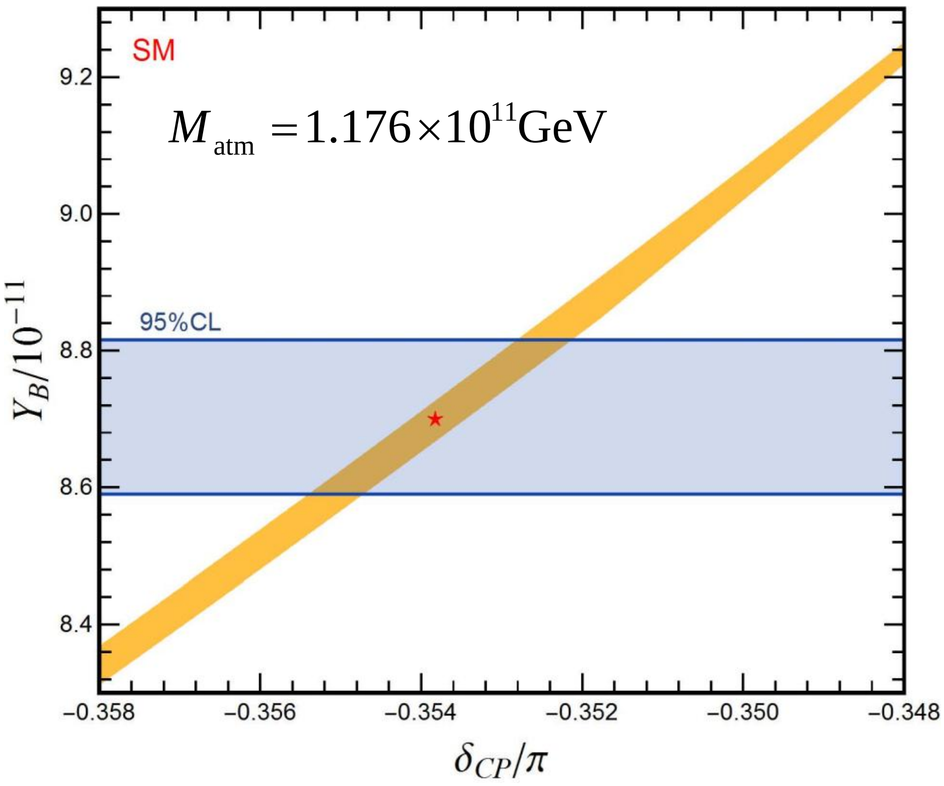


All possible cases derived from Tri-direct CP with S_4

NO:

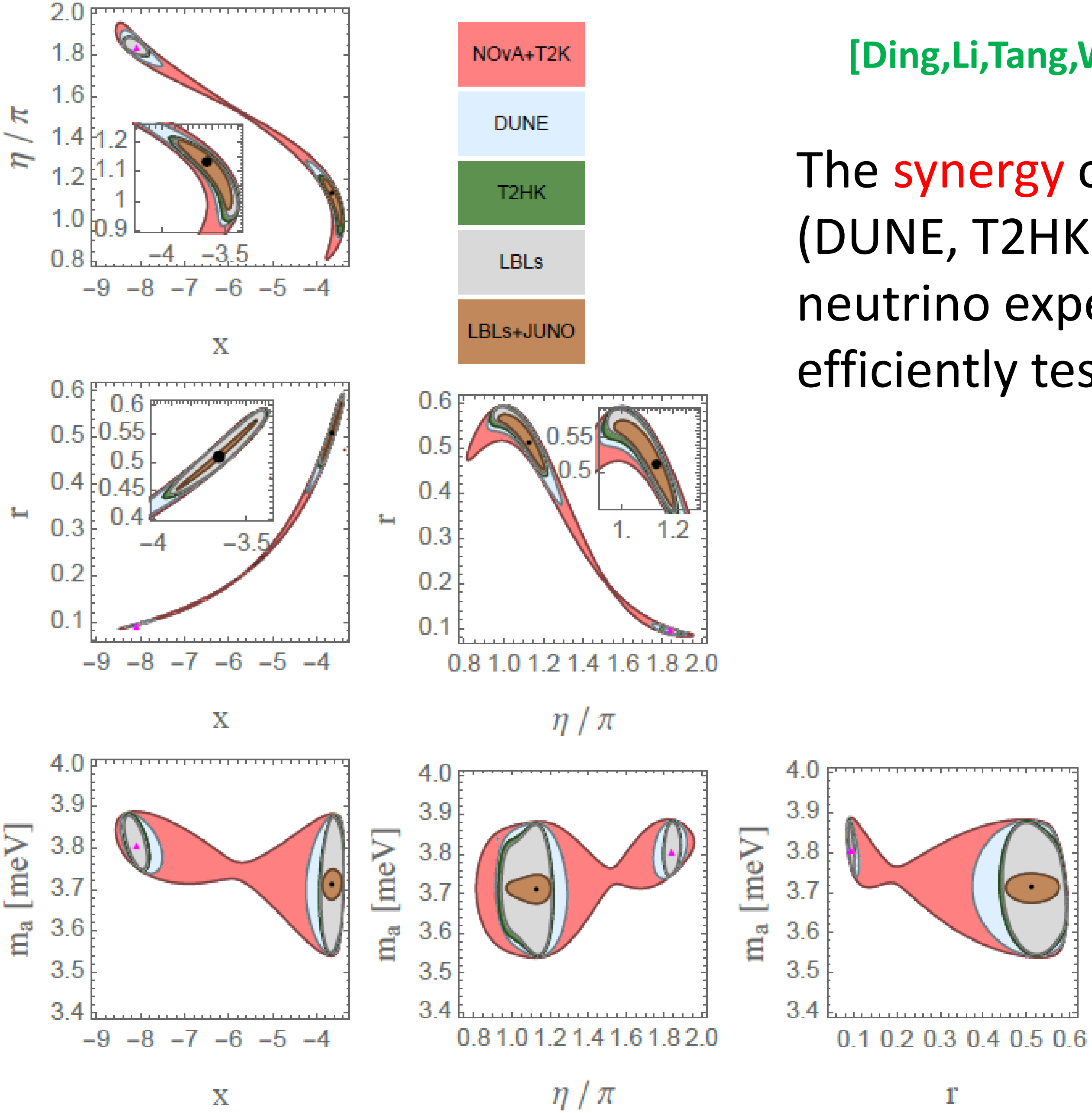
	$(G_l, G_{\rm atm}, G_{\rm sol})$	$X_{\rm sol}$	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ_{CP}/π	β/π	m_2(meV)	m_3(meV)	m_{ee}(meV)
$\mathcal{N}_1$	(Z_3^T, Z_2^U, Z_2^{SU})	1	0.754	0.0221	0.318	0.538	−0.447	−0.458	8.602	49.940	2.394
		U	0.754	0.0221	0.318	0.538	−0.447	0.957	8.602	49.940	3.765
$\mathcal{N}_2$	$(Z_3^T, Z_3^{ST}, Z_2^{SU})$	1	0.754	0.0221	0.318	0.538	−0.447	−0.617	8.602	49.940	2.929
		U	0.754	0.0221	0.318	0.538	−0.447	−0.997	8.603	49.942	3.735
$\mathcal{N}_3$	(Z_3^T, Z_2^S, Z_2^{SU})	U	0.754	0.0221	0.318	0.537	−0.447	0.509	8.605	49.940	3.234
$\mathcal{N}_4$	$(Z_3^T, Z_2^{TST^2}, Z_2^U)$	1	3.957	0.0222	0.332	0.515	0.478	−0.0576	8.606	49.938	1.779
$\mathcal{N}_5$	$(K_4^{(S,U)}, Z_2^{TU}, Z_2^{TU})$	U	21.368	0.0221	0.256	0.417	−1	−0.0616	8.602	49.940	3.243
$\mathcal{N}_6$	$(Z_4^{TSU}, Z_3^T, Z_2^{SU})$	U	6.215	0.0223	0.339	0.509	0.487	−0.0299	8.607	49.934	1.766
$\mathcal{N}_7$	$(K_4^{(S,TST^2)}, Z_3^T, Z_2^{SU})$	1	2.642	0.0221	0.327	0.507	−0.490	0.806	8.604	49.940	3.740
		U	2.520	0.0221	0.327	0.514	0	0	8.603	49.940	3.855
$\mathcal{N}_8$	$(K_4^{(S,TST^2)}, Z_2^U, Z_2^{TU})$	U	0.092	0.0221	0.306	0.517	0.476	−0.394	8.600	49.944	2.612

All possible cases derived from Tri-direct CP with S_4

IO:

	$(G_l, G_{\text{atm}}, G_{\text{sol}})$	X_{sol}	χ^2_{min}	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ_{CP}/π	β/π	m_1(meV)	m_2(meV)	m_{ee}(meV)
$\mathcal{I}_1$	(Z_3^T, Z_3^{ST}, Z_2^U)	1	2.678	0.0223	0.307	0.5	−0.927	0.306	49.193	49.940	43.664
$\mathcal{I}_2$	$(Z_3^T, Z_2^{SU}, Z_2^{TU})$	U	2.678	0.0223	0.307	0.5	−0.676	0.848	49.193	49.940	21.169
$\mathcal{I}_3$	$(K_4^{(S,U)}, Z_2^{TST^2}, Z_2^U)$	1	2.678	0.0223	0.307	0.5	0.495	−0.102	49.193	49.940	47.794
		S	2.678	0.0223	0.307	0.5	−0.495	0.102	49.193	49.940	47.794
$\mathcal{I}_4$	$(K_4^{(S,U)}, Z_2^{TU}, Z_2^{TU})$	U	19.772	0.0224	0.256	0.582	0	0.348	49.193	49.940	42.996
$\mathcal{I}_5$	$(Z_3^T, Z_2^{SU}, Z_2^{SU})$	U	3.412	0.0223	0.318	0.5	0.5	0.991	49.193	49.940	17.274
$\mathcal{I}_6$	$(Z_3^T, Z_2^{TST^2}, Z_2^U)$	1	2.678	0.0223	0.307	0.5	−0.871	0.517	49.193	49.940	35.798
$\mathcal{I}_7$	(Z_3^T, Z_2^U, Z_2^{TU})	U	2.678	0.0223	0.307	0.5	0.975	−0.174	49.193	49.940	46.786
$\mathcal{I}_8$	$(Z_3^T, Z_2^U, Z_2^{STSU})$	U	2.678	0.0223	0.307	0.5	−0.755	0.764	49.190	49.937	24.489
$\mathcal{I}_9$	$(Z_3^T, Z_2^{SU}, Z_2^{STSU})$	U	2.678	0.0223	0.307	0.5	−0.953	0.249	49.193	49.940	45.209
$\mathcal{I}_{10}$	$(Z_4^{TSU}, Z_2^S, Z_2^{TU})$	U	2.678	0.0223	0.307	0.5	−0.995	0.102	49.193	49.940	47.794
		STS	2.678	0.0223	0.307	0.5	−0.995	0.102	49.193	49.940	47.794
$\mathcal{I}_{11}$	$(Z_4^{TSU}, Z_2^S, Z_2^{T^2U})$	U	2.678	0.0223	0.307	0.5	−0.869	0.550	49.193	49.940	34.361
		ST^2S	2.678	0.0223	0.307	0.5	−0.631	−0.550	49.197	49.944	34.364
$\mathcal{I}_{12}$	$(Z_4^{TSU}, Z_2^U, Z_2^{TU})$	U	2.678	0.0223	0.307	0.5	−0.769	0.731	49.193	49.940	25.922
$\mathcal{I}_{13}$	$(Z_4^{TSU}, Z_2^{TU}, Z_2^U)$	1	2.678	0.0223	0.307	0.5	0.830	−0.639	49.193	49.940	30.200
$\mathcal{I}_{14}$	$(K_4^{(S,TST^2)}, Z_3^T, Z_2^{SU})$	1	2.678	0.0223	0.307	0.5	0.106	0.449	49.193	49.940	38.651
		U	5.102	0.0221	0.307	0.606	−0.606	−0.449	49.193	49.940	38.660
$\mathcal{I}_{15}$	$(K_4^{(S,TST^2)}, Z_2^U, Z_2^{TU})$	U	2.678	0.0223	0.307	0.5	−0.670	−0.639	49.193	49.940	30.200
$\mathcal{I}_{16}$	$(K_4^{(S,U)}, Z_2^{TST^2}, Z_2^{TU})$	STS	2.678	0.0223	0.307	0.5	−0.131	−0.550	49.197	49.944	34.364
$\mathcal{I}_{17}$	$(K_4^{(S,U)}, Z_2^{TU}, Z_2^U)$	S	5.410	0.0222	0.307	0.477	0.912	−0.550	49.194	49.940	34.356
$\mathcal{I}_{18}$	$(K_4^{(S,U)}, Z_2^{TU}, Z_2^{T^2U})$	ST^2S	0.895	0.0223	0.307	0.523	0.742	−0.512	49.194	49.940	36.023

Test tri-direct CP models

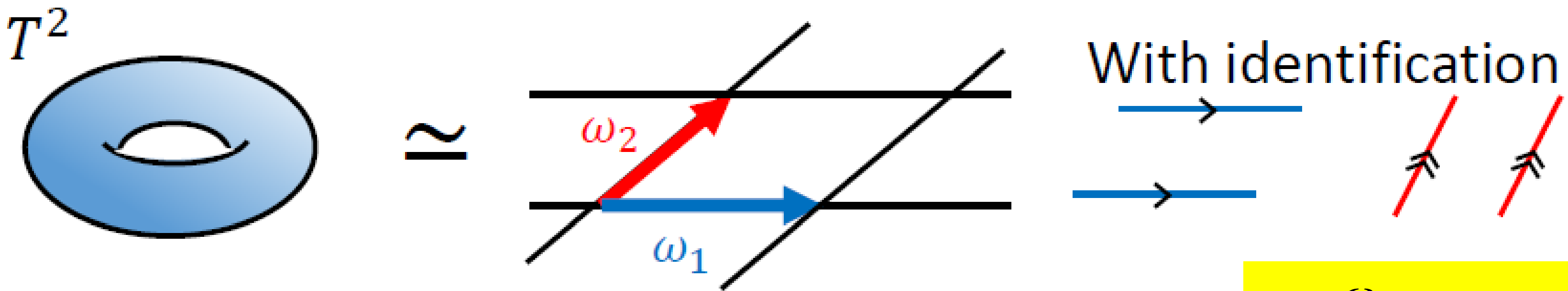


[Ding,Li,Tang,Wang,1905.12939]

The synergy of long baseline (DUNE, T2HK) and reactor (JUNO) neutrino experiments can efficiently test the model

Modular symmetry: a new approach to flavor puzzle

Torus compactification in string theory leads to **Modular Symmetery**

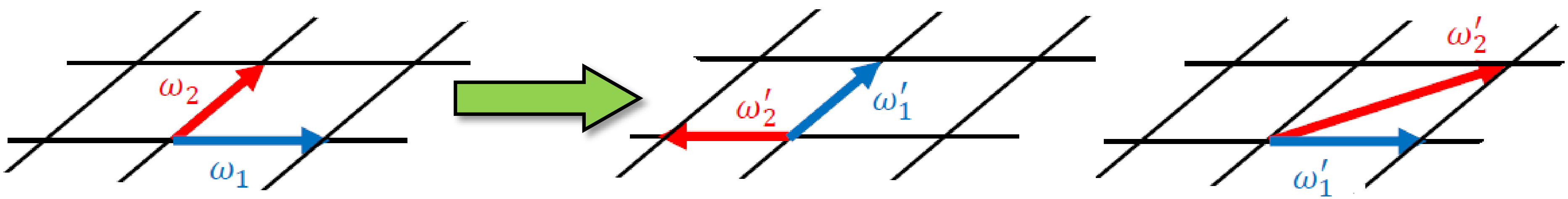


The shape of a torus T^2 is characterized by a modulus $\tau = \frac{\omega_2}{\omega_1}, \ \mathrm{Im}\ \tau > 0$

The lattice is left invariant by modular transformations

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \qquad ad - bc = 1, \quad a, b, c, d \ \text{integers} \qquad \bar{\Gamma} \simeq PSL(2, Z)$$

generated by two independent lattice transformations



$$S^2 = (ST)^3 = 1$$

$$S : \tau \to -1/\tau, \qquad T : \tau \to \tau + 1$$

Crucial element: Modular forms

Modular forms are **holomorphic** functions transforming under

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \forall \gamma \in \overline{\Gamma}(N)$$

N: level, positive integer

k: modular weight, even integer

$$\overline{\Gamma}(N) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \gamma \in \overline{\Gamma},\ \gamma = I (\text{mod N}) \right\}$$

Modular forms of weight k and level N form a linear space, they can be decomposed into irreducible representations of finite modular group ,

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau), \quad \gamma \in \overline{\Gamma}$$

ρ is unitary representation of $\Gamma_N = \overline{\Gamma}/\overline{\Gamma}(N)$

Extension to **odd *k* weight** modular forms

[Liu,Ding,1907.01488]

N	$\dim\mathcal{M}_k(\Gamma(N))$	$\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$	$k=2$
2	$k/2+1$ (k even)	S_3	$\mathbf{2}$
3	$k+1$	A_4	$\mathbf{3}$
4	$2k+1$	S_4	$\mathbf{2} \oplus \mathbf{3}'$
5	$5k+1$	A_5	$\mathbf{3} \oplus \mathbf{3}' \oplus \mathbf{5}$

[Kobayashi et al,1803.10391]

[Feruglio, 1706.08749]

[Petcov et al, 1812.02158]

[Petcov et al, 1812.02158; Ding et al,1903.12588]

Formalism: modular invariant theory

For N=1 global SUSY, the modular invariant action

[Ferrara et al, 1989; Feruglio, 1706.08749]

$$S=\int d^4x d^2\theta d^2\bar{\theta}\; K(\Phi_I,\bar{\Phi}_I,\tau,\bar{\tau})+\int d^4x d^2\theta\, W(\Phi_I,\tau)+\text{h.c.}$$

- Minimal Kahler potential

$$K=-h\ln(-i\tau+i\bar{\tau})+\sum_I(-i\tau+i\bar{\tau})^{-k_I}\,|\Phi_I|^2 \longrightarrow \text{kinetic terms}$$

- Modular invariant superpotential

$$W=\sum_n Y_{I_1 I_2\ldots I_n}(\tau)\Phi_{I_1}\Phi_{I_2}\ldots\Phi_{I_n}$$

$Y_{I_1 I_2\ldots I_n}(\tau)$ are modular forms

$$\tau\to\gamma\tau=\frac{a\tau+b}{c\tau+d},$$

$$\Phi_I\to(c\tau+d)^{-k_I}\rho_I(\gamma)\Phi_I$$

$$Y_{I_1 I_2\ldots I_n}(\tau)\to Y_{I_1 I_2\ldots I_n}(\gamma\tau)=(c\tau+d)^{k_Y}\rho_Y(\gamma)Y_{I_1 I_2\ldots I_n}(\tau)$$

Modular invariance requires

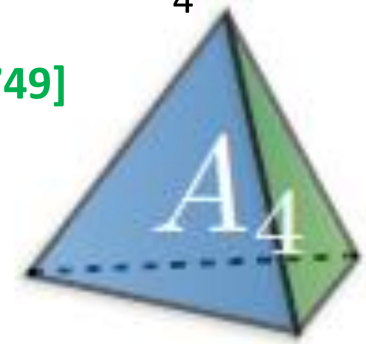
$$\begin{cases} k_Y=k_{I_1}+k_{I_2}+\ldots+k_{I_n} \\ \rho_Y\otimes\rho_{I_1}\otimes\ldots\otimes\rho_{I_n}\supset 1\end{cases}$$

Example: a minimal model based on $\Gamma_3 = A_4$

Γ_3 is isomorphic to A_4, smallest non-abelian finite with 3-dimensional irreducible representation.

- Three weight 2 modular forms transforming as a triplet 3 of A_4

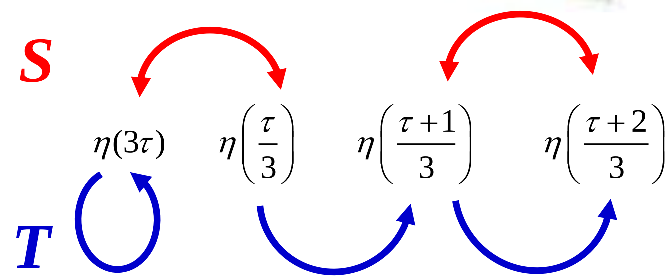
$$Y(\tau) = \left(Y_1(\tau),\quad Y_2(\tau),\quad Y_3(\tau)\right)^T$$ **A_4 triplet** [Feruglio, 1706.08749]



$$Y_1(\tau) = \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right]$$

$$Y_2(\tau) = \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega^2\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]$$

$$Y_3(\tau) = \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega^2\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]$$



Dedekind eta function: $\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)\,,\ \ q \equiv e^{2\pi i\tau}$

$$Y_1(\tau) = 1 + 12q + \ldots,\ \ Y_2(\tau) = -6q^{1/3}(1+7q+\ldots),\ \ Y_3(\tau) = -18q^{2/3}(1+2q+\ldots)$$

Tensor products of $Y_{1,2,3}$ generate higher weight modular forms

- Field content

	N^c	(e^c, μ^c, τ^c)	L	H_d	H_u
$SU(2)_L \times U(1)_Y$	$(1,0)$	$(1,1)$	$(2,-1/2)$	$(2,-1/2)$	$(2,+1/2)$
$\Gamma_3 \cong A_4$	$\mathbf{3}$	$(\mathbf{1}', \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
k_I	1	$(1,3,1)$	1	0	0

Charged lepton mass terms

[Ding, King, Liu, 1907.11714]

$$W_e = \alpha e^c (LY)_{1''} H_d + \beta \mu^c (LY^2)_{1''} H_d + \gamma \tau^c (LY)_{1'} H_d$$

$$M_e = \begin{pmatrix} \alpha Y_3 & \alpha Y_2 & \alpha Y_1 \\ \beta(Y_2^2 - Y_1 Y_3) & \beta(Y_3^2 - Y_1 Y_2) & \beta(Y_1^2 - Y_2 Y_3) \\ \gamma Y_2 & \gamma Y_1 & \gamma Y_3 \end{pmatrix} v_d$$

NO flavons

The couplings α, β and γ are fixed by charged lepton masses.

Neutrino mass terms

$$W_\nu = g_1((N^c L)_{3_S} Y)_1 H_u + g_2((N^c L)_{3_A} Y)_1 H_u + \Lambda (N^c N^c Y)_1$$

$$M_D = \begin{pmatrix} 2g_1 Y_1 & (-g_1 + g_2) Y_3 & (-g_1 - g_2) Y_2 \\ (-g_1 - g_2) Y_3 & 2g_1 Y_2 & (-g_1 + g_2) Y_1 \\ (-g_1 + g_2) Y_2 & (-g_1 - g_2) Y_1 & 2g_1 Y_3 \end{pmatrix} v_u, \quad M_N = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \Lambda$$

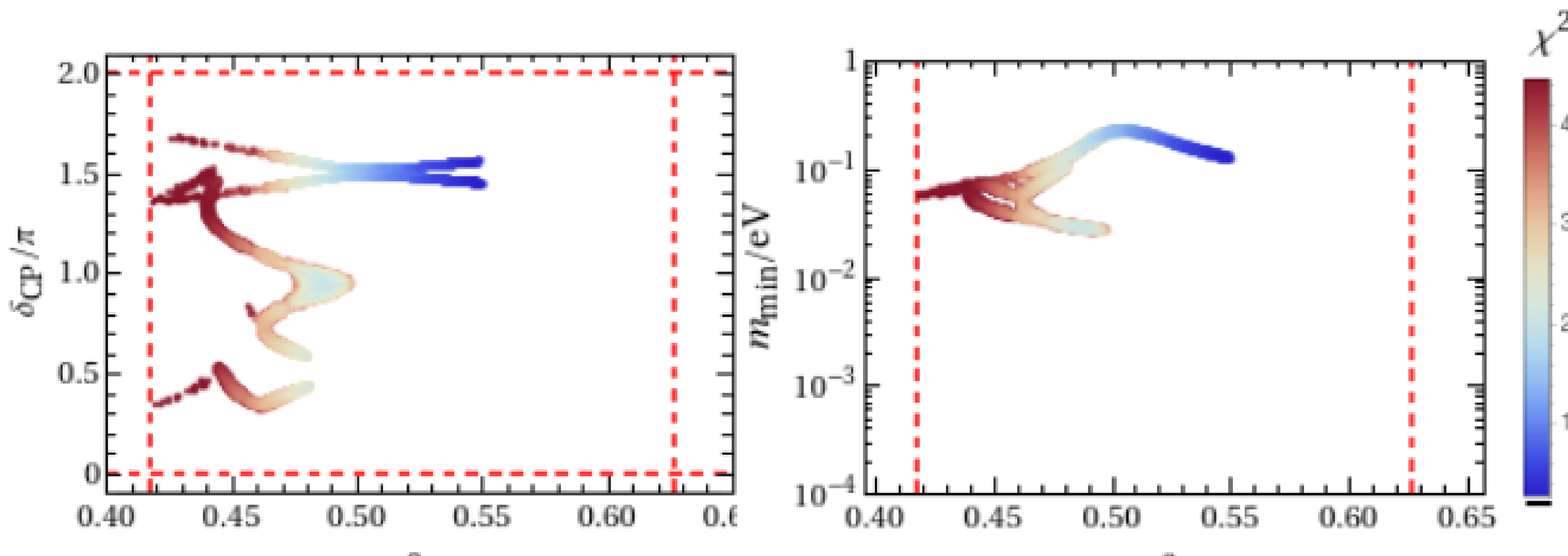
The complex modulus τ is the only source of modular symmetry breaking, best agreement with experimental data can be achieved for

Input parameters: $<\tau> = 0.0428 + 2.105\,i, \quad g_2/g_1 = 1.154 e^{0.625\pi i}$

Predictions:

$$\sin^2\theta_{12} = 0.310, \quad \sin^2\theta_{13} = 0.0224, \quad \sin^2\theta_{23} = 0.580,$$
$$\delta_{CP} = 1.602\pi, \ \alpha_{21} = 1.992\pi, \ \alpha_{31} = 0.986\pi, \quad \chi^2 = 0.0003$$
$$m_1 = 0.0805\text{eV}, \quad m_2 = 0.0810\text{eV}, \ m_3 = 0.0949\text{eV}$$

[Ding, King, Liu, 1907.11714]



Classification of simplest A_4 modular models

charged lepton

$$W_e = E^c L H_d f_E(\tau)$$

neutrino

$$W_\nu = \begin{cases} LLH_u H_u f_W(\tau)/\Lambda, & \text{Weinberg operator} \\ N^c L H_u f_D(\tau) + \Lambda N^c N^c f_N(\tau), & \text{seesaw} \end{cases}$$

Models	mass matrices	A_4	modular weights		
			$k_{E^c_{1,2,3}}$	k_L	k_{N^c}
$\mathcal{A}_1$	W_1, C_1	$1, 1, 1$	$1, 3, 5$	1	—
$\mathcal{A}_2$	W_1, C_2	$1', 1', 1'$	$1, 3, 5$	1	—
$\mathcal{A}_3$	W_1, C_3	$1'', 1'', 1''$	$1, 3, 5$	1	—
$\mathcal{A}_4$	W_1, C_4	$1, 1, 1'$	$1, 3, 1$	1	—
$\mathcal{A}_5$	W_1, C_5	$1, 1, 1''$	$1, 3, 1$	1	—
$\mathcal{A}_6$	W_1, C_6	$1', 1', 1$	$1, 3, 1$	1	—
$\mathcal{A}_7$	W_1, C_7	$1'', 1'', 1$	$1, 3, 1$	1	—
$\mathcal{A}_8$	W_1, C_8	$1'', 1'', 1'$	$1, 3, 1$	1	—
$\mathcal{A}_9$	W_1, C_9	$1', 1', 1''$	$1, 3, 1$	1	—
$\mathcal{A}_{10}$	W_1, C_{10}	$1, 1'', 1'$	$1, 1, 1$	1	—
$\mathcal{B}_1(\mathcal{C}_1)[\mathcal{D}_1]$	$S_1(S_2)[S_3], C_1$	$1, 1, 1$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_2(\mathcal{C}_2)[\mathcal{D}_2]$	$S_1(S_2)[S_3], C_2$	$1', 1', 1'$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_3(\mathcal{C}_3)[\mathcal{D}_3]$	$S_1(S_2)[S_3], C_3$	$1'', 1'', 1''$	$0(3)[1], 2(5)[3], 4(7)[5]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_4(\mathcal{C}_4)[\mathcal{D}_4]$	$S_1(S_2)[S_3], C_4$	$1, 1, 1'$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_5(\mathcal{C}_5)[\mathcal{D}_5]$	$S_1(S_2)[S_3], C_5$	$1, 1, 1''$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_6(\mathcal{C}_6)[\mathcal{D}_6]$	$S_1(S_2)[S_3], C_6$	$1', 1', 1$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_7(\mathcal{C}_7)[\mathcal{D}_7]$	$S_1(S_2)[S_3], C_7$	$1', 1', 1''$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_8(\mathcal{C}_8)[\mathcal{D}_8]$	$S_1(S_2)[S_3], C_8$	$1'', 1'', 1$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_9(\mathcal{C}_9)[\mathcal{D}_9]$	$S_1(S_2)[S_3], C_9$	$1'', 1'', 1'$	$0(3)[1], 2(5)[3], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$
$\mathcal{B}_{10}(\mathcal{C}_{10})[\mathcal{D}_{10}]$	$S_1(S_2)[S_3], C_{10}$	$1, 1'', 1'$	$0(3)[1], 0(3)[1], 0(3)[1]$	$2(-1)[1]$	$0(1)[1]$

40 simplest A_4 modular models without flavons

Models	NO	IO	Models	NO	IO	Models	NO	IO	Models	NO	IO
$\mathcal{A}_1$	✗	✗	$\mathcal{B}_1$	✓	✓	$\mathcal{C}_1$	✗	✗	$\mathcal{D}_1$	✓	✓
$\mathcal{A}_2$	✗	✗	$\mathcal{B}_2$	✓	✓	$\mathcal{C}_2$	✗	✗	$\mathcal{D}_2$	✓	✓
$\mathcal{A}_3$	✗	✗	$\mathcal{B}_3$	✓	✓	$\mathcal{C}_3$	✗	✗	$\mathcal{D}_3$	✓	✓
$\mathcal{A}_4$	✗	✗	$\mathcal{B}_4$	✗	✗	$\mathcal{C}_4$	✗	✗	$\mathcal{D}_4$	✗	✓
$\mathcal{A}_5$	✗	✗	$\mathcal{B}_5$	✗	✗	$\mathcal{C}_5$	✗	✗	$\mathcal{D}_5$	✓	✗
$\mathcal{A}_6$	✗	✗	$\mathcal{B}_6$	✗	✓	$\mathcal{C}_6$	✗	✗	$\mathcal{D}_6$	✓	✗
$\mathcal{A}_7$	✗	✗	$\mathcal{B}_7$	✗	✗	$\mathcal{C}_7$	✗	✗	$\mathcal{D}_7$	✓	✓
$\mathcal{A}_8$	✗	✗	$\mathcal{B}_8$	✗	✗	$\mathcal{C}_8$	✗	✗	$\mathcal{D}_8$	✓	✓
$\mathcal{A}_9$	✗	✗	$\mathcal{B}_9$	✓	✓	$\mathcal{C}_9$	✗	✗	$\mathcal{D}_9$	✓	✓
$\mathcal{A}_{10}$	✗	✗	$\mathcal{B}_{10}$	✓	✓	$\mathcal{C}_{10}$	✗	✗	$\mathcal{D}_{10}$	✓	✓

- ✓ **8** phenomenologically viable minimal models for both NO and IO as compared to **only one D_{10}** in previous analyses
- ✓ **3 free parameters** beside modulus τ for the neutrino sector (3 masses + 3 angles + 3 phases)
- ✓ Dirac CP phase **$\delta_{CP} \approx -\pi/2$** and large neutrino masses still allowed by data

Summary

Neutrino oscillation calls for convincing model of neutrino masses and mixings, with testable and confirmed predictions.

- The tri-direct CP approach is a predictive neutrino model building scheme. The resulting neutrino mass matrix generally only depends on four (or two) parameters. The (new) Littlest seesaw can be naturally produced.

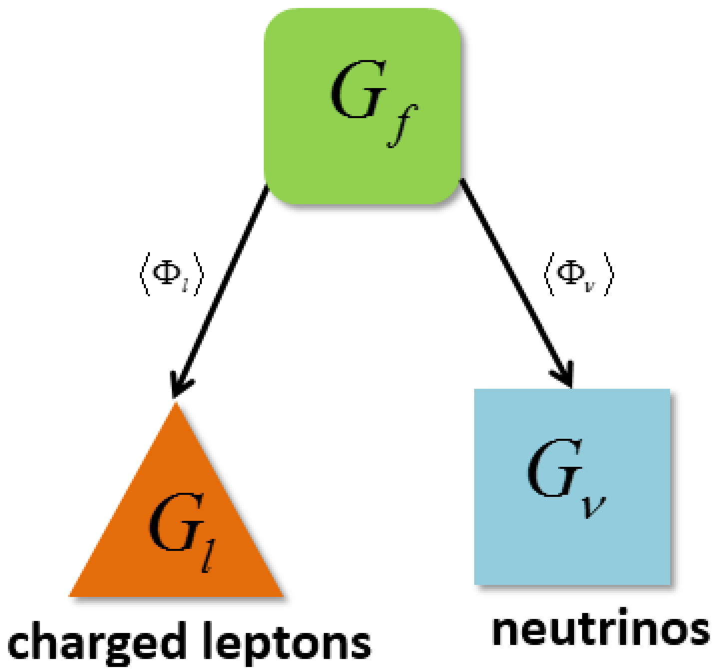
- Modular symmetry is a new promising approach to the fermion masses and flavor mixing puzzles. A systematic method of constructing simplest modular symmetry models is proposed, A_4 as an example for illustration.

Thank you for your attention!

Backup

The drawback of the usual flavor symmetry approach

➢ Vacuum alignment



- ✓ extra symmetry Z_n or $U(1)_R$
- ✓ alignment of flavon VEVs?

➢ predictability

- ✓ large number of free parameters
- ✓ complicate flavor breaking sector
- ✓ higher order corrections