

# Status and prospects of $\mu - \tau$ symmetry

**Newton Nath**



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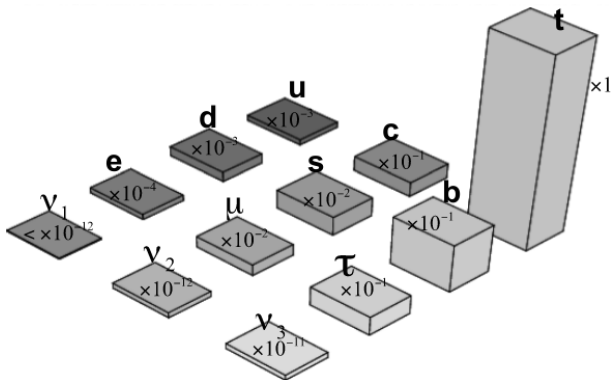
Mini-Workshop on “New Physics at the Tera Scale” (2019)

TDLI, Shanghai

August 3 - 5

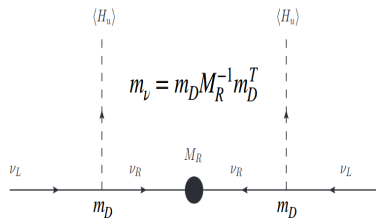
# Neutrino Mass:

**Tiny Neutrino mass: a long standing issue.**



Cont...

## Type-I seesaw



[ Minkowski'77, Yanagida'79, Gell-Mann/Slansky/Ramond'79, Mohapatra/Senjanovic'80, Schecter/Valle'80 ]

# Neutrino Mixing

## Standard 3-flavour $\nu$ -mixing patterns:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & e^{-i\delta} \sin \theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} \sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric,  
K2K, MINOS, T2K, etc.

Reactor  
Accelerator

Solar  
KamLAND

- 3-mixing angles, 1 CP-phase.

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- 3-mixing angles, 1 CP-phase.
- 2-additional Majorana phases.

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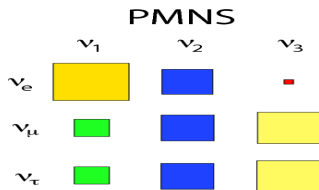
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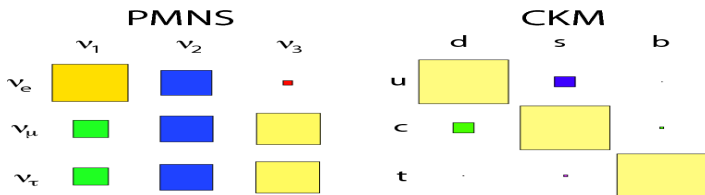
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Why this form ?

# Tri-Bi-Maximal Mixing:

- ▶ Before Daya-Bay results, [PRL'13]:

$$\mathbf{U}_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/3} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/3} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}.$$

- $U_{TBM}$  was 1st proposed by Harrison, Perkins & Scott

arXiv:hep-ph/0202074, PLB530 (2002)

- ▶  $U_{TBM} \Rightarrow$ 
  - $\sin \theta_{12} = \frac{1}{\sqrt{3}} \Rightarrow$  'trimaximal mixing',
  - $\sin \theta_{23} = \frac{1}{\sqrt{2}} \Rightarrow$  'bimaximal mixing'
  - and  $\theta_{13} = 0^\circ$ .



# Current Status:

Parameter	Best fit $\pm 1\sigma$	$2\sigma$ range	$3\sigma$ range	
$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	$7.55^{+0.20}_{-0.16}$	7.20-7.94	7.05-8.14	
$ \Delta m_{21}^2  [10^{-3} \text{eV}^2]$ (NO)	$2.50 \pm 0.03$	2.44-2.57	2.41-2.60	
$ \Delta m_{31}^2  [10^{-3} \text{eV}^2]$ (IO)	$2.42^{+0.03}_{-0.04}$	2.34-2.47	<u>2.31-2.51</u>	
$\sin^2 \theta_{12}/10^{-1}$	$3.20^{+0.20}_{-0.16}$	2.89-3.59	2.73-3.79	
$\theta_{12}/^\circ$	$34.5^{+1.2}_{-1.0}$	32.5-36.8	31.5-38.0	
$\sin^2 \theta_{23}/10^{-1}$ (NO)	$5.47^{+0.20}_{-0.30}$	4.67-5.83	4.45-5.99	
$\theta_{23}/^\circ$	$47.7^{+1.2}_{-1.7}$	43.1-49.8	<u>41.8-50.7</u>	
$\sin^2 \theta_{23}/10^{-1}$ (IO)	$5.51^{+0.18}_{-0.30}$	4.91-5.84	<u>4.53-5.98</u>	
$\theta_{23}/^\circ$	$47.9^{+1.0}_{-1.7}$	44.5-48.9	42.3-50.7	
$\sin^2 \theta_{13}/10^{-2}$ (NO)	$2.160^{+0.083}_{-0.069}$	2.03-2.34	1.96-2.41	
$\theta_{13}/^\circ$	$8.45^{+0.16}_{-0.14}$	8.2-8.8	8.0-8.9	
$\sin^2 \theta_{13}/10^{-2}$ (IO)	$2.220^{+0.074}_{-0.076}$	2.07-2.36	1.99-2.44	
$\theta_{13}/^\circ$	$8.53^{+0.14}_{-0.15}$	8.3-8.8	8.1-9.0	
$\delta/\pi$ (NO)	$1.32^{+0.21}_{-0.15}$	1.01-1.75	0.87-1.94	
$\delta/^\circ$	$238^{+38}_{-27}$	182-315	<u>157-349</u>	
$\delta/\pi$ (IO)	$1.56^{+0.13}_{-0.15}$	1.27-1.82	1.12-1.94	
$\delta/^\circ$	$281^{+23}_{-27}$	229-328	202-349	

**NO or IO ?**

**LO or HO ?**

**CPV**

- At present  $\theta_{13} = 0$  is excluded at more than  $5\sigma$ .

# $\mu - \tau$ reflection symmetry

Originally proposed by Harrison & Scott, PLB547 (2002)

- ▶  $M_\nu$  is unchanged under:

$$\nu_e \leftrightarrow \nu_e^c, \quad \nu_\mu \leftrightarrow \pm \nu_\tau^c, \quad \nu_\tau \leftrightarrow \pm \nu_\mu^c.$$

where,

$$M_\nu = \begin{pmatrix} D & A & \pm A^* \\ A & B & C \\ \pm A^* & C & B^* \end{pmatrix}; \quad C, D \in \mathbb{R} \text{ \& } A, B \in \mathbb{C}.$$

- ▶  $M_\nu$  can be diagonalized by

$$U = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ \pm v_1^* & \pm v_2^* & \pm v_3^* \end{pmatrix} \Rightarrow |U_{\mu i}| = |U_{\tau i}|, i = 1, 2, 3$$

- ▶ 2-well known predictions:  $\theta_{23} = 45^\circ$ ,  $s_{13} \cos \delta = 0$ .
- ▶ Allows non-zero  $\theta_{13}$  for  $\delta = \pm 90^\circ$  (...**wow**).
- ▶ Also predicts trivial Majorana phases,  $\rho, \sigma = 0^\circ, 90^\circ$ .

Recent review: Xing and Zhao, RPP79 (2016)

## Cont...

- ▶ **Embedded  $\mu - \tau$  reflection symmetry in minimal seesaw**  
 $\Rightarrow m_{light} = 0$  (still allowed by latest data).
- ▶ **To address both the  $\nu$ -mass & mixing patterns.**
- ▶ **We assume,**

$$\nu_L \rightarrow \mathbf{S}\nu_L^c, \quad \mathbf{N}_R \rightarrow \mathbf{S}'\mathbf{N}_R^c$$

where  $\nu_L^c = C\bar{\nu}_L^T$  and  $\mathbf{N}_R^c = C\bar{\mathbf{N}}_R^T$  and

$$\mathbf{s} = \begin{pmatrix} 1 & 0 \\ 0 & S' \end{pmatrix}, \quad \mathbf{S}' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

- ▶  $M_D, M_R$  obey,

$$M_D = \mathbf{S}M_D^*\mathbf{S}', \quad M_R = \mathbf{S}'M_R^*\mathbf{S}'.$$

- ▶ One gets,

$$M_D = \begin{pmatrix} |b|e^{i\phi_b} & |b|e^{-i\phi_b} \\ |c|e^{i\phi_c} & |d|e^{i\phi_d} \\ |d|e^{-i\phi_d} & |c|e^{-i\phi_c} \end{pmatrix}, \quad M_R = \begin{pmatrix} |m_{22}|e^{i\phi_m} & m_{23} \\ m_{23} & |m_{22}|e^{-i\phi_m} \end{pmatrix},$$

# Cont...

- ▶ In type-I seesaw,

$$-M_\nu = M_D M_R^{-1} M_D^T = \begin{pmatrix} A & B & B^* \\ B & C & D \\ B^* & D & C^* \end{pmatrix}.$$

- ▶  $M_\nu$  can be diagonalized by,

$$V = P_l \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (1)$$

where  $P_l = \text{diag}(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})$  and  $P_\nu = \text{diag}(1, e^{i\rho}, e^{i\sigma})$ .

- ▶ **6-predictions:**

$$\phi_e = 90^\circ, \quad \phi_\mu \equiv -\phi_\tau = \phi, \quad \theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ, \quad \rho, \sigma = 0 \text{ or } 90^\circ.$$

- ▶ Also,

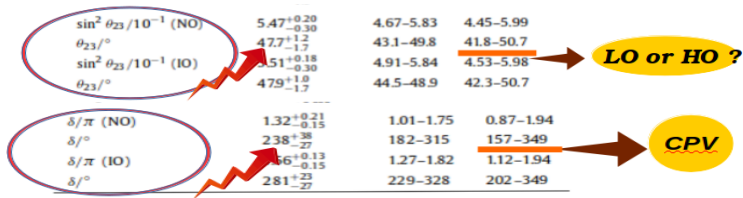
$$\tan \theta_{13} = \mp \frac{1}{\sqrt{2}} \frac{\text{Im}(C')}{\text{Im}(B')} \quad (C' = Ce^{-2i\phi}, B' = Be^{-i\phi}),$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2} \cos 2\theta_{13} \text{Im}(B')}{c_{13} [(\text{Re}(C') - D) \cos 2\theta_{13} - (\text{Re}(C') + D)s_{13}^2 + Ac_{13}^2]}; \quad \text{for NH}$$

- ▶ Excellent agreement with the latest data.

Cont...

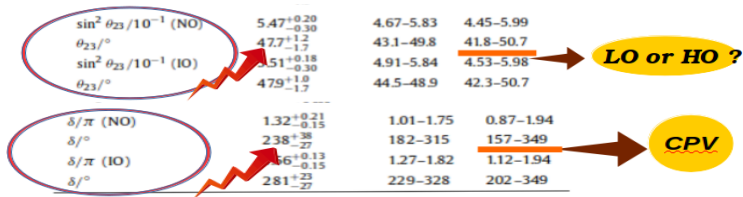
Reminder: Best-fit preferences



Looking for more realistic model

Cont...

Reminder: Best-fit preferences



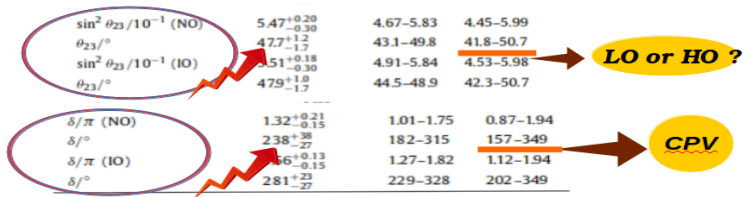
## Looking for more realistic model

- Break  $\mu - \tau$  reflection symmetry:

**RG-running**

Cont...

Reminder: Best-fit preferences



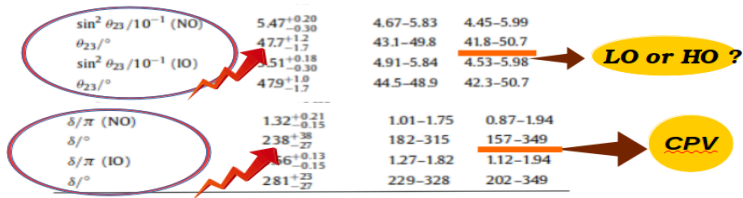
## Looking for more realistic model

- Break  $\mu - \tau$  reflection symmetry:

**RG-running** or

Cont...

Reminder: Best-fit preferences



## Looking for more realistic model

- Break  $\mu - \tau$  reflection symmetry:

**RG-running** or **Explicit**

Recent topics: **Generalized CP symmetry, Modular symmetry, Bi-large ansatz, Tri-direct CP approaches etc...**



# RGE effect:

- ▶ **REG effect works as a bridge between the high-energy predictions and the low-energy measurements.**
- ▶ At the one-loop level, the energy dependence of  $M_\nu$  is given by

$$16\pi^2 \frac{dM_\nu}{dt} = C \left( Y_l^\dagger Y_l \right)^T M_\nu + C M_\nu \left( Y_l^\dagger Y_l \right) + \alpha M_\nu .$$

[Chankowski, Pluciennik, PLB316(1993) ]

- ▶ With

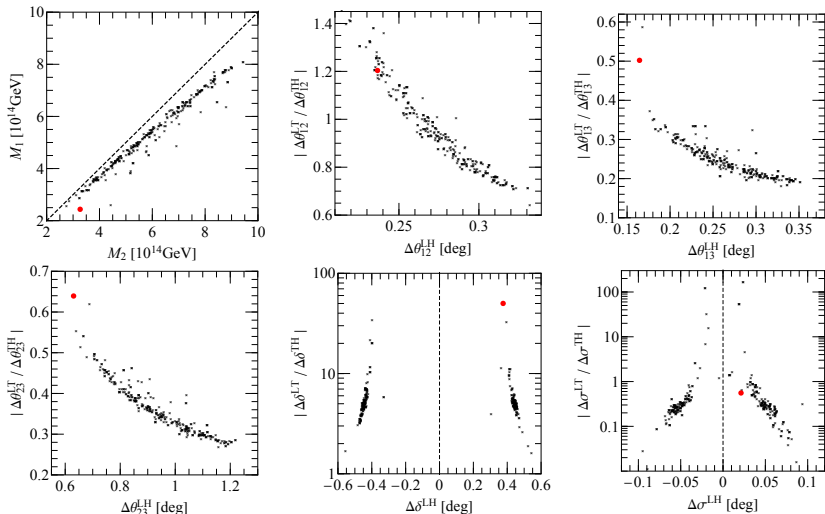
$$M_\nu(\Lambda_{EW}) = I_\alpha I_\tau^\dagger M_\nu(\Lambda_{\mu\tau}) I_\tau^* ,$$

where one defines  $I_\tau \simeq \text{diag}\{1, 1, 1 - \Delta_\tau\}$  along with

$$I_\alpha = \exp \left( \frac{\mathbf{1}}{16\pi^2} \int_{\ln \Lambda_{\mu\tau}}^{\ln \Lambda_{EW}} \alpha dt \right) , \quad \Delta_\tau = \frac{\mathbf{C}}{16\pi^2} \int_{\ln \Lambda_{EW}}^{\ln \Lambda_{\mu\tau}} \mathbf{y}_\tau^2 dt .$$

Cont...

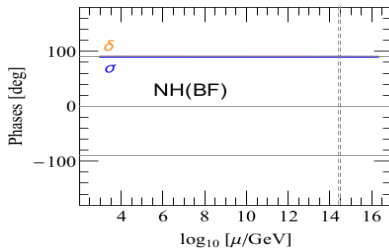
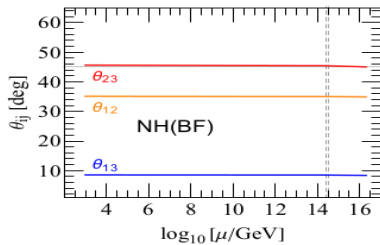
- RGE predictions for NO (in MSSM with  $\tan\beta = 30$ ):



[NN, Xing & Zhang, 1801.09931/hep-phEPJC78 (2018).]

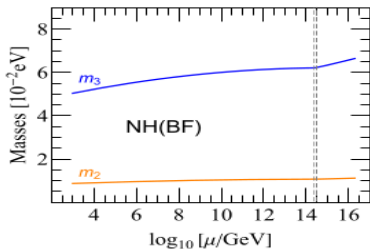
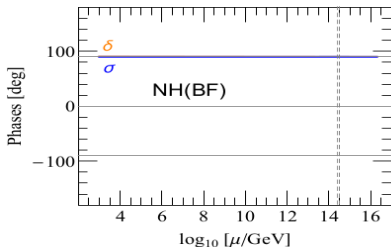
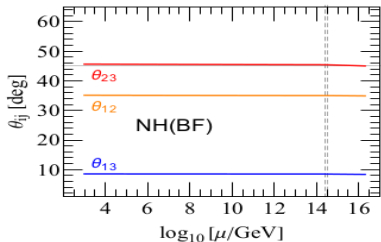
Cont...

- Impact of RG running:



# Cont...

- Impact of RG running:



## Summarizing all explicit-breaking scenarios:

Breaking Scenarios	$\theta'_{23}$ [deg]	$\delta'_{CP}$ [deg]	$\Delta\theta'_{12}$ [deg]	$\Delta\theta'_{13}$ [deg]	$\sum m_\nu$ [eV]	$m_{ee}$ [meV]
<b>S1</b>	44.3 $\rightarrow$ 45.7	-180 $\rightarrow$ 180	-15 $\rightarrow$ 10	-1 $\rightarrow$ 9	0.0575 $\rightarrow$ 0.061	1 $\rightarrow$ 4.2
<b>S2</b>	35 $\rightarrow$ 46	-100 $\rightarrow$ -88	-18 $\rightarrow$ 1	-0.1 $\rightarrow$ 1.3	0.057 $\rightarrow$ 0.061	3 $\rightarrow$ 4.5
	40 $\rightarrow$ 45	-90 $\rightarrow$ -70	0 $\rightarrow$ 9	0 $\rightarrow$ 1.2	-	-
<b>S3</b>	37.5 $\rightarrow$ 47	-98 $\rightarrow$ -88	2 $\rightarrow$ 7	-1.4 $\rightarrow$ 0.2	0.057 $\rightarrow$ 0.0615	3 $\rightarrow$ 4.5
	46 $\rightarrow$ 47	-94 $\rightarrow$ -56	-20 $\rightarrow$ 3	-1.7 $\rightarrow$ 0.3	-	-
<b>S4</b>	43 $\rightarrow$ 46	-100 $\rightarrow$ -88	-0.2 $\rightarrow$ 0.7	-3 $\rightarrow$ 1	0.0575 $\rightarrow$ 0.061	3.1 $\rightarrow$ 4.4
<b>S5</b>	39 $\rightarrow$ 46.5	-120 $\rightarrow$ -84	-1 $\rightarrow$ 2.6	-8 $\rightarrow$ 8	0.057 $\rightarrow$ 0.061	3 $\rightarrow$ 4.5

Scenarios  $\Rightarrow M_D(12) \rightarrow$  **S1**,  $M_D(22) \rightarrow$  **S2**,  $M_D(32) \rightarrow$  **S3**,  
 $M_R(22) \rightarrow$  **S4**,  $M_R(12) \rightarrow$  **S5**

NN, Xing & Zhang, EPJC78 (2018)

# Minimal seesaw @ TeV Scale:

Xing & Zhou, PLB '07

- ▶  $2-N_R$  with highly degenerate masses of  $\mathcal{O}(1)$  TeV are added.
- ▶ Along with  $\nu$ -oscillation parameters, it also explains baryon number asymmetry through “resonant leptogenesis”. [Pilaftsis, Underwood '04]
- ▶  $M_D$  has been parameterized as,

$$M_D = V_0 \begin{pmatrix} 0 & 0 \\ x & 0 \\ 0 & x \end{pmatrix} U; \quad (M_I, M_R \text{ are diag}).$$

where  $V_0, U$  are

$$V_0 = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/3} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/3} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}, \quad U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{+i\alpha} \end{pmatrix}$$

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**TBM pattern**

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**TBM pattern**

- ▶ **Note:**  $\alpha$  is the only phase, explains CPV in  $\nu$ -oscillation &  $N_i$  decays.
- ▶ This model leads to nearly TBM pattern:

$$\theta_{23} = \pi/4, \quad \delta = \pm\pi/2, \quad \sin^2 \theta_{12} = (1 - 2 \tan^2 \theta_{13})/3.$$



# Cont...

- ▶ The flavor-dependent CP-violating asymmetry  $\varepsilon_{i\alpha}$ :

$$\varepsilon_{ie} = \frac{\omega^2}{3(\omega^2 - 1)} \varepsilon_i, \quad \varepsilon_{i\mu} = \varepsilon_{i\tau} = \frac{2\omega^2 - 3}{6(\omega^2 - 1)} \varepsilon_i$$

where  $\varepsilon_i$  is given by:

$$\varepsilon_i = \frac{-32\pi v^2 y^2 (1 - \omega^2)^2}{(1 + \omega^2) [1024\pi^2 v^4 r^2 + y^4 (1 + \omega^2)^2]} r \sin 4\alpha,$$

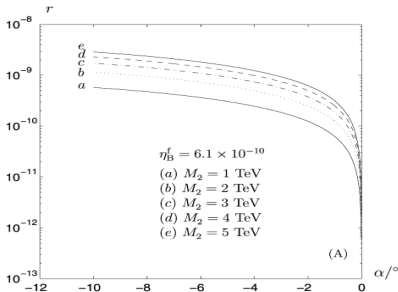
with  $r = (M_2 - M_1)/M_2$ .

- ▶ **Note:** for  $\varepsilon_i \neq 0$ ,  $r, \alpha$  can't be zero.
- ▶ The final baryon number asymmetry  $\eta_B^f$ :

$$\eta_B^f \approx -0.96 \times 10^{-2} \sum_{i,\alpha} (\varepsilon_{i\alpha} \kappa_{i\alpha});$$

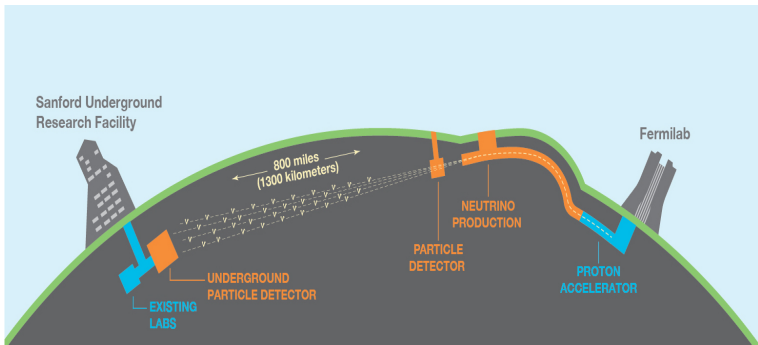
$\kappa_{i\alpha} \Rightarrow$  efficiency factor

$\Rightarrow$



# Testing $\mu - \tau$ reflection symmetry @ DUNE:

- ▶ **DUNE** : 'Deep Underground Neutrino Experiment' is a proposed long baseline experiment at Fermilab, USA.



- ▶ DUNE:  $\nu_s$  travel from Fermilab to Sanford Underground Research Facility (SURF), 1300 km, 2.3 GeV, 1.07 MW,  $4 \times 10$  kt-LArTPC detector.  
[\[Alio et al.\(DUNE\), arXiv:1601.09550\].](#)
- ▶ Their first 2-modules are expected to be completed in 2024, with the beam operational in 2026.
- ▶ To simulate the data we use **GLoBES** package.

[arXiv:hep-ph/0407333, 0701187.](#)

# Framework:

- ▶ We consider,

$$M_D = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & be^{-i\phi_b} \end{pmatrix}, M_R = \text{diag}(M_1, M_1).$$

- ▶ Within type-I seesaw:

$$\begin{aligned} -M_\nu &= M_D M_R^{-1} M_D^T, \\ &= \frac{1}{M_1} \begin{pmatrix} 2a^2 \cos 2\phi_a & abe^{i(\phi_a+\phi_b)} + ace^{-i(\phi_a-\phi_c)} & abe^{-i(\phi_a+\phi_b)} + ace^{i(\phi_a-\phi_c)} \\ - & b^2 e^{2i\phi_b} + c^2 e^{2i\phi_c} & 2bc \cos(\phi_b - \phi_c) \\ - & - & b^2 e^{-2i\phi_b} + c^2 e^{-2i\phi_c} \end{pmatrix}. \end{aligned}$$

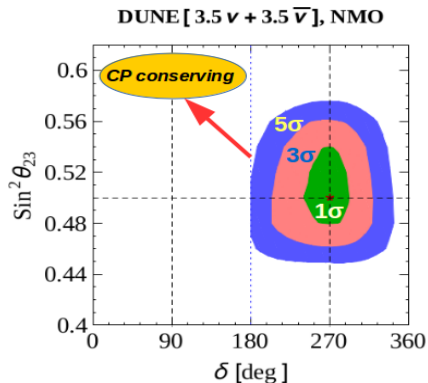
$$\bullet M_{ee} = M_{ee}^*, \quad M_{\mu\tau} = M_{\mu\tau}^*, \quad M_{e\mu} = M_{e\tau}^*, \quad M_{\mu\mu} = M_{\tau\tau}^*$$

- ▶ Predicts non-zero  $\theta_{13}$  with,

$$\theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ.$$

Cont...

► DUNE's Potential:



● CP-conservation hypothesis can be ruled out around 5 $\sigma$

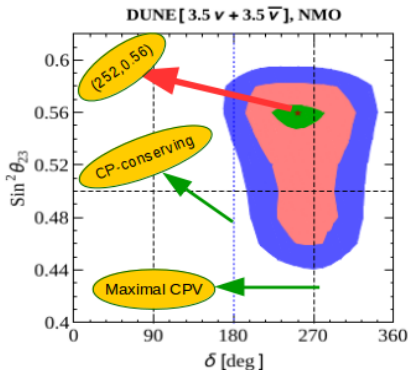
NN, arXiv: 1805.05823, PRD98 (2018)

# Cont...

Break  $M_D$ :

$$\hat{M}_D = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & b(1+\epsilon)e^{-i\phi_b} \end{pmatrix}.$$

$$\hat{M}_\nu \simeq M_\nu - \epsilon \frac{be^{-i\phi_b}}{M_1} \begin{pmatrix} 0 & 0 & ae^{-i\phi_a} \\ 0 & 0 & ce^{i\phi_c} \\ be^{-i\phi_a} & ce^{i\phi_c} & 2be^{-2i\phi_b} \end{pmatrix} + \mathcal{O}(\epsilon^2).$$



- **Best fit:**  $(238_{-27}^{+38}, 0.547_{-0.03}^{+0.02})$
- **Predicted  $\delta, \theta_{23}$  are well within  $1\sigma$**
- **Maximal  $\theta_{23}$  is ruled at  $> 1\sigma$**

# Wrap-up Comments:

- ▶ At present,  $\mu - \tau$  reflection symmetry has an excellent agreement with the latest data, it predicts  $\theta_{23} = \pi/4$ ,  $\delta = \pm\pi/2$  and  $\theta_{13} \neq 0$  along with  $\rho, \sigma = 0, \pi/2$ .
- ▶ We embed  $\mu - \tau$  reflection symmetry in minimal seesaw to explain non-zero neutrino mass as well as their mixings.
- ▶ Considering RG-running equations, we discuss the breaking of the  $\mu - \tau$  reflection symmetry
- ▶ Also, discuss a TeV scale minimal seesaw model which explains baryon asymmetry of the universe through the resonant leptogenesis.
- ▶ Impact of  $\mu - \tau$  reflection symmetry on DUNE has been examined.
- ▶ DUNE with its high statistics and ability to measure  $(\theta_{23}, \delta)$  with high precision, will serve as an excellent experiment to test these different mixing patterns.

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thank you

Back-Up

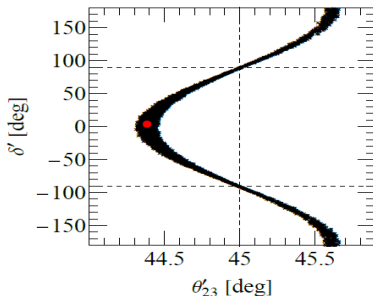


# Explicit Breaking

- Modify (12)-position of  $M_D$ ,

$$\mathbf{S1} : M'_D = \begin{pmatrix} b & b^*(1 + \epsilon) \\ c & d \\ d^* & c^* \end{pmatrix}, \quad (2)$$

“ $\epsilon$  breaks  $\mu - \tau$  Reflection Symmetry”



## Cont...

- ▶ Modify (22)-position of  $M_R$ ,

$$\mathbf{S4} : M'_R = \begin{pmatrix} m_{22} & m_{23} \\ m_{23} & m_{22}^*(1 + \epsilon) \end{pmatrix}, \quad (3)$$

“ $\epsilon$  breaks  $\mu - \tau$  Reflection Symmetry”

