Probing New Physics Scale via nTGC at e^+e^- Colliders

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Introduction

- Currently there is no confirmed BSM evidence. Possible deviations from SM in terms of higher-dimensional effective operators are suppressed by some power of an underlying new physics scale $\Lambda \gg 100\,\mathrm{GeV}$.
- SMEFT approach has mainly been applied with the assumption that only dimension-6 operators. Dimension-6 contributions are absent in some instances, then experimental measurements are sensitive are those of higher dimensions.
- Probe directly dimension-8 operators is via the $ZZ\gamma$ and $Z\gamma\gamma$ couplings which are absent in SM and dimension-6 operators.
  We study here how these dimension-8 operators can be probed via the $e^+e^- \to Z\gamma$ process at future e^+e^- colliders. (ILC, CEPC, FCC-ee and CLIC)
- We make use of angular distributions in the e^+e^- centre-of-mass frame and Z decay frame to separate the SM contribution to $Z\gamma$ final states and distinguish dimension-8 contribution from SM background via Z decays into $\bar{f}f$ pairs.

CP-conserving effective operators

$$\Delta\mathcal{L}(\text{dim-8}) = \sum_{j=1}^{4} \frac{c_j}{\tilde{\Lambda}^4}\mathcal{O}_j = \sum_{j=1}^{4} \frac{\text{sign}(c_j)}{\Lambda_j^4}\mathcal{O}_j\,,$$

Operators with Higgs fields

$$\begin{aligned}
\mathcal{O}_{\widetilde{B}W} &= \mathrm{i}\, H^\dagger \widetilde{B}_{\mu\nu} W^{\mu\rho}\{D_\rho, D^\nu\}H + \text{h.c.},\\
\mathcal{O}_{B\widetilde{W}} &= \mathrm{i}\, H^\dagger B_{\mu\nu} \widetilde{W}^{\mu\rho}\{D_\rho, D^\nu\}H + \text{h.c.},\\
\mathcal{O}_{\widetilde{W}W} &= \mathrm{i}\, H^\dagger \widetilde{W}_{\mu\nu} W^{\mu\rho}\{D_\rho, D^\nu\}H + \text{h.c.},\\
\mathcal{O}_{\widetilde{B}B} &= \mathrm{i}\, H^\dagger \widetilde{B}_{\mu\nu} B^{\mu\rho}\{D_\rho, D^\nu\}H + \text{h.c.},
\end{aligned}$$

$\mathcal{O}_{B\widetilde{W}}$ is equivalent to $\mathcal{O}_{\widetilde{B}W}$
$\mathcal{O}_{\widetilde{W}W}$ and $\mathcal{O}_{\widetilde{B}B}$ do not contribute to $ZV\gamma$ coupling for on-shell Z and γ

With equation of motion, operators with pure gauge fields can be converted into operators with Higgs plus extra operators involving the gauge current of fermions

$$\mathrm{i}\,\Gamma^{\mu\nu\alpha}_{Z\gamma Z^*}(q_1, q_2, q_3) = \text{sign}(c_j)\frac{vM_Z(q_3^2 - M_Z^2)}{\Lambda^4}\epsilon^{\mu\nu\alpha\beta}q_{2\beta}\,,$$

$\mathbf{Z\gamma}$ Production at $\mathbf{e^+e^-}$ Colliders

$e^-(p_1)e^+(p_2) \to Z(q_1)\gamma(q_2)$

$$p_1 = E_1(1,0,0,1), \quad p_2 = E_1(1,0,0,-1), \quad q_1 = (E_Z,\ q\sin\theta,\ 0,\ q\cos\theta), \quad q_2 = q(1,\ -\sin\theta,\ 0,\ -\cos\theta),$$

$$\begin{aligned}
\overline{|\mathcal{T}_{\rm sm}|^2}[Z_L\gamma_T] &= e^4\left(8s_W^4 - 4s_W^2+1\right)\frac{M_Z^2 s}{c_W^2 s_W^2\left(s-M_Z^2\right)^2} \propto s^{-1}\,, \\
\overline{|\mathcal{T}_{\rm sm}|^2}[Z_T\gamma_T] &= e^4\left(8s_W^4 - 4s_W^2+1\right)\frac{\left(1+\cos^2\theta\right)\left(s^2+M_Z^4\right)}{2s_W^2 c_W^2\sin^2\theta\left(s-M_Z^2\right)^2} \propto s^0\,, \\
2\Re\mathrm{e}\left(\overline{\mathcal{T}_{\rm sm}\mathcal{T}^*_{(8)}}\right)[Z_L\gamma_T] &= \pm\frac{e^2\left(1-4s_W^2\right)}{2s_W c_W}\,\frac{M_Z^2\,s}{\Lambda^4} \propto s^1\,, \\
2\Re\mathrm{e}\left(\overline{\mathcal{T}_{\rm sm}\mathcal{T}^*_{(8)}}\right)[Z_T\gamma_T] &= \pm\frac{e^2\left(1-4s_W^2\right)}{2s_W c_W}\,\frac{M_Z^4}{\Lambda^4} \propto s^0\,, \\
\overline{|\mathcal{T}_{(8)}|^2}[Z_L\gamma_T] &= \frac{(8s_W^4-4s_W^2+1)(\cos2\theta+3)}{32}\,\frac{M_Z^2(s-M_Z^2)^2 s}{\Lambda^8} \propto s^3\,, \\
\overline{|\mathcal{T}_{(8)}|^2}[Z_T\gamma_T] &= \frac{(8s_W^4-4s_W^2+1)\sin^2\theta}{8}\,\frac{M_Z^4(s-M_Z^2)^2}{\Lambda^8} \propto s^2\,.
\end{aligned} \tag{2}$$

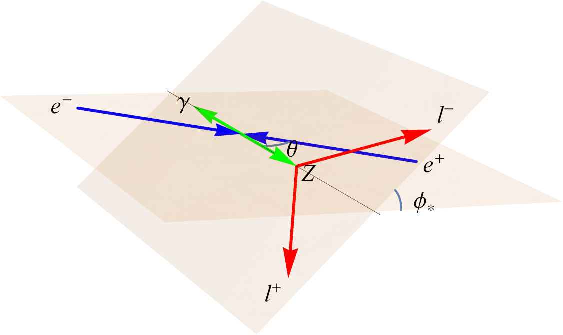
$$\begin{aligned}
\sigma(Z\gamma) &= \frac{e^4(1-4s_W^2+8s_W^4)\left[-(s-M_Z^2)^2-2(s^2+M_Z^4)\ln\left(\sin\frac{\delta}{2}\right)\right]}{32\pi s_W^2 c_W^2(s-M_Z^2)s^2} \\
&\quad \pm\frac{e^2(1-4s_W^2)M_Z^2\left(s-M_Z^2\right)\left(s+M_Z^2\right)}{32\pi s_W c_W\Lambda^4 s^2} + \frac{(1-4s_W^2+8s_W^4)M_Z^2\left(s+M_Z^2\right)\left(s-M_Z^2\right)^3}{192\pi\Lambda^8 s^2} + O(\delta)\,,
\end{aligned}$$

Analysis of Angular Observables

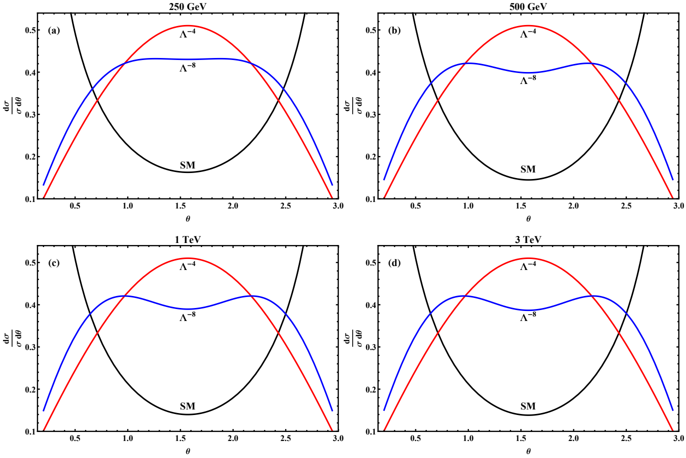
Lepton momenta in the Z rest frame:

$$k_1 = \frac{M_Z}{2}\,(1,\ \sin\theta_*\cos\phi_*,\ \sin\theta_*\sin\phi_*,\ \cos\theta_*)\,,$$

$$k_2 = \frac{M_Z}{2}\,(1,\ -\sin\theta_*\cos\phi_*,\ -\sin\theta_*\sin\phi_*,\ -\cos\theta_*)\,.$$



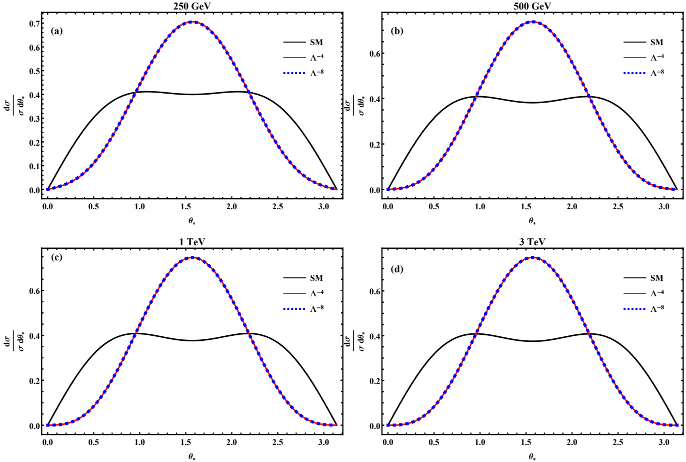
Define angular distribution function $f_\xi^j = \dfrac{\mathrm{d}\sigma_j}{\sigma_j \mathrm{d}\xi}$



$$f_\theta^0 \quad = \quad -\frac{\csc\theta\left[3s^2 + \cos 2\theta (s - M_Z^2)^2 + 2M_Z^2 s + 3M_Z^4\right]}{4\left[(s - M_Z^2)^2 + 2(s^2 + M_Z^4)\ln\left(\sin\frac{\delta}{2}\right)\right]} \,,$$

$$f_\theta^1 \quad = \quad \frac{1}{2}\sin\theta \,,$$

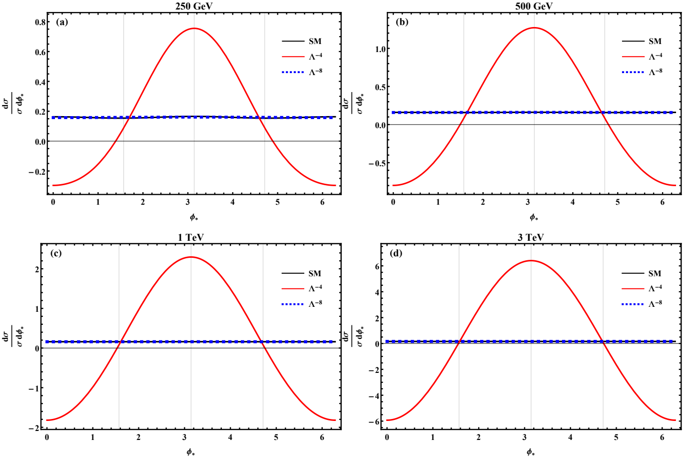
$$f_\theta^2 \quad = \quad \frac{3\sin\theta\left[3s + \cos 2\theta (s - 2M_Z^2) + 2M_Z^2\right]}{16(s + M_Z^2)} \,;$$



$$f^0_{\theta_*} = \frac{3\sin\theta_*(3+\cos 2\theta_*)}{16} + \frac{3\sin\theta_*(1+3\cos 2\theta_*)M_Z^2\, s}{8\left[(s-M_Z^2)^2+2(s^2+M_Z^4)\ln\left(\sin\frac{\delta}{2}\right)\right]} + O(\delta)\,,$$

$$f^1_{\theta_*} = \frac{3\sin\theta_*\left[2s-\cos 2\theta_*(2s-M_Z^2)+3M_Z^2\right]}{16(s+M_Z^2)} + O(\delta)\,,$$

$$f^2_{\theta*} = \frac{3\sin\theta_*\left[2s-\cos 2\theta_*(2s-M_Z^2)+3M_Z^2\right]}{16(s+M_Z^2)} + O(\delta)\,.$$



$$f^0_{\phi_*} = \frac{1}{2\pi} + \frac{3\pi^2 (c_L^2 - c_R^2)^2 M_Z \sqrt{s}\,(s+M_Z^2)\cos\phi_* - 8(c_L^2+c_R^2)^2 M_Z^2\, s\cos 2\phi_*}{16\pi (c_L^2+c_R^2)^2 \left[(s-M_Z^2)^2 + 2(s^2+M_Z^4)\ln\left(\sin\frac{\delta}{2}\right)\right]} + O(\delta),$$

$$f^1_{\phi_*} = \frac{1}{2\pi} - \frac{9\pi^2\sqrt{s}\,(s+M_Z^2)\cos\phi_* - 32 M_Z\, s\cos 2\phi_*}{128\pi M_Z (s+M_Z^2)} + O(\delta),$$

$$f^2_{\phi_*} = \frac{1}{2\pi} - \frac{9\pi (c_L^2 - c_R^2)^2 M_Z \sqrt{s}\cos\phi_*}{128 (c_L^2+c_R^2)^2 (s+M_Z^2)} + O(\delta)\,,$$

Analysis of the $\mathcal{O}(\Lambda^{-4})$ Contribution

We divide the range of ϕ_* into two regions — region (a) and region (b). Region (a) includes the ranges $\left[0, \frac{\pi}{2}\right] \bigcup \left[\frac{3\pi}{2}, 2\pi\right)$ and the region (b) is the range $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

$$\mathbb{O}_1^c = |\sigma_1| \left(\int_{\pi-\phi_c}^{\pi+\phi_c} - \int_0^{\phi_c} - \int_{2\pi-\phi_c}^{2\pi} \right) f_{\phi_*}^1 \, \mathrm{d}\phi_*$$

$$\begin{aligned} B &= N_a - N_b \,, \\ \Delta_B &= \sqrt{\Delta_a^2 + \Delta_b^2} = \sqrt{N_0^a + N_0^b} = \sqrt{\sigma_0 \times \mathcal{L} \times \epsilon} \,, \end{aligned}$$

$$\mathcal{Z}_4 = \frac{S}{\Delta_B} = \frac{\mathbb{O}_1^c(Z\gamma)}{\sqrt{\sigma_0^c(Z\gamma)}} \times \sqrt{\mathrm{Br}(Z \to \ell\ell) \times \mathcal{L} \times \epsilon} \,.$$

$$\mathbb{O}_1^c \simeq \frac{3\alpha(1-4s_W^2)M_Z(s-M_Z^2)\left[3(\pi-2\delta)(s+M_Z^2) - (s-3M_Z^2)\sin 2\delta\right]\sin\phi_c}{256\, s_W c_W \,\Lambda^4\, s^{\frac{3}{2}}} \,,$$

$$\sigma_0^c \simeq \frac{2\phi_c}{\pi}\sigma_0 = \frac{\alpha^2(1-4s_W^2+8s_W^4)\left[-\cos\delta(s-M_Z^2)^2 + 2(s^2+M_Z^4)\ln\left(\cot\frac{\delta}{2}\right)\right]\phi_c}{c_W^2 s_W^2 (s-M_Z^2) s^2} \,,$$

$$\sqrt{s} = 250\,\text{GeV}, \qquad (\sigma_0^c,\ \mathbb{O}_1^c) = \left(3936,\ 0.913\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,, \quad \mathcal{Z}_4 = 3.29\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 500\,\text{GeV}, \qquad (\sigma_0^c,\ \mathbb{O}_1^c) = \left(860,\ 1.85\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,, \quad \mathcal{Z}_4 = 2.18\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 1\,\text{TeV}, \qquad (\sigma_0^c,\ \mathbb{O}_1^c) = \left(209,\ 3.71\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,, \quad \mathcal{Z}_4 = 3.62\left(\frac{\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 3\,\text{TeV}, \qquad (\sigma_0^c,\ \mathbb{O}_1^c) = \left(23.1,\ 11.1\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,, \quad \mathcal{Z}_4 = 2.05\left(\frac{2\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 5\,\text{TeV}, \qquad (\sigma_0^c,\ \mathbb{O}_1^c) = \left(8.30,\ 18.5\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,, \quad \mathcal{Z}_4 = 2.33\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,.$$

$\mathcal{L} = 2ab^{-1}$ and $\epsilon = 100\%$.

At high energies $s \gg M_Z^2$, we have

$$\mathcal{Z}_4 \propto \frac{M_Z\, s}{\Lambda^4}\sqrt{\mathcal{L}\times\epsilon}\,.$$

$$\Lambda \propto \left(\frac{M_Z\sqrt{\mathcal{L}\times\epsilon}}{\mathcal{Z}_4}\right)^{\frac{1}{4}} \times (\sqrt{s})^{\frac{1}{2}}\,.$$

Analysis Including the $\mathcal{O}(\Lambda^{-8})$ Contribution

$$\sigma_c^0(Z\gamma) = \frac{e^4\left(8s_W^4-4s_W^2+1\right)}{32\pi s_W^2 c_W^2\left(s-M_Z^2\right)s^2} \times \frac{1}{16}\Big[4\cos\delta\,(9\cos\delta_* - \cos3\delta_*)M_Z^2\, s - (15\cos\delta_* + \cos3\delta_*)(s^2+M_Z^4)\left(\cos\delta + 2\ln\tan\frac{\delta}{2}\right)\Big],$$

$$\sigma_c^1(Z\gamma) = \pm\frac{e^2(1-4s_W^2)M_Z^2\left(s-M_Z^2\right)}{32\pi s_W c_W \Lambda^4 s^2}\,\frac{\left[2(5-\cos2\delta_*)s+(\cos2\delta_*+7)M_Z^2\right]\cos\delta\cos\delta_*}{8},$$

$$\sigma_c^2(Z\gamma) = \frac{\left(8s_W^4-4s_W^2+1\right)M_Z^2\left(s-M_Z^2\right)^3}{192\pi\Lambda^8 s^2} \times\frac{\cos\delta}{64}\Big[(7+\cos2\delta)(9\cos\delta_*-\cos3\delta_*)s+(5-\cos2\delta)(15\cos\delta_*+\cos3\delta_*)M_Z^2\Big],$$

$$\mathcal{Z}_8 = \frac{S}{\Delta_B} = \frac{|\sigma_c^1(Z\gamma)+\sigma_c^2(Z\gamma)|}{\sqrt{\sigma_c^0(Z\gamma)}} \times \sqrt{\mathrm{Br}(Z\to\ell\ell)\times\mathcal{L}\times\epsilon}\ .$$

$$\sqrt{s} = 250\ \text{GeV}, \qquad \sigma(Z\gamma) = \left[2427 \pm 6.62\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 + 1.39\left(\frac{0.5\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 1.90\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 + 0.400\left(\frac{0.5\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}$$

$$\sqrt{s} = 500\ \text{GeV}, \qquad \sigma(Z\gamma) = \left[417 \pm 0.996\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 + 0.624\left(\frac{0.8\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.689\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 + 0.432\left(\frac{0.8\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 1\ \text{TeV}, \qquad \sigma(Z\gamma) = \left[94.0 \pm 0.404\left(\frac{\text{TeV}}{\Lambda}\right)^4 + 1.73\left(\frac{\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.589\left(\frac{\text{TeV}}{\Lambda}\right)^4 + 2.53\left(\frac{\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 3\ \text{TeV}, \qquad \sigma(Z\gamma) = \left[10.1 \pm 0.0252\left(\frac{2\text{TeV}}{\Lambda}\right)^4 + 0.554\left(\frac{2\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.112\left(\frac{2\ \text{TeV}}{\Lambda}\right)^4 + 2.46\left(\frac{2\ \text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 5\ \text{TeV}, \qquad \sigma(Z\gamma) = \left[3.63 \pm 0.0103\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 + 0.718\left(\frac{2.5\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

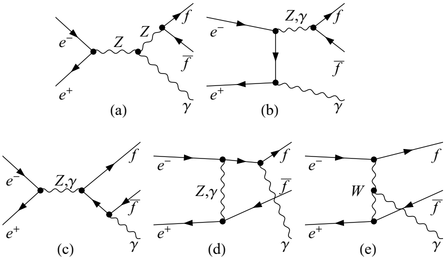
$$\mathcal{Z}_8 = \left|\pm 0.0764\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 + 5.32\left(\frac{2.5\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,.$$

At high energies $s \gtrsim (1\text{TeV})^2 \gg M_Z^2$,

$$\mathcal{Z}_8 \propto \frac{M_Z^2(\sqrt{s})^5}{\Lambda^8}\sqrt{\mathcal{L}\times\epsilon}\,,$$

$$\Lambda \propto \left(\frac{M_Z^2\sqrt{\mathcal{L}\,\epsilon}}{\mathcal{Z}_8}\right)^{\frac{1}{8}}(\sqrt{s})^{\frac{5}{8}}\,.$$

Including reducible Background and Monte Carlo Simulation



16 diagrams for electrons backgrounds and 8 diagrams for other charged fermions backgrounds. Each diagram has 8 helicity combinations.

We use `FeynArts` to generate all the background diagrams and then compute them by `FeynCalc`. Finally we use `CUDAlink` to do numerical integration.

For $Z \to e^- e^+$

$$\sqrt{s} = 250\,\text{GeV}, \qquad (\sigma_0^c,\, \mathbb{O}_1^c) = \left(141,\ 0.0256\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,,\ \mathcal{Z}_4^e = 1.54\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 500\,\text{GeV}, \qquad (\sigma_0^c,\, \mathbb{O}_1^c) = \left(26.6,\ 0.0524\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,,\ \mathcal{Z}_4^e = 1.11\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 1\,\text{TeV}, \qquad (\sigma_0^c,\, \mathbb{O}_1^c) = \left(6.15,\ 0.109\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,,\ \mathcal{Z}_4^e = 1.97\left(\frac{\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 3\,\text{TeV}, \qquad (\sigma_0^c,\, \mathbb{O}_1^c) = \left(0.691,\ 0.340\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,,\ \mathcal{Z}_4^e = 1.14\left(\frac{2\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 5\,\text{TeV}, \qquad (\sigma_0^c,\, \mathbb{O}_1^c) = \left(0.250,\ 0.567\left(\frac{\text{TeV}}{\Lambda}\right)^4\right)\text{fb}\,,\ \mathcal{Z}_4^e = 1.30\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 \times \sqrt{\epsilon}\,.$$

at $\mathcal{O}(\Lambda^{-8})$

$$\sqrt{s} = 250\,\text{GeV}, \qquad \mathcal{Z}_8^e = \left|\pm 0.96\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 + 0.203\left(\frac{0.5\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 500\,\text{GeV}, \qquad \mathcal{Z}_8^e = \left|\pm 0.38\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 + 0.232\left(\frac{0.8\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 1\,\text{TeV}, \qquad \mathcal{Z}_8^e = \left|\pm 0.31\left(\frac{\text{TeV}}{\Lambda}\right)^4 + 1.38\left(\frac{\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 3\,\text{TeV}, \qquad \mathcal{Z}_8^e = \left|\pm 0.06\left(\frac{2\,\text{TeV}}{\Lambda}\right)^4 + 1.35\left(\frac{2\,\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 5\,\text{TeV}, \qquad \mathcal{Z}_8^e = \left|\pm 0.03\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 + 2.90\left(\frac{2.5\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,.$$

combined signal significance

$$\mathcal{Z}_{\ell\ell} = \sqrt{(\mathcal{Z}_4^e)^2+(\mathcal{Z}_4^\mu)^2+(\mathcal{Z}_4^\tau)^2+(\mathcal{Z}_8^e)^2+(\mathcal{Z}_8^\mu)^2+(\mathcal{Z}_8^\tau)^2}\,,$$

Analysis of Invisible Decay Channel $\mathbf{Z \to \nu\bar{\nu}}$

For invisible channel, we could only impose cuts on the scattering angle θ of the final state mono-photon

$$\sqrt{s} = 250\ \text{GeV}, \qquad \sigma(Z\gamma) = \left[3236 \pm 7.25\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 + 1.53\left(\frac{0.5\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 2.55\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4 + 0.538\left(\frac{0.5\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}$$

$$\sqrt{s} = 500\ \text{GeV}, \qquad \sigma(Z\gamma) = \left[656 \pm 1.13\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 + 0.709\left(\frac{0.8\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.884\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4 + 0.554\left(\frac{0.8\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 1\ \text{TeV}, \qquad \sigma(Z\gamma) = \left[156 \pm 0.465\left(\frac{\text{TeV}}{\Lambda}\right)^4 + 2.00\left(\frac{\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.744\left(\frac{\text{TeV}}{\Lambda}\right)^4 + 3.20\left(\frac{\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 3\ \text{TeV}, \qquad \sigma(Z\gamma) = \left[17.1 \pm 0.0291\left(\frac{2\text{TeV}}{\Lambda}\right)^4 + 0.640\left(\frac{2\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.141\left(\frac{2\,\text{TeV}}{\Lambda}\right)^4 + 3.10\left(\frac{2\,\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,,$$

$$\sqrt{s} = 5\ \text{TeV}, \qquad \sigma(Z\gamma) = \left[6.15 \pm 0.0119\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 + 0.830\left(\frac{2.5\text{TeV}}{\Lambda}\right)^8\right]\text{fb}\,,$$

$$\mathcal{Z}_8 = \left|\pm 0.0961\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4 + 6.69\left(\frac{2.5\text{TeV}}{\Lambda}\right)^8\right| \times \sqrt{\epsilon}\,.$$

$\sqrt{s}$ (GeV)	250	500	1000	3000	5000
$\Lambda_{\bar{l}l}^{2\sigma}$ (TeV)	0.57(0.56)	0.82(0.80)	1.2	2.1	2.9
$\Lambda_{\bar{l}l}^{5\sigma}$ (TeV)	0.46(0.44)	0.67(0.64)	0.98(0.95)	1.9	2.5
$\Lambda_{\nu\bar{\nu}}^{2\sigma}$ (TeV)	0.55(0.32)	0.75(0.62)	1.1	2.1	2.9
$\Lambda_{\nu\bar{\nu}}^{5\sigma}$ (TeV)	0.45(0.32)	0.65(0.57)	0.97(0.93)	1.9	2.6
$\Lambda_{l\nu}^{2\sigma}$ (TeV)	0.61(0.59)	0.85(0.80)	1.2	2.3	3.0
$\Lambda_{l\nu}^{5\sigma}$ (TeV)	0.49(0.46)	0.70(0.64)	1.0	2.0	2.7

Sensitivity reaches of the new physics scale Λ from $e^-e^+ \to \nu\bar{\nu}\gamma$ *channel, and from combining both* $\ell^-\ell^+\gamma$ *and* $\nu\bar{\nu}\gamma$ *channels, at the* 2σ *and* 5σ *levels, for different collider energies. Here the two numbers in the parentheses correspond to the case of the dimension-8 operator whose coefficient has a minus sign, while in all other entries the effects due to the coefficient having a minus sign are negligible.*

Initial electron polarization

The leading term of differential cross section at $\mathcal{O}(\Lambda^{-4})$ is proportional to

$$\Re\left[\mathcal{T}^{L}_{(8)}(0\pm)\mathcal{T}^{T*}_{sm}(\mp\pm)\right]\sin\theta\sin\theta_*$$
$$\propto \frac{v^2\sqrt{s}}{\Lambda^4 M_Z}\left[(e_L^2+e_R^2)(f_L^2-f_R^2)(1+\cos^2\theta)+2(e_L^2-e_R^2)(f_L^2+f_R^2)\cos\theta\cos\theta_*\right]\sin^2\theta_*\cos\phi_*,$$

The $\mathbb{O}_1^c$ term is suppressed by the factor $f_L^2 - f_R^2 \propto 1-4s_w^2$. If electrons and positrons are fully polarized and electrons are left-handed, we can redefine $\mathbb{O}_1^c$ as

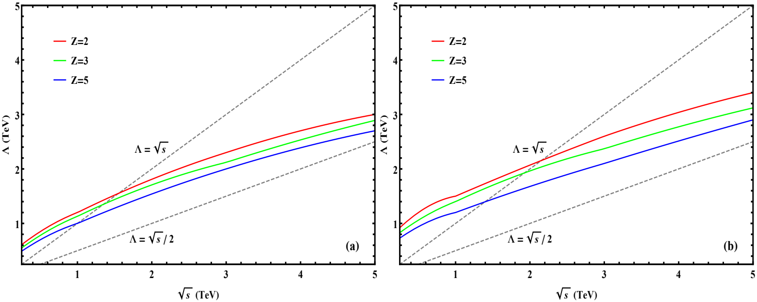
$$\mathbb{O}_1^c = \left|\sigma_1\int f^1\frac{|\theta-\pi/2|}{\theta-\pi/2}\frac{|\theta_*-\pi/2|}{\theta_*-\pi/2}\frac{|\cos\phi_*|}{\cos\phi_*}d\theta d\theta_* d\phi_* dM_*\right|$$
$$f^i = \frac{d^4\sigma^i}{\sigma^i d\theta d\theta_* d\phi_* dM_*},$$

$$\sqrt{s}=250\,\text{GeV},\qquad \mathcal{Z}_4^e = 4.46\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,\ \mathcal{Z}_4^\mu = 4.86\left(\frac{0.5\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,$$
$$\sqrt{s}=500\,\text{GeV},\qquad \mathcal{Z}_4^e = 3.64\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,\ \mathcal{Z}_4^\mu = 3.78\left(\frac{0.8\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,$$
$$\sqrt{s}=1\,\text{TeV},\qquad \mathcal{Z}_4^e = 6.40\left(\frac{\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,\ \mathcal{Z}_4^\mu = 6.47\left(\frac{\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,$$
$$\sqrt{s}=3\,\text{TeV},\qquad \mathcal{Z}_4^e = 3.80\left(\frac{2\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,\ \mathcal{Z}_4^\mu = 3.80\left(\frac{2\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,$$
$$\sqrt{s}=5\,\text{TeV},\qquad \mathcal{Z}_4^e = 4.32\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,,\ \mathcal{Z}_4^\mu = 4.33\left(\frac{2.5\text{TeV}}{\Lambda}\right)^4\times\sqrt{\epsilon}\,.$$

$\sqrt{s}$ (GeV)	250	500	1000	3000	5000
$\Lambda_{l\bar{l}}^{2\sigma}$ (TeV)	0.81	1.1	1.5(1.4)	2.5	3.2
$\Lambda_{l\bar{l}}^{5\sigma}$ (TeV)	0.64	0.87(0.85)	1.2(1.1)	2.1(2.0)	2.7
$\Lambda_{\nu\bar{\nu}}^{2\sigma}$ (TeV)	0.87	1.1	1.3(0.87)	2.3(2.1)	3.0(2.9)
$\Lambda_{\nu\bar{\nu}}^{5\sigma}$ (TeV)	0.69	0.86(0.83)	1.1(0.84)	2.0(1.9)	2.7(2.6)
$\Lambda_{l\nu}^{2\sigma}$ (TeV)	0.92	1.2	1.5	2.6(2.5)	3.4(3.3)
$\Lambda_{l\nu}^{5\sigma}$ (TeV)	0.73	0.94(0.92)	1.2	2.1	2.9(2.8)

Polarized(90% left-handed electrons and 65% right-handed positrons) sensitivity reaches of the new physics scale Λ from $e^-e^+ \to l\bar{l}\gamma$ channel, and from combining both $\ell^-\ell^+\gamma$ and $\nu\bar{\nu}\gamma$ channels, at the 2σ and 5σ levels, for different collider energies. Here the two numbers in the parentheses correspond to the case of the dimension-8 operator whose coefficient has a minus sign, while in all other entries the effects due to the coefficient having a minus sign are negligible.

Reaches on Λ for unpolarized and polarized $(P_L^e, P_R^{\bar{e}}) = (90\%, 65\%)$ beams



Summary

- $e^+e^- \to Z\gamma$ provides a rare opportunity to probe an effective dimension-8 operator in the SMEFT.
- We have used a general analysis of the angular distributions to identify particular angular distributions and cuts that maximize the statistical sensitivity to the possible new physics scale Λ.
- The prospective sensitivities increase with the collision energies but more slowly than $\sqrt{s}$.
- Future e^+e^- colliders (such as the ILC, CEPC, FCC-ee, and CLIC) may be able to provide very competitive sensitivities to probing the scale of new physics.