Wall Speed and Shape in Singlet-Assisted Strong Electroweak Phase Transitions

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In collaboration with Ian Banta, James Cline, and David Tucker-Smith

Based on: arxiv:2009.14295

Overview

- Electroweak Baryogenesis
- The Scalar Singlet Model
- The Phase Transition
- Friction
- Determining the Wall Velocity
- Results
- Strange Transitions

Electroweak Baryogenesis

- If the EWPT was first order, sphaleron and CP-violating interaction around the wall could produce the matter-antimatter assymmetry

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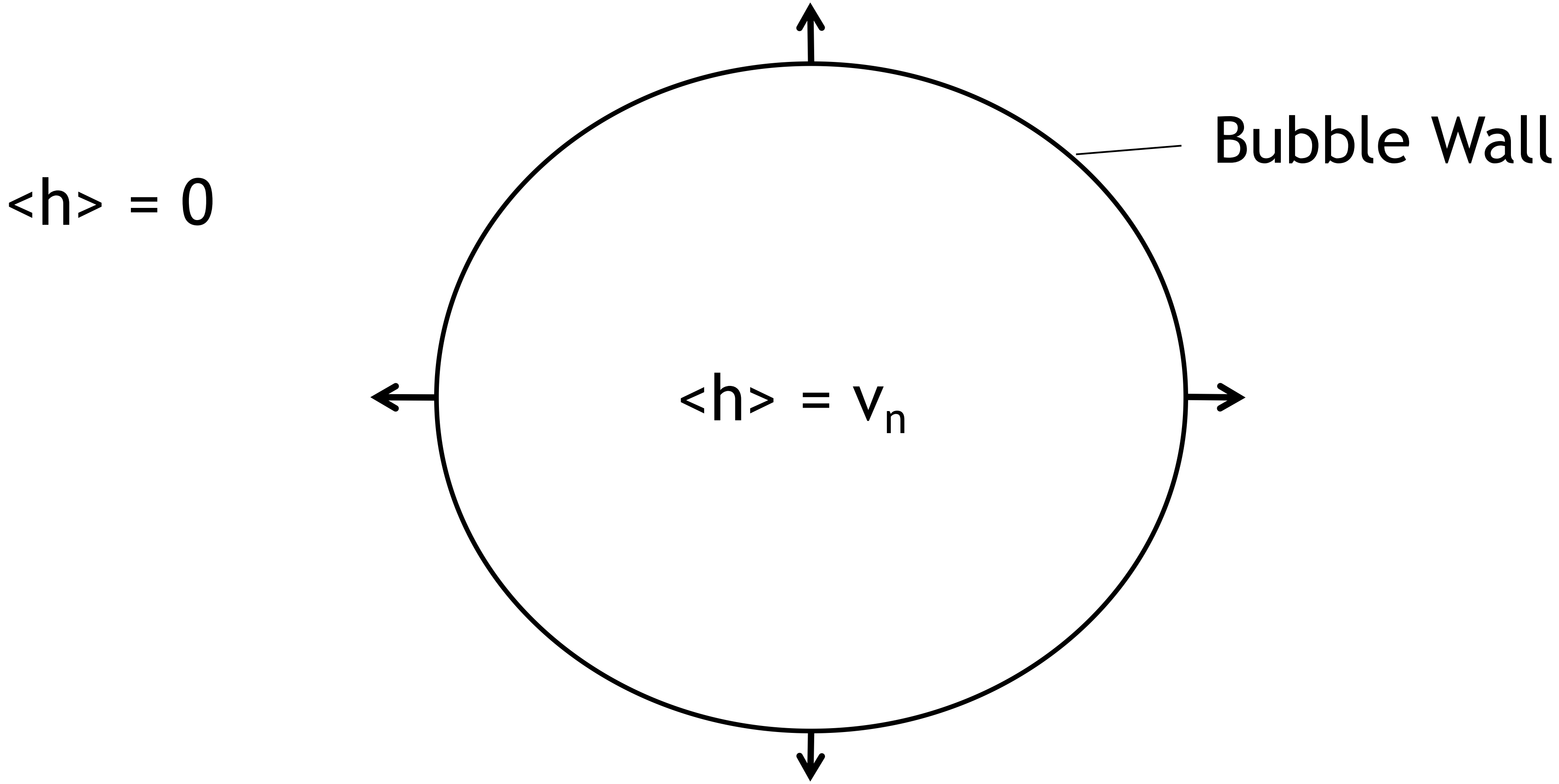
- Not possible in the Standard Model because the EWPT is a smooth cross-over

Electroweak Phase Transition



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- Generally slow walls are preferred for baryogenesis
- Difficult to compute

Singlet Scalar Model

- Add a singlet scalar field with a Z_2 symmetry

$$V_0 = \lambda_h(|H|^2 - \tfrac{1}{2}v_0^2)^2 + \frac{\lambda_s}{4}(s^2 - w_0^2)^2 + \frac{\lambda_{hs}}{2}|H|^2 s^2$$

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- When electroweak symmetry is broken

$$s = 0,\, H = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2, h + i\chi_3)^T$$

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- When electroweak symmetry is broken

$$s = 0,\, H = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2, h + i\chi_3)^T$$

- Singlet mass in the broken phase is

$$m_s^2 = -\lambda_s w_0^2 + \tfrac{1}{2}\lambda_{hs} v_0^2 s^2$$

The Effective Potential

- The effective potential was calculated to one loop

$$V_{eff} = V_0 + V_1 + V_{CT} + V_T$$

- V_0 is tree level potential from previous slide

$$V_0 = \lambda_h(|H|^2 - \frac{1}{2}v_0^2)^2 + \frac{\lambda_s}{4}(s^2 - w_0^2)^2 + \frac{\lambda_{hs}}{2}|H|^2 s^2$$

The Effective Potential

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$$V_{eff} = V_0 + V_1 + V_{CT} + V_T$$

- V_1 is Coleman-Weinberg Potential including thermal mass ressumation

$$V_1 = \sum_{i=h,s,\chi,t,W,Z,\gamma} \frac{n_i m_i^4(h,s,T)}{64\pi^2}\left[ln\left(\frac{m_i^2(h,s,T)}{v_0^2}\right) - c_i\right]$$

The Effective Potential

- The effective potential was calculated to one loop

$$V_{eff} = V_0 + V_1 + V_{CT} + V_T$$

- Counterterms (V_{CT}) set to preserve three physical quantities: λ_{hs}, w_0, and m_s

$$\frac{\partial V}{\partial s}|_{h=0,s=w_0} = 0 \qquad \frac{\partial^2 V}{\partial s^2}|_{h=v_0,s=0} = m_s^2$$

$$\frac{\partial^4 V}{\partial h^2 s^2}|_{h=v_0,s=0} = \lambda_{hs}$$

The Effective Potential

- The effective potential was calculated to one loop

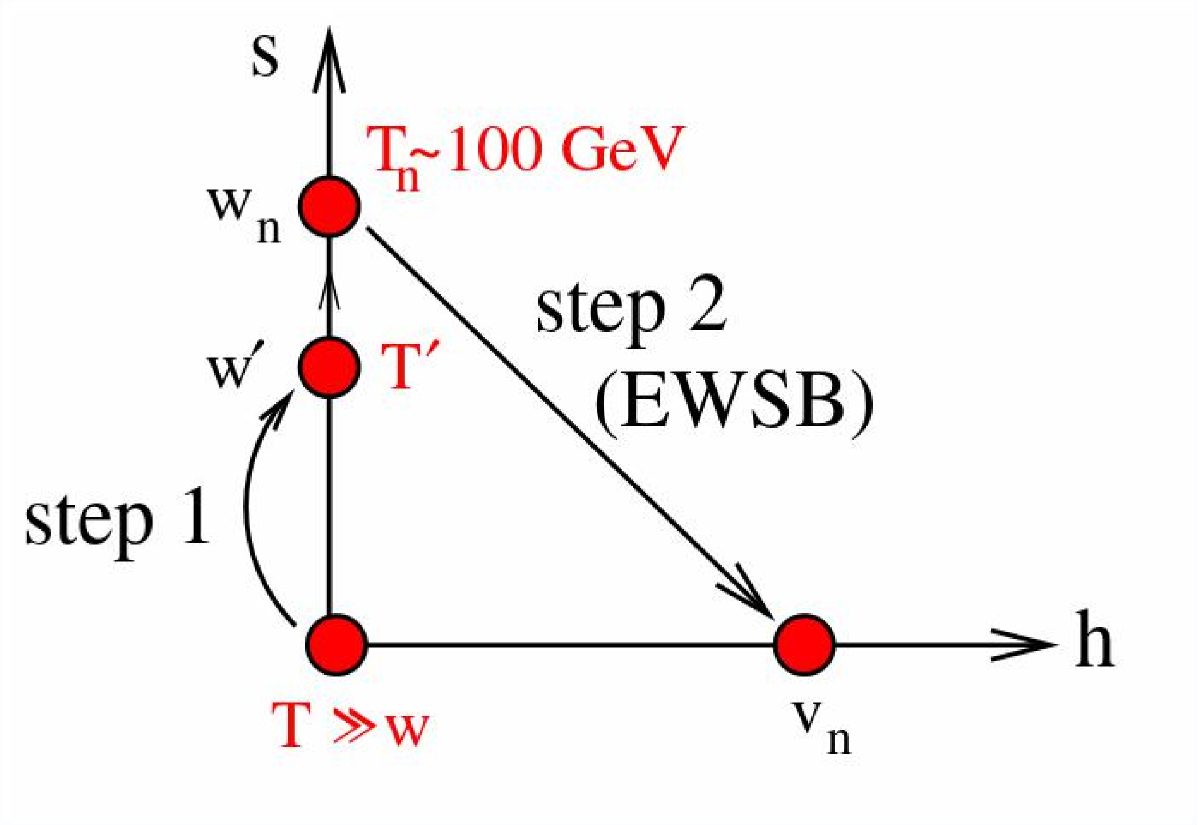
$$V_{eff} = V_0 + V_1 + V_{CT} + V_T$$

- V_T includes one-loop thermal potential

$$V_T = -\frac{12T^4}{2\pi^2} J_F\left(\frac{m_t(h)}{T^2}\right) + \sum_{i=h,s,\chi,W,Z} \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2(h,s,T)}{T^2}\right)$$

Two Step Transition

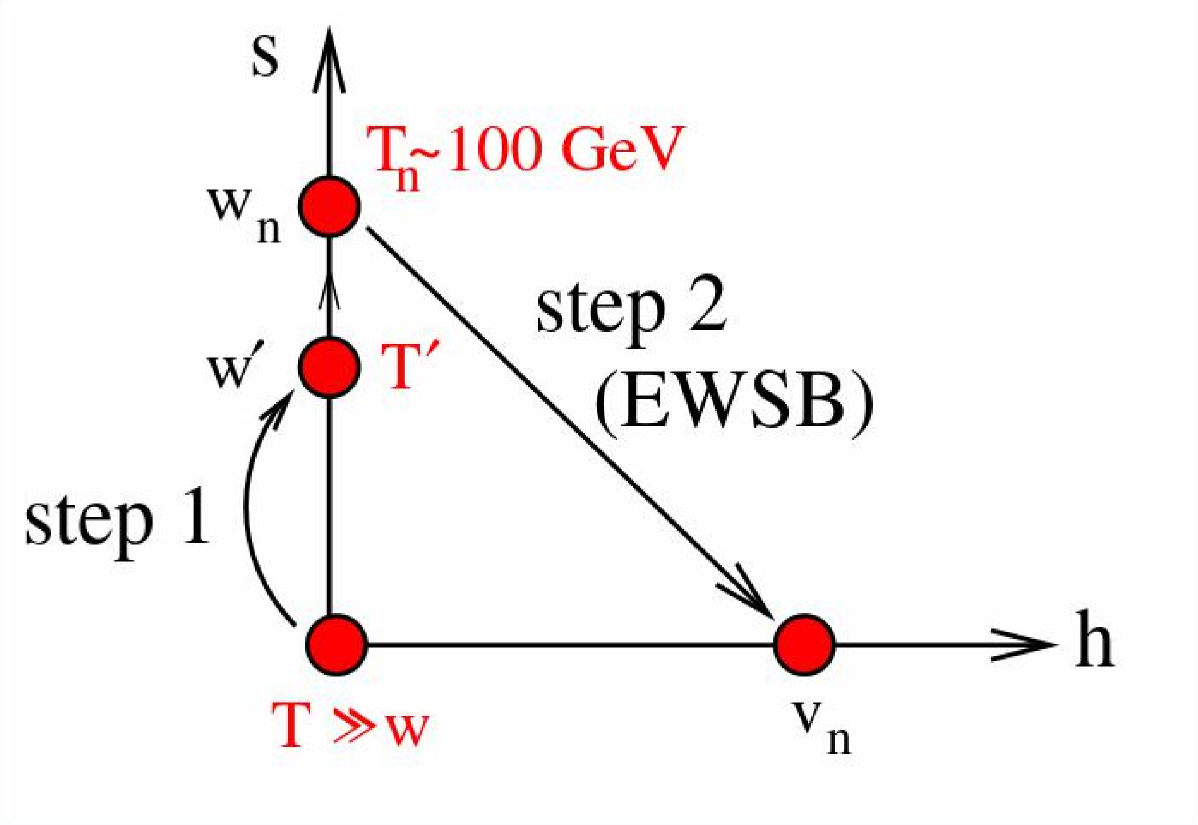
- Universe starts in EW and Z_2 symmetric phase



Two Step Transition

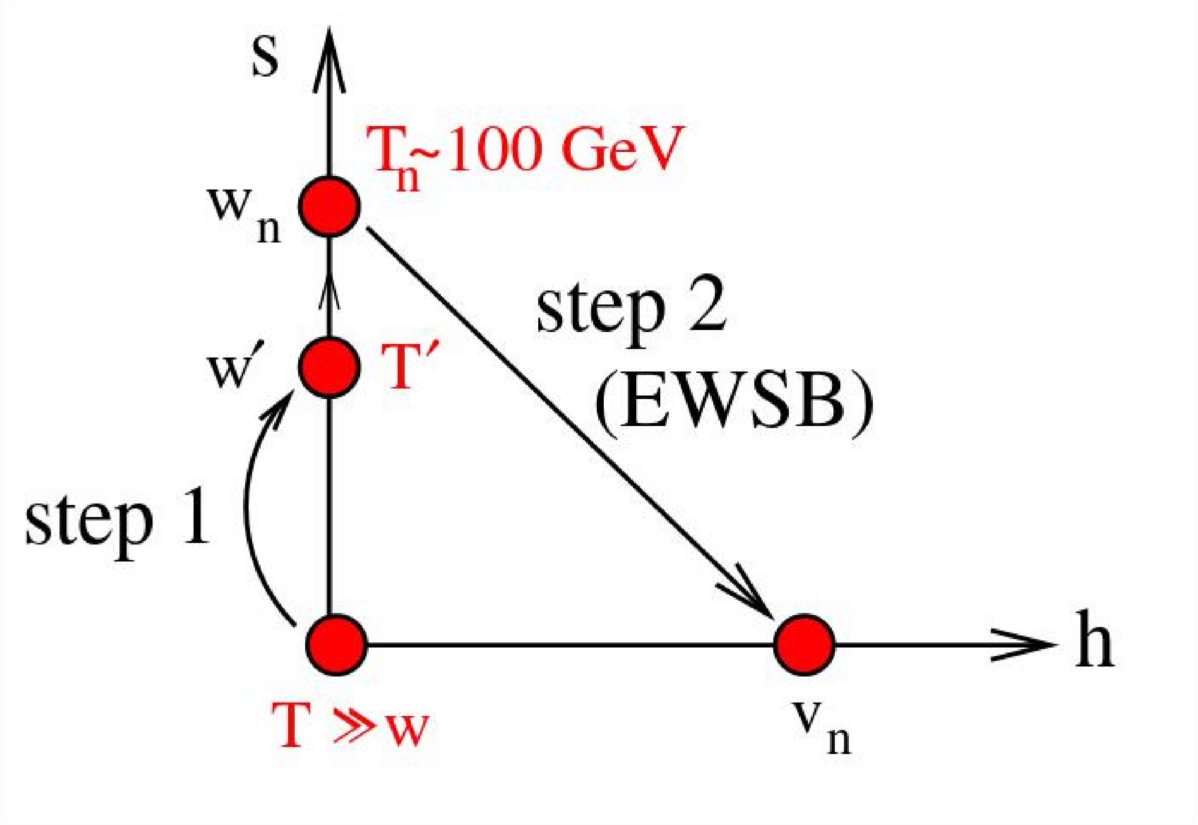
- Universe starts in EW and Z_2 symmetric phase

- First transition breaks Z_2 symmetry



Two Step Transition

- Universe starts in EW and Z_2 symmetric phase
- First transition breaks Z_2 symmetry
- Second transition breaks electroweak and restores Z_2 symmetry



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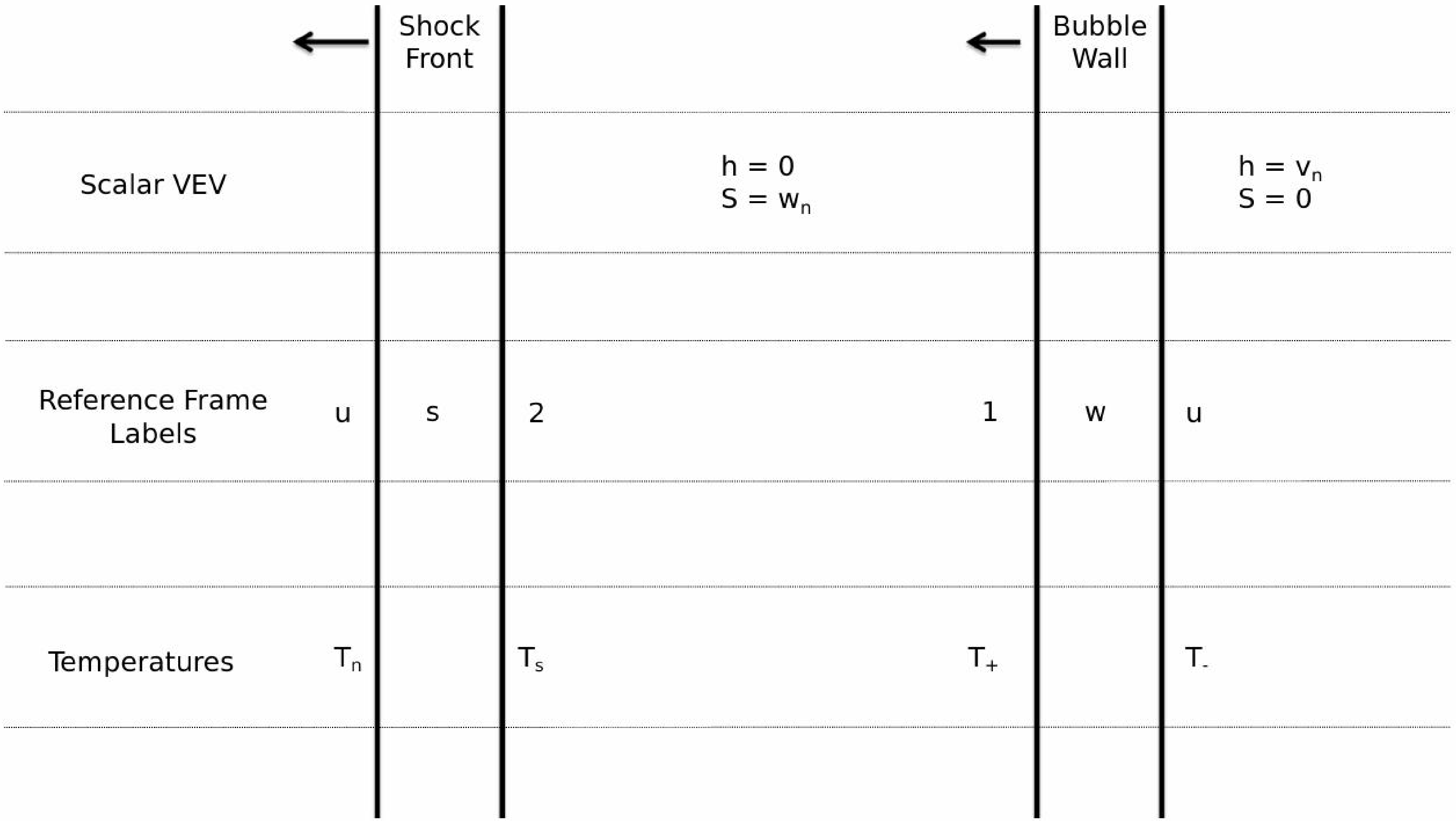
- v_n Higgs VEV at temperature T_n
  - $v_n/T_n > 1.1$ to avoid washout of baryon assymetry

Deflagrations

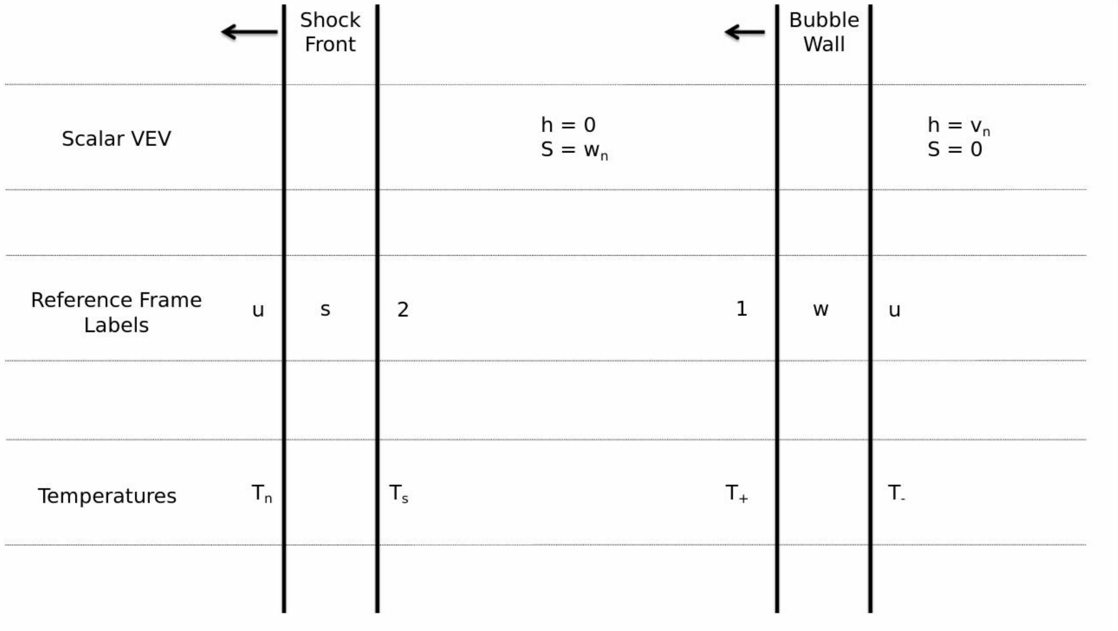


- Treated as perfect fluid
- Fluid Velocity and temperature change as wall and shock front pass

Determining the Wall Temperature

- T_+ found from system of 8 equations

- 6 come from integrating $T_{\mu\nu}$ across 3 regions:
  - Across the wall
  - Across the shock front
  - From the wall to the shock front



- 2 come from lorentz transforms between fluid reference frames

Assumptions in Determining T_+

- Subsonic walls
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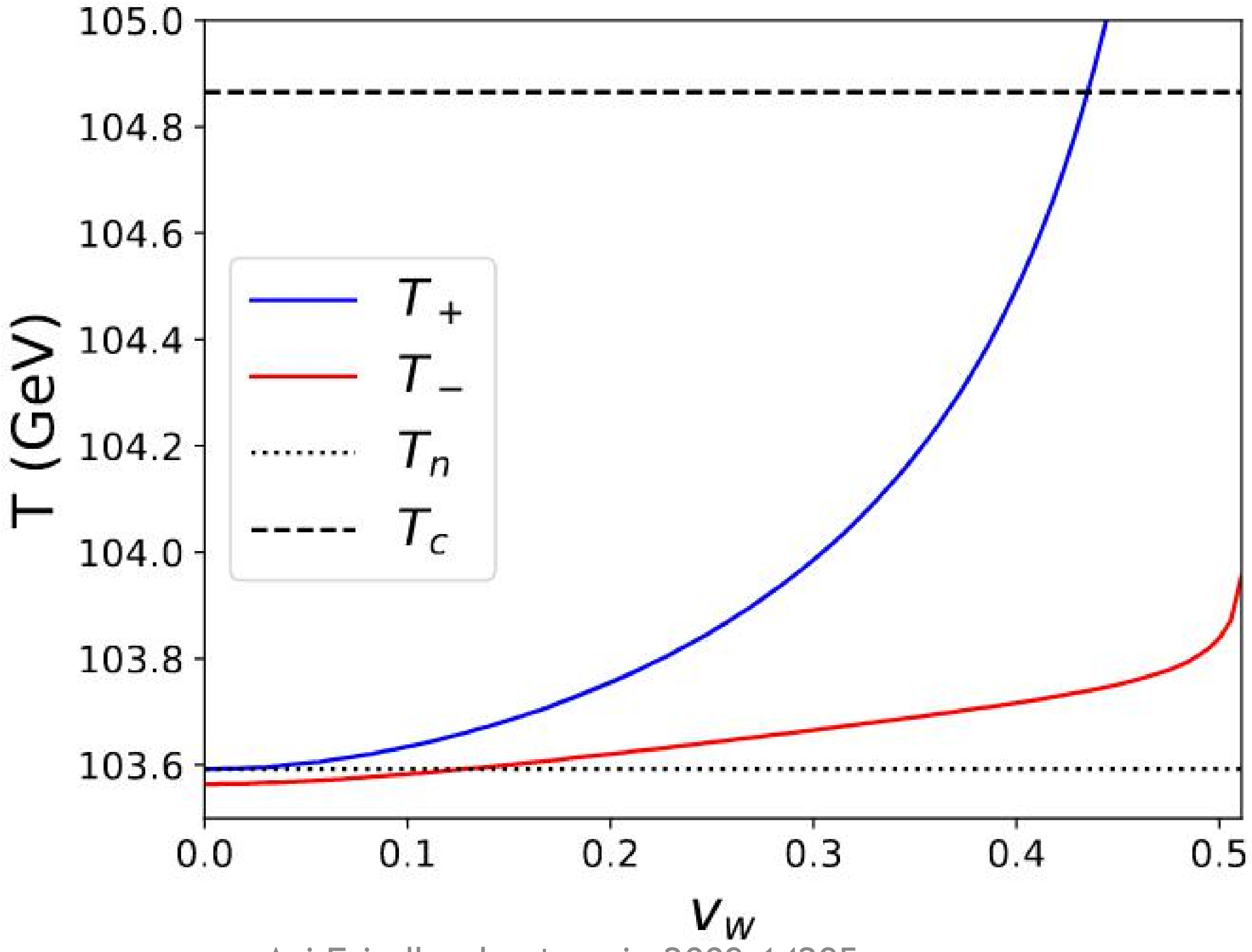
Assumptions in Determining T_+

- Subsonic walls
  - Equations are singular when walls break sound barrier
- Fluid velocity in universe frame are small
- Not too much supercooling
  - Allows density and pressure dependence on temperature to simply to T^4
  - Allows speed of sound to be $1/\sqrt{3}$

Wall Temperature



- v_w defined in reference frame of fluid in front of wall
- Solutions blow up when wall velocity in universe frame approaches c_s

Equations of Motion

- Treated as scalar fields coupled to perfect fluids

$$-s''(z) + \frac{\partial V_{eff}(h,s,T)}{\partial s} = 0$$

$$-h''(z) + \frac{\partial V_{eff}(h,s,T)}{\partial h} + \sum_{i=t,W,Z} n_i \frac{dm_i^2}{dh} \int \frac{d^3p}{(2\pi)^3 2E} \delta f_i(p,z) = 0$$

Equations of Motion

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Friction

- We assume dominant friction comes from top quark and gauge bosons

Friction

$$\sum_{i=t,W,Z} n_i \frac{dm_i^2}{dh} \int \frac{d^3p}{(2\pi)^3 2E} \delta f_i(p,z)$$

- Requires determining deviation from equillibrium

- Treat as 3 fluids
  - Top quark
  - Gauge Boson (Combines W and Z fluids)
  - Background (all other particles which are treated as massless)

Fluid Approximation

- Only consider fluid excitations with $p \gg 1/L_W$
  - We confirmed IR excitations are subdominant

- Parameterize phase space as:

$$f_i(E,z) = \frac{1}{e^{(E+\delta_i(z))/T} \pm 1}$$

where perturbation is described by

$$\delta_i(z) = -\left[T(\delta\mu_i + \delta\mu_{bg})(z) + E(\delta\tau_i + \delta\tau_{bg})(z) + p_z(\delta v_i + \delta v_{bg})(z)\right]$$

Determining the Perturbations

- Perturbations described by Boltzmann equation

$$\frac{d}{dt} f_i(E, z) = -C[f_i(E, z)]$$

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- Perturbations described by Boltzmann equation

$$\frac{d}{dt} f_i(E, z) = -C[f_i(E, z)]$$

- Boltzmann eq. linearized and turned into ODEs by taking three moments

$$\int d^3p/(2\pi)^3 \qquad \int p_z \, d^3p/(2\pi)^3$$

$$\int (E/T) \, d^3p/(2\pi)^3$$

Perturbation Equations

- Resulting ODEs are:

$$A_W(\vec{q}_W + \vec{q}_{bg})' + \Gamma_W \vec{q}_W = S_W$$

$$A_t(\vec{q}_t + \vec{q}_{bg})' + \Gamma_t \vec{q}_t = S_t$$

$$A_{bg}\vec{q}\,'_{bg} + \Gamma_{bg,W}\vec{q}_W + \Gamma_{bg,t}\vec{q}_t = 0$$

$$\vec{q}_i^{\,T} = (\delta\mu_i, \delta\tau_i, \delta v_i)$$

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$$\vec{q}_i^{\,T} = (\delta\mu_i, \delta\tau_i, \delta v_i)$$

$$A_i \equiv \begin{bmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3} d_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3} d_4^i \\ \frac{1}{3} d_3^i & \frac{1}{3} d_4^i & \frac{1}{3} v_w d_4^i \end{bmatrix}$$

$$c_j^i\left(\frac{m_i}{T}\right) \equiv \int \frac{d^3p}{(2\pi)^3}\left(-f'_{0,i}\right)\frac{E^{j-2}}{T^{j+1}}$$

$$d_j^i\left(\frac{m_i}{T}\right) \equiv \int \frac{d^3p}{(2\pi)^3}\left(-f'_{0,i}\right)\frac{p^2 E^{j-4}}{T^{j+1}}$$

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$$S_i \equiv \frac{m_i' m_i}{T^2} \begin{bmatrix} v_w c_1^i \\ v_w c_2^i \\ 0 \end{bmatrix}$$

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- Γ_i matrices describes fluid interaction rate

$$A_i \equiv \begin{bmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3} d_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3} d_4^i \\ \frac{1}{3} d_3^i & \frac{1}{3} d_4^i & \frac{1}{3} v_w d_4^i \end{bmatrix}$$

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m/T approximation

- A_i and S_i depends on particle mass via c_i/d_i

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m/T approximation

- A_i and S_i depends on particle mass via c_i/d_i
- Often solved to lowest order in m/T where $c_i=d_i$
  - Only true for EWPT with small v_n/T_n

$$A_i \equiv \begin{bmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3} d_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3} d_4^i \\ \frac{1}{3} d_3^i & \frac{1}{3} d_4^i & \frac{1}{3} v_w d_4^i \end{bmatrix}$$

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m/T approximation

- A_i and S_i depends on particle mass via c_i/d_i

- Often solved to 1st order in m/T where $c_i=d_i$
  - Only true for EWPT with small v_n/T_n

- We use full m/T dependence because we find slow walls for phase transitions where $m_t/T > 1$

$$A_i \equiv \begin{bmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3} d_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3} d_4^i \\ \frac{1}{3} d_3^i & \frac{1}{3} d_4^i & \frac{1}{3} v_w d_4^i \end{bmatrix}$$

$$S_i \equiv \frac{m_i' m_i}{T^2} \begin{bmatrix} v_w c_1^i \\ v_w c_2^i \\ 0 \end{bmatrix}$$

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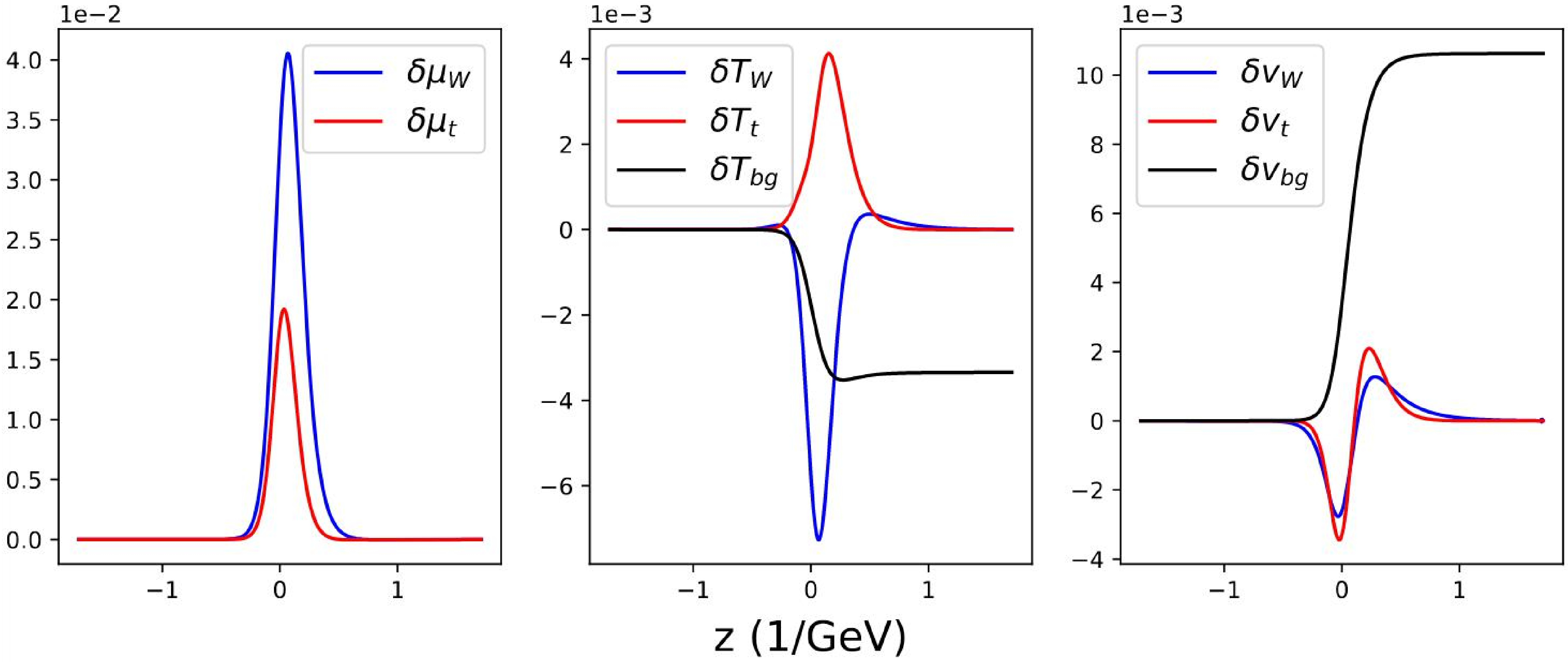
Perturbation Solutions



Equations of Motion Revisited

- Friction can now be calculated with

$$\int \frac{d^3p}{(2\pi)^3 2E}\,\delta f_i(\vec{p},z) \approx \frac{T^2}{2}\left[c_1^i(z)\delta\mu_i(z) + c_2^i(z)\left(\delta\tau_i(z) + \delta\tau_{bg}(z)\right)\right]$$

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- Then the equations of motion are

$$-h''(z) + \frac{\partial V_{eff}(h,s,T_+)}{\partial h} + \frac{n_t T_+}{2}\frac{dm_t^2}{dh}\left[c_1^t\delta\mu_t + c_2^t(\delta\tau_t + y\delta\tau_{bg})\right] + \frac{n_W T_+}{2}\frac{dm_W^2}{dh}\left[c_1^W\delta\mu_W + c_2^W(\delta\tau_W + y\delta\tau_{bg})\right] = 0$$

$$-s''(z) + \frac{\partial V_{eff}(h,s,T)}{\partial s} = 0$$

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$$+\frac{n_t T_+}{2}\frac{dm_t^2}{dh}\left[c_1^t\delta\mu_t + c_2^t(\delta\tau_t + y\delta\tau_{bg})\right]$$
$$+\frac{n_W T_+}{2}\frac{dm_W^2}{dh}\left[c_1^W\delta\mu_W + c_2^W(\delta\tau_W + y\delta\tau_{bg})\right] = 0$$
$$-s''(z) + \frac{\partial V_{eff}(h,s,T)}{\partial s} = 0$$

Set so friction cancels out potential term

Equations of Motion Revisited

$$-h''(z) + \frac{\partial V_{eff}(h,s,T_+)}{\partial h} + \frac{n_t T_+}{2}\frac{dm_t^2}{dh}\left[c_1^t \delta\mu_t + c_2^t(\delta\tau_t + y\delta\tau_{bg})\right] + \frac{n_W T_+}{2}\frac{dm_W^2}{dh}\left[c_1^W \delta\mu_W + c_2^W(\delta\tau_W + y\delta\tau_{bg})\right] = 0$$

$$-s''(z) + \frac{\partial V_{eff}(h,s,T)}{\partial s} = 0$$

Must have correct v_w, h(z), and s(z) in order to solve

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1. First use tanh ansatz to find velocity and shape guess

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-Find v_w, L_w that minimize EOM moments

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   -Find v_w, L_w that minimize EOM moments

2. Use tanh ansatz as initial guess for full solution

   -Alternate between relaxing wall shape and resolving friction equation

   -Converges to EOM solution if v_w is correct

The Parameter Space

- Scanned the parameter space with

$$0.1 \leq \lambda_{hs} \leq 1.5$$

$$63 \text{ GeV} \leq m_s \leq 114 \text{ GeV}$$

$$100 \text{ GeV} \leq w_0 \leq 170 \text{ GeV}$$

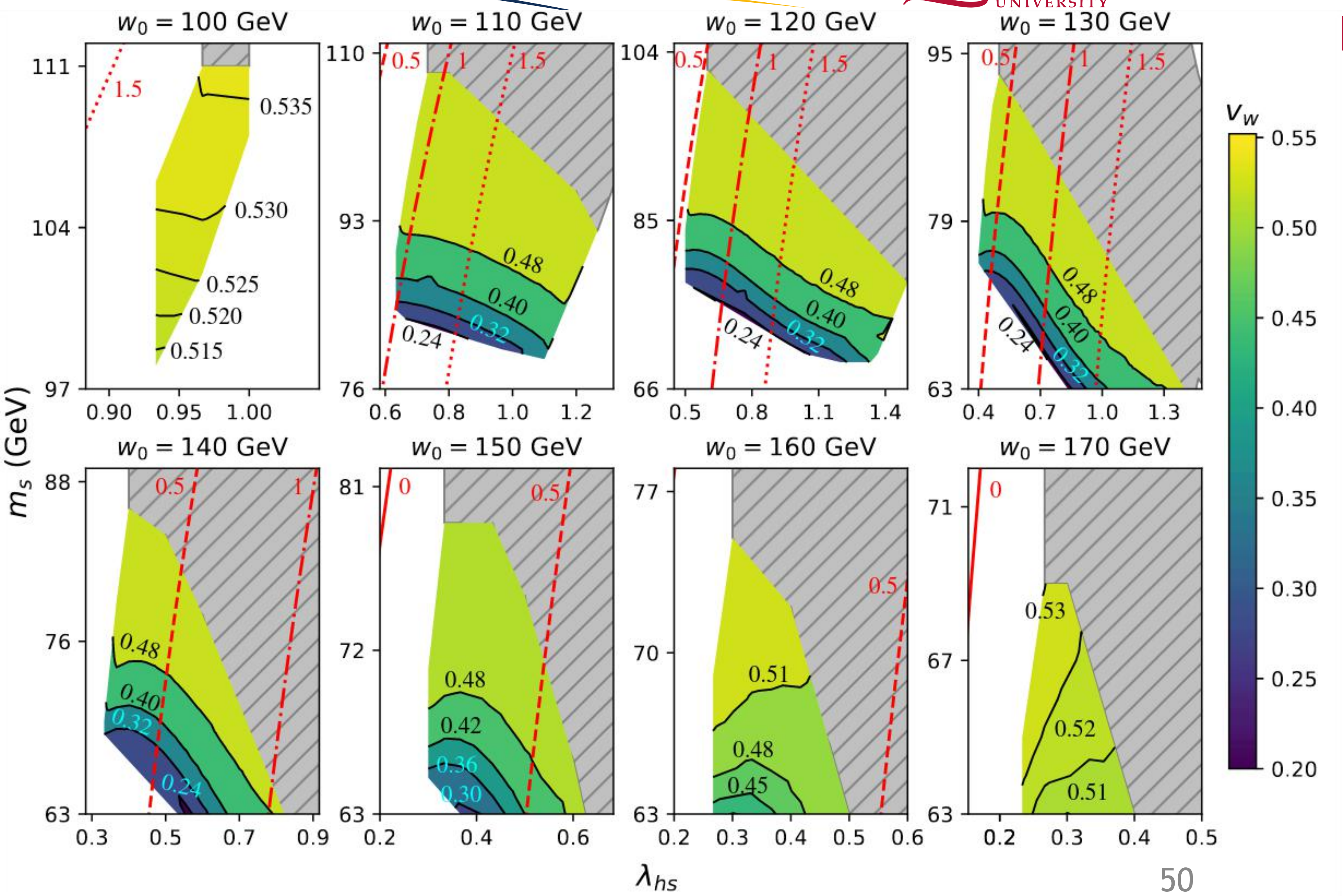
- This appears to cover the full viable space relevant for subsonic walls

Wall Velocities

-Red contours indicate λ_s values

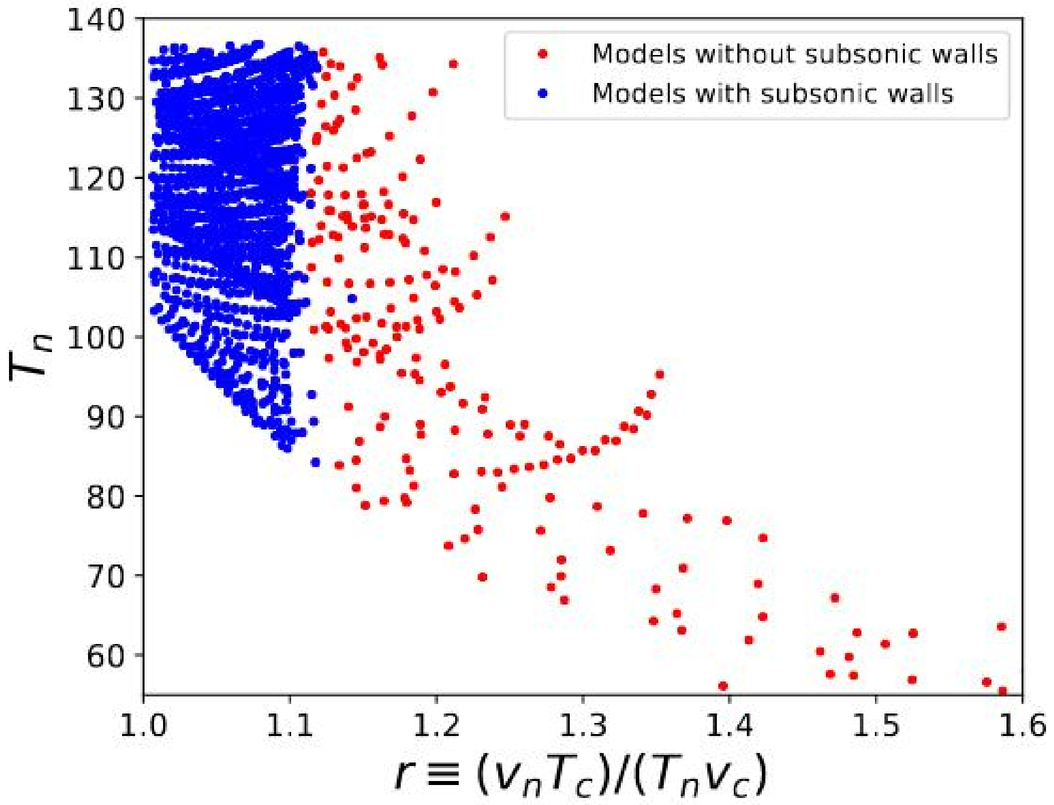
-Black contours indicate v_w values

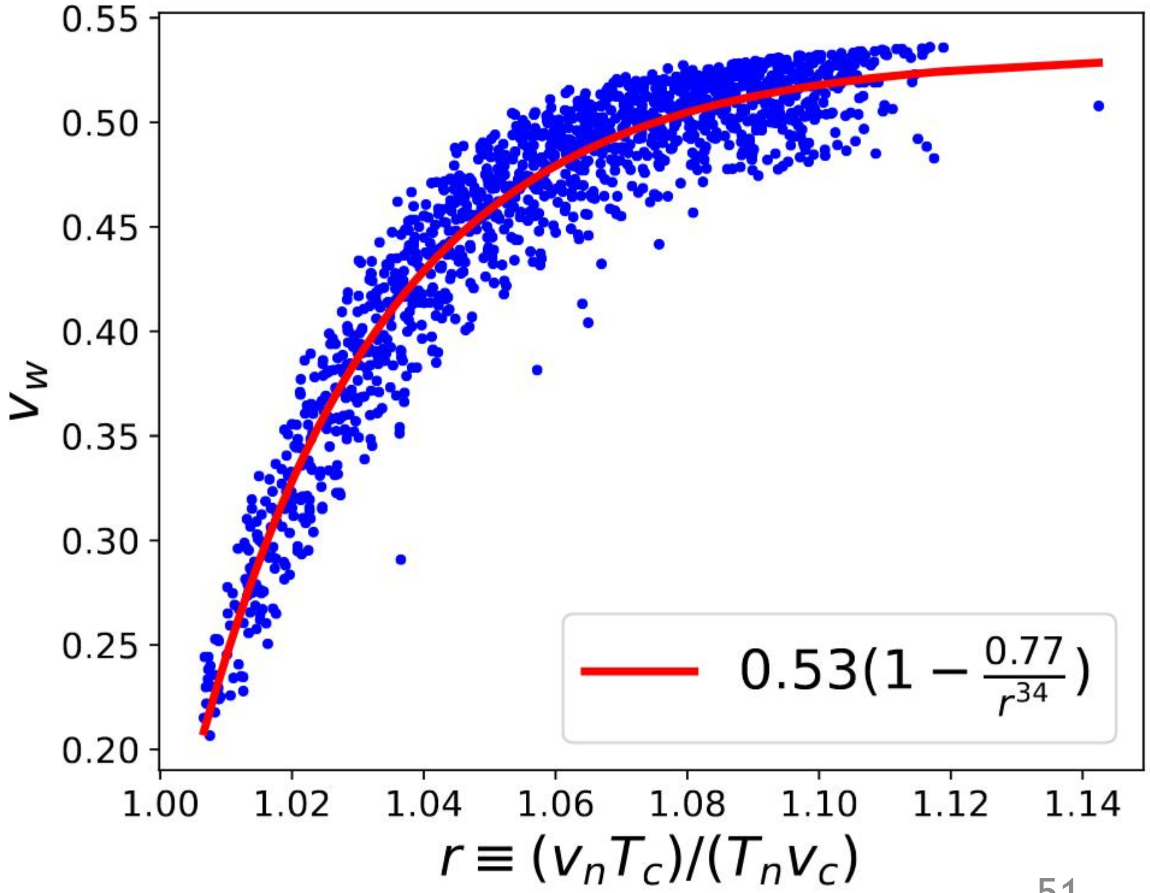
-Grey region has no subsonic solution



Supercooling Parameter

- v_w is strongly correlated with super cooling paraemter, r





Wall Shape Parameterization

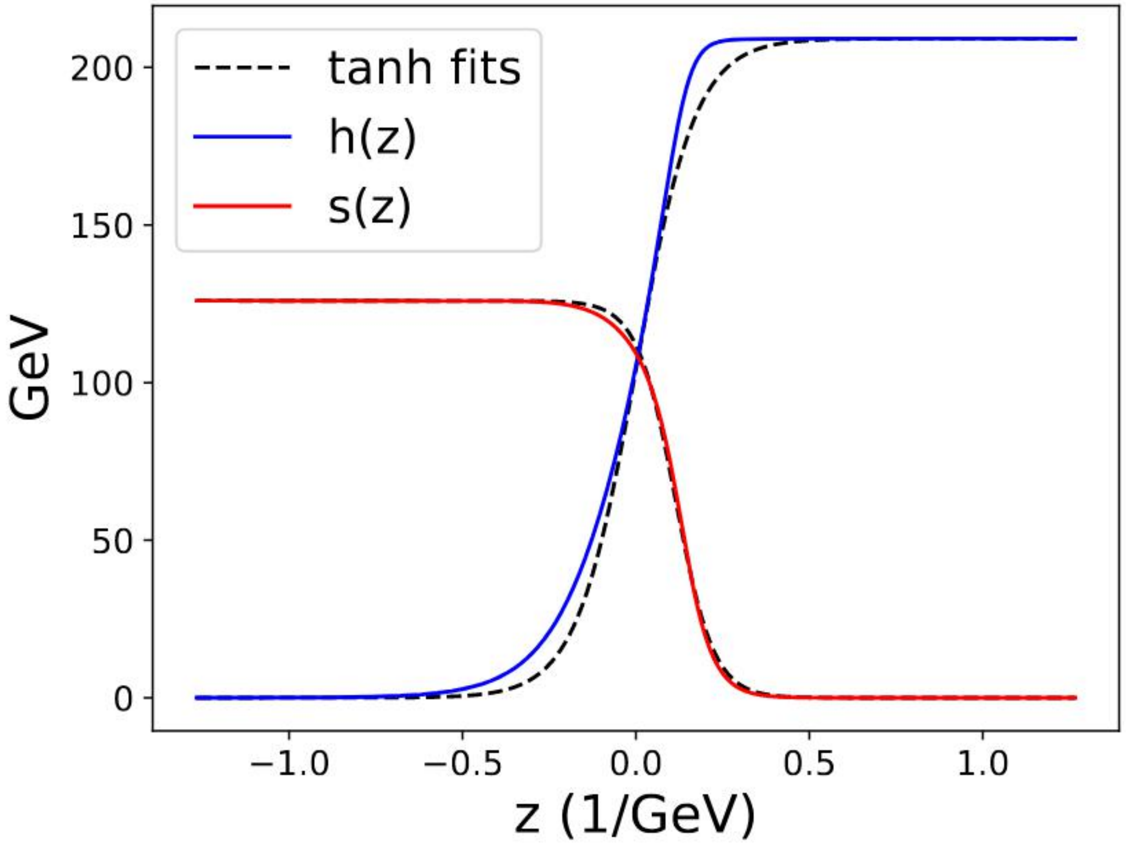
- To describe the trends in wall shapes we use fits to tanh profiles

$$h_{fit}(z) = \frac{h_0}{2}\left(1 + tanh\left(\frac{z}{L_h}\right)\right)$$

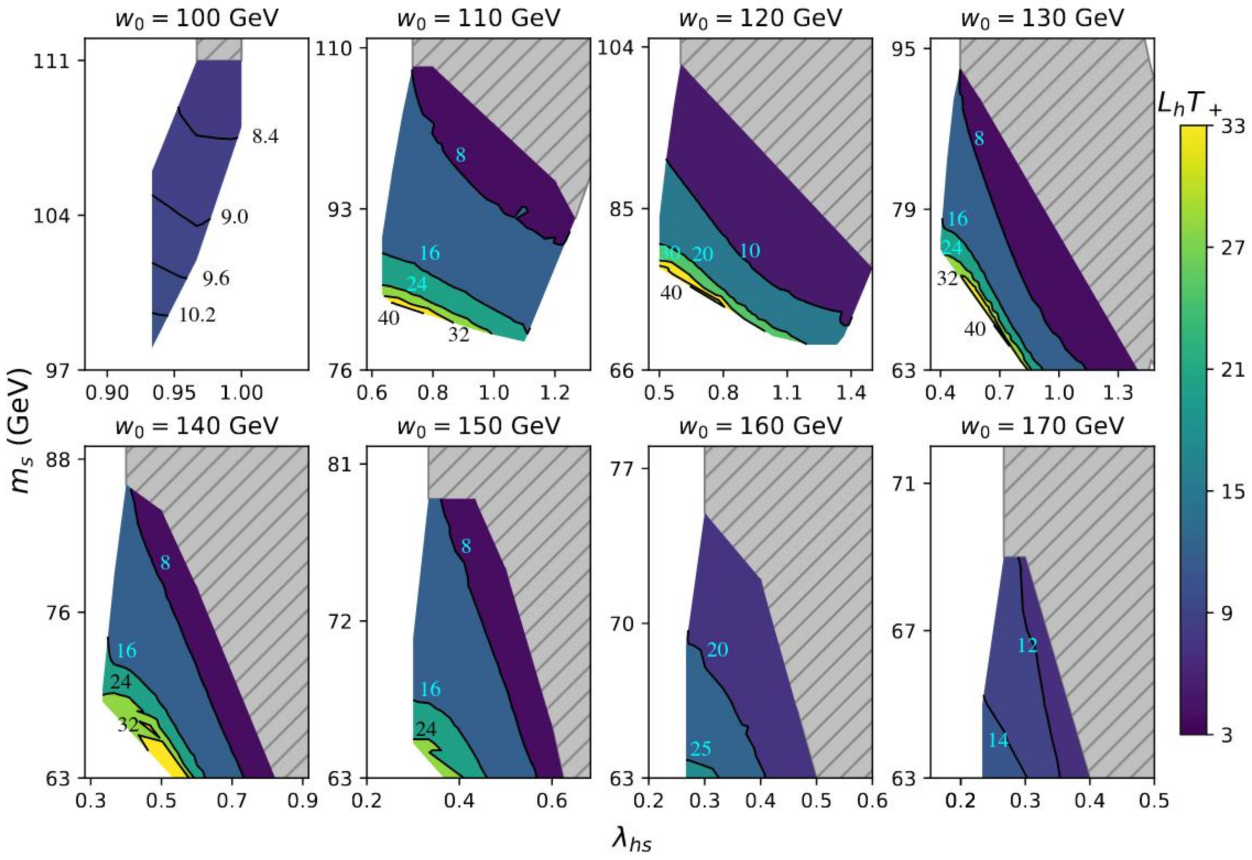
$$s_{fit}(z) = \frac{s_0}{2}\left(1 + tanh\left(\frac{z-\delta_z}{L_s}\right)\right)$$

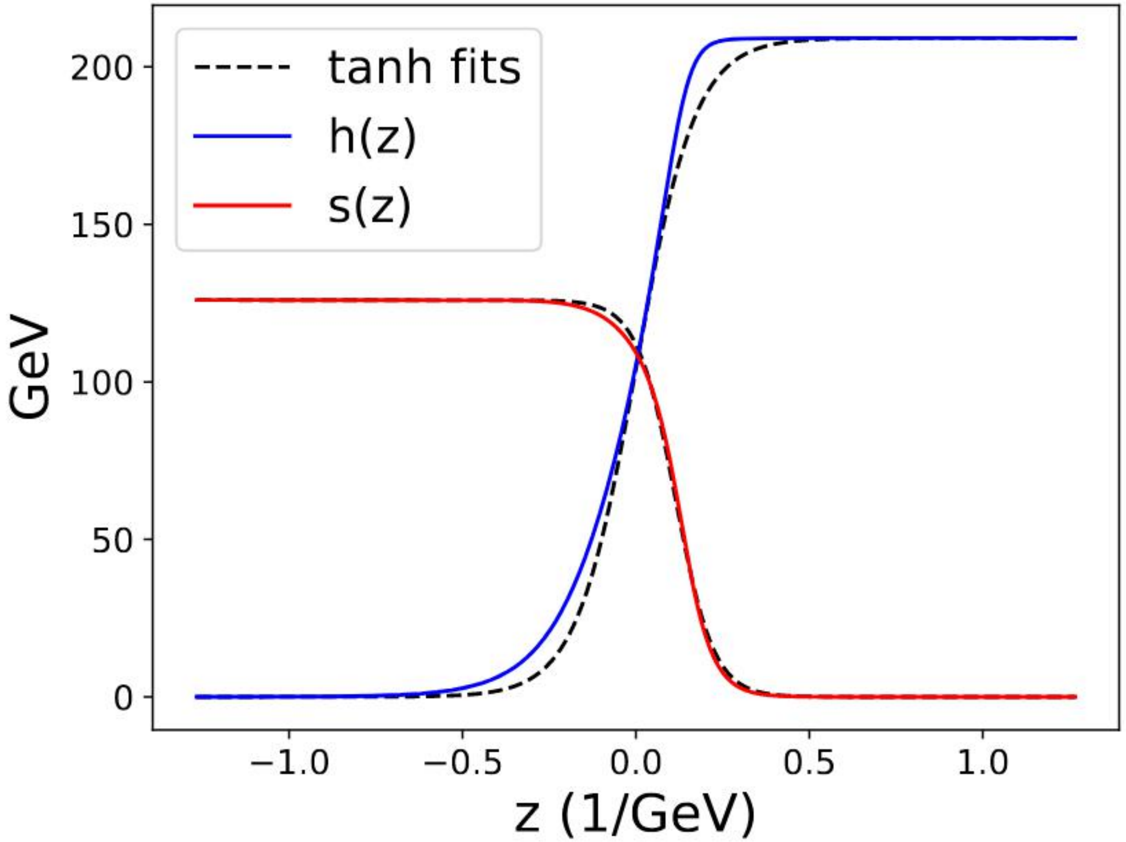
- Wall shape therefore described by 3 parameters:
  - L_h, L_s, δ_z
  - Expressed either in units of GeV^{-1} or T_+^{-1}

Wall Shape Results



Wall Shape Results





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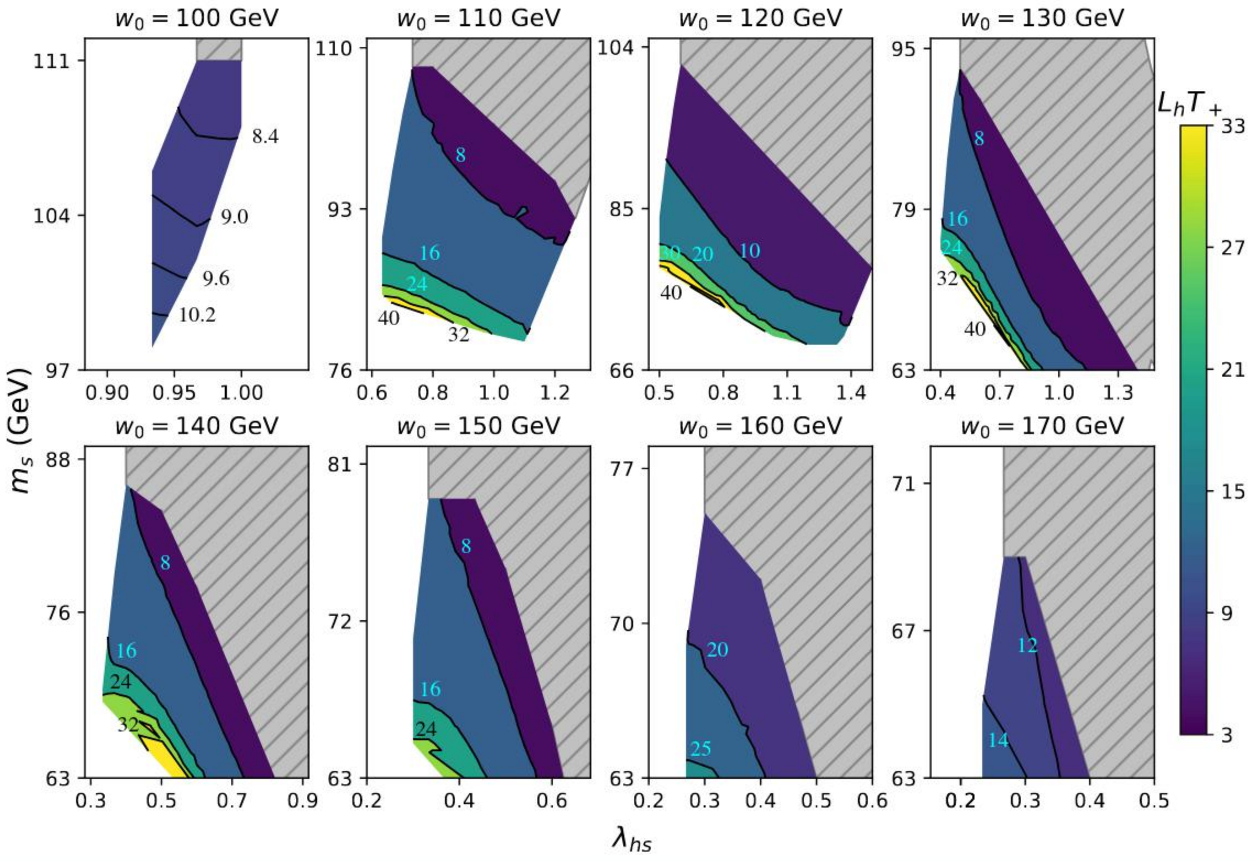

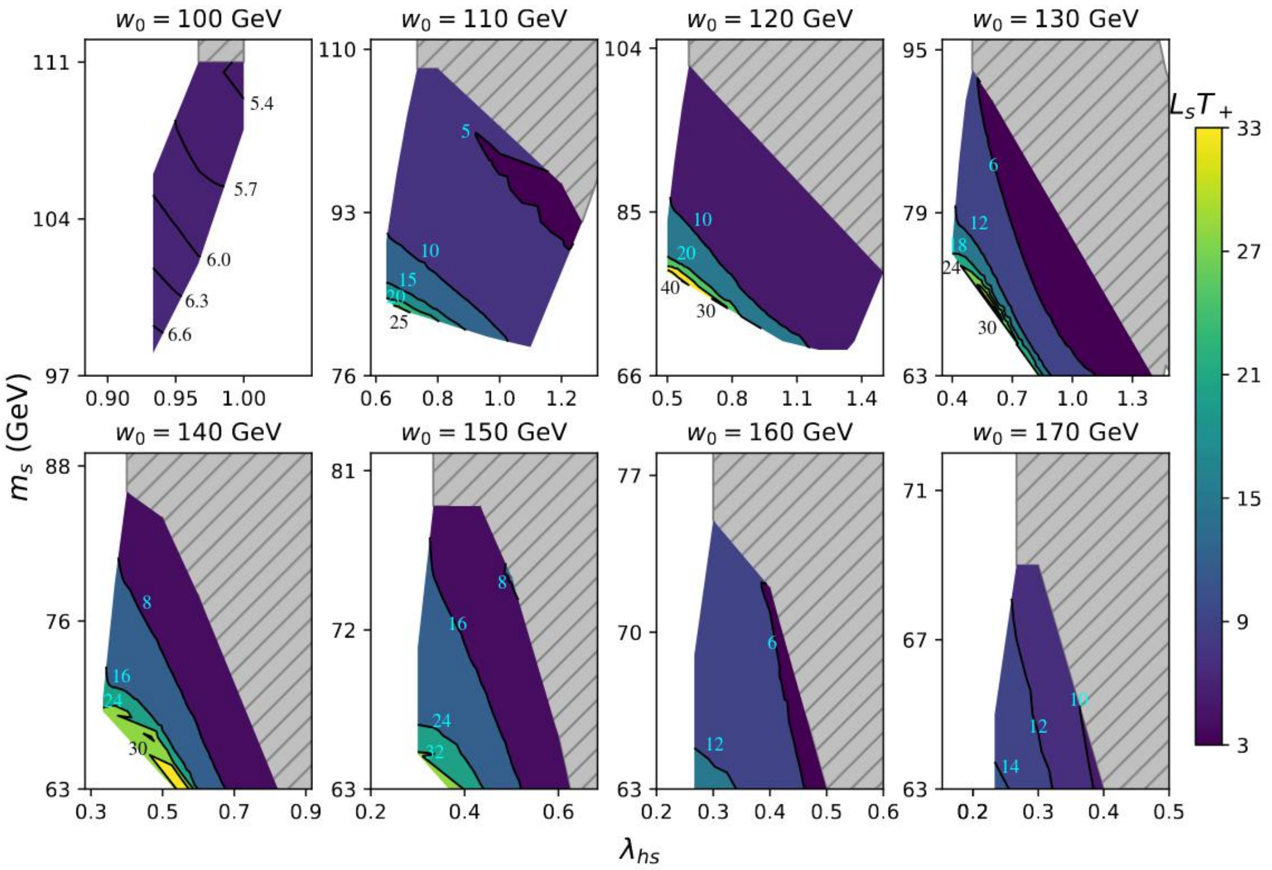

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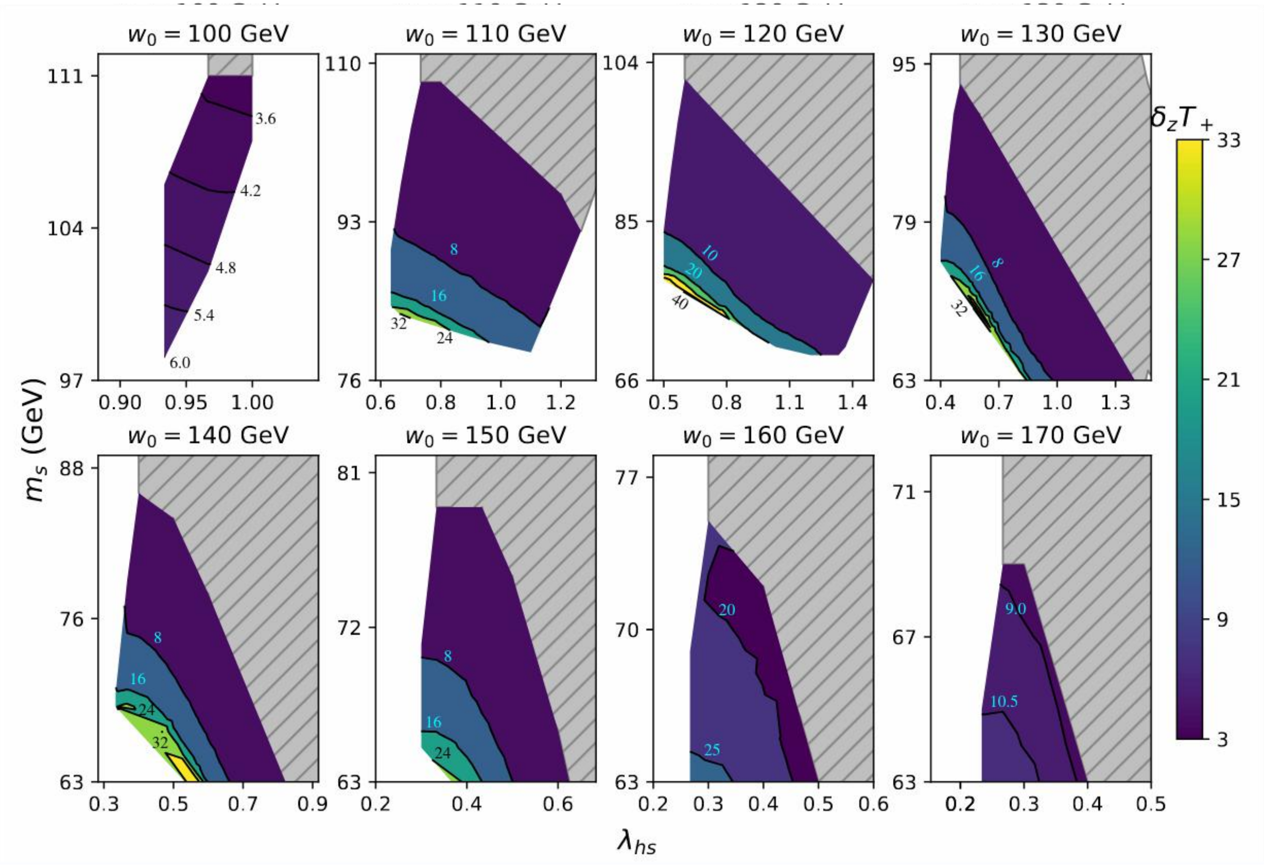
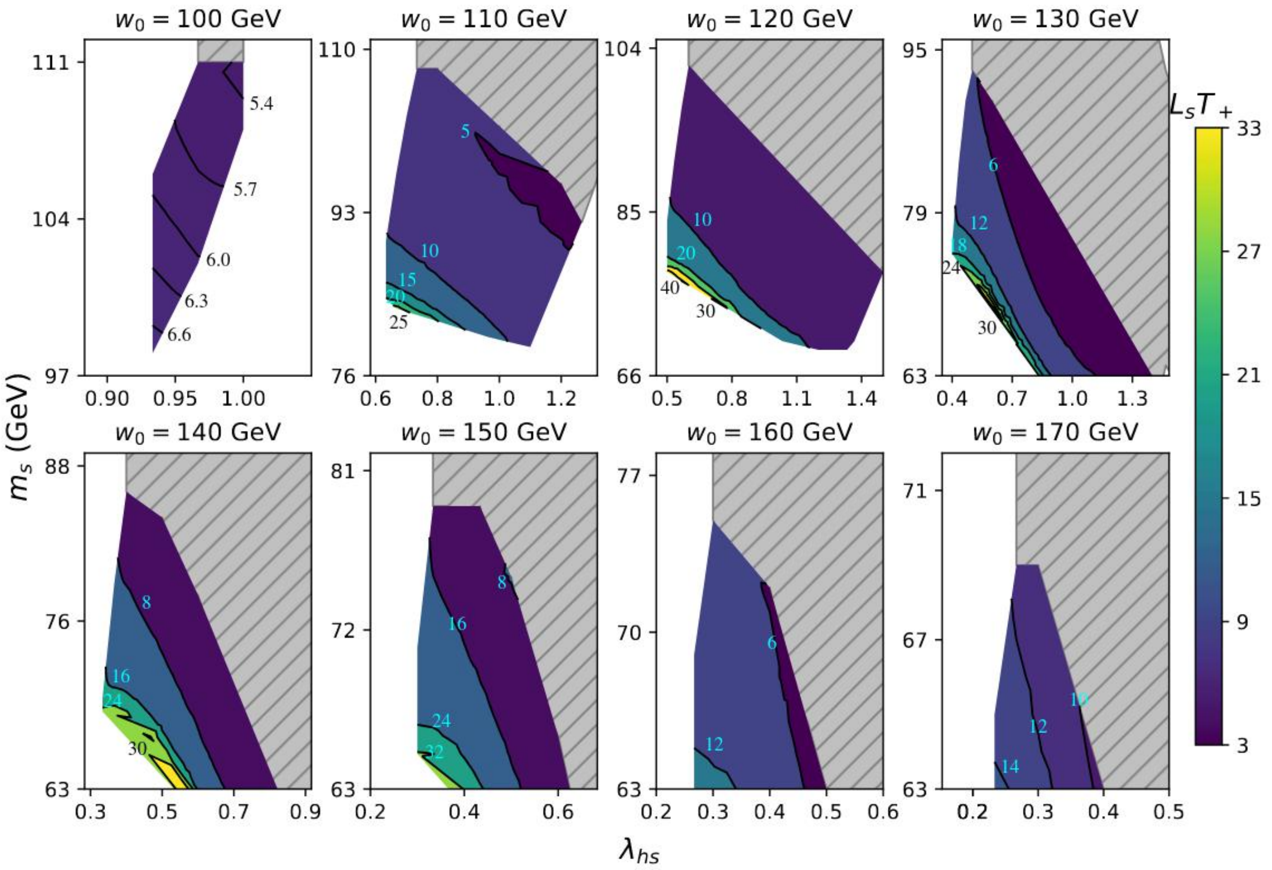
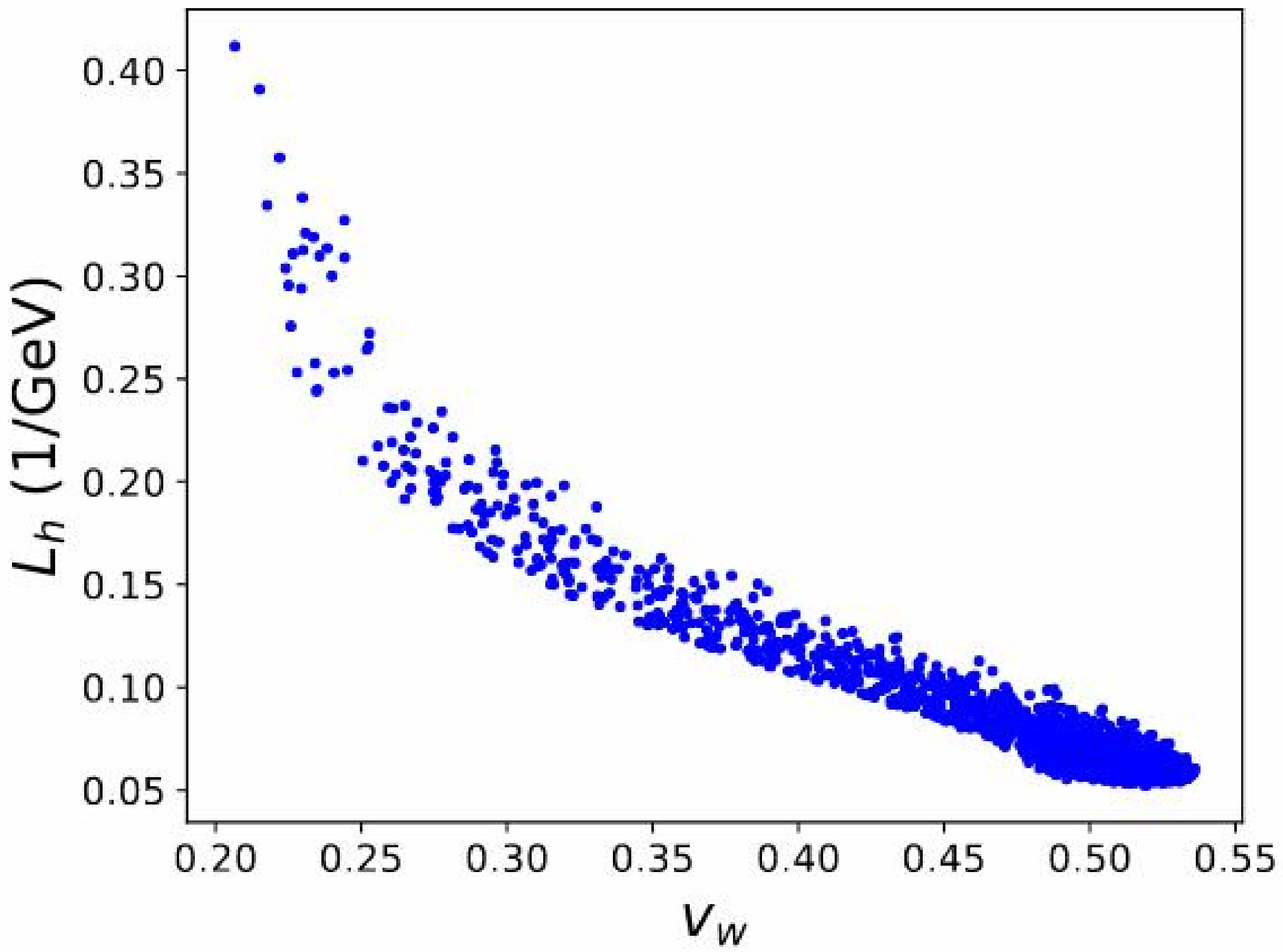
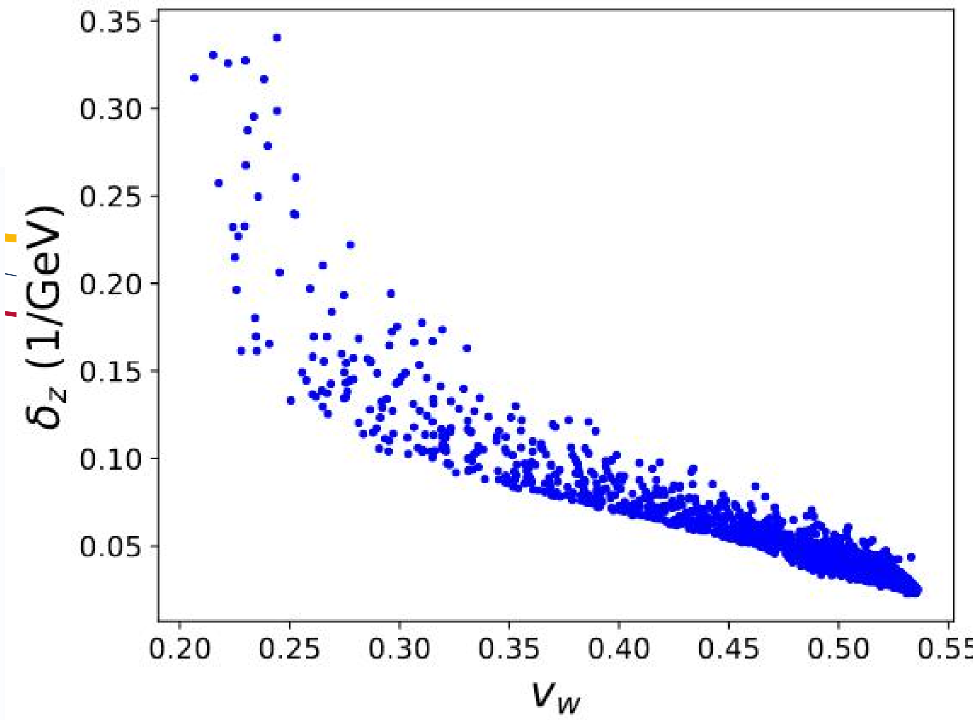
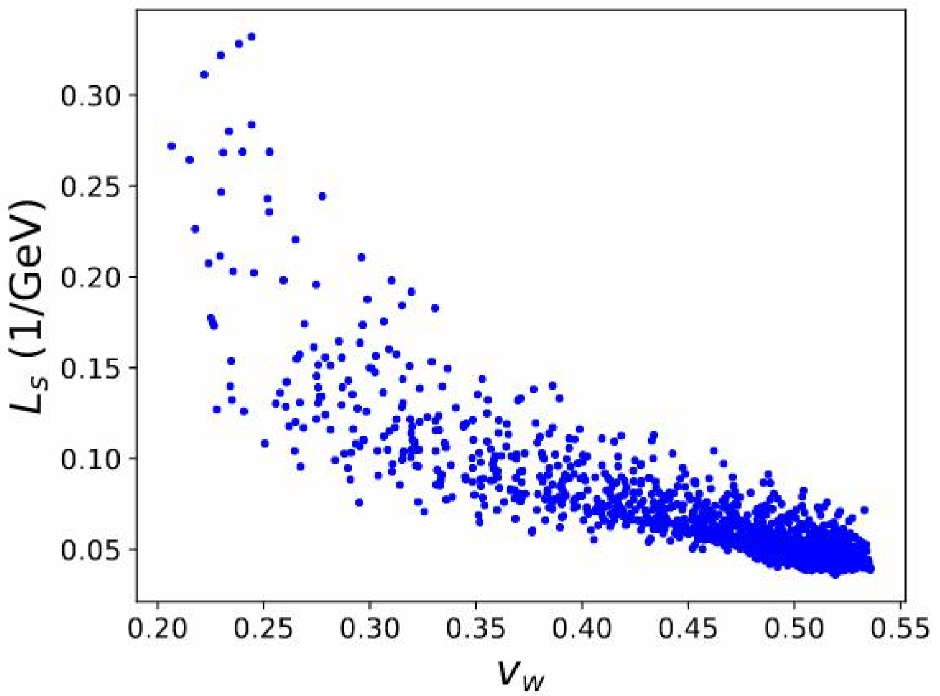




Shape-Velocity Correlation



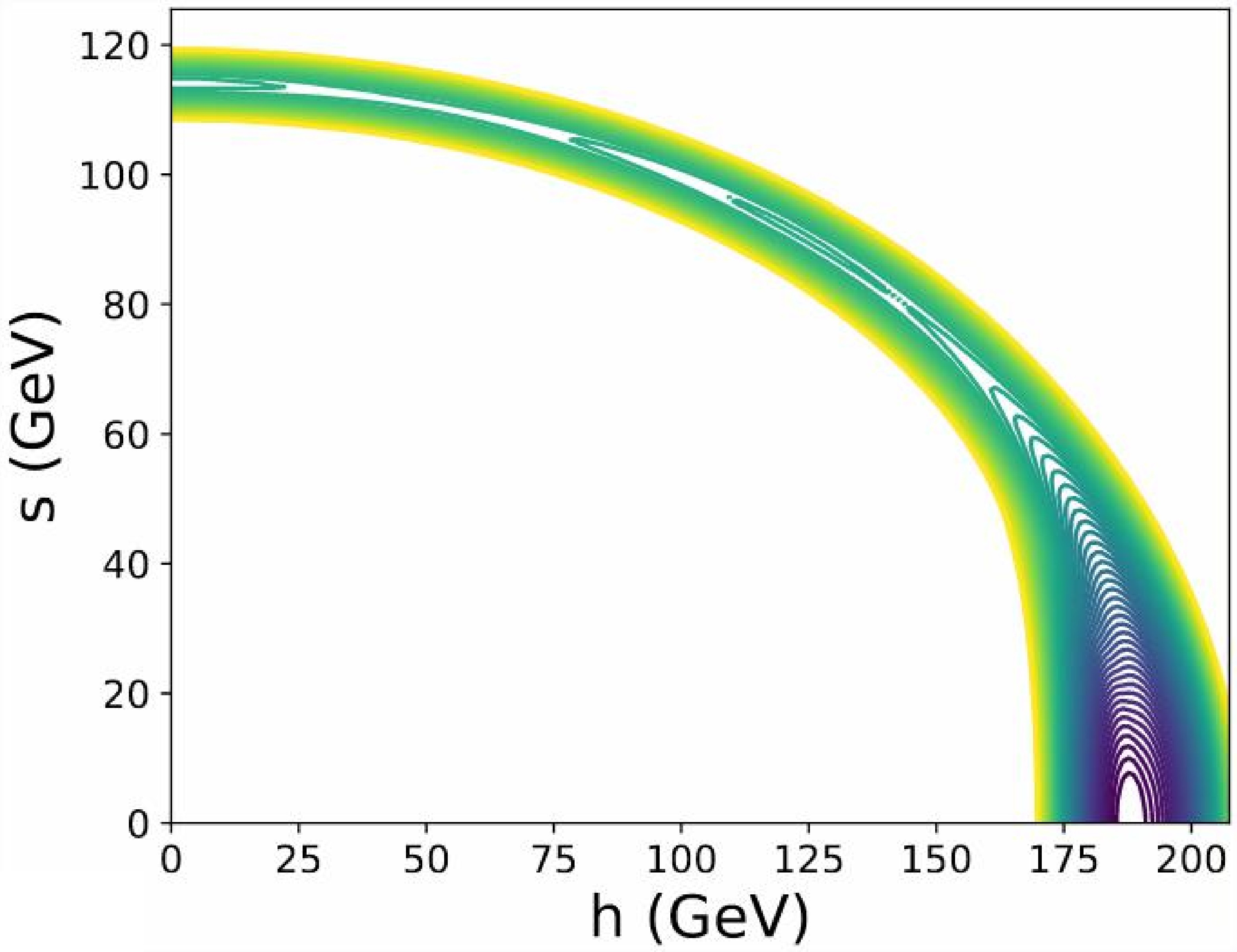




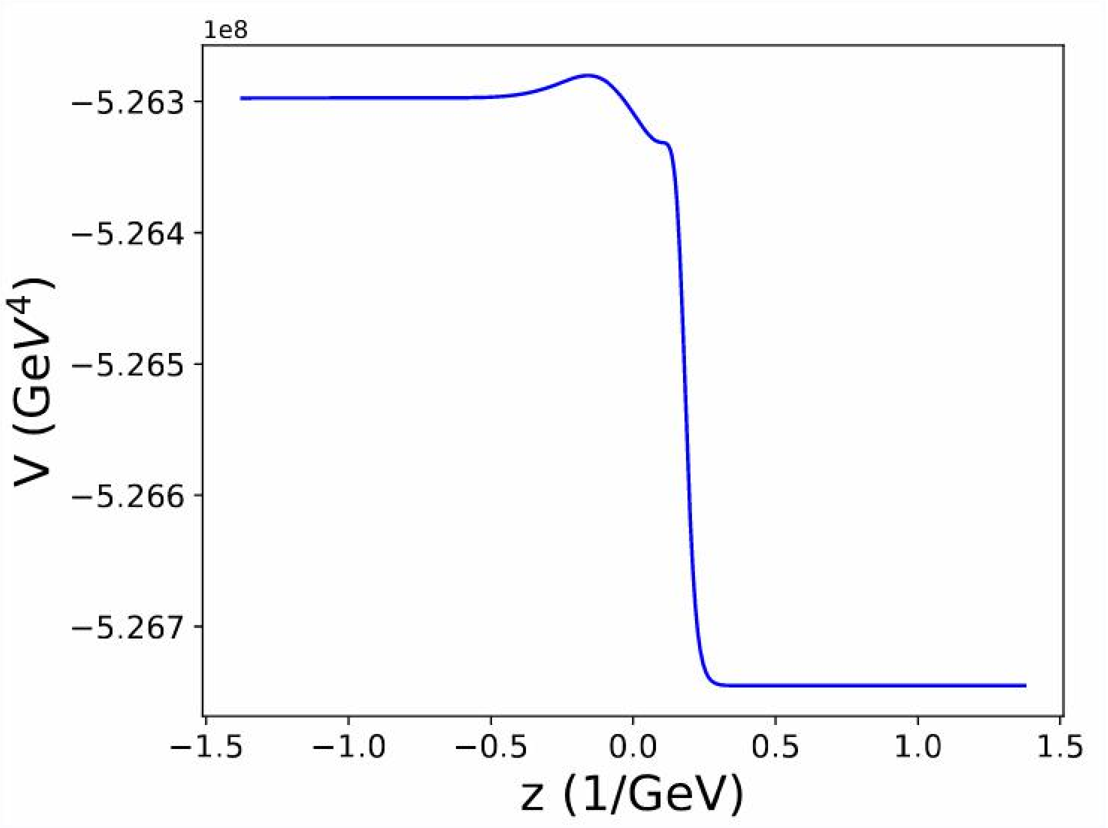
Strange Transitions

- Some potentials have extra minima or plateau in their potential

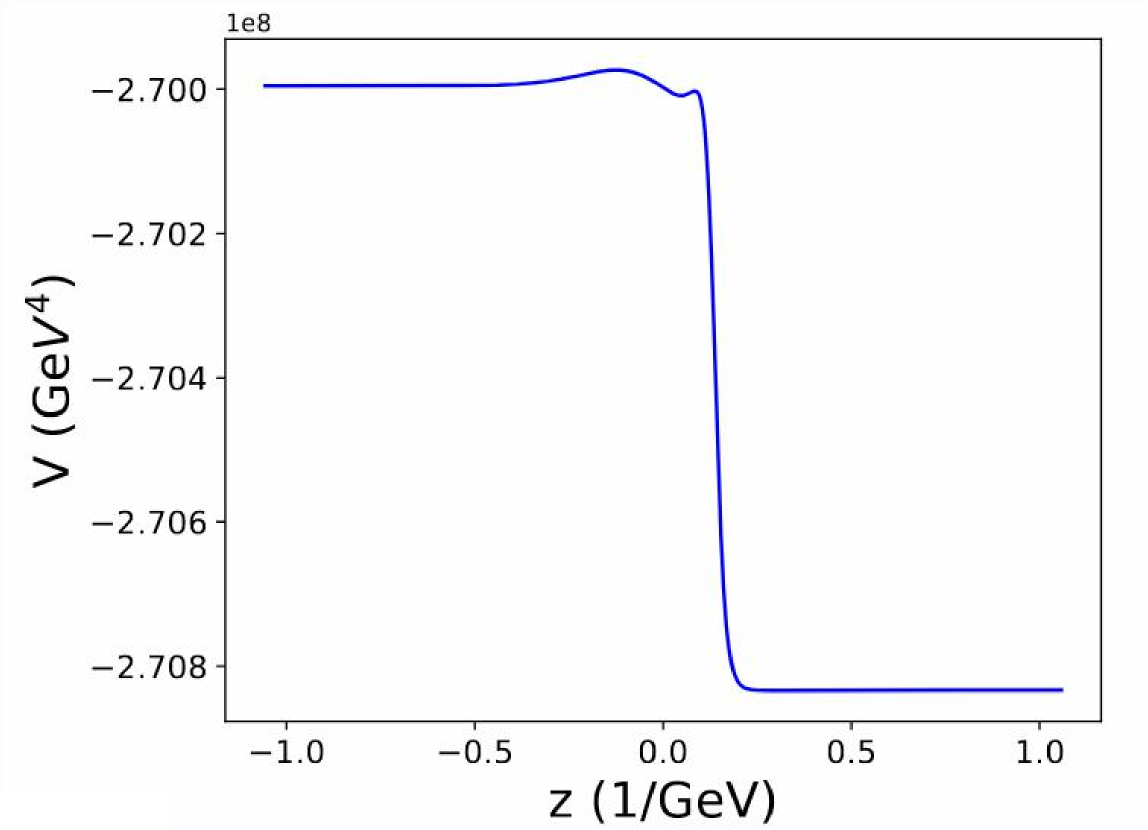
- Sensitive to IR corrections to the potential



Strange Transitions Profiles



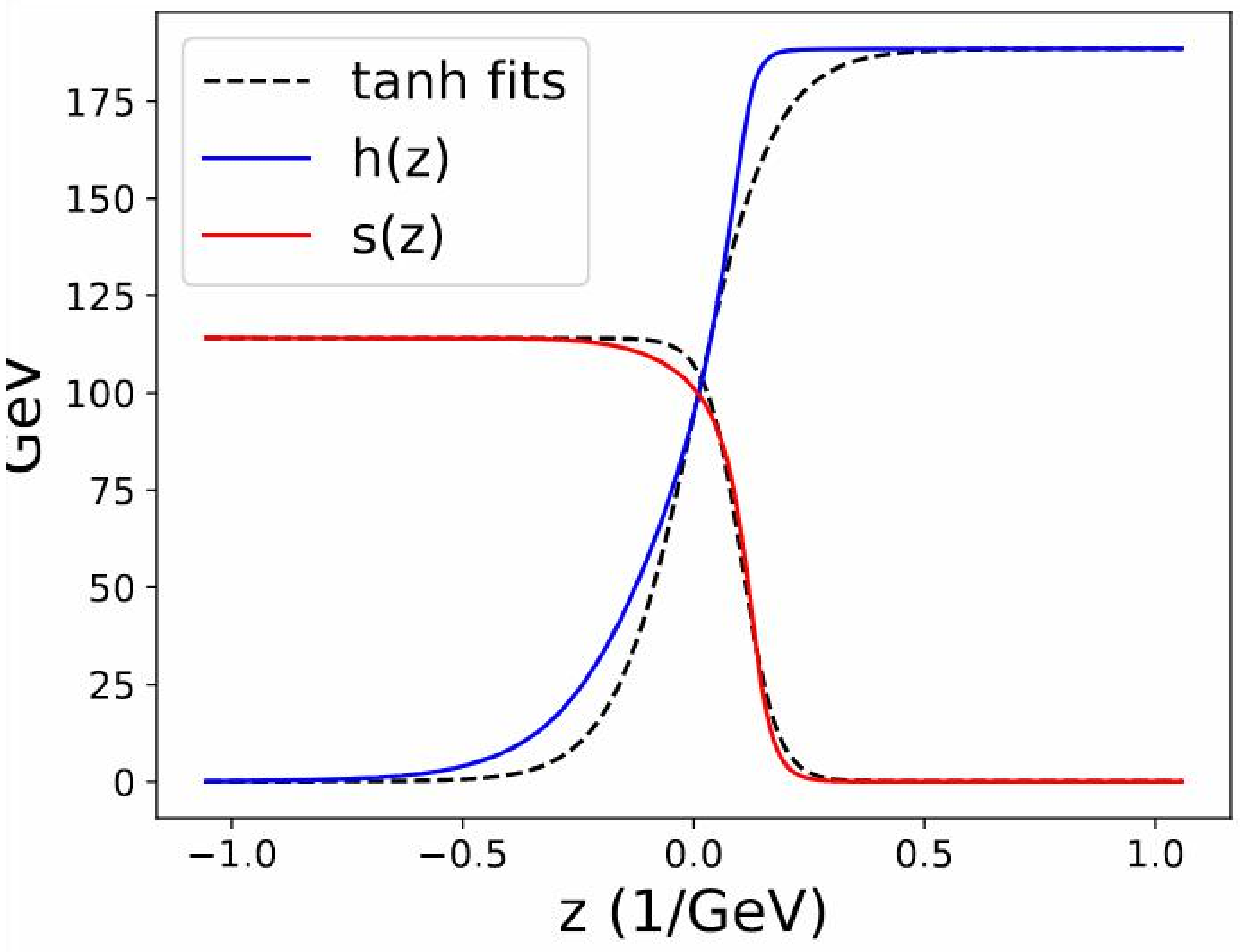
Plateau



Minima

Strange Transitions

- The bubble walls in these transitions have larger deviations from a tanh profile



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- Some transition results are **sensitive to IR corrections** to the effective potential

Questions?