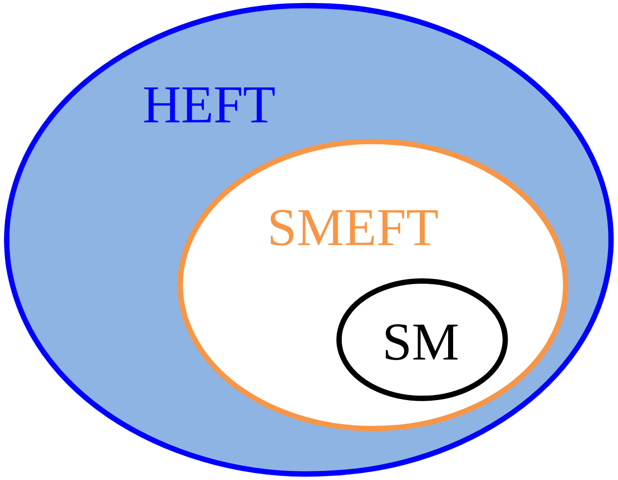
What is HEFT?

TDLI and INPAC Joint Theory Seminar, Nov 3, 2020

Xiaochuan Lu

University of Oregon

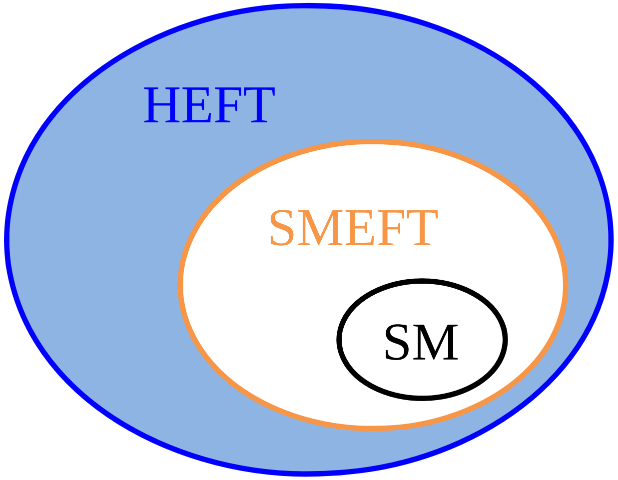
arXiv: 2008.08597,
with Timothy Cohen, Nathaniel Craig, and Dave Sutherland



H $\qquad$ $SU(2)_W$ doublet

$$\begin{cases} h & \text{physical Higgs} \\ U \equiv e^{i\pi^a t^a / v} & \text{Goldstones} \end{cases}$$

$$\mathcal{L}_{\text{SMEFT}}(H) = \mathcal{L}_{\text{SM}} + \frac{C_H}{\Lambda^2}|H|^6 + \frac{C_{H\Box}}{\Lambda^2}|H|^2 \partial^2 |H|^2 + \frac{C_R}{\Lambda^2}|H|^2 |DH|^2 + \cdots$$



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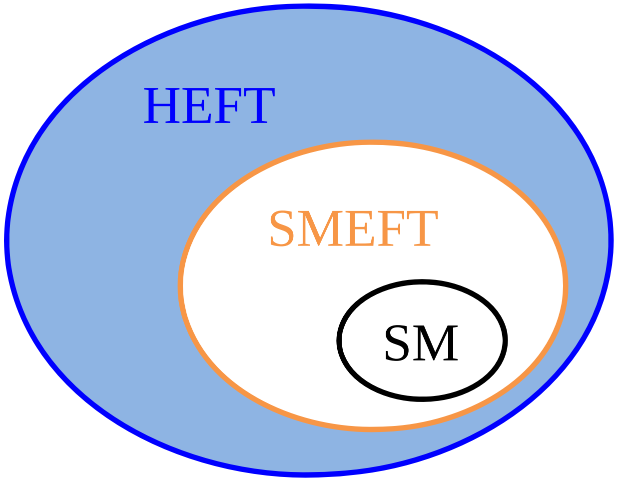

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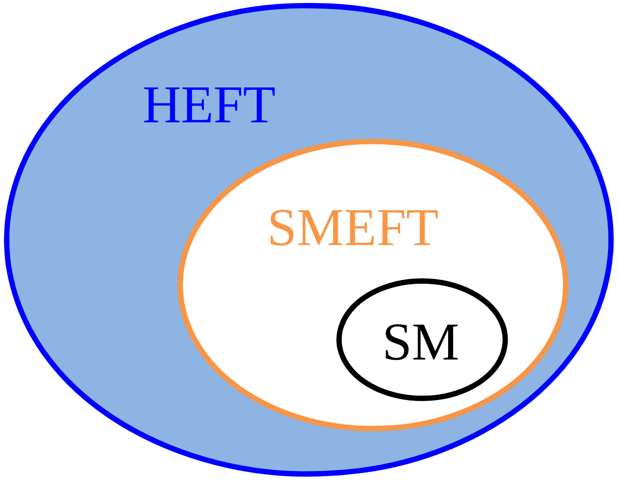

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Nonlinear

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SMEFT $\Rightarrow$ HEFT

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$$\begin{cases} \text{SMEFT:} \ (x,y) \\ \text{HEFT:} \ (r,\theta) \end{cases} \Rightarrow \quad r = \sqrt{x^2+y^2}$$

Outline

- HEFT $\Leftrightarrow$ non-analyticities

- Field redefinitions: physical vs unphysical non-analyticities

  --- Geometric criteria

- What UV Physics lead to HEFT?

  --- Integrating out extra electroweak breaking

  --- Integrating out “massless” states

Non-analyticity is an all-order effect

$$v + h = \sqrt{2|H|^2} = \sqrt{v^2 + \left(2|H|^2 - v^2\right)}$$

$$= v + \frac{1}{2v}\left(\underline{2|H|^2 - v^2}\right) + \cdots$$

$$2vh + h^2$$

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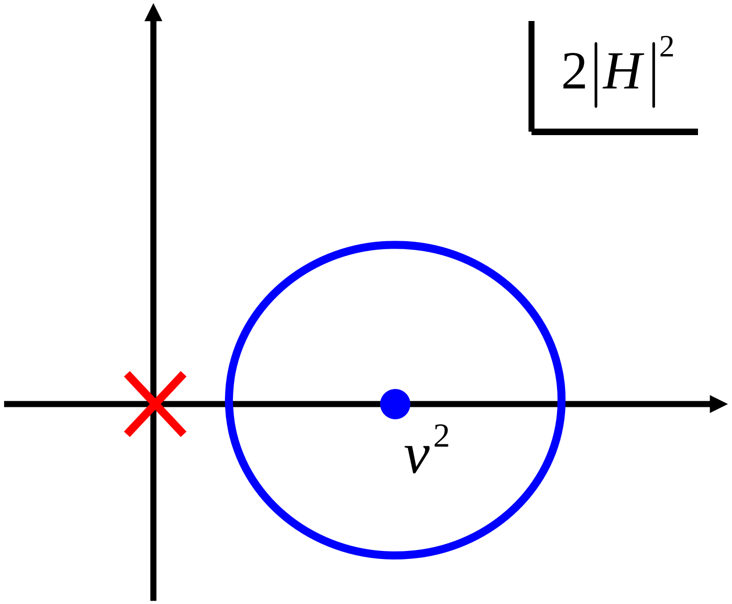

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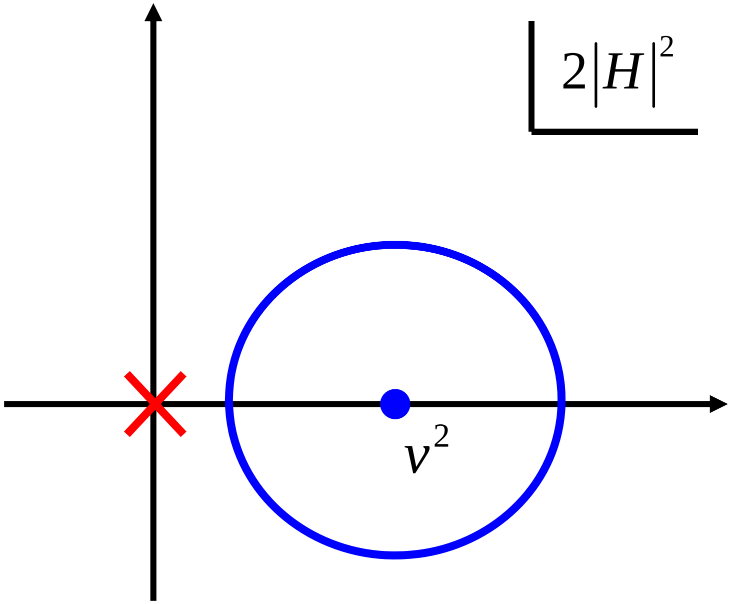

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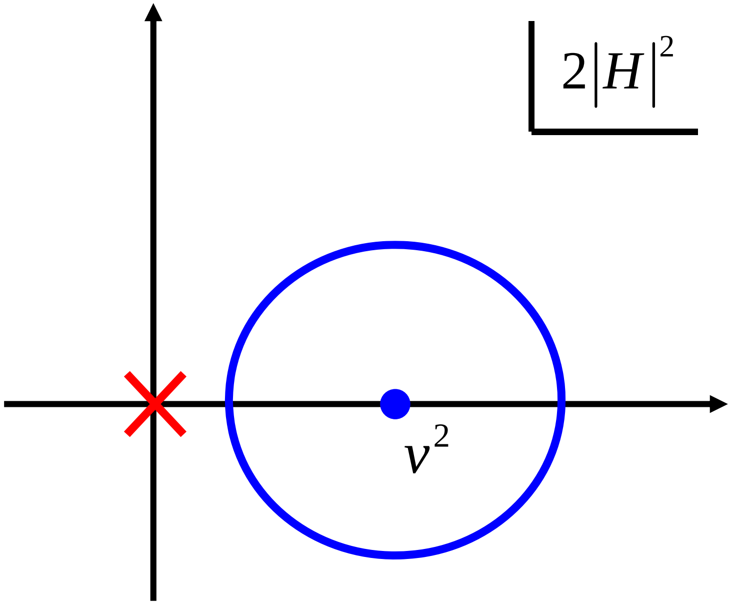

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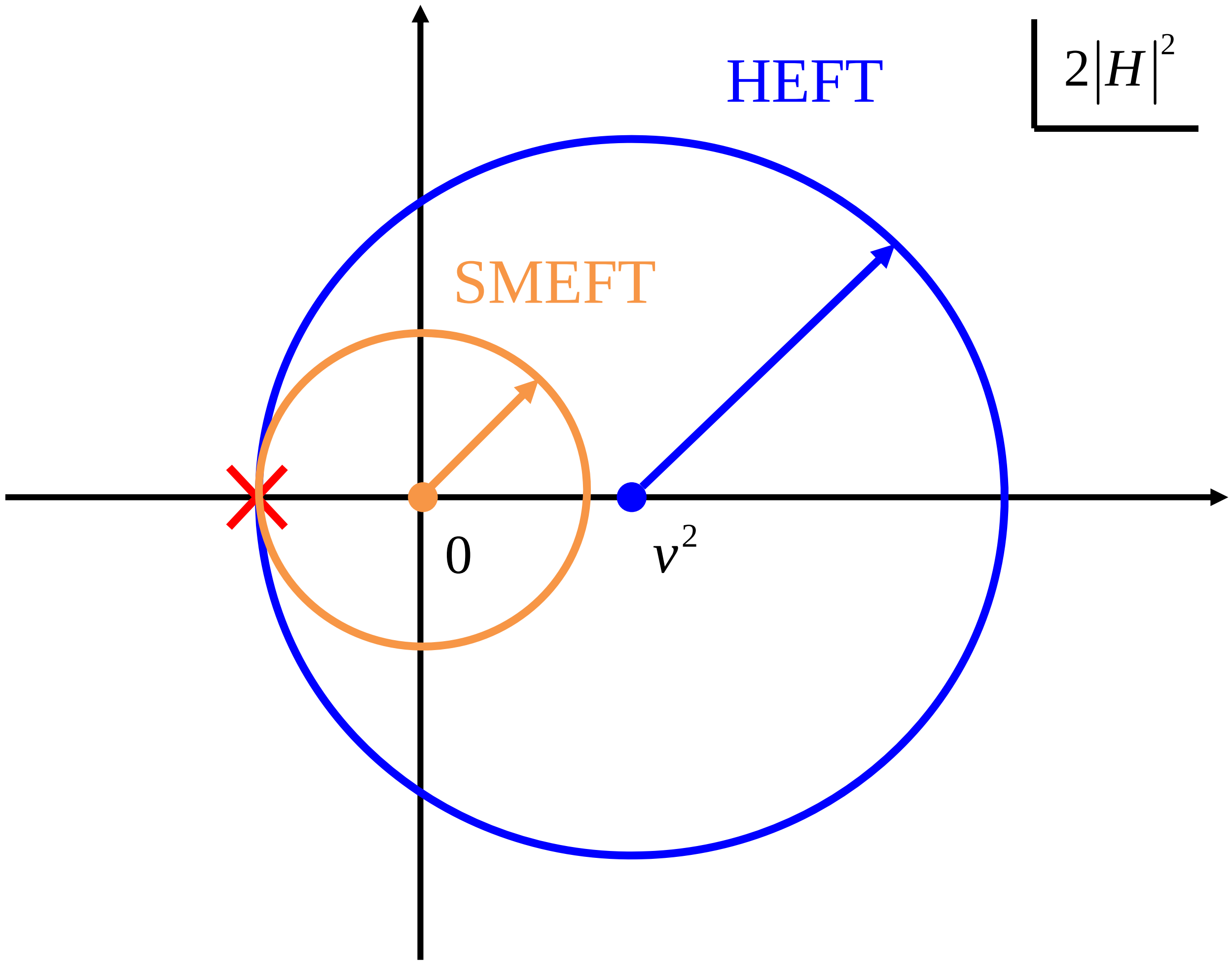
Need to know to all powers


$2|H|^2$
HEFT
SMEFT
0
v^2

Adam Falkowski and Riccardo Rattazzi: (arXiv: 1902.05936)

UV cut-off. Our distinction between analytic and non-analytic lagrangians coincides with the distinction, in use in the Higgs EFT community, between linear (so-called SMEFT) and non-linear (so-called HEFT) effective theory, or equivalently between h being or not being part of a $SU(2)_W$ doublet. We however believe our classification is more adequate and enlightening from a physical point of view.

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Field redefinition

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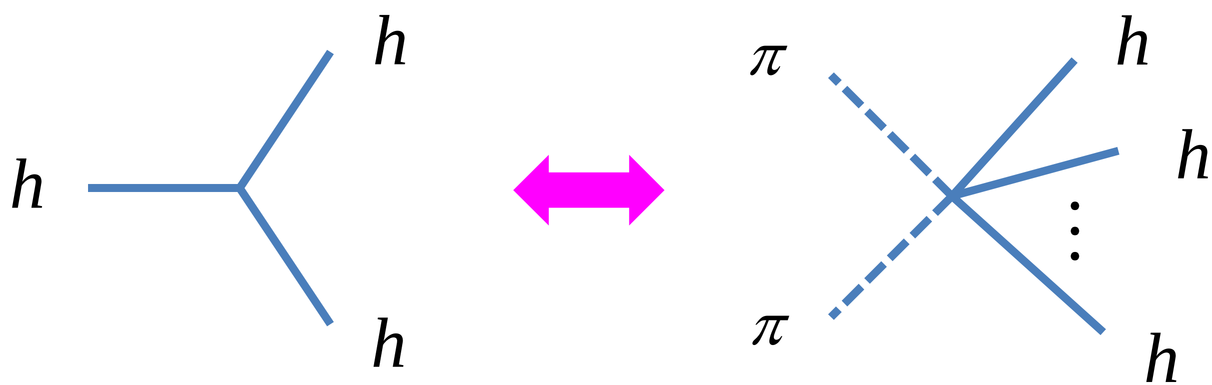

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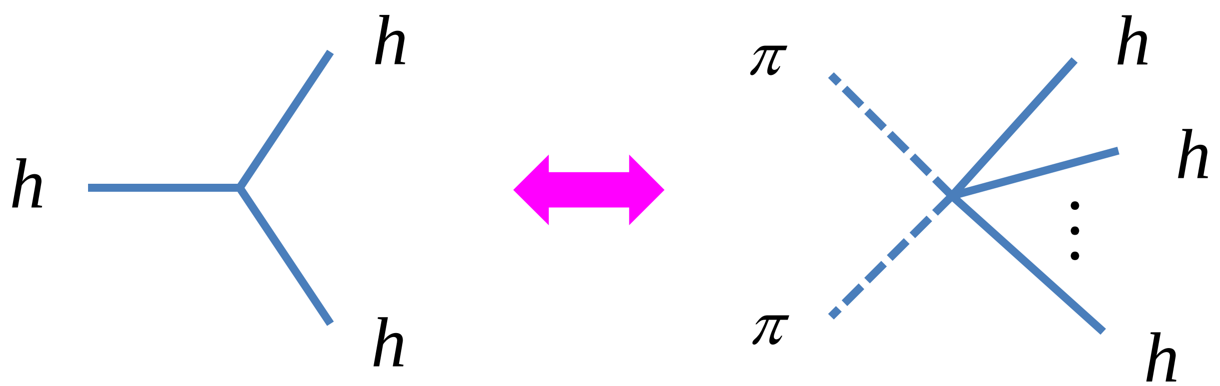

Rodrigo Alonso, Elizabeth E. Jenkins, Aneesh V. Manohar: (arXiv: 1605.03602)

One can convert the SMEFT Lagrangian to HEFT form using Eq. (2.11) to switch from Cartesian and polar coordinates. One can attempt to convert from HEFT to SMEFT form using

$$\frac{\phi}{(\phi \cdot \phi)^{1/2}} = \boldsymbol{n} \tag{2.30}$$

with $(\phi \cdot \phi)^{1/2}$ some function of h. This substitution gives a Lagrangian $L(\phi)$ that need not be analytic in ϕ. However, if there is an $O(4)$ fixed point, then there is a suitable change of variables such that the resulting Lagrangian is analytic in ϕ.

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Geometric invariants

$$ds^2 \equiv g_{ab}(\phi)\,d\phi_a d\phi_b$$

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = (v+h)\vec{n} \quad , \quad \vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ \sqrt{1-n_1^2-n_2^2-n_3^2} \end{pmatrix} \quad , \quad \vec{n}\cdot\vec{n} = 1$$

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\left[K(h)\right]^2(\partial h)^2 + \frac{1}{2}\left[vF(h)\right]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

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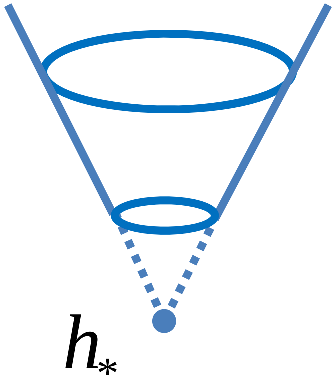
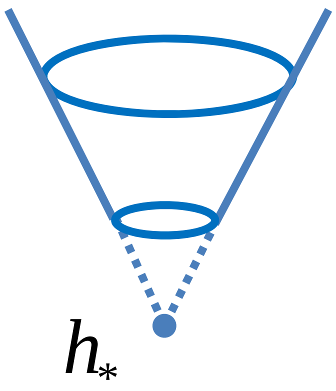
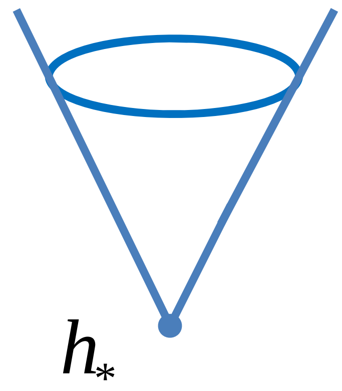


AJM (arXiv: 1605.03602)

$\exists\, h_* \quad \text{such that} \quad F\left(h_*\right) = 0$





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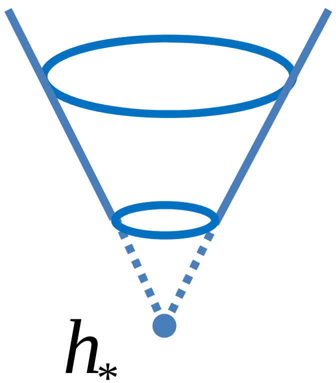

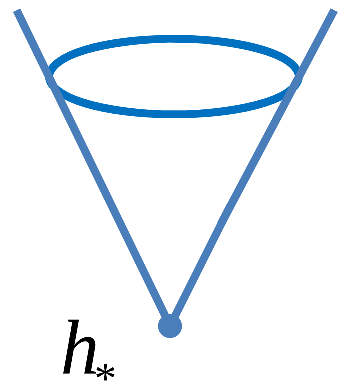

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Our work (arXiv: 2008.08597)

$$F = 0$$

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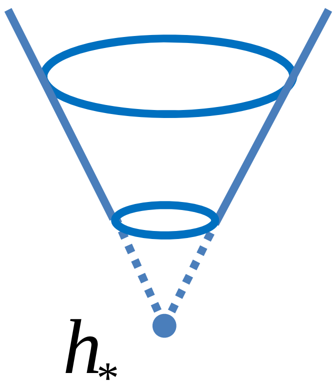

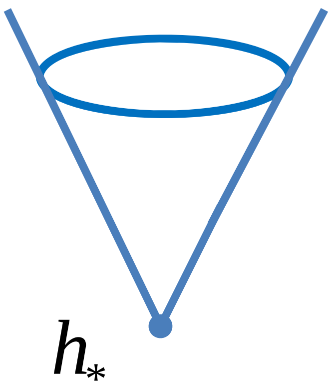

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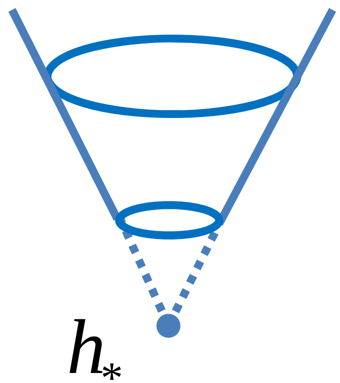
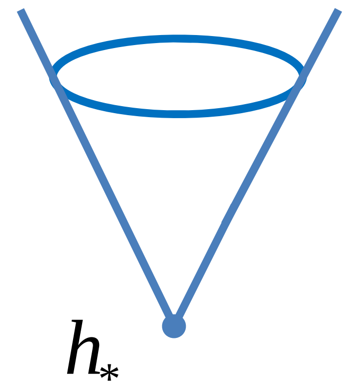
UV theories that will generate HEFT?





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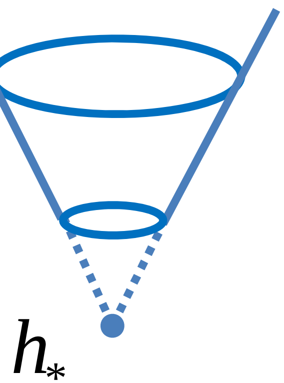

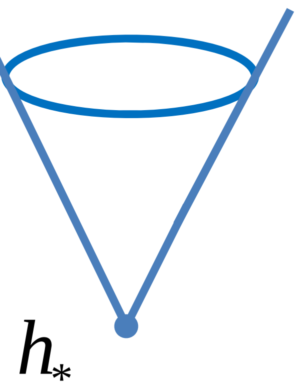

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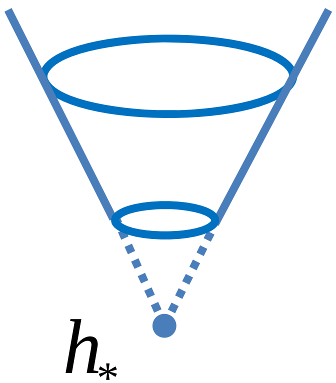
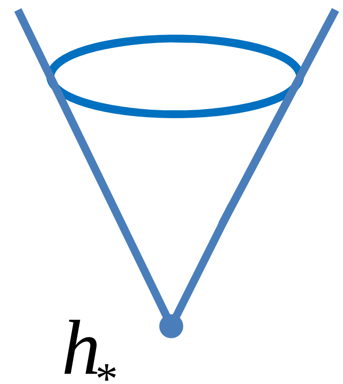


Mass fully from electroweak symmetry breaking



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Insight from tree-level matching

$$\mathcal{L}_{\mathrm{UV}}\left[\phi,\Phi\right] \;\Rightarrow\; \mathcal{L}_{\mathrm{EFT}}\left[\phi\right]=\mathcal{L}_{\mathrm{UV}}\left[\phi,\Phi_c\left[\phi\right]\right] \quad , \quad \left.\frac{\delta S_{\mathrm{UV}}}{\delta\Phi}\right|_{\Phi_c[\phi]}=0$$

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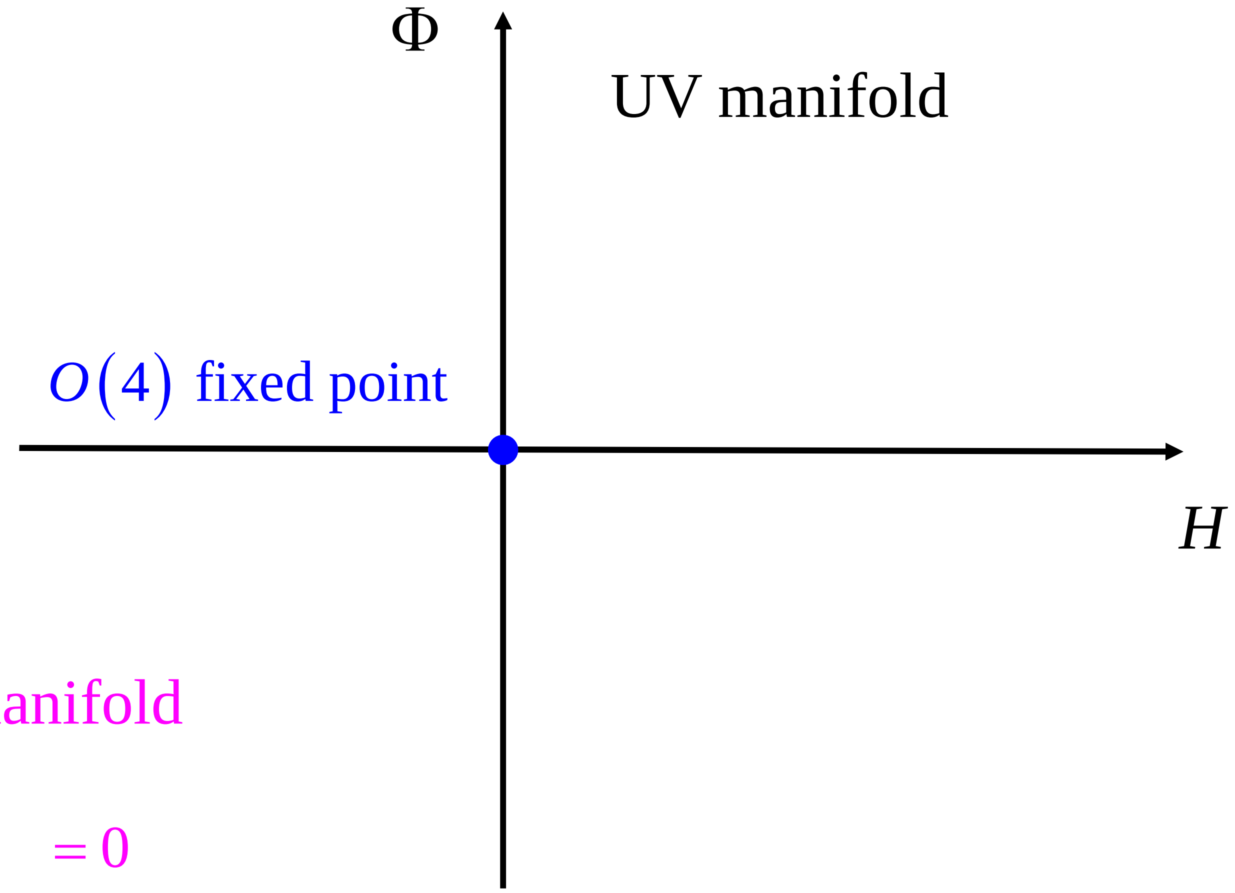

EFT submanifold

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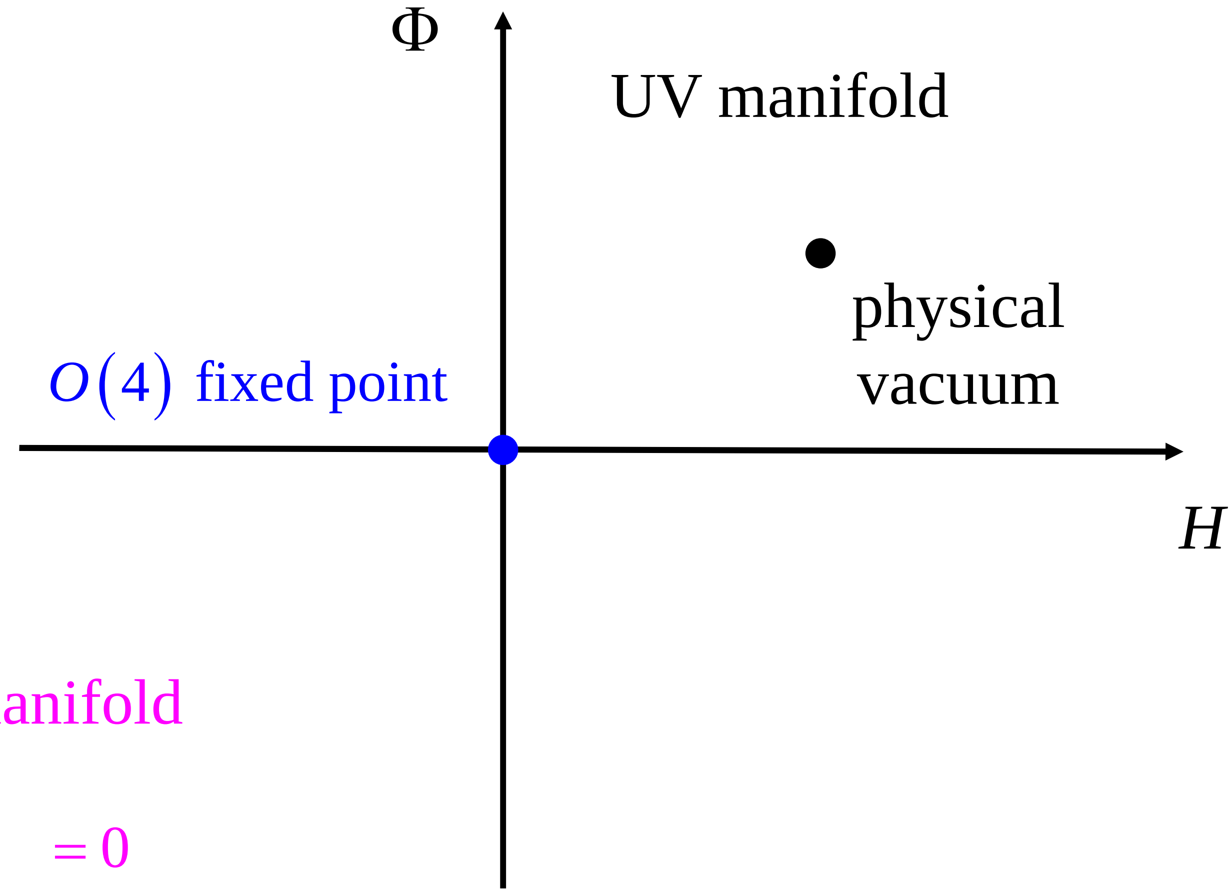

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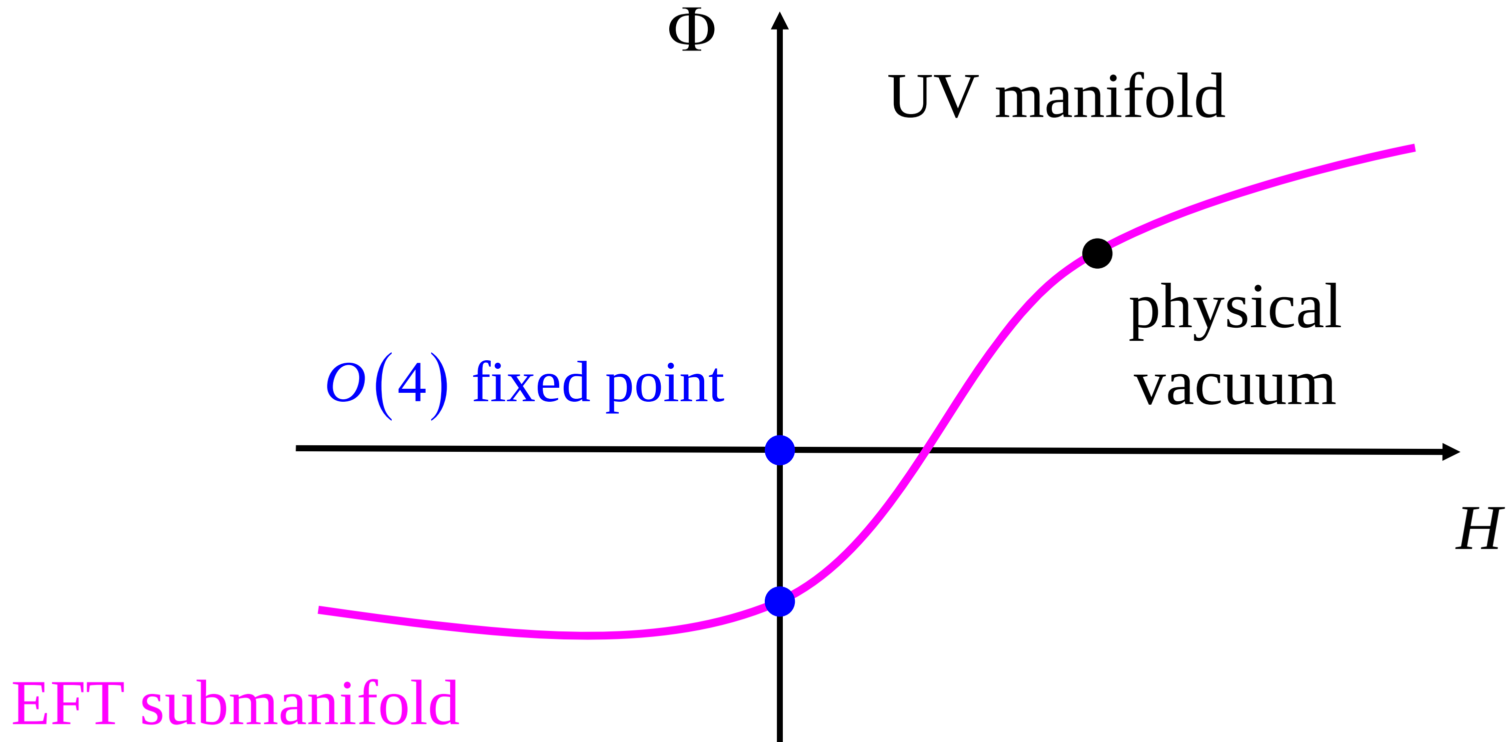

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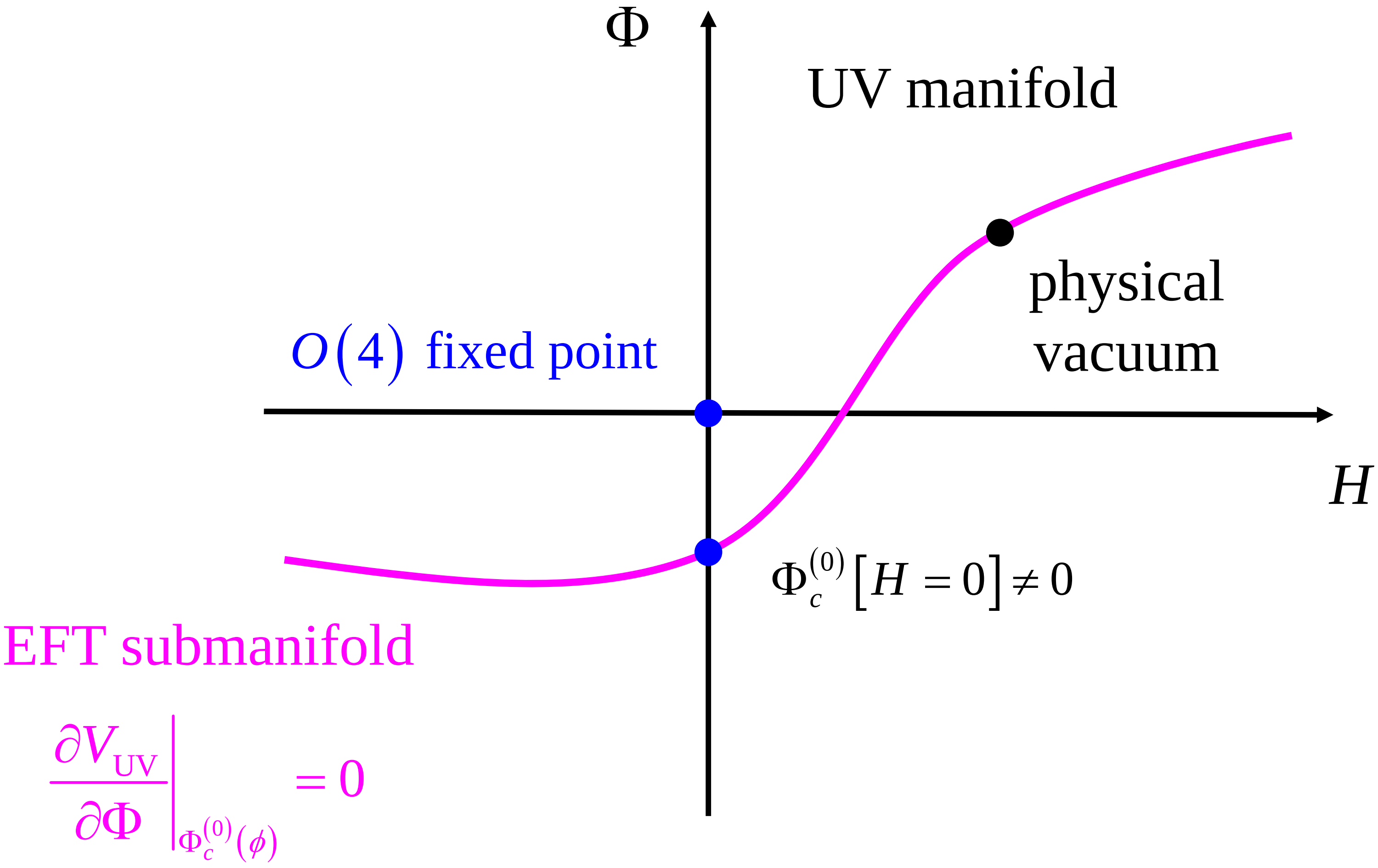

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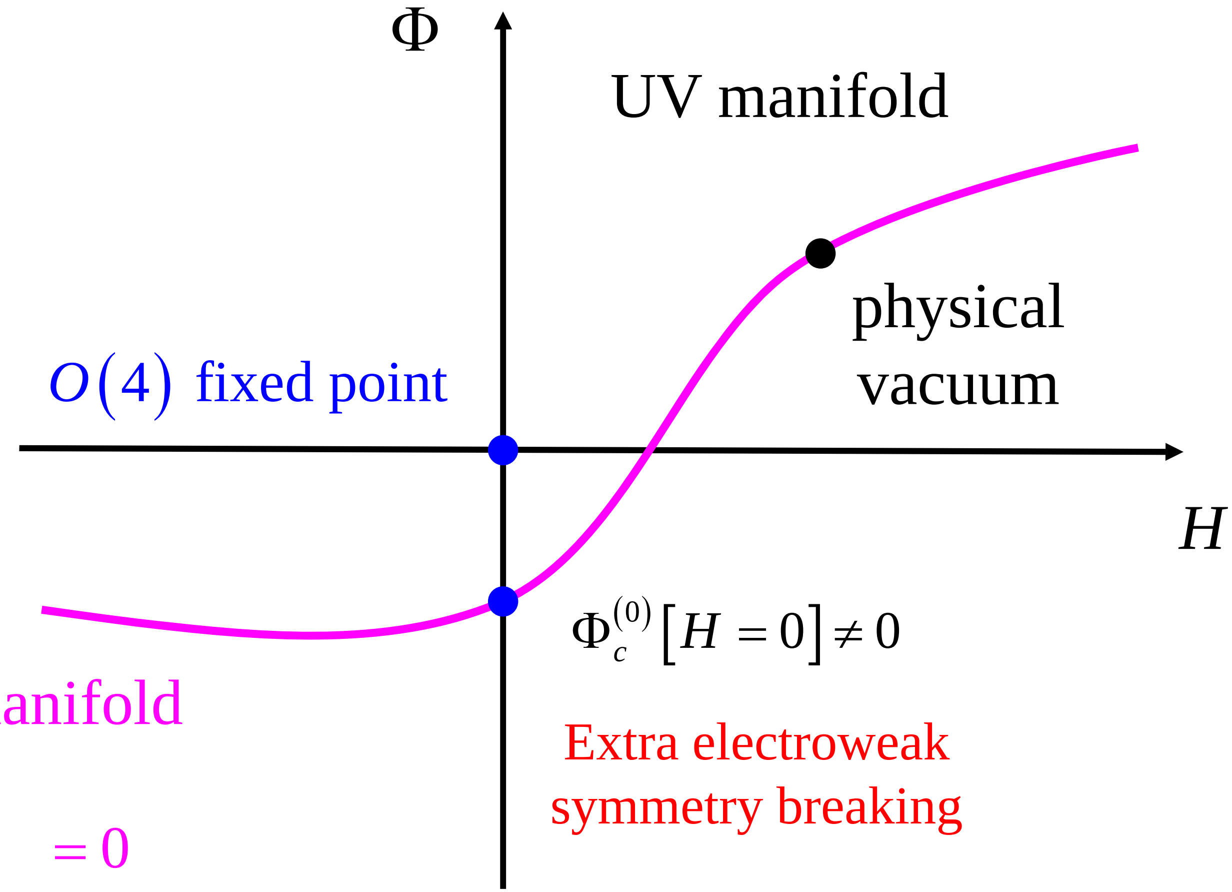

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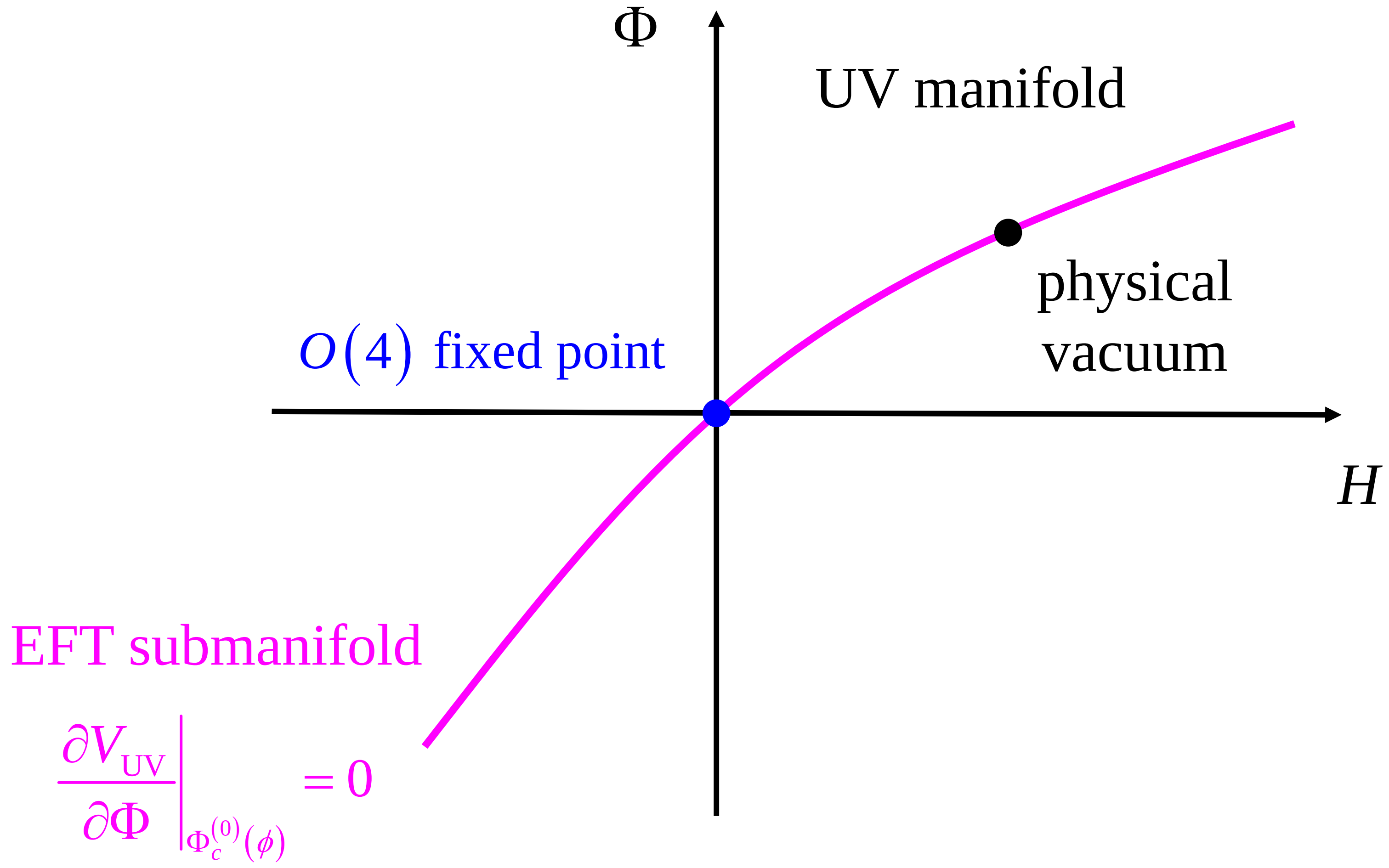

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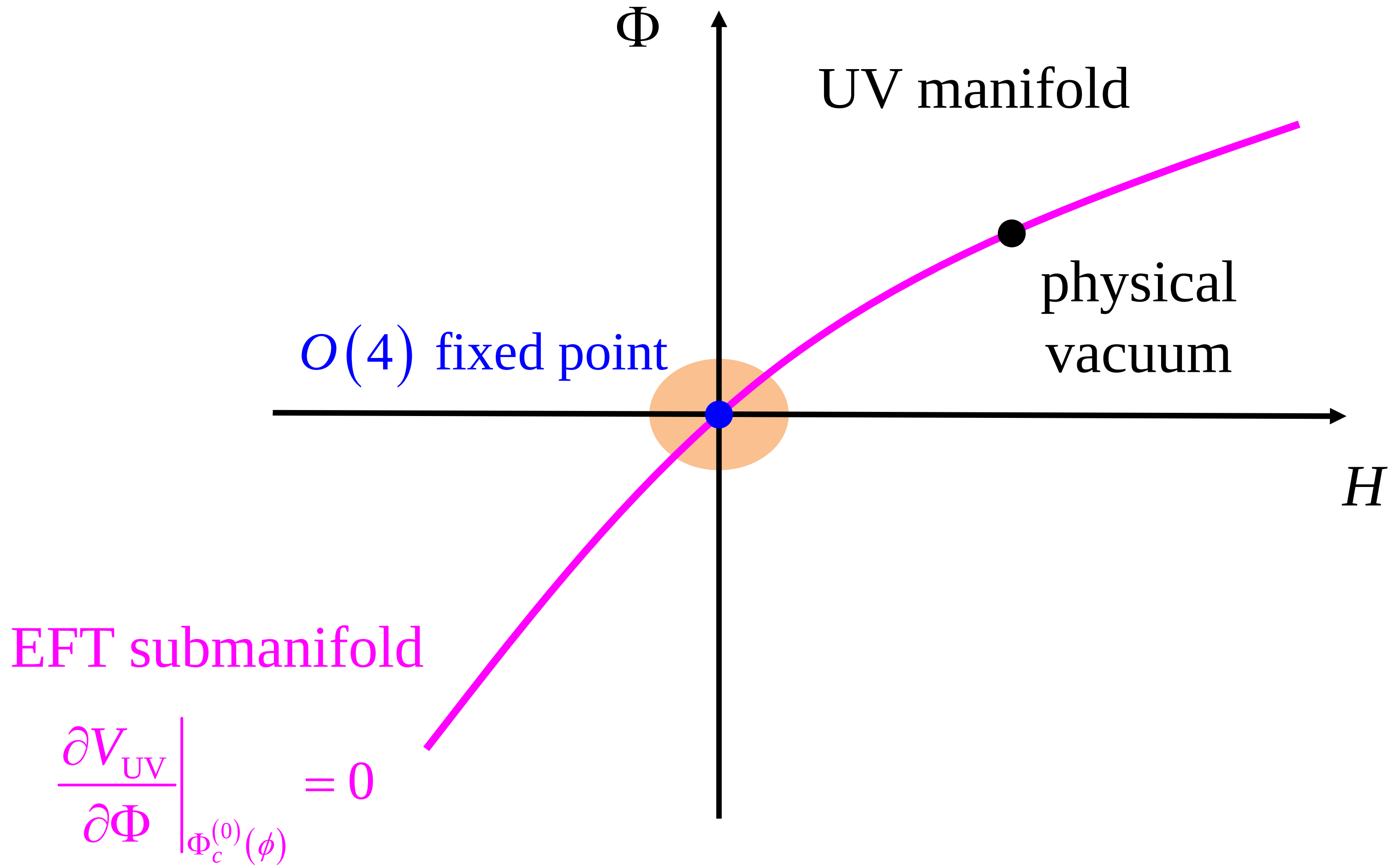

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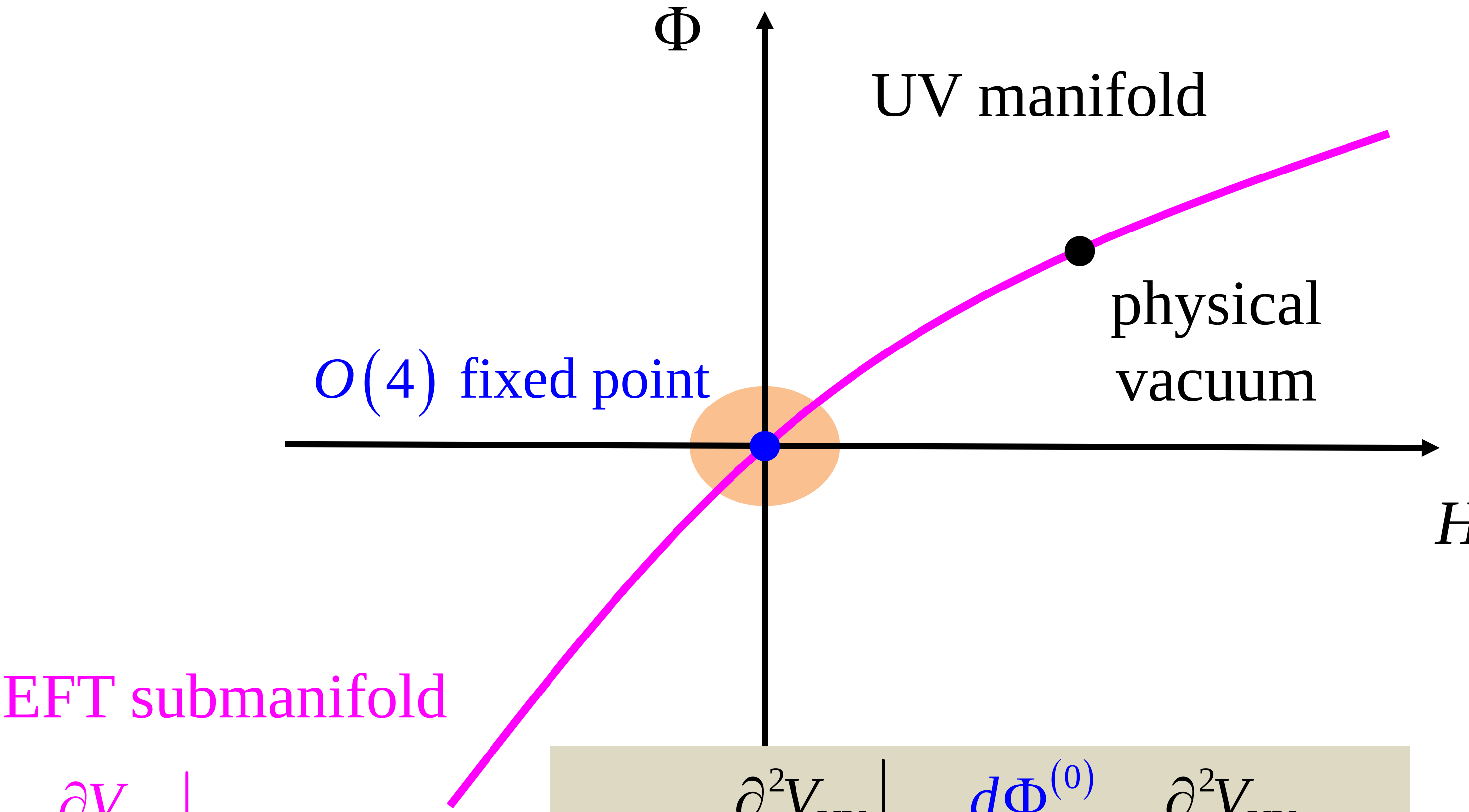

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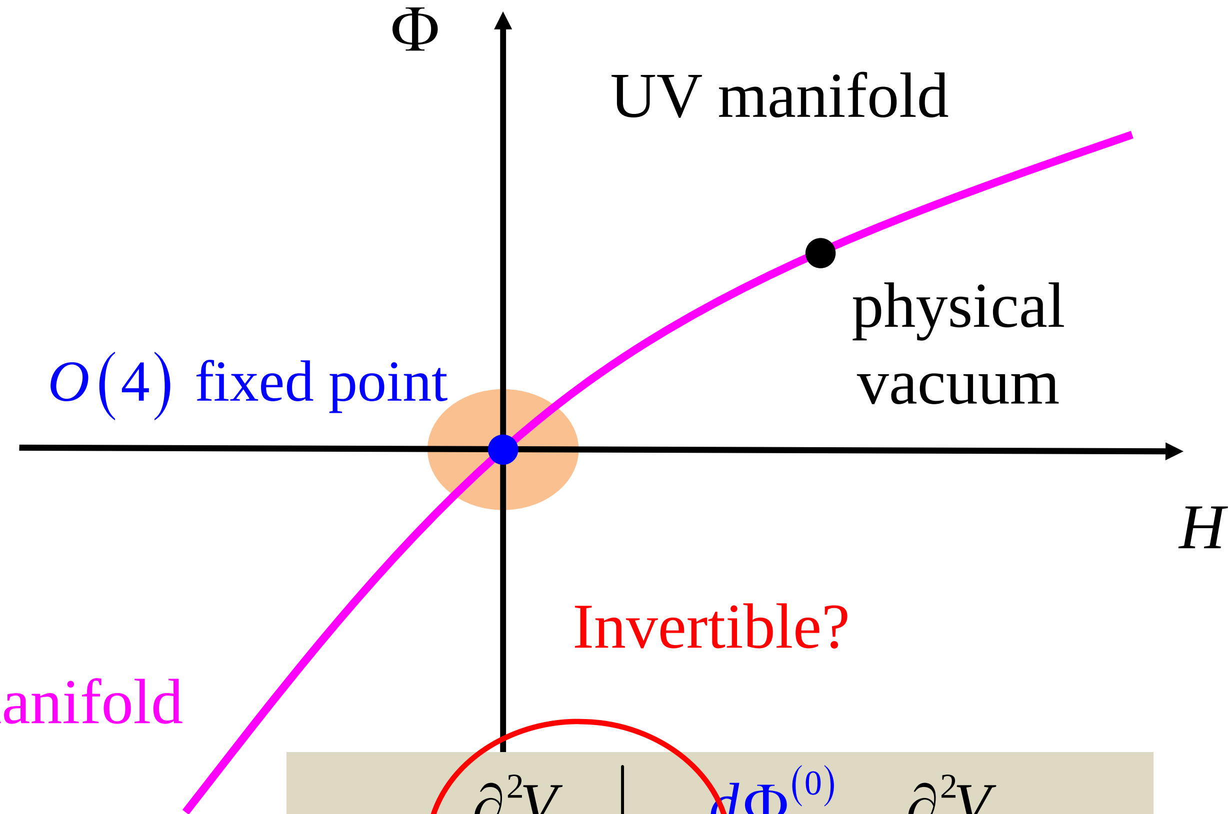

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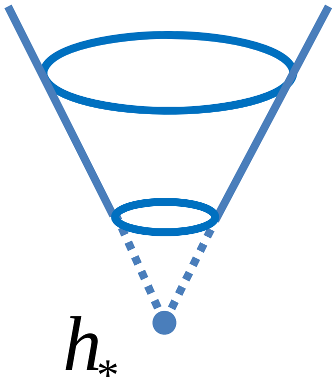
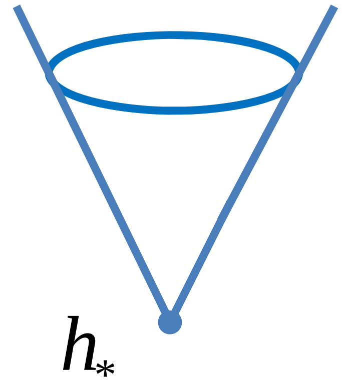
Extra electroweak symmetry breaking



$$F(h) \neq 0$$

Mass fully from electroweak symmetry breaking



$$F(h_*) = 0$$

$R(h_*)$ ill defined

Matching to all-order in fields

Coleman-Weinberg: $$\mathcal{L}_{\text{UV}}[\phi,\Phi] \supset -\frac{1}{2}\Phi\left[\partial^2 + M^2 + U(\phi)\right]\Phi$$

Matching to all-order in fields

Coleman-Weinberg: $\mathcal{L}_{\text{UV}}[\phi,\Phi]\supset-\frac{1}{2}\Phi\left[\partial^2+M^2+U(\phi)\right]\Phi$

$$\mathcal{L}_{\text{EFT}}[\phi]=\frac{i}{2}\ln\det\left(\partial^2+M^2+U\right)=\frac{i}{2}\text{Tr}\ln\left(\partial^2+M^2+U\right)$$

$$=\frac{i}{2}\int d^4x\int\frac{d^4p}{(2\pi)^4}\text{tr}\left[\ln\left(-p^2+M^2+U\right)+\frac{1}{6\left(p^2-M^2-U\right)^3}(\partial U)^2\right]$$

$$=\frac{1}{2}\int d^4x\frac{1}{16\pi^2}\text{tr}\left[\frac{1}{2}\left(M^2+U\right)^2\left(\ln\frac{\mu^2}{M^2+U}+\frac{3}{2}\right)+\frac{1}{12}\frac{1}{M^2+U}(\partial U)^2+\cdots\right]$$

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$$= \frac{1}{2}\int d^4x \frac{1}{16\pi^2}\operatorname{tr}\left[\frac{1}{2}\left(M^2 + U\right)^2\left(\ln\frac{\mu^2}{M^2 + U} + \frac{3}{2}\right) + \frac{1}{12}\frac{1}{M^2 + U}(\partial U)^2 + \cdots\right]$$

If $\left[U, \partial_\mu U\right] \neq 0$: see App. D in 2008.08597

A heavy singlet at one-loop level

$$\mathcal{L}_{\mathrm{UV}} = \mathcal{L}_{\mathrm{SM}} - \frac{1}{2} S\left(\partial^2 + M^2 + \kappa |H|^2\right) S \quad , \quad m_S^2 = M^2 + \frac{1}{2}\kappa v^2$$

$M^2 > 0$, SMEFT $\qquad$ $M^2 = 0$, HEFT

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$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{16\pi^2}\left[\frac{1}{4}\left(M^2 + \kappa |H|^2\right)^2 \left(\ln \frac{\mu^2}{M^2 + \kappa |H|^2} + \frac{3}{2}\right) + \frac{1}{24} \frac{\kappa^2}{M^2 + \kappa |H|^2}\left(\partial |H|^2\right)^2\right]$$

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$$\boxed{\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\left[K(h)\right]^2 (\partial h)^2 + \frac{1}{2}\left[vF(h)\right]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)}$$

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$$vF(h) = v + h$$

$$K^2 = 1 + \frac{1}{96\pi^2} \frac{\kappa^2 |H|^2}{M^2 + \kappa |H|^2}$$

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$$R = \frac{1}{16\pi^2}\left[\frac{1}{2K^4}\frac{\kappa^2 M^2}{\left(M^2 + \kappa |H|^2\right)^2} + \frac{1}{2K^2}\frac{\kappa^2}{M^2 + \kappa |H|^2}\right]$$

An abelian toy model at tree level

$$\mathcal{L}_{\text{UV}} = |\partial H_A|^2 + |\partial H_B|^2 - V$$

$$V = m_A^2 |H_A|^2 + m_B^2 |H_B|^2 + \lambda_A |H_A|^4 + \lambda_B |H_B|^4$$
$$+ 2\kappa |H_A|^2 |H_B|^2 + \left(\mu H_A^* H_B^2 + h.c.\right)$$

Field	Q
H_A	$+2$
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$$|H_A|^2 \equiv \frac{1}{2} f^2$$

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$$f(r)$$

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$$H_A = \frac{-\mu}{m_A^2 + 2\lambda_A|H_A|^2 + 2\kappa|H_B|^2}H_B^2 = \frac{-\mu}{m_A^2 + \lambda_A f^2 + \kappa r^2}\frac{1}{2}r^2 e^{2i\theta}$$

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$$f(r)$$

$$\mathcal{L}_{\text{EFT}} \supset |\partial H_A|^2 + |\partial H_B|^2 = \frac{1}{2}\left[1+\left(\frac{df}{dr}\right)^2\right](\partial r)^2 + \frac{1}{2}\left(r^2 + 4f^2\right)(\partial\theta)^2$$

A heavy triplet at tree level

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(\partial\Phi_a\right)^2 - V$$

$$V = \frac{1}{2}M^2\Phi_a\Phi_a - \mu H^\dagger t^a H\Phi_a + \kappa\left|H\right|^2\Phi_a\Phi_a + \frac{\lambda_\Phi}{4}\left(\Phi_a\Phi_a\right)^2$$

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Summary

- A geometric criterion to tell a HEFT Lagrangian from SMEFT
- An understanding of what UV theories would generate HEFT
- A functional method to match to all order in *fields* (not *derivatives* yet)
- A few UV examples show that the leading order criterion works