Geomagnetic Signal of Millicharged Dark Matter

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Main structure in one slide

Objective:

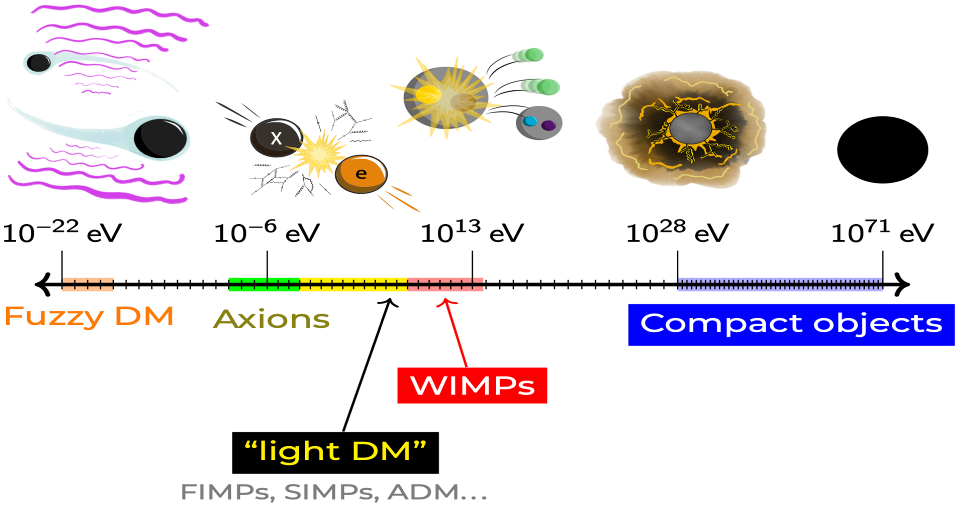
Presenting a novel mechanism utilizing a magnetic signal to detect ultralight millicharged dark matter, employing the Earth as a cavity (Arza, Gong, Shu, Wu, Zhu, arXiv:2501.14949)





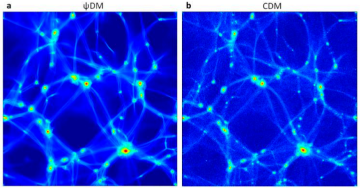
Dark Matter Landscape

From Benjamin V. Lehmann

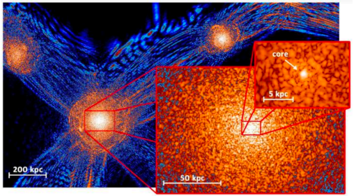


Why Ultralight Dark Matter

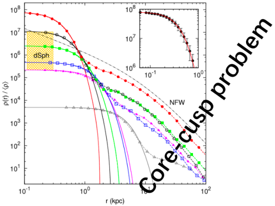
Small Scale Problem: Fuzzy Dark Matter Candidate



Comparison of cosmological large-scale structures formed by standard CDM and by wavelike DM



A slice of density field of wave DM simulation



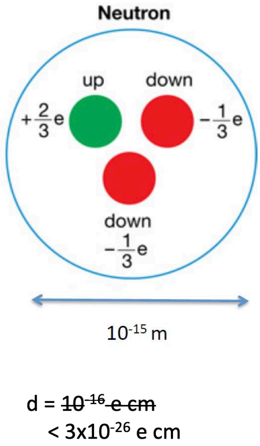
Radial density profiles of haloes formed in the wave DM model

Schive, Chiueh, Broadhurst, 1406.6586

Why Ultralight Dark Matter

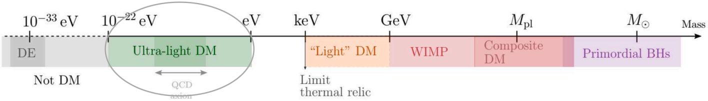
Strong CP Problem

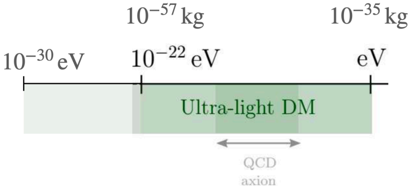
- Naive estimation: $10^{-16} e$ cm, Exp: $3 \times 10^{-26} e$ cm
- The best explanation: New U(1) axial symmetry, that when broken, cancels CP violation in the strong sector (Pecci, Quinn, 1977)
- Consequence: New particle, called the axion (Weinberg, Wilczek, 1978)



What is Ultralight Dark Matter

We define ultralight dark matter (ULDM) as bosonic DM candidates with $m < \mathrm{eV}$



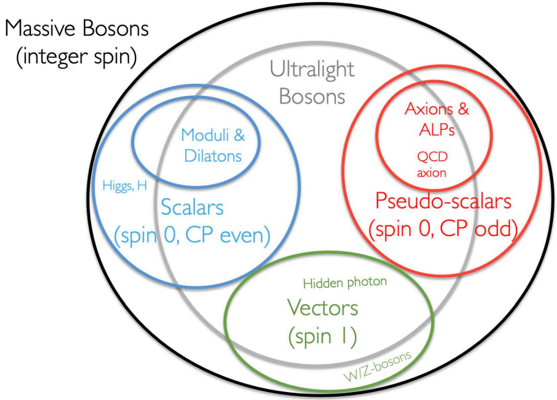


→ Bosons

Non-thermally produced

ULDM Candidates

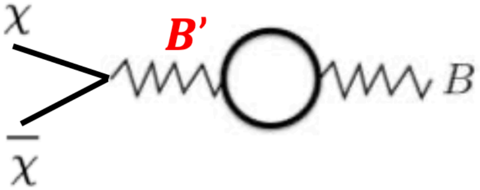
Many extensions of the Standard Model predict additional massive bosons, Ref.: Chadha-Day et al 2022



Why and What is millicharged scalar dark matter (mCP)?

Why mCP?

- A long-standing question: Is electric charge quantized, mCP is used to testing whether or not $e/3$ is the minimal charge.
- mCP could have natural link to dark sector (massless dark photon, etc.)



- Used for the cooling of gas temperature to explain the EDGES anomaly

mCP 101

- mCP is some relic charged under a dark $U(1)$.

$$f_\chi = \frac{\rho_{\rm mCP}}{\rho_{\rm DM}} \sim 0.0001 - 1$$

- Through kinetic mixing $\left(F_{\mu\nu}F^{\mu\nu} + \epsilon\,(F')_{\mu\nu}\,F^{\mu\nu} + (F')_{\mu\nu}\,(F')^{\mu\nu}\right)$ with our own photon, MCDM acquires an effective charge

$$q_{\rm eff} = Q \propto \epsilon$$

- Minimal assumptions for interaction, gauge invariance. Most robust constraints.

$$\mathcal{L} = D_\mu\phi(D^\mu\phi)^* - m_\phi^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Production Mechanism

How to naturally generate ultralight mCP in the cosmos

- Freeze-in works for keV to TeV-scale mCP (X. Chu, T. Hambye and M.H.G. Tytgat, The Four Basic Ways of Creating Dark Matter Through a Portal, [arXiv:1112.0493])
- Misalignment works for sub-eV MCDM
  Zachary Bogorada and Natalia Toro, arXiv:2112.11476

$$\rho_0 \sim 6 \times 10^{-30} \frac{\text{GeV}}{\text{cm}^3} \left(\frac{\mu}{1\text{eV}}\right)^{1/2} \left(\frac{v}{1\text{GeV}}\right)^2 |\pi_0|^2$$

Our Detection Approach for mCP

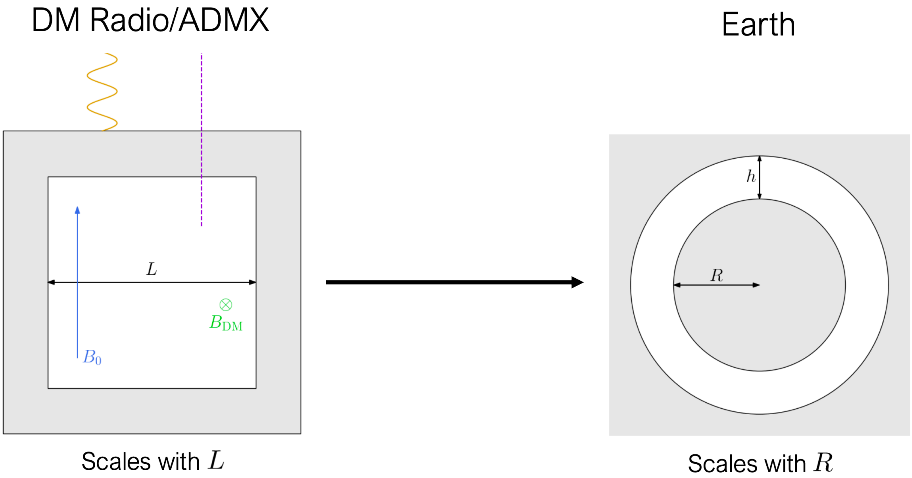
- Shielded/Cavity experiments search for ultralight EM-coupled DM
  - axion: $g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$
  - dark photon: $\epsilon \tilde{F}_{\mu\nu} F^{\mu\nu}$
  - mCP: $D_\mu \phi = (\partial_\mu + i e_m A_\mu)\phi$
- Signal scales with size of apparatus

Motived by the scaling law

We use the Earth as our apparatus!

Earth as a transducer

A natural vacuum cavity: Formed between the inner conducting sphere of the Earth and the conducting ionospheric layer



mCP Electrodynamics

- mCP interacts with the photon via the Lagrangian

$$\mathcal{L} = D_\mu \phi \left(D^\mu \phi\right)^* - m_\phi^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- Equation of motion for mCP field in an external electromagnetic field $\mathcal{A}_\mu = \left(\mathcal{A}_0, -\vec{\mathcal{A}}\right)$

$$\begin{aligned} \partial_\mu F^{\mu\nu} =& J_m^\nu - 2e_m^2 \mathcal{A}^\nu |\phi|^2 \equiv J_{\text{eff}}^\nu \,, \\ \left(\Box + m_\phi^2\right)\phi =& - ie_m \partial_\mu \mathcal{A}^\mu \phi - 2ie_m \partial_\mu \phi \mathcal{A}^\mu \\ & + e_m^2 \mathcal{A}_\mu \mathcal{A}^\mu \phi . \end{aligned}$$

Gauge Invariance

- The correct way is to include the mCP current

$$J_m^\nu = ie_m \left(\phi^* \partial^\nu \phi - \phi \partial^\nu \phi^*\right).$$

- The mCP can be solved via equation of motion

$$\Box \phi = -\, ie_m \partial_\mu \mathcal{A}^\mu \phi - 2ie_m \partial_\mu \phi \mathcal{A}^\mu + e_m^2 \mathcal{A}_\mu \mathcal{A}^\mu \phi$$

- For a different gauge choice $\mathcal{A}'_\mu = \mathcal{A}_\mu + \partial_\mu \Lambda$, correspondingly $\phi' = \phi\, e^{-ie_m \Lambda}$, so that

$$J_m^{\mu\prime} + J_{\text{eff}}^{\mu\prime} = J_m^\mu + J_{\text{eff}}^\mu$$

Perturbation Theory

Linearize the EoM

- Choose Coulomb gauge

$$\vec{\nabla} \cdot \vec{A} = 0, \quad A_0 = 0$$

- Expand the mCP to first order in e_m: $\phi = \phi_0 + \phi_1$ so that

$$\left(\Box + m_\phi^2\right) \phi_0 = 0, \quad \left(\Box + m_\phi^2\right) \phi_1 = -2ie_m \vec{\nabla}\phi_0 \cdot \vec{A}$$

- ϕ_0 is order $\mathcal{O}((e_m)^0)$ and charge symmetric , ϕ_1 is order $\mathcal{O}(e_m)$.

$$\phi_0 = \frac{\sqrt{2\rho}}{m_\phi} \cos\left(\vec{k}_\phi \cdot \vec{x} - m_\phi t\right)$$

- Once ϕ_1 is known, we can reduce EoM to

$$\Box \vec{A} = 2e_m \phi_0 \, \mathrm{Im}\left(\vec{\nabla}\phi_1\right) - 2e_m^2 \vec{A}\phi_0^2$$

Effective Current

Just a EM problem with a background current!

In non-relativistic limit, EoM becomes

$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J}_{\text{eff}}$$

When $\lambda_\phi > R_e$, the electric field becomes negligible, leaving only a magnetic field signal.

- For dark-photon dark matter,

$$\mathbf{J}_{\text{eff}} = -\varepsilon m_{A'}^2 \mathbf{A}' \sim \mathcal{O}(m_{A'})$$

- For axion-like dark matter,

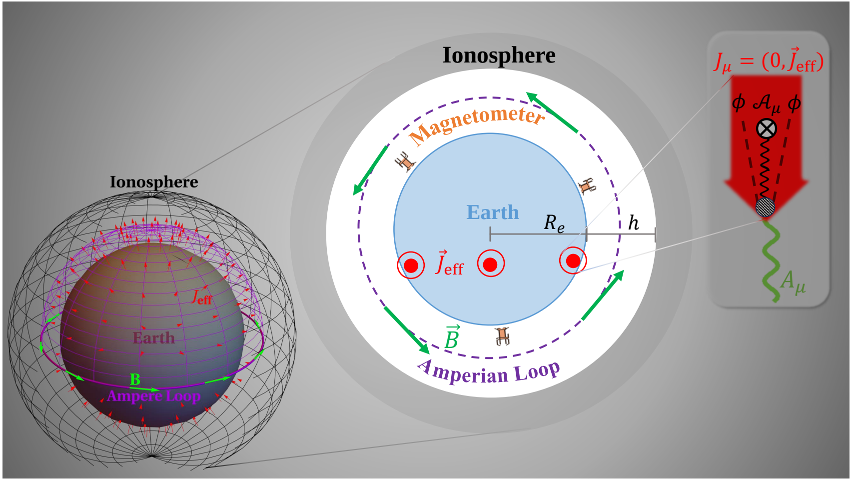
$$\mathbf{J}_{\text{eff}} = i g_{a\gamma} m_a a \mathbf{B}_0 \sim \mathcal{O}((m_a)^0)$$

- For mCP, The effective current from mCPs scales inversely with the square of their mass, enhancing sensitivity to ultralight mCPs

$$\mathbf{J}_{\text{eff}} = 2 e_m^2 \vec{A} \phi_0^2 \sim \mathcal{O}((m_\phi)^{-2})$$

Experimental Setup

a monochromatic magnetic signal with spatial dependence of a particular vector spherical harmonics



Rough Estimate

Why mCP has much better sensitivity than dark photon and axion

- Ampere law

$$\int \mathbf{B} \cdot \mathbf{d}\ell = \iint \mathbf{dA} \cdot \mathbf{J}$$

- Simple dimensional analysis

$$BR \approx R^2 e_m^2 (B_0 R) \phi_0^2 \rightarrow B \sim 100 \text{ pT}$$

- Rough sensitivity for Vector Magnetoresistive (VMR) sensors

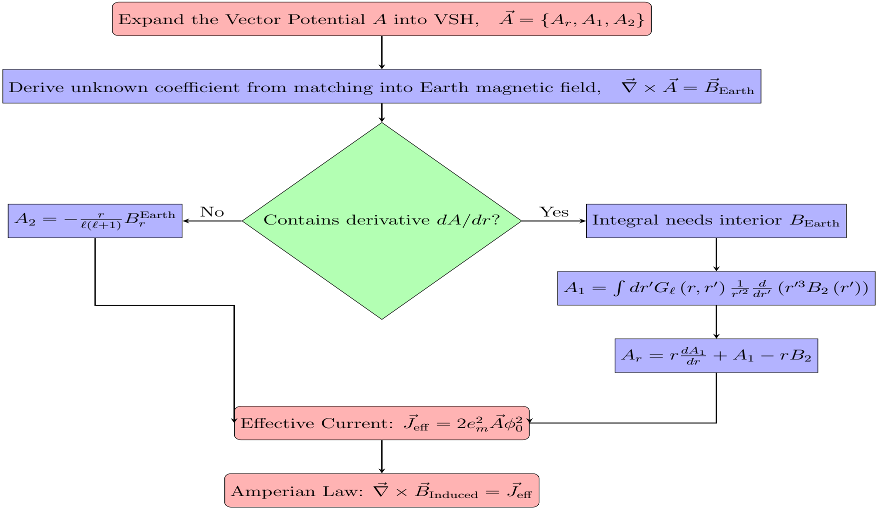
$$300\text{pT}/\sqrt{\text{Hz}} \quad \text{over the frequency } 0.1 - 100 \text{ Hz}$$

- While for dark photon (proportional to mass) and axion (independent to mass)

The lower mass, the better sensitivity for mCP

Our basic motivation for mCP

Logic Flow



Expand the Vector Potential A into VSH, $\vec{A} = \{A_r, A_1, A_2\}$
Derive unknown coefficient from matching into Earth magnetic field, $\vec{\nabla} \times \vec{A} = \vec{B}_{\text{Earth}}$
Contains derivative dA/dr?
No
$A_2 = -\frac{r}{\ell(\ell+1)} B_r^{\text{Earth}}$
Yes
Integral needs interior B_{Earth}
$A_1 = \int dr' G_\ell(r, r') \frac{1}{r'^2} \frac{d}{dr'} (r'^3 B_2(r'))$
$A_r = r\frac{dA_1}{dr} + A_1 - rB_2$
Effective Current: $\vec{J}_{\text{eff}} = 2e_m^2 \vec{A} \phi_0^2$
Amperian Law: $\vec{\nabla} \times \vec{B}_{\text{Induced}} = \vec{J}_{\text{eff}}$

Computational Framework

Vector potential in the vector spherical harmonics bases

- Expand Earth magnetic field and vector potential in terms of VSH

$$\vec{\mathcal{B}}(\vec{x}) = \sum_{\ell,m} \Big(\mathcal{B}_r(r)\vec{Y}_{\ell m}(\theta,\varphi) + \mathcal{B}_1(r)\vec{\Psi}_{\ell m}(\theta,\varphi) + \mathcal{B}_2(r)\vec{\Phi}_{\ell m}(\theta,\varphi) \Big)$$

$$\vec{\mathcal{A}}(\vec{x}) = \sum_{\ell,m} \Big(\mathcal{A}_r(r)\vec{Y}_{\ell m}(\theta,\varphi) + \mathcal{A}_1(r)\vec{\Psi}_{\ell m}(\theta,\varphi) + \mathcal{A}_2(r)\vec{\Phi}_{\ell m}(\theta,\varphi) \Big)$$

  $\mathcal{B}_{r,1,2}$ are the known coefficients of Earth magnetic field, $\mathcal{A}_{r,1,2}$ are our desire.
- Vector spherical harmonics

$$\boldsymbol{Y}_{\ell m} = Y_{\ell m}\hat{\boldsymbol{r}}, \quad \boldsymbol{\Psi}_{\ell m} = r\boldsymbol{\nabla} Y_{\ell m}, \quad \boldsymbol{\Phi}_{\ell m} = \boldsymbol{r} \times \boldsymbol{\nabla} Y_{\ell m}$$

  $\hat{\boldsymbol{r}}$ is radial unit vector, the other two point tangentially to a constant radius sphere.

Build Consistent Equations for Unknown Coefficients

In terms of $\vec{\nabla} \times \vec{\mathcal{A}} = \vec{\mathcal{B}}$

$$\vec{\nabla} \times \overrightarrow{\mathcal{A}} = \sum_{\ell,m} \left(-\frac{\ell(\ell+1)}{r} \mathcal{A}_2 \vec{Y}_{\ell m} - \left(\frac{d\mathcal{A}_2}{dr} + \frac{\mathcal{A}_2}{r} \right) \vec{\Psi}_{\ell m} + \left(-\frac{\mathcal{A}_r}{r} + \frac{d\mathcal{A}_1}{dr} + \frac{\mathcal{A}_1}{r} \right) \vec{\Phi}_{\ell m} \right).$$

Comparison yields this set of first-order ordinary differential equations for the vector potential components

$$-\frac{\ell(\ell+1)}{r} \mathcal{A}_2 = \mathcal{B}_r$$
$$\frac{d\mathcal{A}_2}{dr} + \frac{\mathcal{A}_2}{r} = \mathcal{B}_1$$
$$\frac{d\mathcal{A}_1}{dr} + \frac{\mathcal{A}_1}{r} - \frac{\mathcal{A}_r}{r} = \mathcal{B}_2$$
$$\frac{d\mathcal{A}_r}{dr} + \frac{2\mathcal{A}_r}{r} - \ell(\ell+1)\frac{\mathcal{A}_1}{r} = 0$$

The final equation is the Coulomb gauge $\vec{\nabla} \cdot \overrightarrow{\mathcal{A}} = 0$, the second a trivial function $\nabla \cdot \overrightarrow{\mathcal{B}} = 0$

Still three unknowns for three equations!

Computational Framework

Solving for the vector potential

- The first equation is easy to solve

$$A_{\ell m}^{(2)} = -\frac{r}{\ell(\ell+1)} B_{\ell m}^{(r)}(r)$$

  local magnetic field is good enough.

- For the last two equations

$$\frac{\left(r^2 (r A_{\ell m}^{(1)})'\right)'}{r^3} - \frac{\ell(\ell+1)}{r^2} A_{\ell m}^{(1)} = \frac{1}{r^3}\left(r^3 B_{\ell m}^{(2)}\right)'$$

- Due to Green function

$$A_{\ell m}^{(1)} = \int dr' G_\ell(r, r') \frac{1}{r'^3}\left(r'^3 B_{\ell m}^{(2)}(r')\right)'$$

  with

$$G_\ell(r, r') = -\frac{1}{2\ell+1} \begin{cases} \frac{r^{\ell-1}}{r'^{\ell-2}}, & r < r' \\ \frac{r'^{\ell+3}}{r^{\ell+2}}, & r > r' \end{cases}$$

  Need interior Earth magnetic field

Geomagnetic Model

- Exterior $r > R_{\text{cmb}}$, IGRF model

$$\vec{\mathcal{B}}(\vec{x}) = \sum_{\ell,m} C_{\ell m} \left(\frac{R_e}{r}\right)^{\ell+2} \Big((\ell+1)\vec{Y}_{\ell m}(\theta,\varphi) - \vec{\Psi}_{\ell m}(\theta,\varphi)\Big)$$

- Interior $R_{\text{icb}} < r < R_{\text{cmb}}$, no model for now. We have to compute it by hand

Geomagnetic Signal

It is $\vec{\Phi}_{10} mode$

- The external vector potential above the Earth's surface

$$\vec{\mathcal{A}} = -B_0 R_e \left(1.01\beta \left(\frac{R_e}{r}\right)^3 \left(2\vec{Y}_{10} - \vec{\Psi}_{10}\right) - \sqrt{\frac{4\pi}{3}} \left(\frac{R_e}{r}\right)^2 \vec{\Phi}_{10}\right)$$

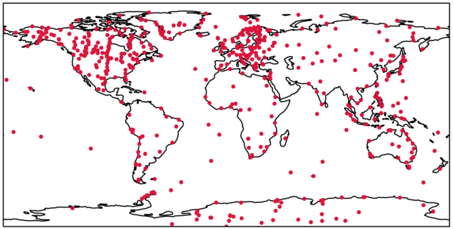
- mCP current

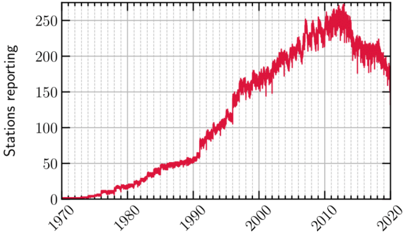
$$\vec{J}_{\text{eff}} = \frac{2e_m^2 \rho B_0 R_e}{m_\phi^2} \left(1.01\beta \left(\frac{R_e}{r}\right)^3 \left(2\vec{Y}_{10} - \vec{\Psi}_{10}\right) - \sqrt{\frac{4\pi}{3}} \left(\frac{R_e}{r}\right)^2 \vec{\Phi}_{10}\right) e^{-2im_\phi t}$$

- The magnetic field signal at the Earth's surface

$$\vec{B}(t) = \frac{e_m^2 \rho B_0 R_e^2}{m_\phi^2} \left(\sqrt{\frac{4\pi}{3}} \frac{h}{R_e} \vec{\Psi}_{10} - 2.02\beta \vec{\Phi}_{10}\right) e^{-i2m_\phi t}$$

SuperMAG Dataset



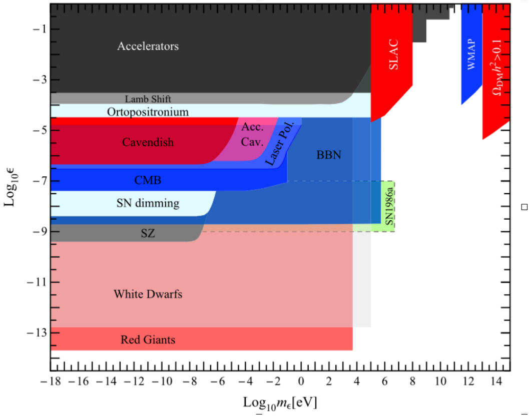


- 500+ ground-based magnetometers
- 50 years of data
- 1-minute resolution
- Typical frequency: $10^{-3} - 10^{-2}\text{Hz}$

SNIPE Hunt

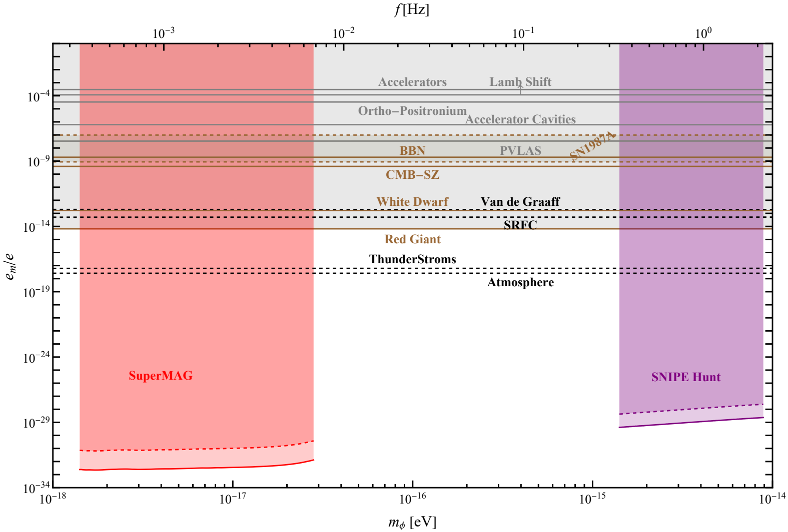
- Magnetometers are located away from man-made magnetic noise sources: Hayward, Lewisburg , Oberlin
- Measurement campaign over 3 days in July
- Typical frequency: 1 - 1000 Hz band

Previous Search



- **Big-bang nucleosynthesis**
  Particles with small electric charge will interact with the plasma in the early universe contributing to $\Delta N_{\rm eff}$
- **Stellar evolution of Red Giant**
  Plasma decay process affect the Stellar evolution
- **Territorial Experiment**
  Lamb Shift, Coulomb's Law, invisible decay of ortho-positronium

Result


f[Hz]
10^{-3}
10^{-2}
10^{-1}
10^{0}
Accelerators
Lamb Shift
Ortho−Positronium
Accelerator Cavities
SN1987A
BBN
PVLAS
CMB−SZ
White Dwarf
Van de Graaff
SRFC
Red Giant
ThunderStroms
Atmosphere
SuperMAG
SNIPE Hunt
e_m/e
10^{-4}
10^{-9}
10^{-14}
10^{-19}
10^{-24}
10^{-29}
10^{-34}
10^{-18}
10^{-17}
10^{-16}
10^{-15}
10^{-14}
m_ϕ [eV]