Dark matter and neutrino focus week, Tsung-Dao LEE Institute, Shanghai Jiaotong University 2025.18~24, Shanghai

Gravitational wave from dark matter

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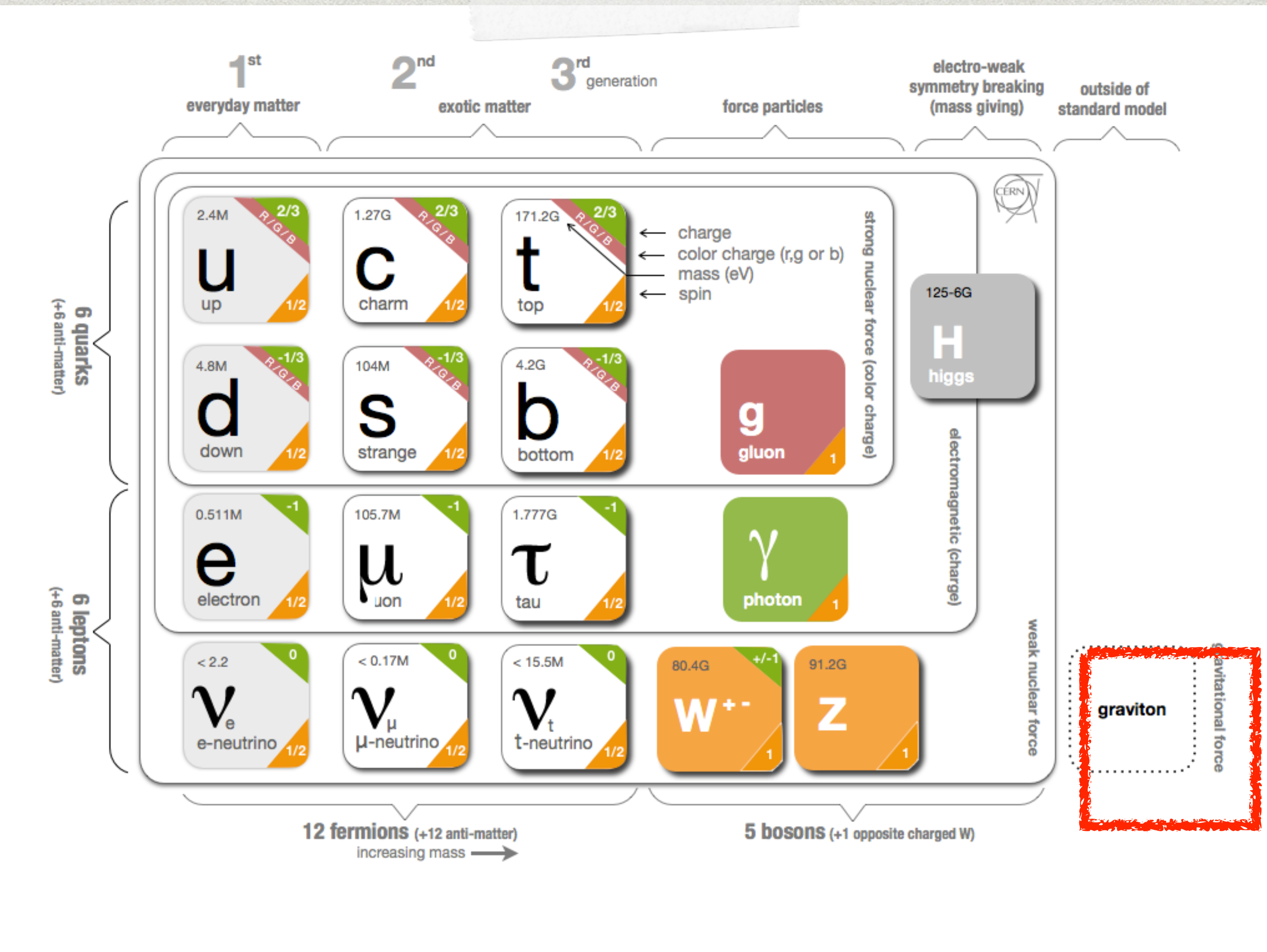
New physics beyond the standard model

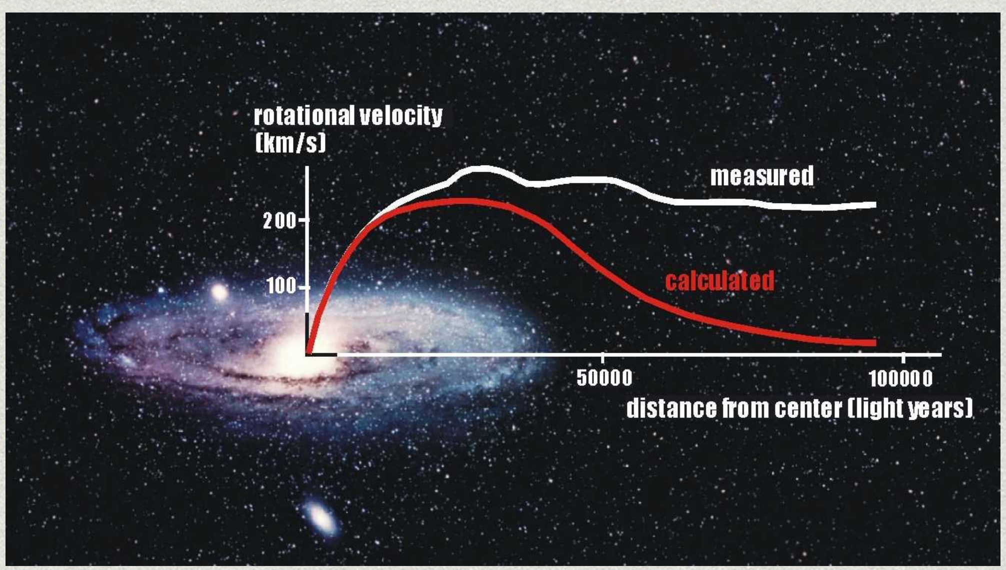


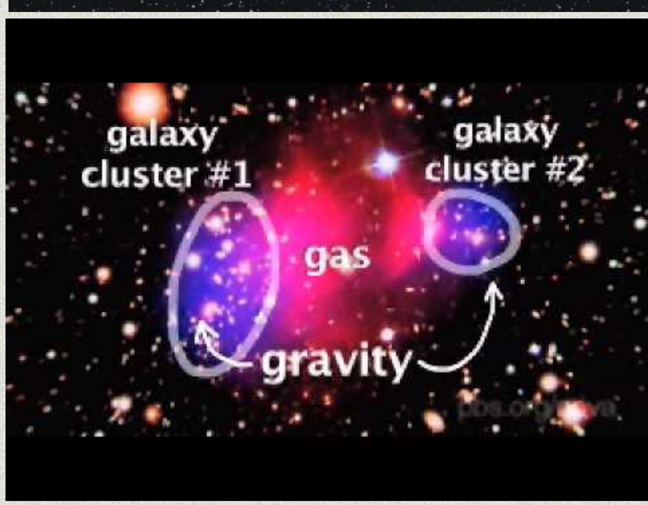
- Neutrino masses
- Dark matter
- Baryon asymmetry

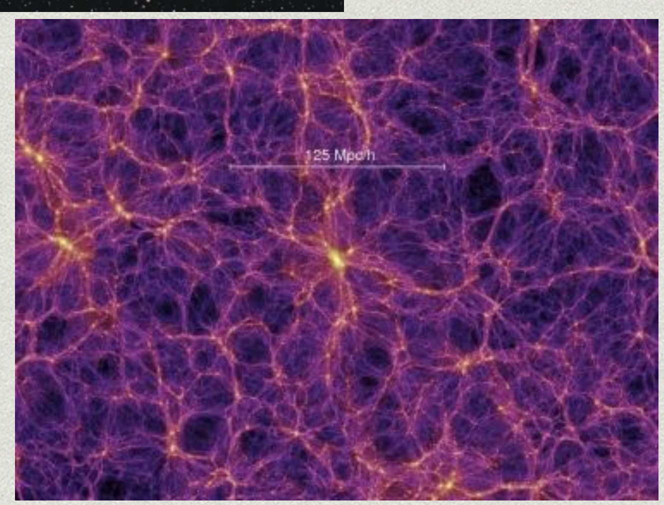
What is Dark matter

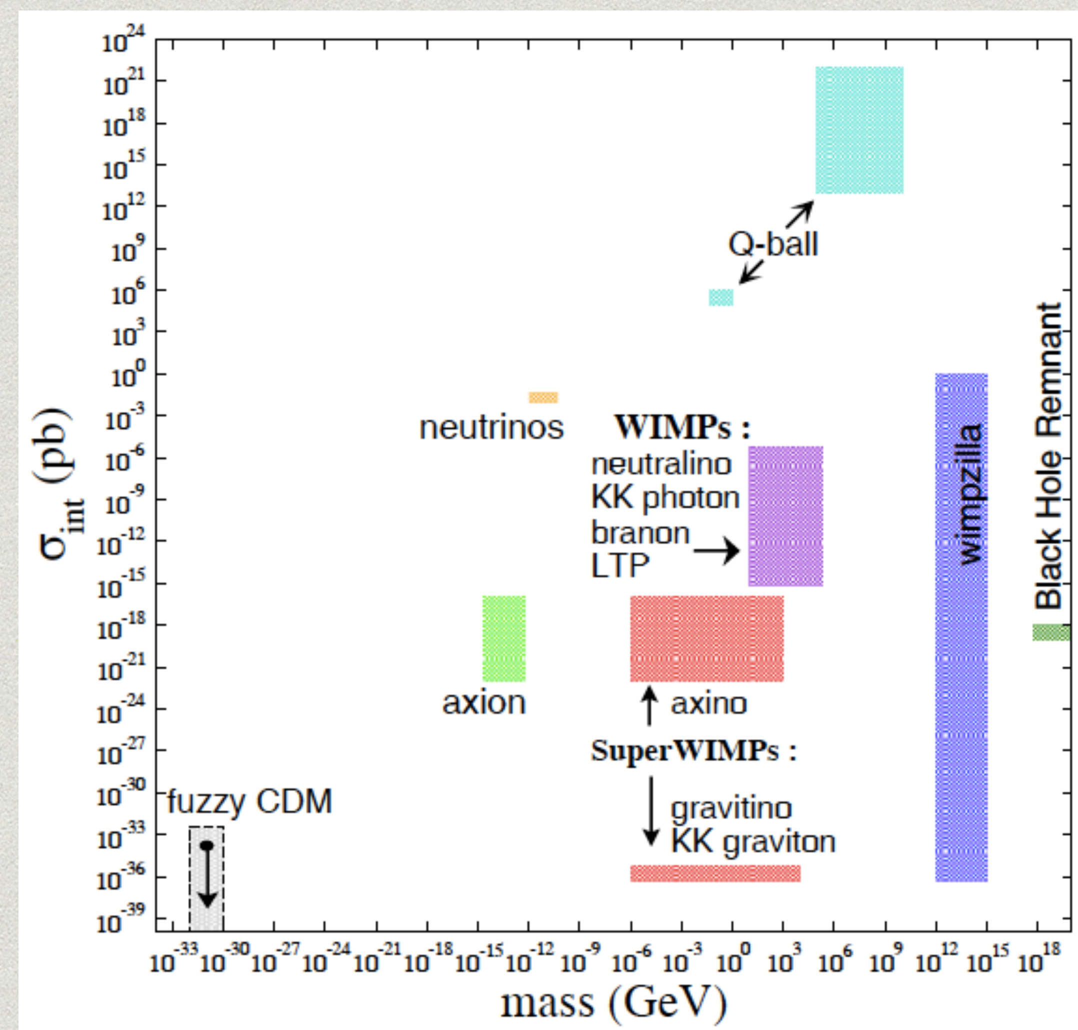
Mass ✗ Spin ✗ Interactions ✗

Neutral, non-baryonic, weakly interacting particle!

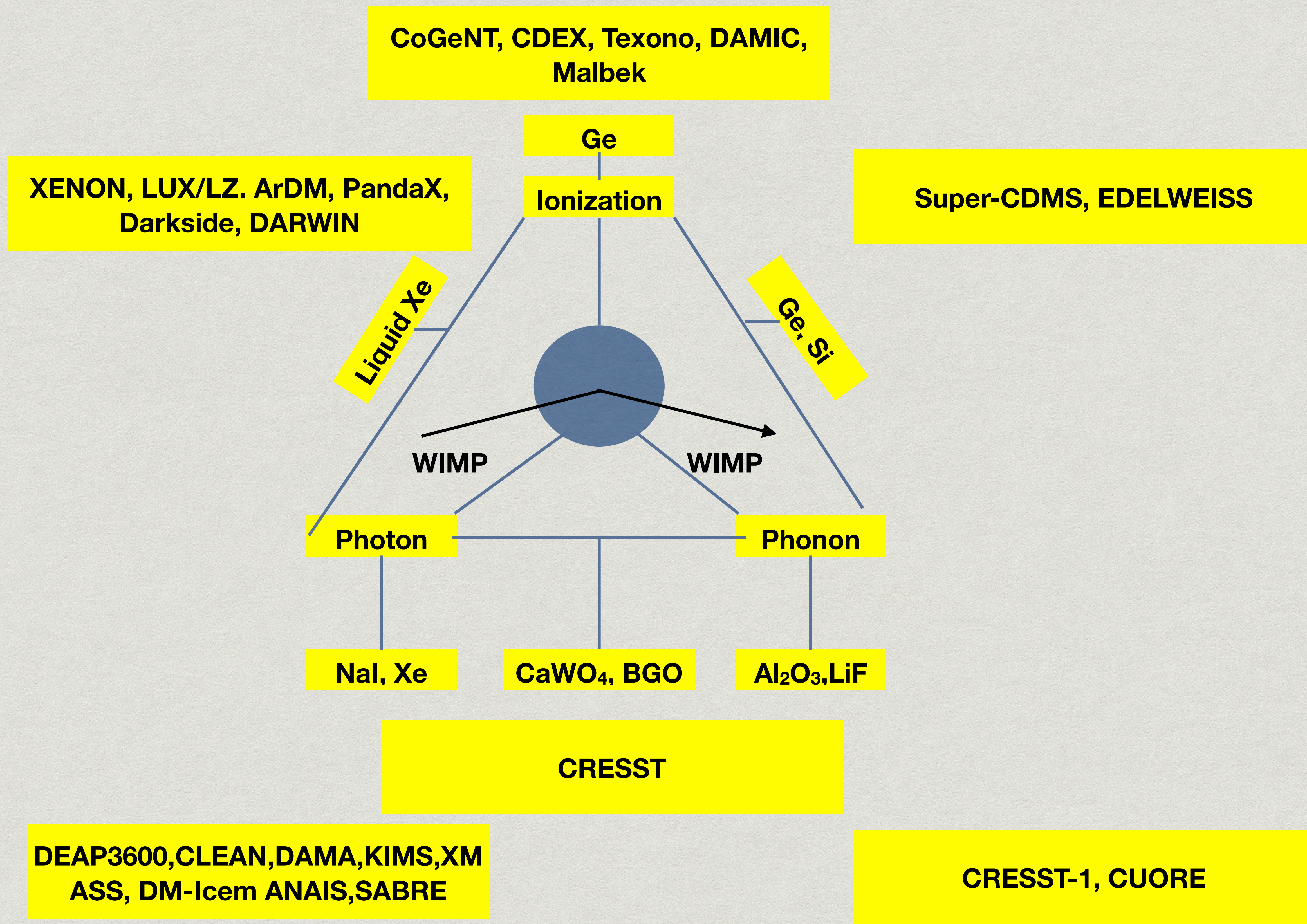




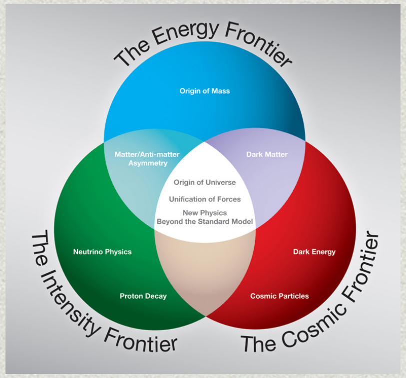




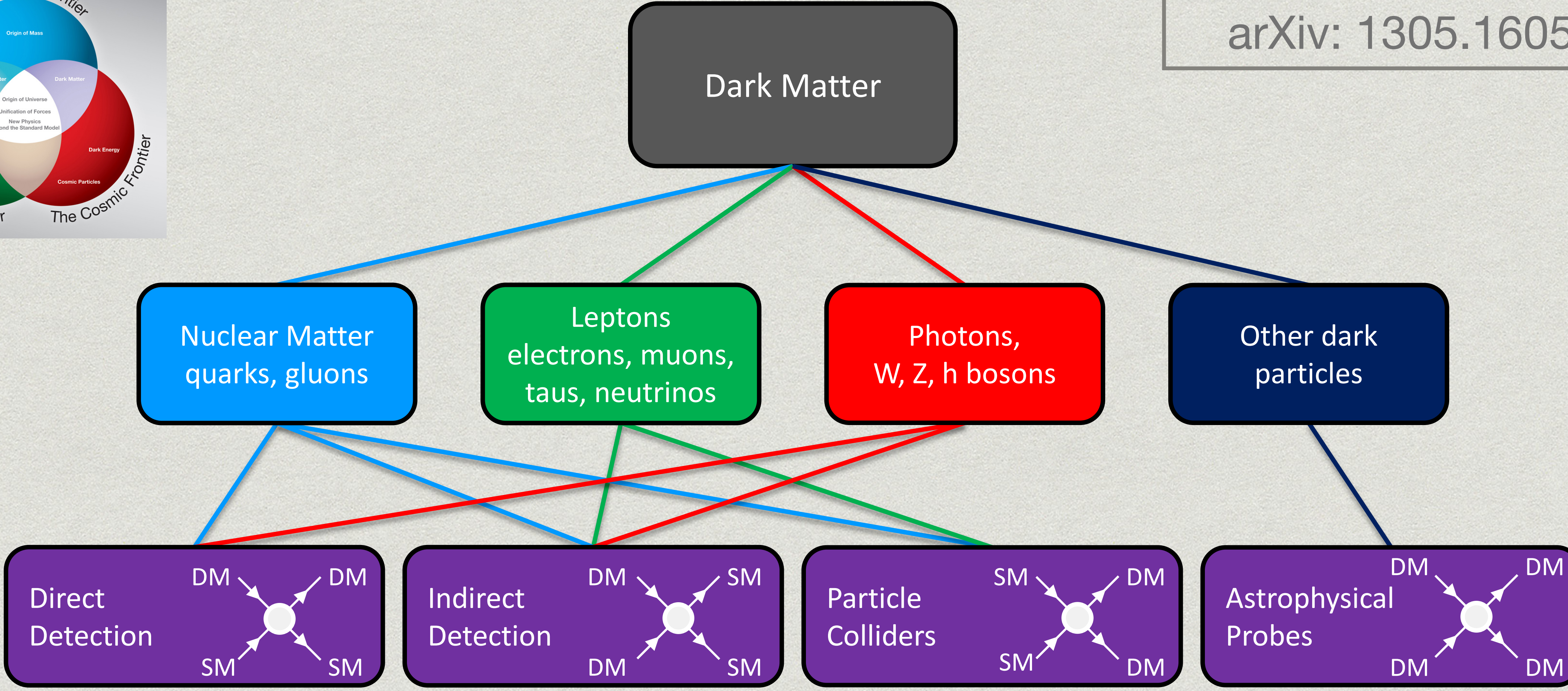
Traditional ways to directly detect dark matter

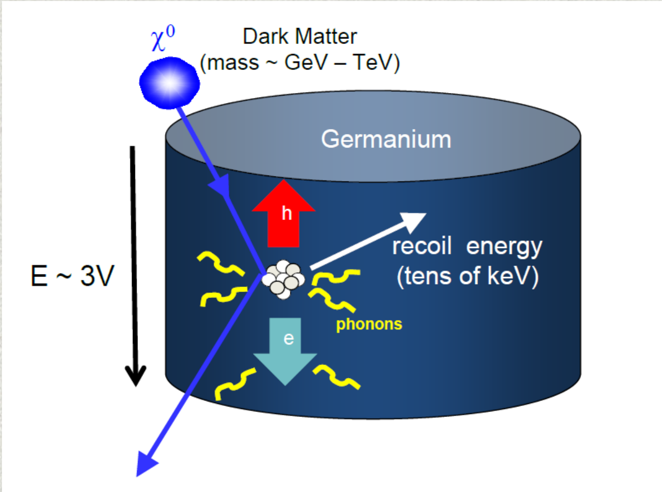


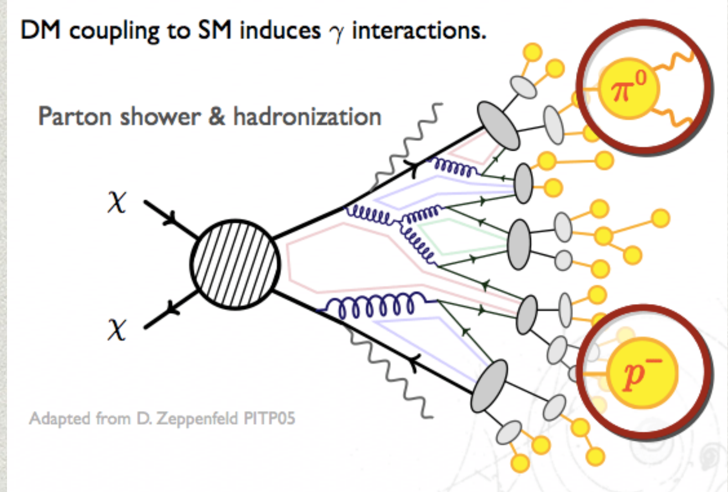
Traditional ways to directly detect dark matter

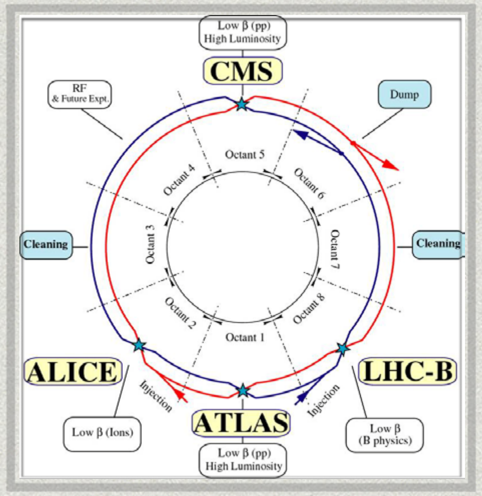


arXiv: 1305.1605

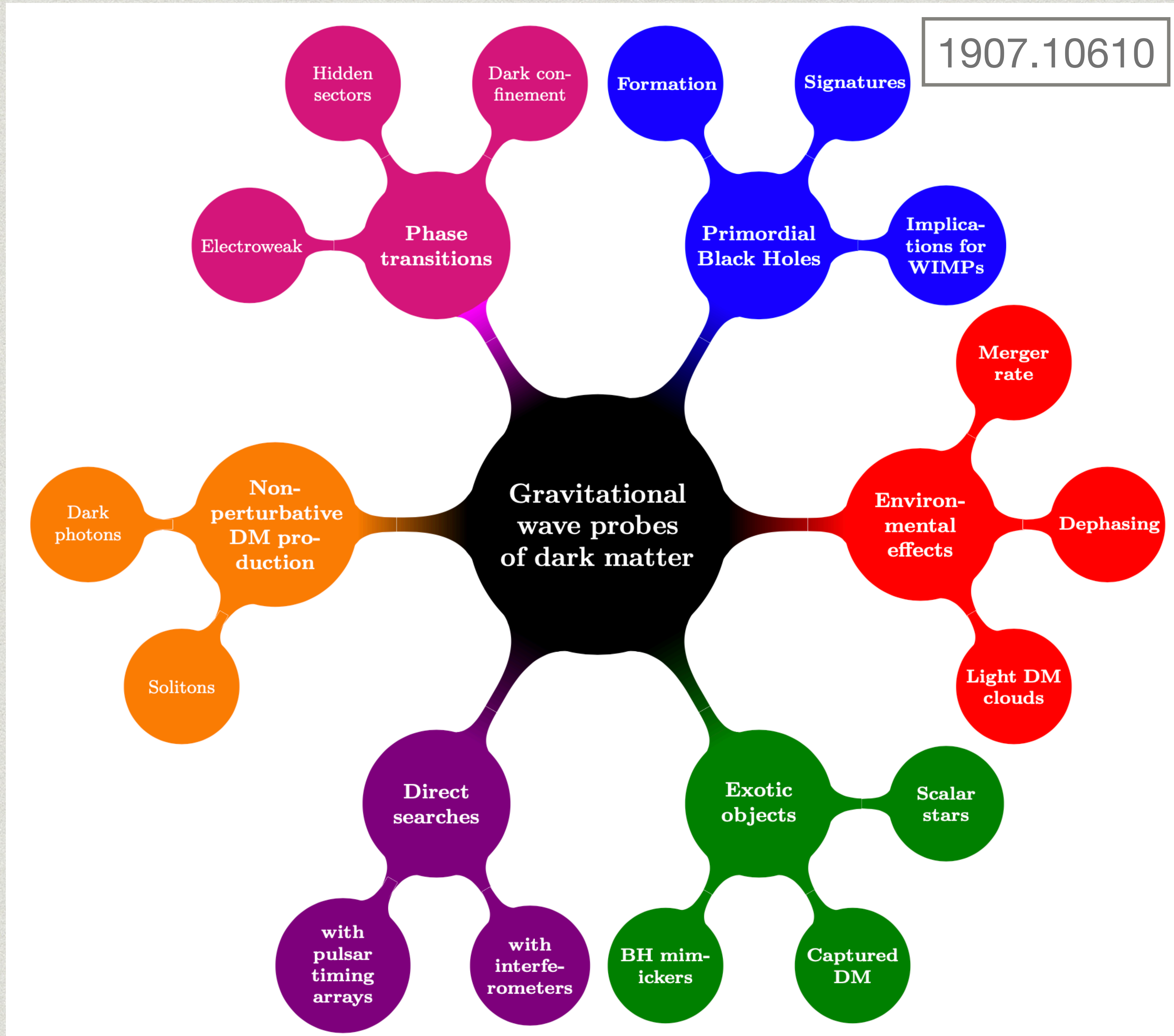


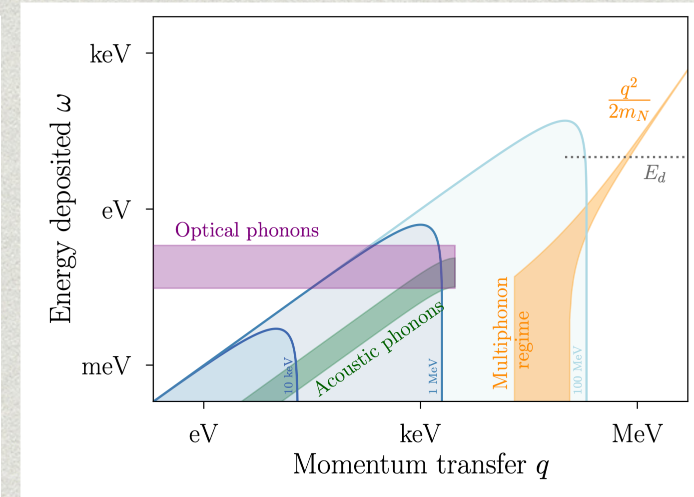


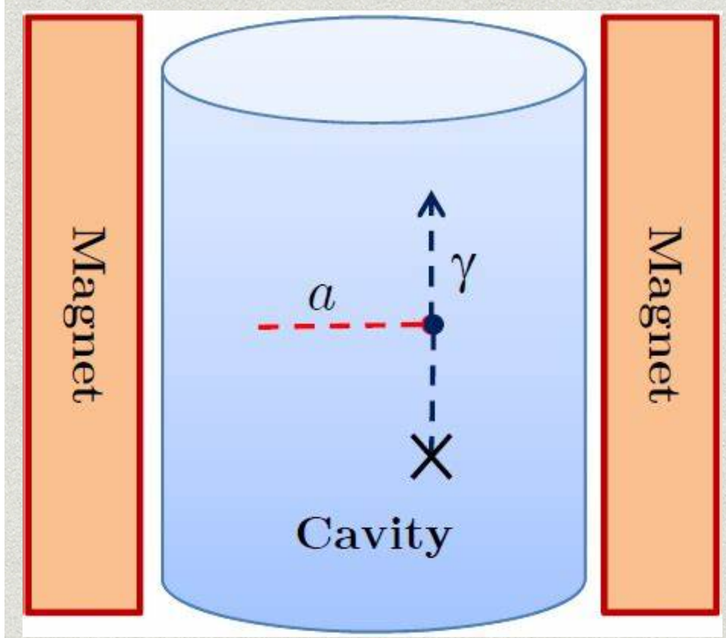




Modern strategies of probing dark matter







Outline

- GW from oscillations of ultralight DM
- GW from the scalar DM triggered EWPT
- GW from graviton bremsstrahlung in DM production via Freeze-in

Ultralight DM oscillating into GWs

Dark photon DM

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2}M_{pl}^2R-\frac{1}{4}\hat{F}_{\mu\nu}g^{\alpha\mu}\hat{F}_{\alpha\beta}g^{\beta\nu}-\frac{\varepsilon}{2}F_{\mu\nu}g^{\alpha\mu}\hat{F}_{\alpha\beta}g^{\beta\nu}\right)$$

g: **Determinant of the background metric**

$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ **with** $|h_{\mu\nu}\ll 1|$

ε : **Mixing between photon and dark photon**

Equation of motion

$$\partial_\nu\hat{F}^{\mu\nu}=\hat{j}^\mu+\partial_\nu\left(-\frac{1}{2}h\hat{F}^{\mu\nu}+\hat{F}^{\mu\alpha}h_\alpha^\nu-\hat{F}^{\nu\alpha}h_\alpha^\mu\right)+\varepsilon\partial_\nu\left(-\frac{1}{2}hF^{\mu\nu}+F^{\mu\alpha}h_\alpha^\nu-F^{\nu\alpha}h_\alpha^\mu\right)$$

Ultralight DM oscillating into GWs

Dark photon DM

Schrodinger-like equation

$$i\frac{\partial}{\partial z}\begin{pmatrix}\gamma_{\|}\\ \gamma'_{\|}\\ G_{\times}\end{pmatrix} = \left[\omega + \begin{pmatrix}\Delta_{\gamma\gamma} & \Delta_{\gamma\gamma'} & \Delta_{\gamma h}\\ \Delta_{\gamma\gamma'} & \Delta_{\gamma'\gamma'} & \Delta_{\gamma' h}\\ \Delta_{\gamma h} & \Delta_{\gamma' h} & 0\end{pmatrix}\right]\begin{pmatrix}\gamma_{\|}\\ \gamma'_{\|}\\ G_{\times}\end{pmatrix}$$

$$\Delta_{\gamma\gamma} = \Delta_{\text{pla}} + \Delta_{\text{vac}} + \Delta_{\text{CM}},$$

Plasma effect QED vacuum effect Cotton-Mouton birefringence

$$\Delta_{\gamma' h} = \frac{\varepsilon}{2}\kappa\left|B_T\right| + \frac{1}{2}\kappa\left|B'_T\right|,$$

Transverse to the GW propagation

$$\Delta_{\gamma'\gamma'} \approx \frac{m^2_{A'}}{2\omega},$$

DP mass

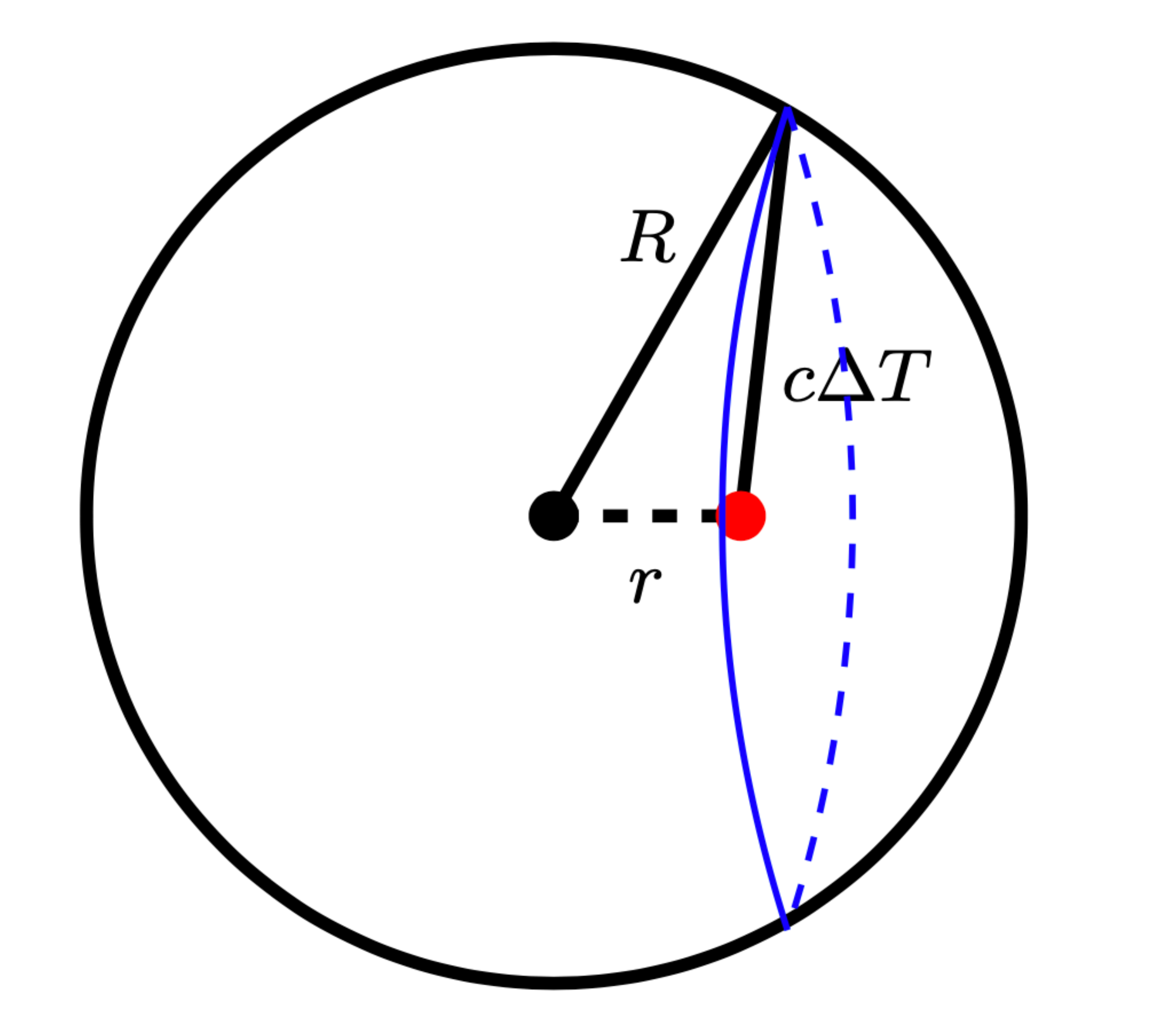
$$\Delta_{\gamma h} = \frac{1}{2}\kappa\left|B_T\right|,$$

Oscillation probability

$$P(i\to j) = \left|\sum_{\alpha}\mathcal{U}^*_{i\alpha}e^{-i\rho_\alpha z}\mathcal{U}_{j\alpha}\right|^2 \quad , \quad i,j\in\gamma,\gamma',G_{\times}\ .$$

Ultralight DM oscillating into GWs

Dark photon DM: Local relic density



Survival probability in the galaxy cluster

$$f(r,\,\Delta T)=\begin{cases}1\,, & c\Delta T\le R-r\,,\\ \frac{1}{2}\left(1-\frac{r^2+c^2\Delta T^2-R^2}{2rc\Delta T}\right)\,, & c\Delta T> R-r\,,\\ 0\,, & c\Delta T\ge R+r\,,\end{cases}$$

Energy density

$$\rho_{\rm LGW}(t_0)=\frac{1}{V_{\rm GC}}\int_{t_0-2T}^{t_0}dt\int_0^R dr\mathscr{L}(r)\cdot f(r,t_0-t)4\pi r^2\,.$$

$$\mathscr{L}(r)=\rho_{\rm DM}(r)\mathscr{R}\quad {\rm GeV}\cdot{\rm cm}^{-3}\cdot{\rm s}^{-1}\,,$$

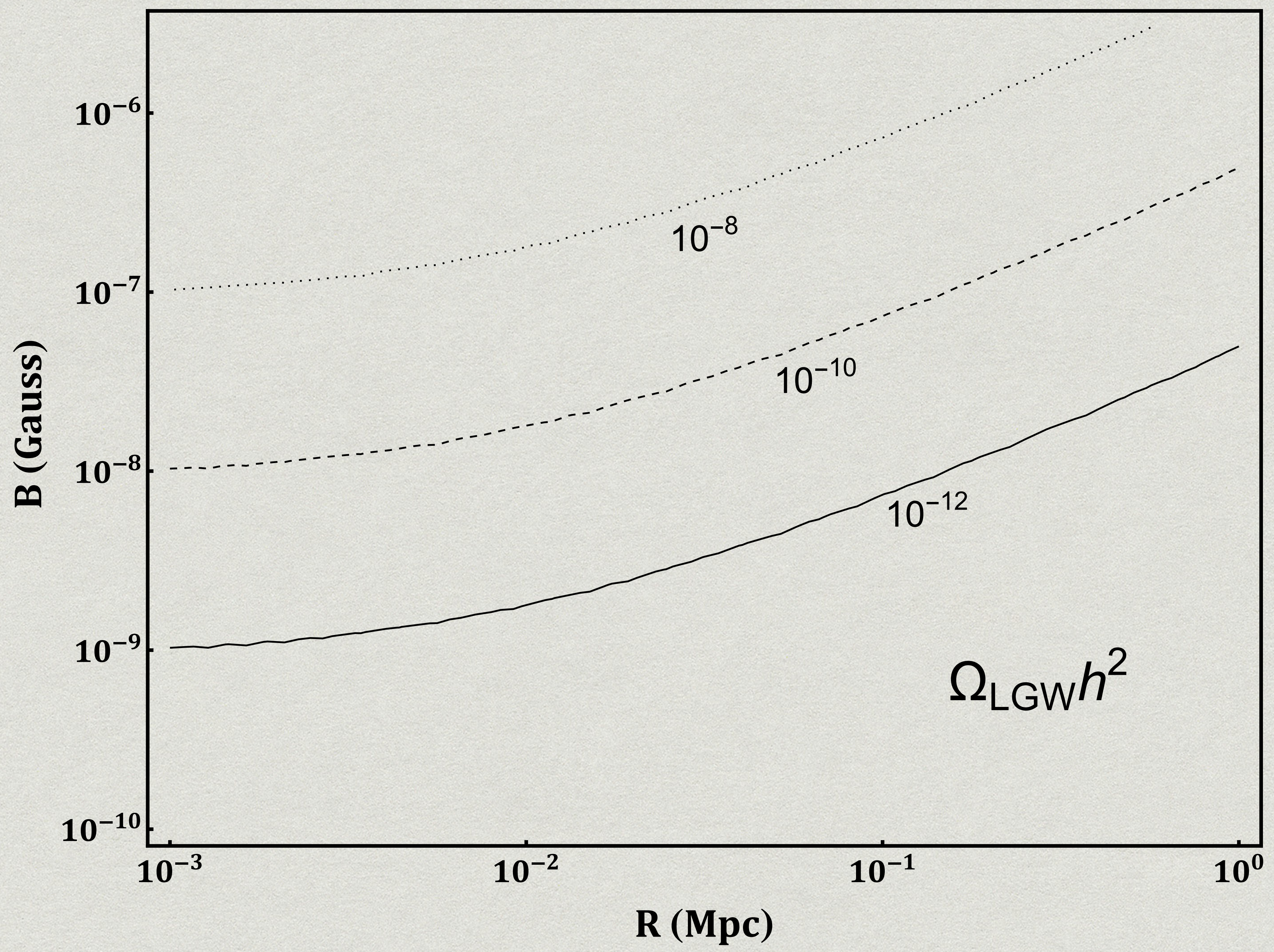
Luminosity of the oscillation

R: conversion rate per ULDP per second

GW flux $$\mathscr{D}_{\rm GW}=\int\frac{dn_{\rm DM}(r)}{dE}\frac{\mathscr{R}d^3r}{4\pi|\vec{r}-\vec{d}|^2}\sim 1.72\times10^{-7}/m_{\rm ULDP}^2\ {\rm cm}^{-2}\cdot{\rm s}^{-1}\cdot{\rm eV}^{-1}$$

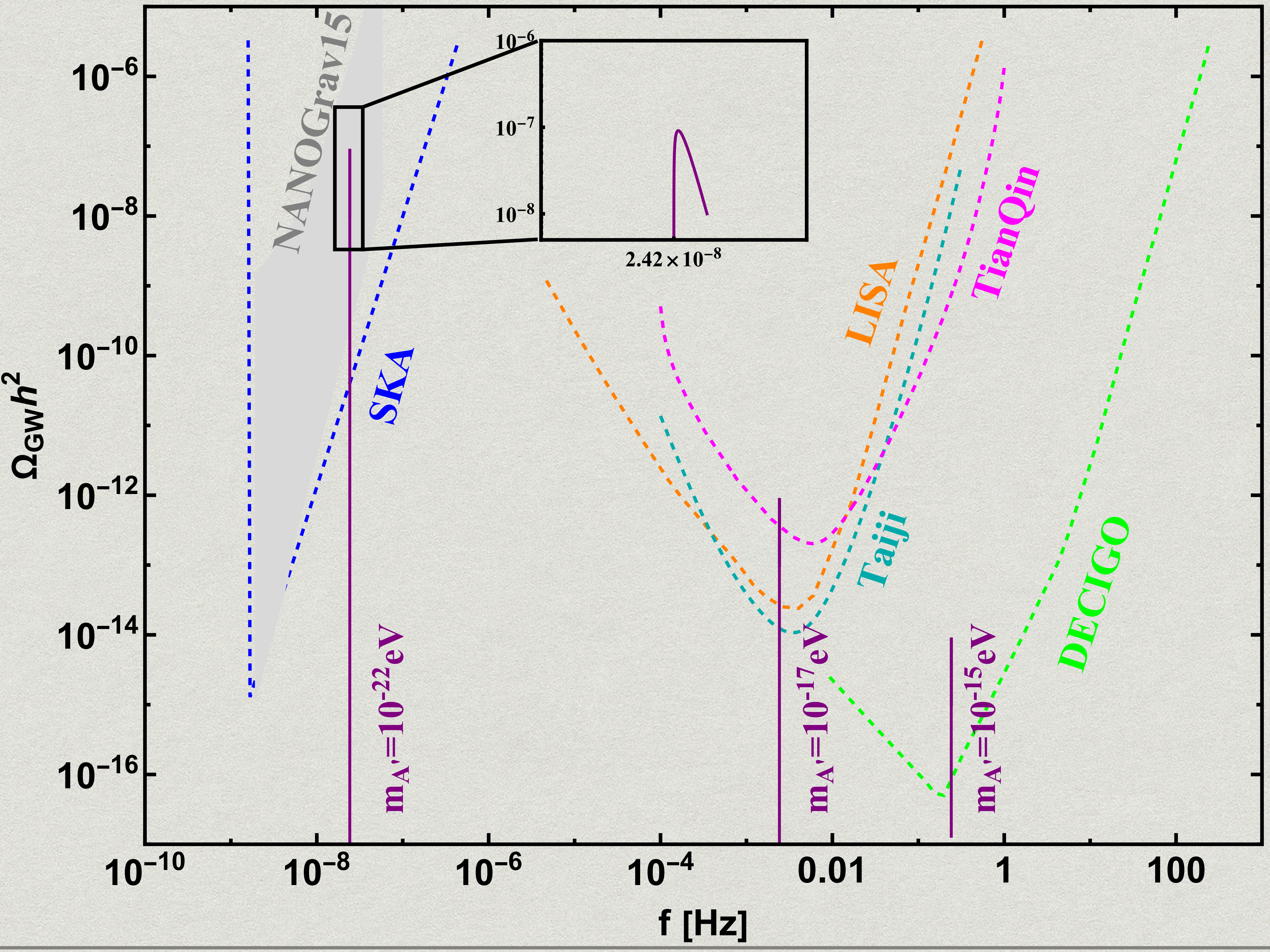
Ultralight DM oscillating into GWs

Dark photon DM



Ultralight DM oscillating into GWs

Dark photon DM



Ultralight DM oscillating into GWs

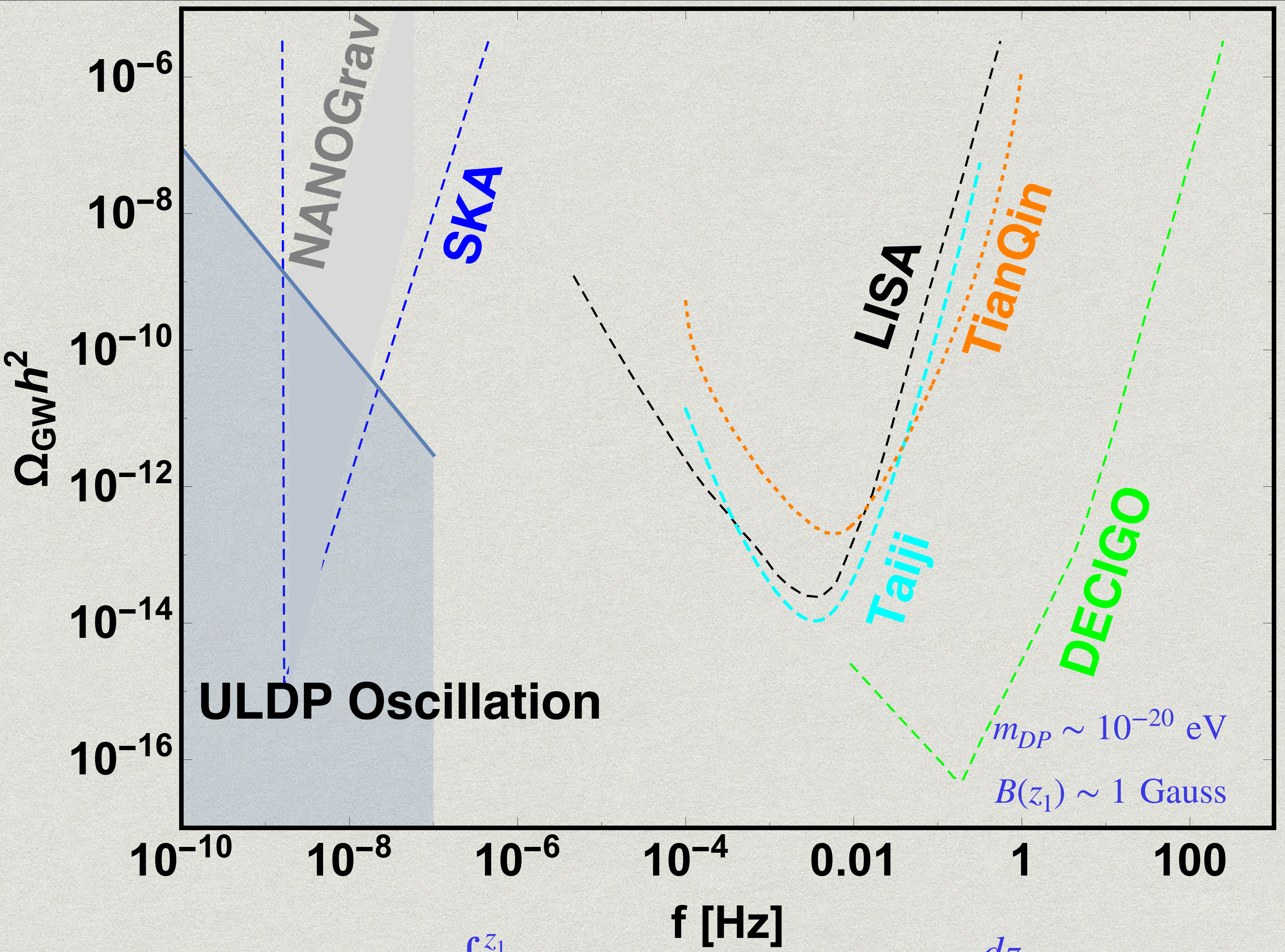
Dark photon oscillations in the early universe

Possible mechanisms for the production of primordial dark magnetic field

- PDMF can be generated during the pseudo-scalar inflation, ϕ, with the Chern-Simons interaction, $\frac{\phi}{4\pi f_a} F \widetilde{F}$.
- PDMF can be produced from the axion oscillations.
- PDMF can be generated from the oscillation of photon in the axion or some other exotic background.

Ultralight DM oscillating into GWs

Dark photon oscillations in the early universe



$$\rho_{\rm SGW} = \int_{z_0}^{z_1} n_{\rm ULDP}(z_0)\mathscr{R}(z)m_{\rm ULDP}\frac{dz}{(1+z)^2H(z)}$$

Outline

- GW from oscillations of ultralight DM
- GW from the scalar DM triggered EWPT
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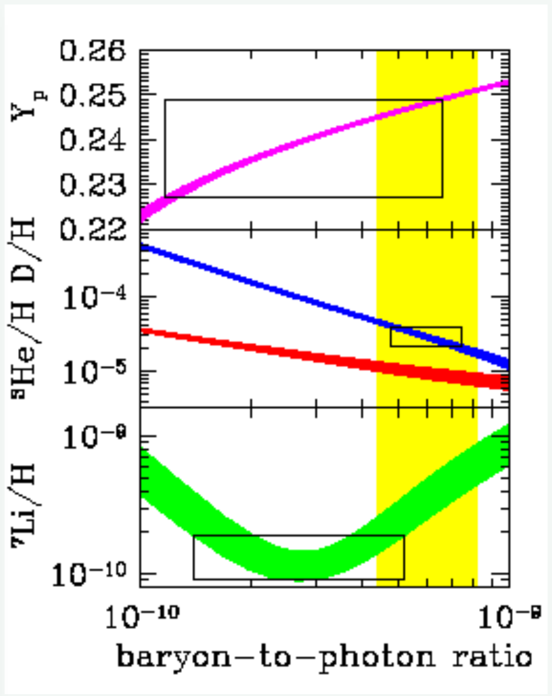
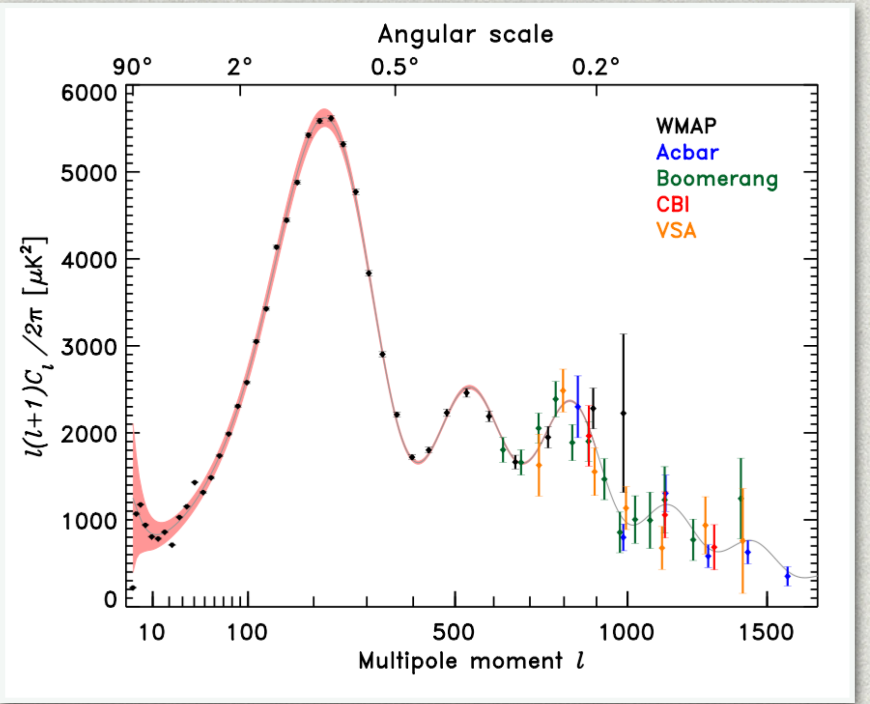
Why First order EWPT

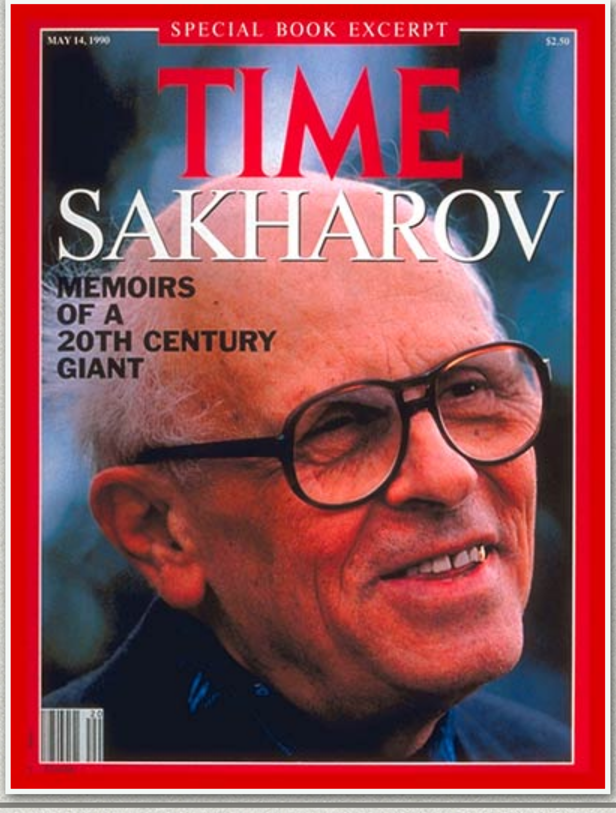
Excess of matter over anti-matter in the Universe!

Both Big bang Nucleosynthesis and measurements of CMB have:

$$\eta_B = \frac{\rho_B}{s} = (8.61 \pm 0.09) \times 10^{-11}$$





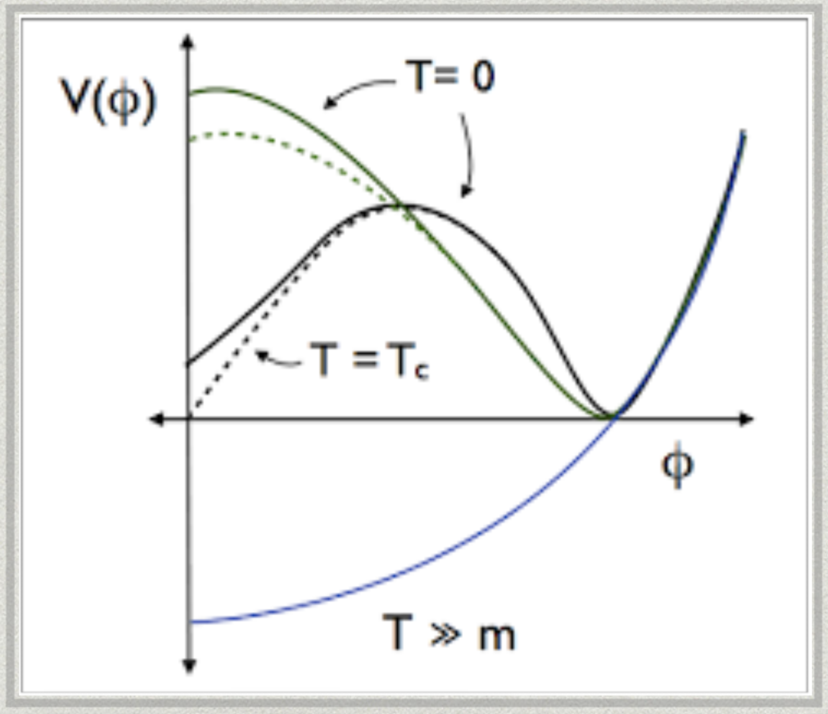


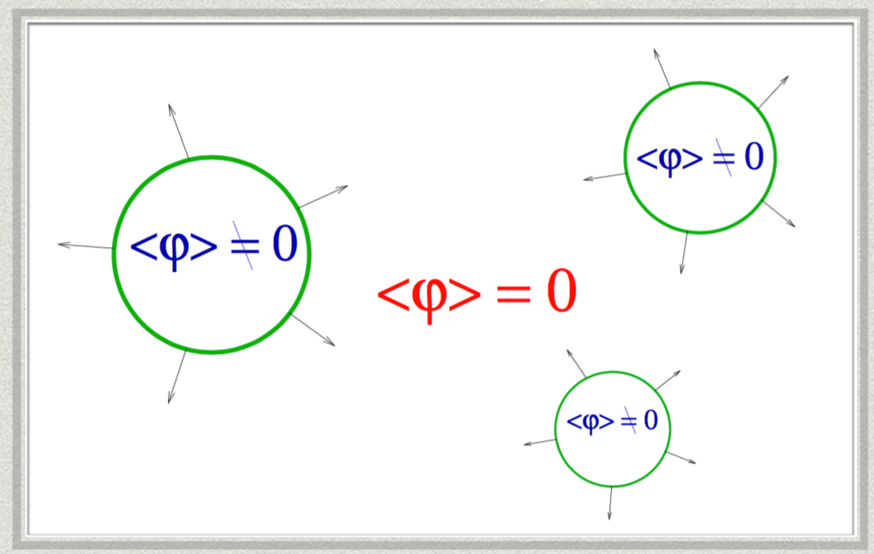
- ★Baryon number violation
- ★C&CP violation
- ★Departure from thermal equilibrium

First order electroweak phase transition if baryon asymmetry is generated during the EWPT without CPT violation.

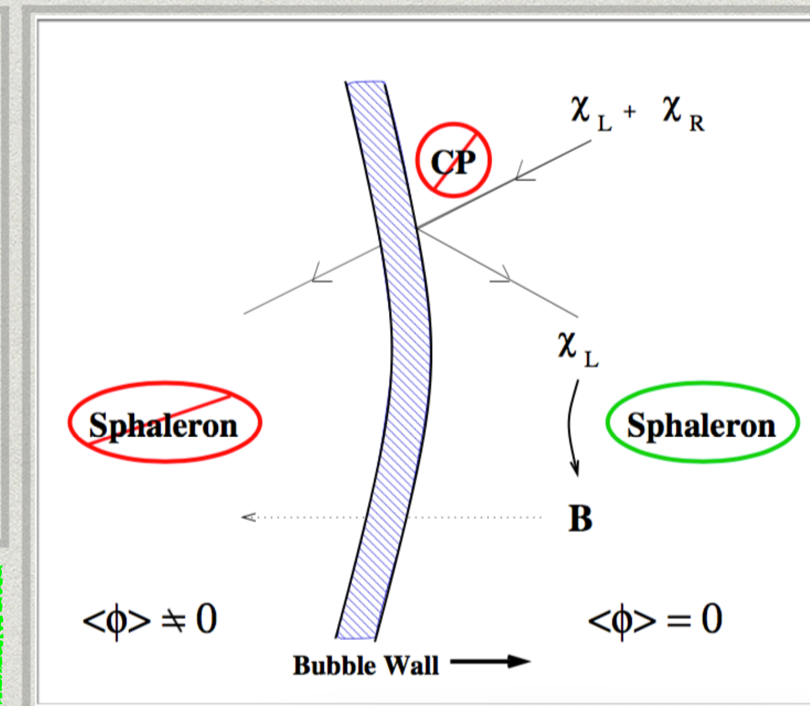
Electroweak Baryogenesis

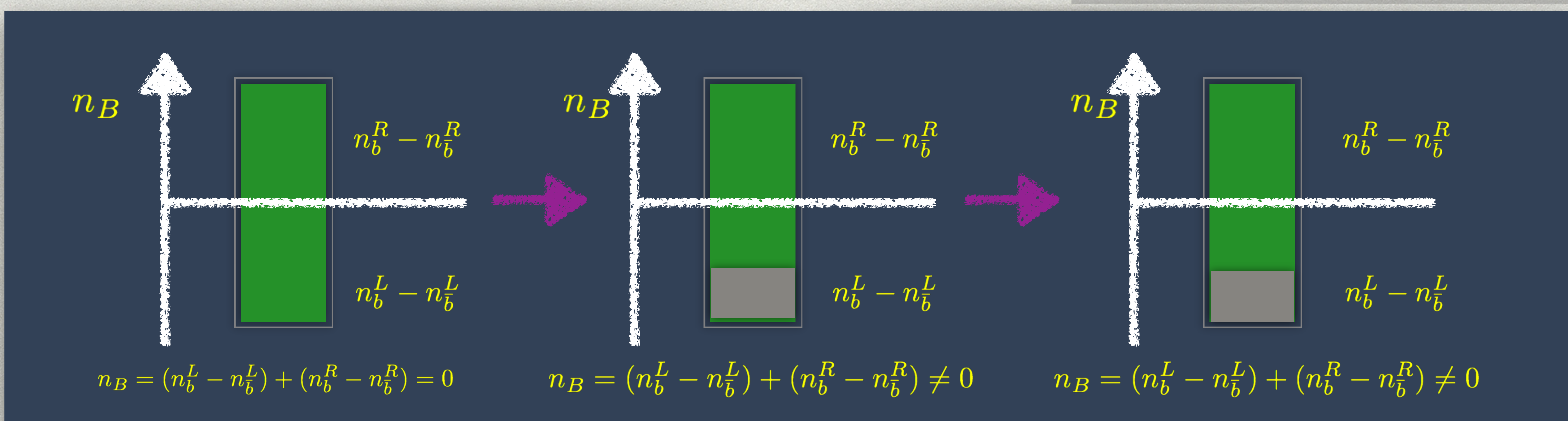
* **Generate BAU during the electroweak phase transition**





Basic description



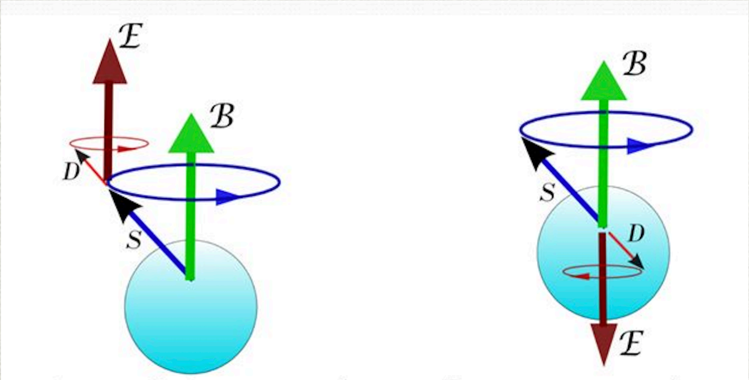


Fate of the EWBG

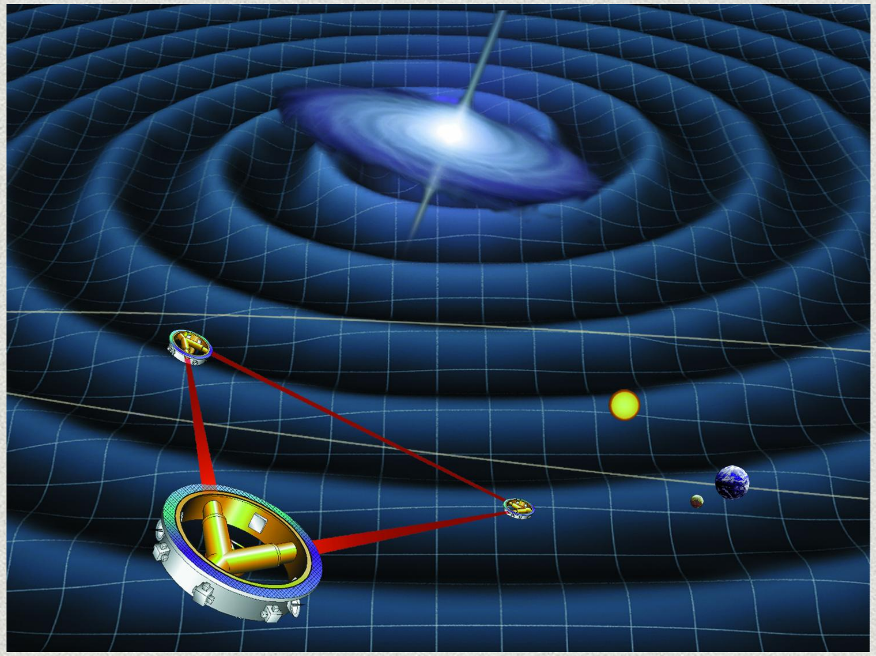
Three Detectives

LHC

EDM



GW



Conventional EWBG mechanism might be found or excluded in the near future when these three detection methods are combined.

A typical example: Wino-catalyzed EWBG is excluded by the ACME result(intensity frontier) and the Higgs search results at the LHC(energy frontier).

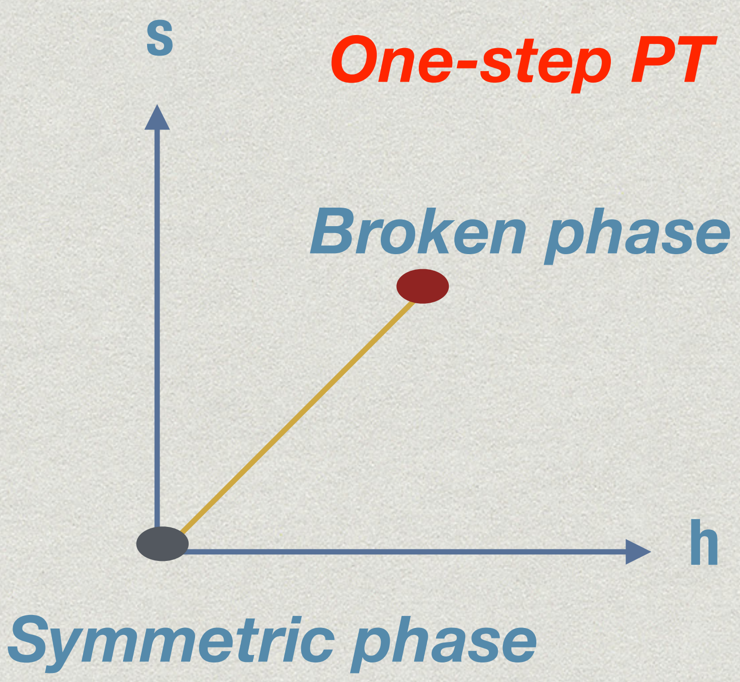
Questions: Is there a mechanism of electroweak baryogenesis that can escape from these hunters?

Bubble dynamics

1. Patterns of PT.

SM Higgs is too heavy to saturate first order EWPT

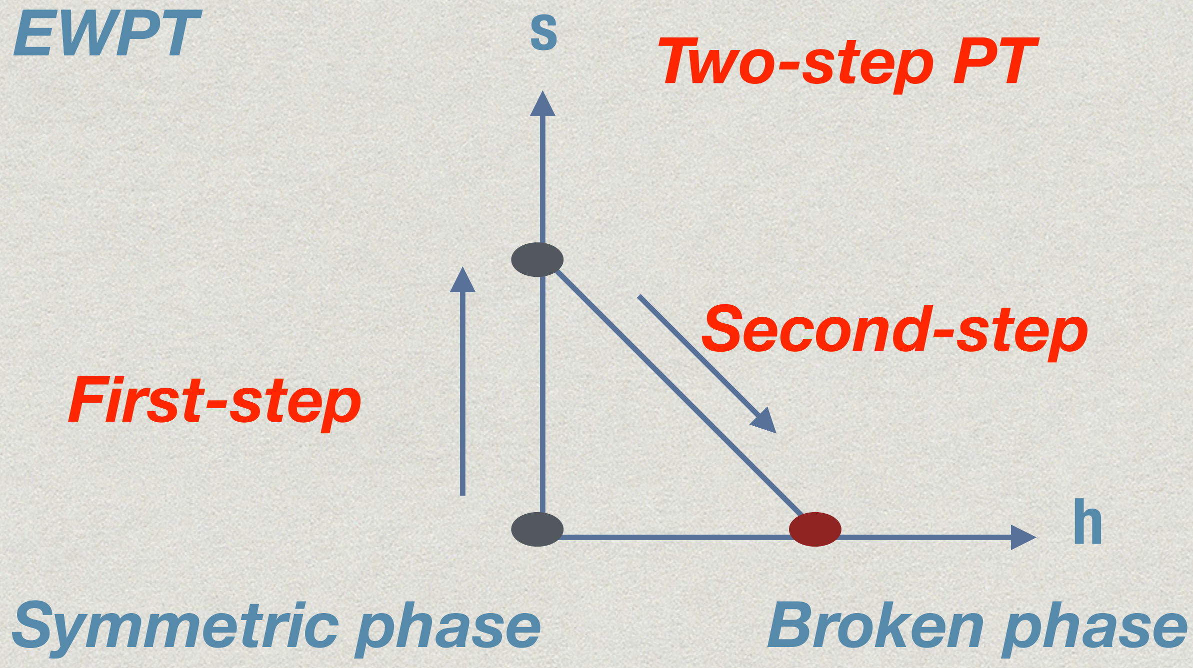
One-step PT



The barrier between the symmetric and the broken phase usually comes from the gauge fields

$$V_{\rm eff}(\phi,T)=\mathcal{A}(T)\phi^2+\mathcal{B}(T)\phi^3+\mathcal{C}(T)\phi^4+\cdots$$

Two-step PT



The barrier exists at the tree-level

Merits:

1. No mixing with the SM Higgs
2. Correlated with the dark matter
3. Avoid the gauge dependent problem.

GW from the PT

Basics of gravitational wave from EWPT

Gravitational waves are described by a transverse-traceless gauge invariant perturbation, h_{ij}, in a FRW metric,	$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$
Einstein eq for transverse-traceless part	$h''_{ij} - \nabla^2 h_{ij} = 16\pi G\{(e+p)\gamma^2 v_i v_j + \partial_i \phi \partial_j\}$
Gravitational wave energy density	$\rho_{gw} = \frac{1}{32\pi G}\frac{1}{2\pi^2}\int dk k^2 P_{\dot{h}}(k)$
Energy spectrum	$h^2\Omega_{\rm GW}(f) = \frac{h^2}{\rho_c}\frac{d\rho_{\rm gw}}{d\log f}$

GW from the PT

Sources of GW from EWPT:

- ✦ *Collisions of bubble wall and shocks in the plasma*
- ✦ *Sound wave after the collision but before the expansion has dissipated the kinetic energy.*
- ✦ *Magnetohydrodynamic turbulence : percolation can also induce MHD turbulence since the plasma is fully ionized.*

Fitted results of GW spectrum

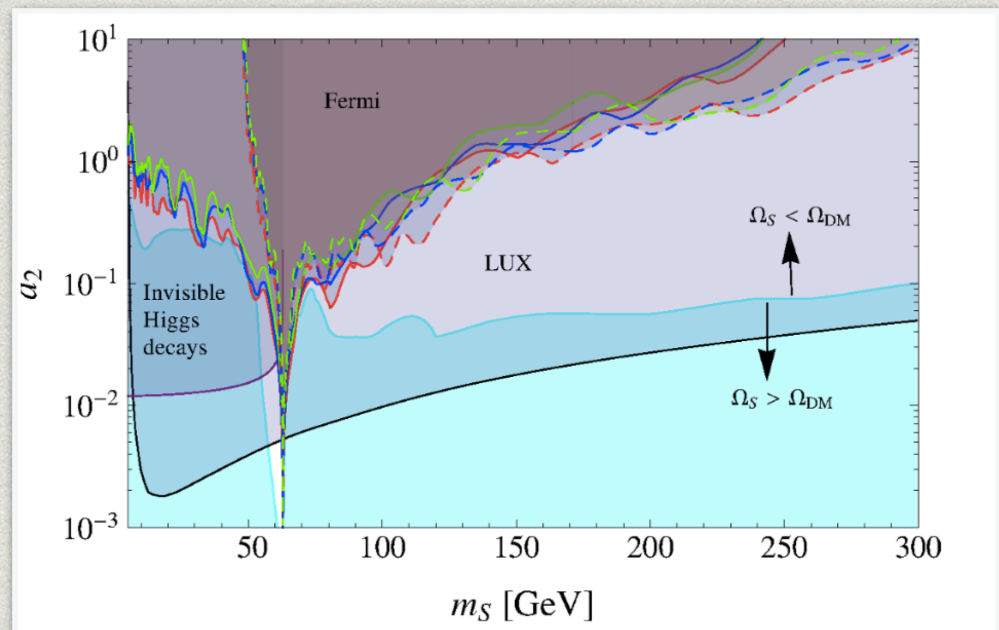
Bubble collision	$h^2\Omega_{\rm coll}(f) = 1.67\times 10^{-5}\left(\frac{H_n}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\times\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)\left[\frac{3.8(f/f_{\rm coll})^{2.8}}{1+2.8(f/f_{\rm coll})^{3.8}}\right],$
Sound wave	$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{H_n}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2}$
MHD turbulence	$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_n}{\beta}\right)\left(\frac{\kappa_{\rm tu}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\times v_w\frac{(f/f_{\rm tu})^3}{(1+f/f_{\rm tu})^{11/3}(1+8\pi f/h_n)}$

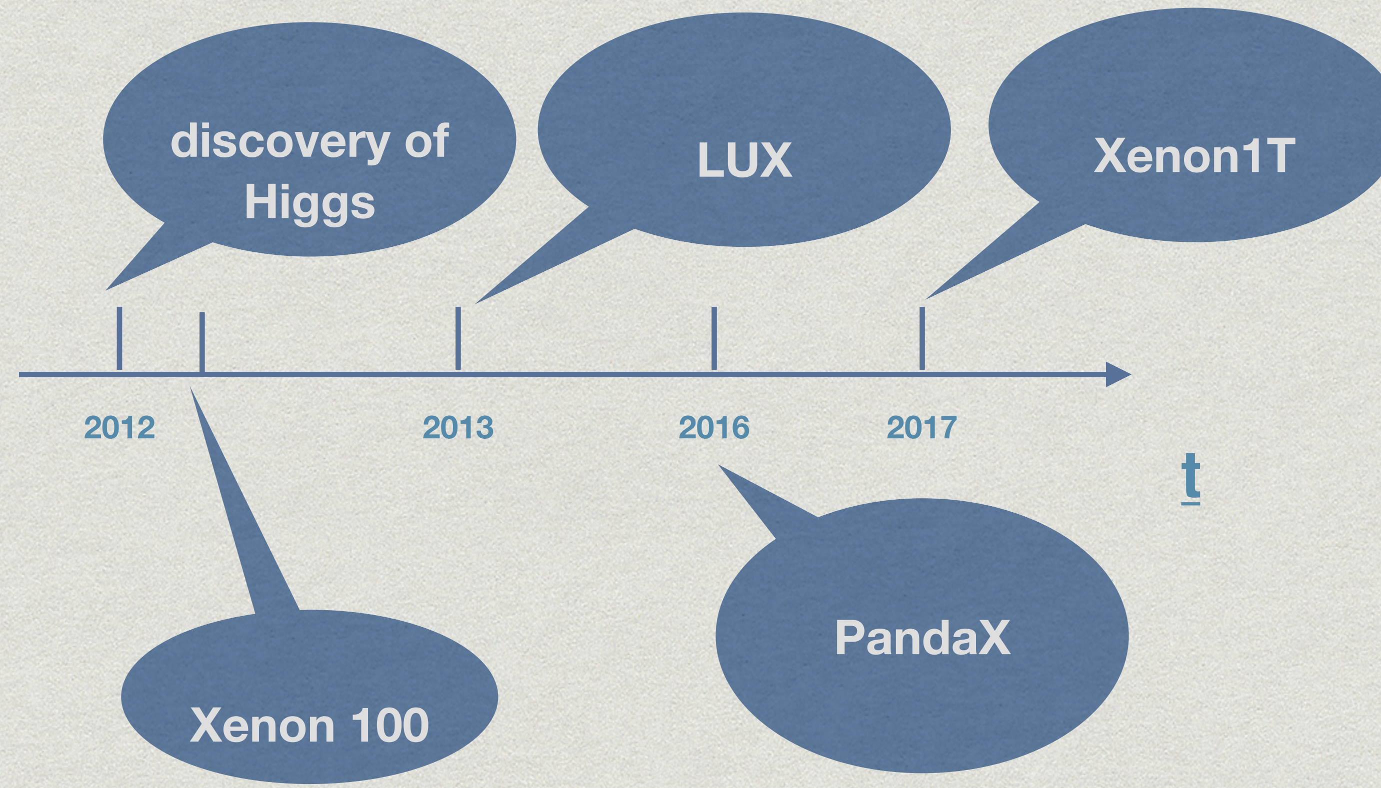
Total energy spectrum:

$$h^2\Omega_{\rm GW}\approx h^2\Omega_{\rm coll}+h^2\Omega_{\rm sw}+h^2\Omega_{\rm turb}$$

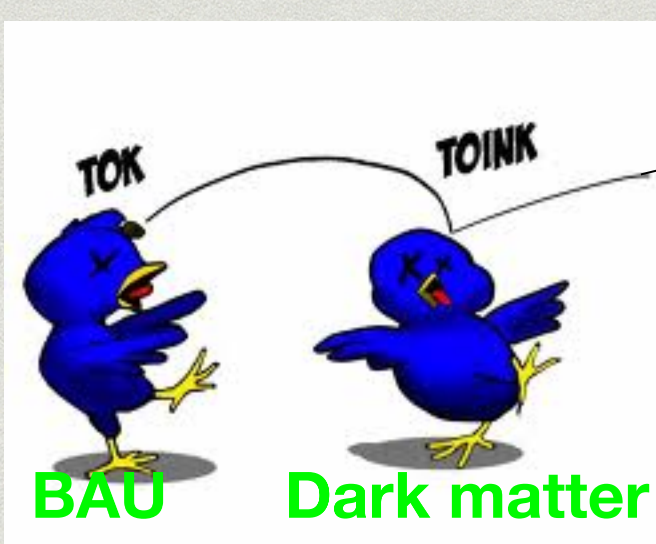
GW from the PT

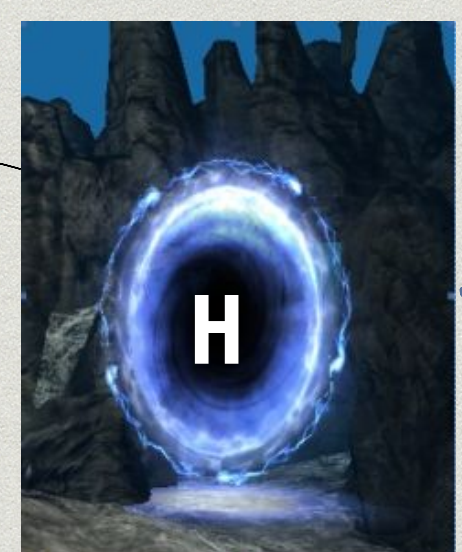
GW from EWPT, a typical model:





Ordinary Higgs portal is dead since 2013!





GW from the PT

The model: SM + two real scalar singlets

$$V_0 = -\frac{1}{2}\mu_\Phi^2\Phi^2 + \frac{1}{4}\lambda_\Phi\Phi^4 - \frac{1}{2}\mu_S^2 S^2 + \frac{1}{4}\lambda_S S^4 - \mu^2 H^\dagger H + \lambda(H^\dagger H)^2 + \lambda_1 S^2 H^\dagger H + \lambda_2 \Phi^2 H^\dagger H + \lambda_3 S^2\Phi^2,$$

Real singlet

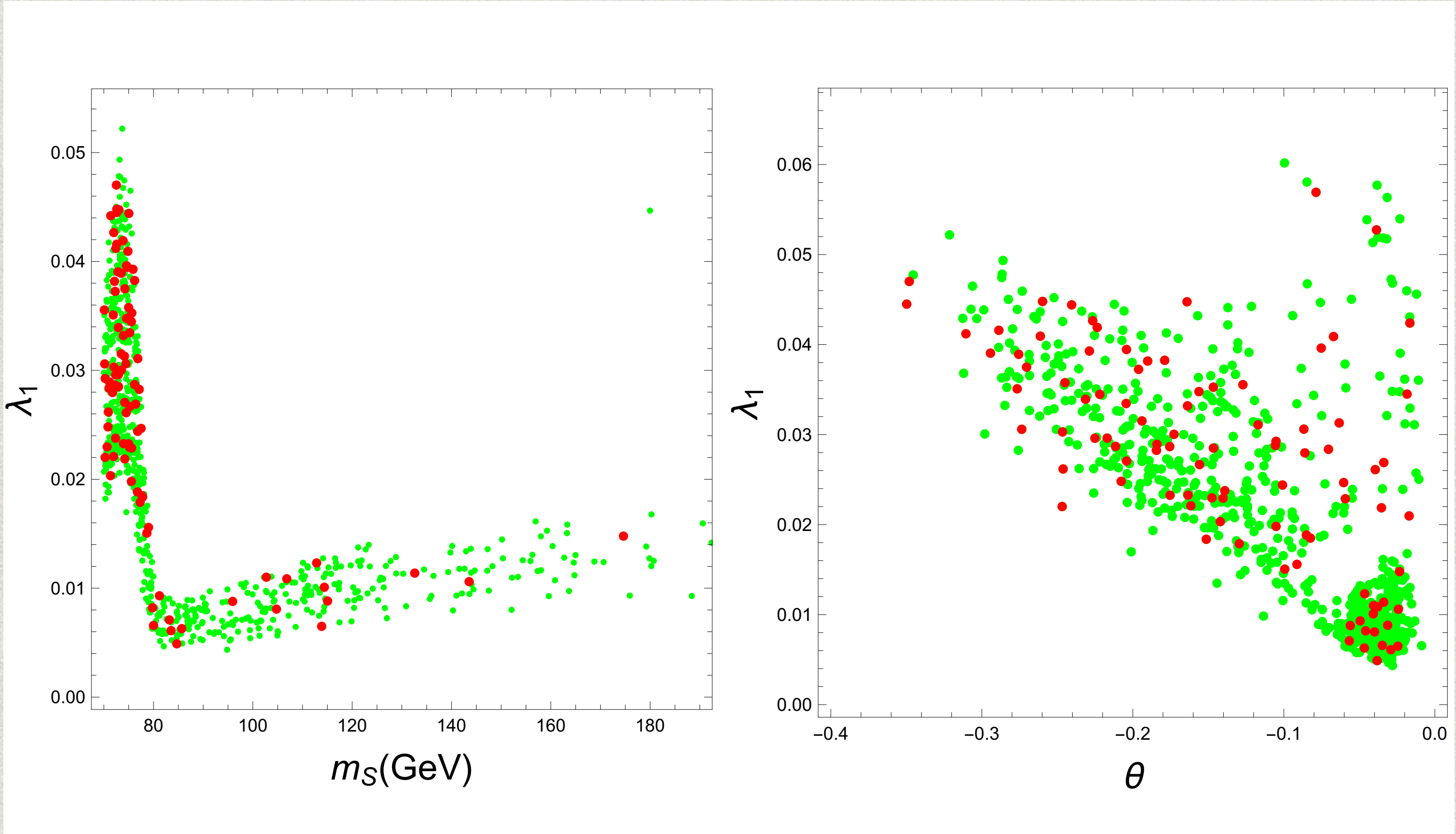
Dark matter

SM Higgs

Physical parameters: $v_\Phi,\ m_\Phi,\ m_S,\ \lambda_S,\ \theta,\ \lambda_1,\ \lambda_3$

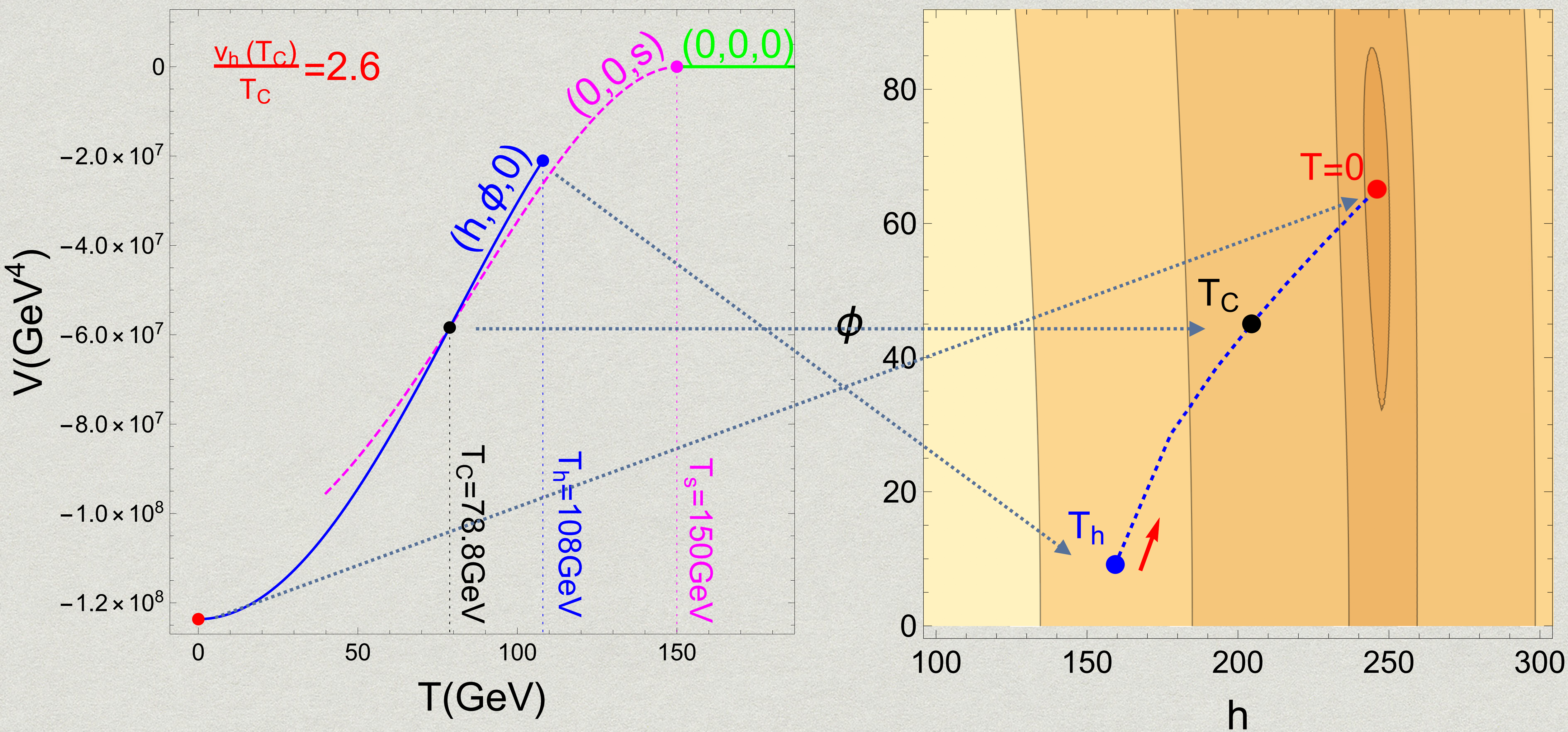
Conditions for a negligible DD cross section:

$$\lambda_3 = \frac{v_{\rm EW}\lambda_1(m_{\hat h}^2\tan\theta + m_{\hat\phi}^2\cot\theta)}{2v_\Phi(m_{\hat h}^2 - m_{\hat\phi}^2)}$$



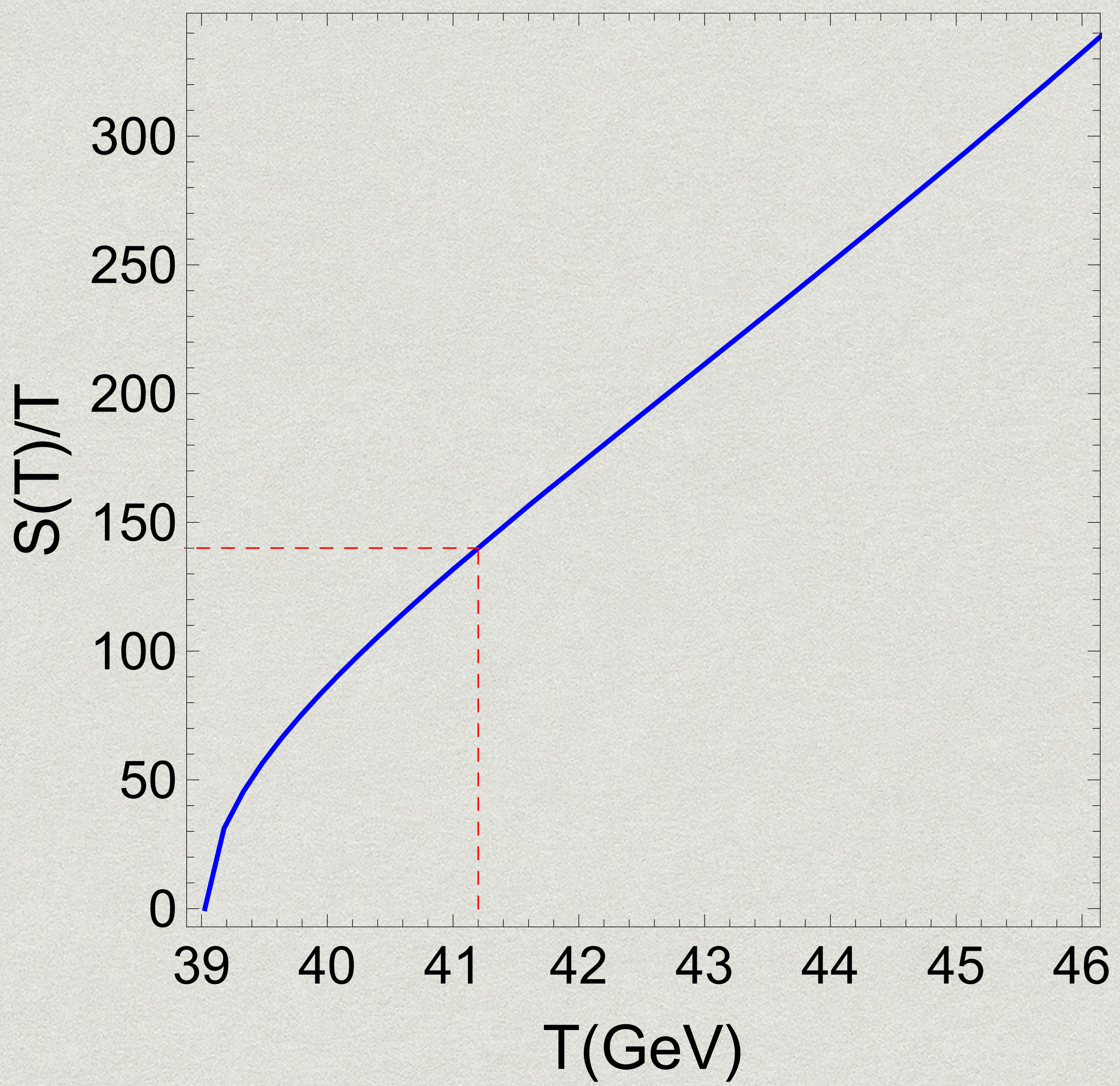
GW from the PT

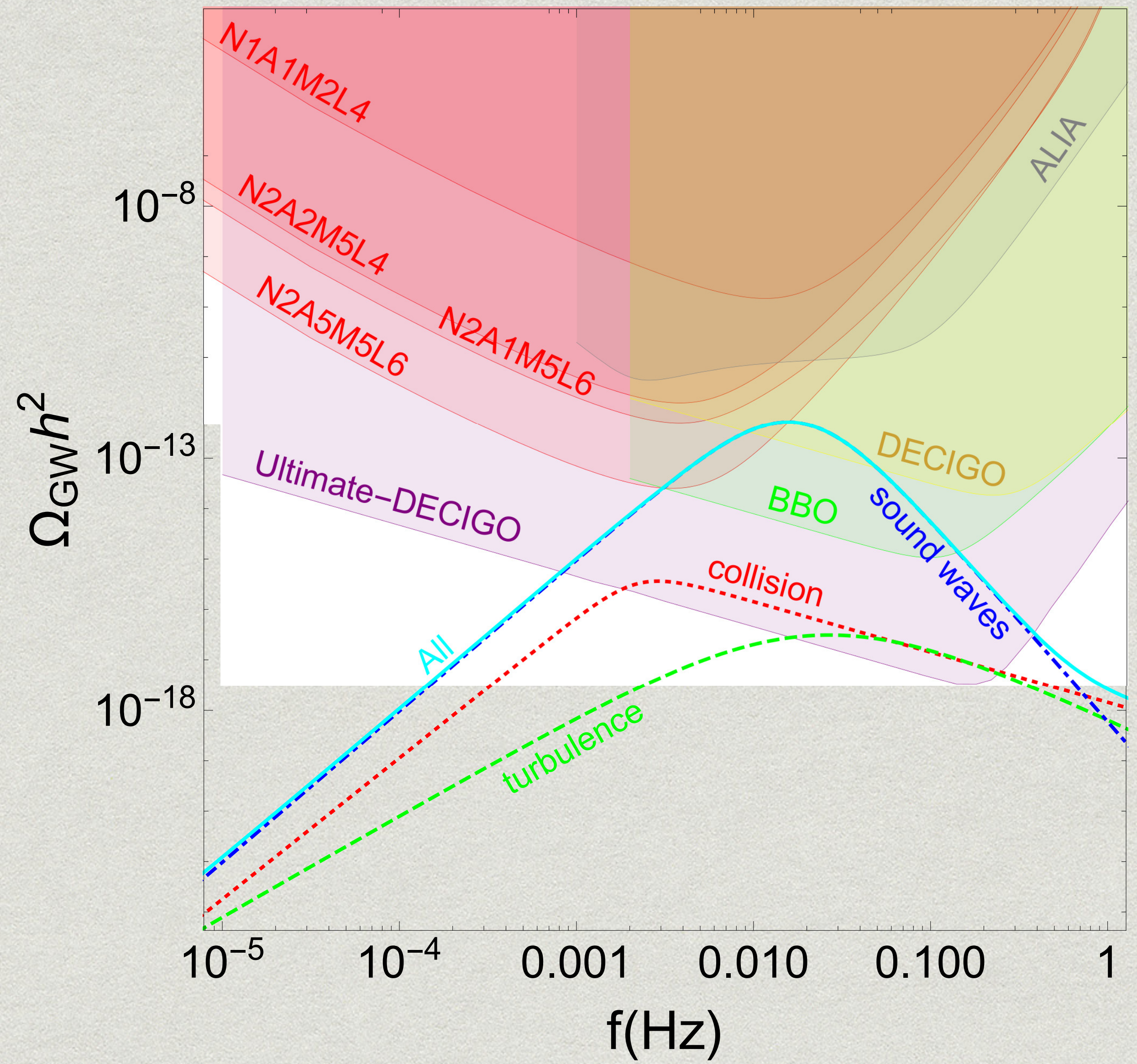
PT details:



GW from the PT

GW spectrum:





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- GW from oscillations of ultralight DM
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Freeze-in Mechanism

L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, JHEP 03, 080 (2010), arXiv:0911.1120 [hep-ph]

DM with negligible coupling with the plasma: $\mathscr{L} \supset -\beta\,\phi\,\bar{\psi}\psi,$

Boltzmann equation:

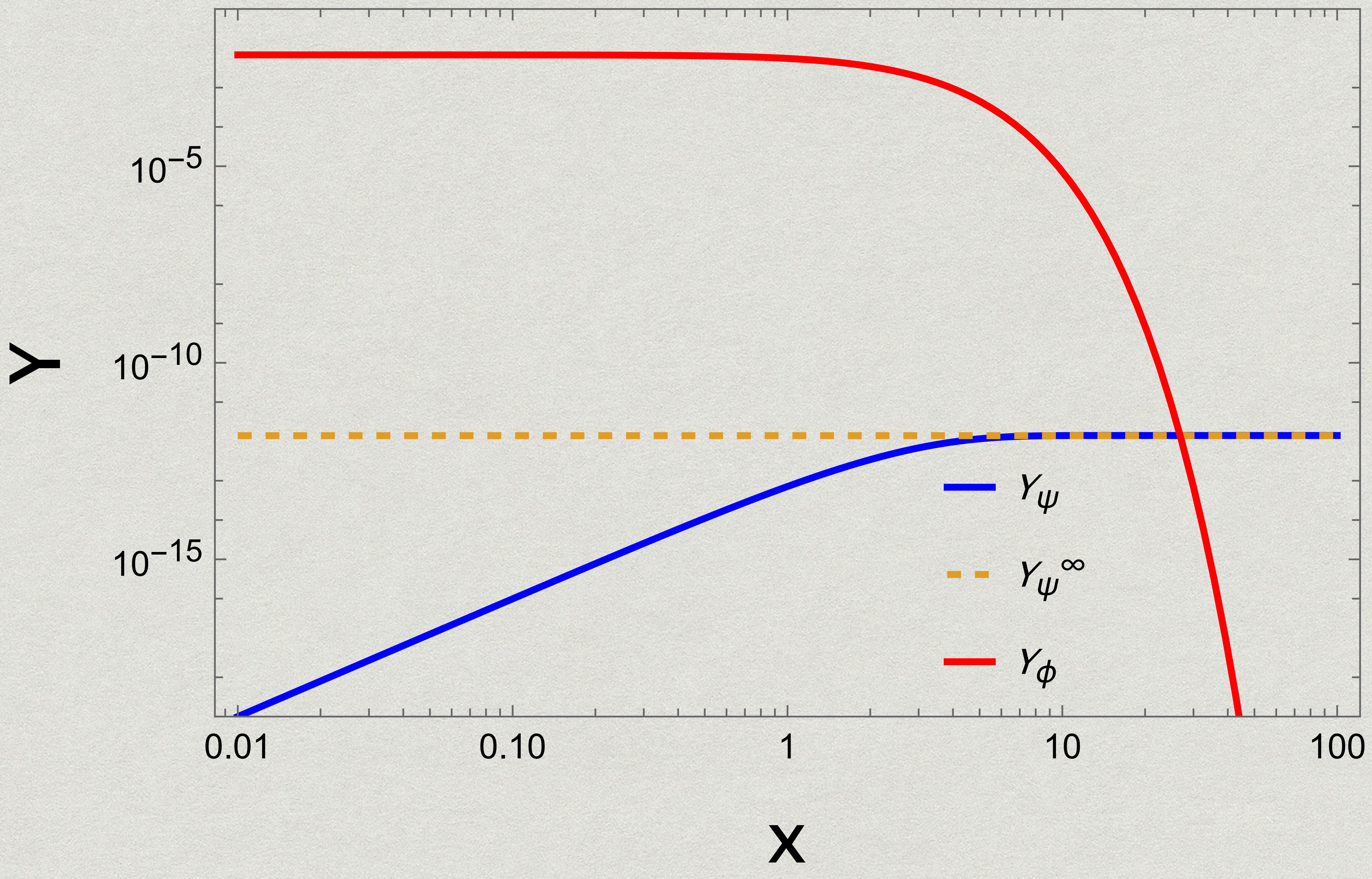
$$\begin{aligned}\frac{dn_\psi}{dt} + 3Hn_\psi &= g_\phi \int d\Pi_\phi\, d\Pi_\psi\, d\Pi_{\bar{\psi}}\, (2\pi)^4 \delta^4(p_\phi - p_\psi - p_{\bar{\psi}})\, |\mathscr{M}|^2_{\phi\to\psi\bar{\psi}}\, f_\phi \\ &= \frac{1}{2\pi^2} m_\phi^2 T\, K_1(m_\phi/T)\, \Gamma_{\phi\to\psi\bar{\psi}},\end{aligned}$$

Boltzmann equation of comoving number density:

$$\frac{dY_\psi}{dx} = \frac{90\sqrt{90}}{(2\pi)^6} \frac{M_P}{\sqrt{g_*^\rho g_*^S}} \frac{\beta^2}{m_\phi} x^3 K_1(x), \longrightarrow Y_\psi^\infty = \frac{135\sqrt{90}}{2(2\pi)^5} \frac{M_P}{\sqrt{g_*^\rho g_*^S}} \frac{\beta^2}{m_\phi}.$$

Freeze-in Mechanism

L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, JHEP 03, 080 (2010), arXiv:0911.1120 [hep-ph]



Ultraviolet Freeze-in Mechanism

F. Elahi, C. Kolda, and J. Unwin, Journal of High Energy Physics 2015, 1 (2015).

DM with high dimensional coupling with the plasma: $\mathscr{L} \supset -\frac{1}{\Lambda}\chi\chi^{\dagger}\bar{f}f,$

Boltzmann equation:

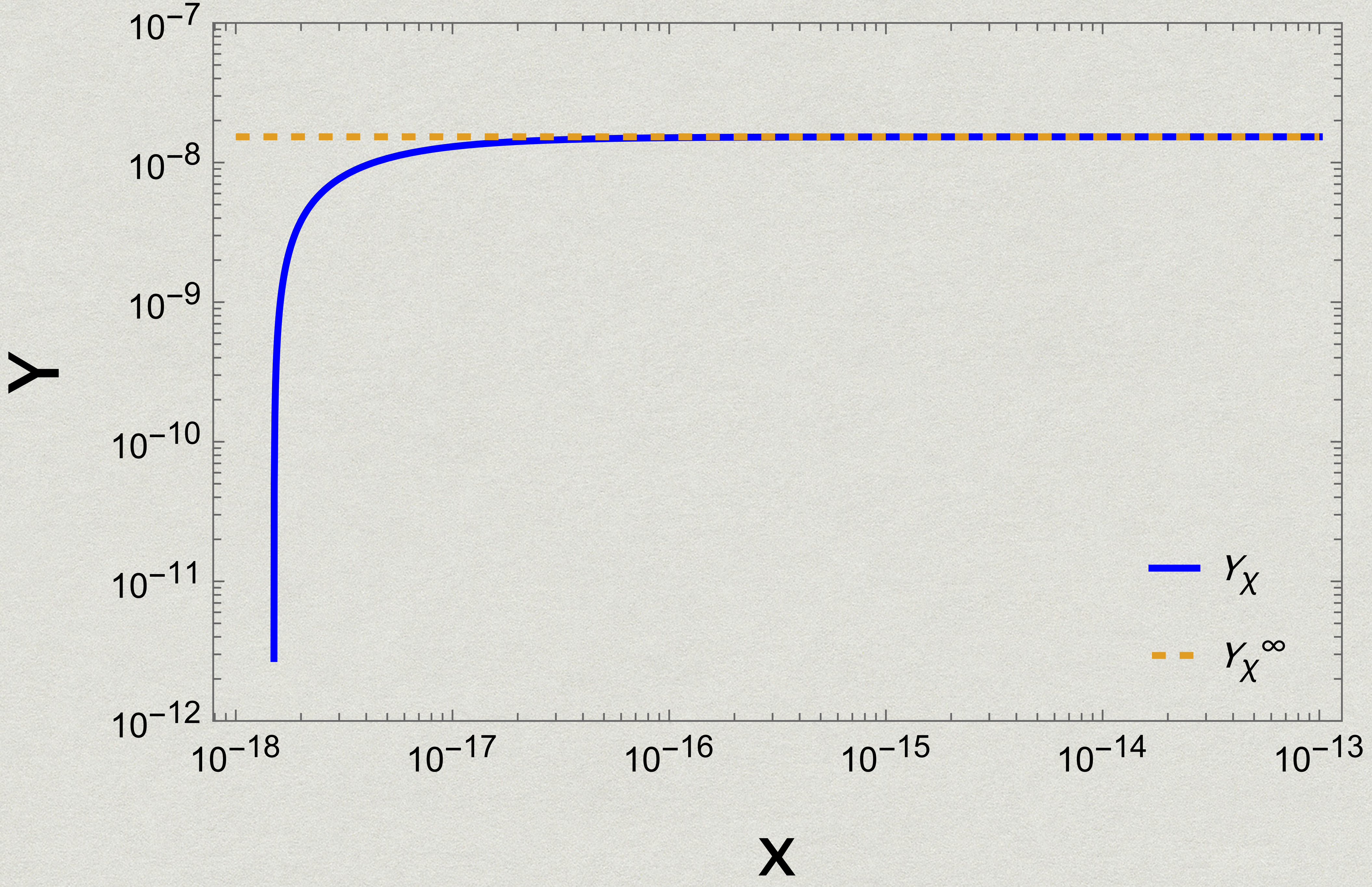
$$\frac{dn_\chi}{dt} + 3Hn_\chi = g_f g_{\bar{f}} \int d\Pi_f \, d\Pi_{\bar{f}} \, d\Pi_\chi \, d\Pi_{\chi^\dagger} \, (2\pi)^4 \delta^4(p_f + p_{\bar{f}} - p_\chi - p_{\chi^\dagger}) \, |\mathscr{M}|^2_{f\bar{f}\to\chi\chi^\dagger} \mathscr{F}_f \mathscr{F}_{\bar{f}},$$

For comoving number density:

$$\frac{dY_\chi}{dT} = -\frac{45\sqrt{90}}{2\pi^3(2\pi)^5} \frac{1}{g_*^S\sqrt{g_*^\rho}} \frac{M_P}{\Lambda^2} \longrightarrow Y_\chi^\infty \simeq \frac{180\sqrt{90}}{(2\pi)^8} \frac{1}{g_*^S\sqrt{g_*^\rho}} \frac{M_P T_R}{\Lambda^2}$$

Ultraviolet Freeze-in Mechanism

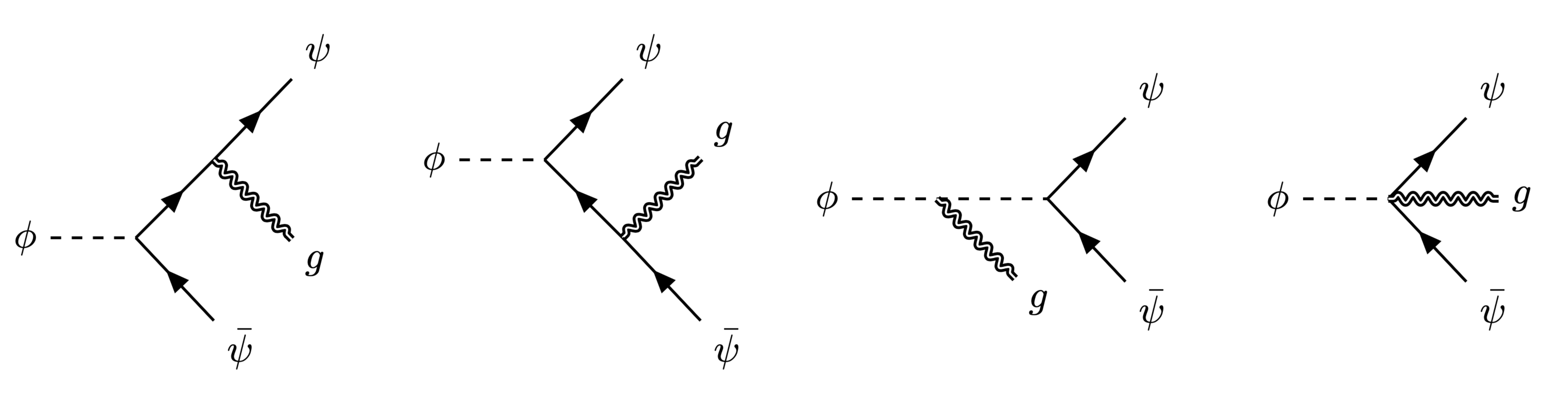
F. Elahi, C. Kolda, and J. Unwin, Journal of High Energy Physics 2015, 1 (2015).



1. A nice mechanism to address the DM production; 2. It is difficult to be tested in the laboratory

Graviton bremsstrahlung in the freeze-in process

IR freeze-in case:



Boltzmann equation:

$$\frac{d\rho_{GW}}{dt} + 4H\rho_{GW} = g_\phi \int d\Pi_\phi d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_g (2\pi)^4 \delta^4(p_\phi - p_\psi - p_{\bar{\psi}} - p_g) |\mathcal{M}|^2 f_\phi \omega$$

$$\frac{d}{dx}\left(\frac{dY_{GW}}{d\omega}\right) = \frac{T^2}{m_\phi}\frac{1}{HTS^{4/3}}\frac{dC_{\phi\to\psi\bar{\psi}g}}{d\omega} = \left(\frac{45}{2\pi^2 g_*^S(T)}\right)^{4/3} \frac{\beta^2\sqrt{90}}{4(2\pi)^6\sqrt{g_*^\rho(T)}}\frac{\omega_D^2}{M_P T_D^2}$$

$$\times K_1(x)\, x^2 \left(1 - \frac{2\omega_D}{T_D}\frac{1}{x}\right)\left[2 - \frac{2T_D}{\omega_D}x + \left(\frac{T_D}{\omega_D}x\right)^2\right]$$

Graviton bremsstrahlung in the freeze-in process

IR freeze-in case:

Redshift of graviton

$$\omega(T) = \omega_D \frac{a(T_D)}{a(T)} = \left(\frac{g_*^S(T)}{g_*^S(T_D)}\right)^{1/3} \frac{T}{T_D}\omega_D$$

GW Spectrum

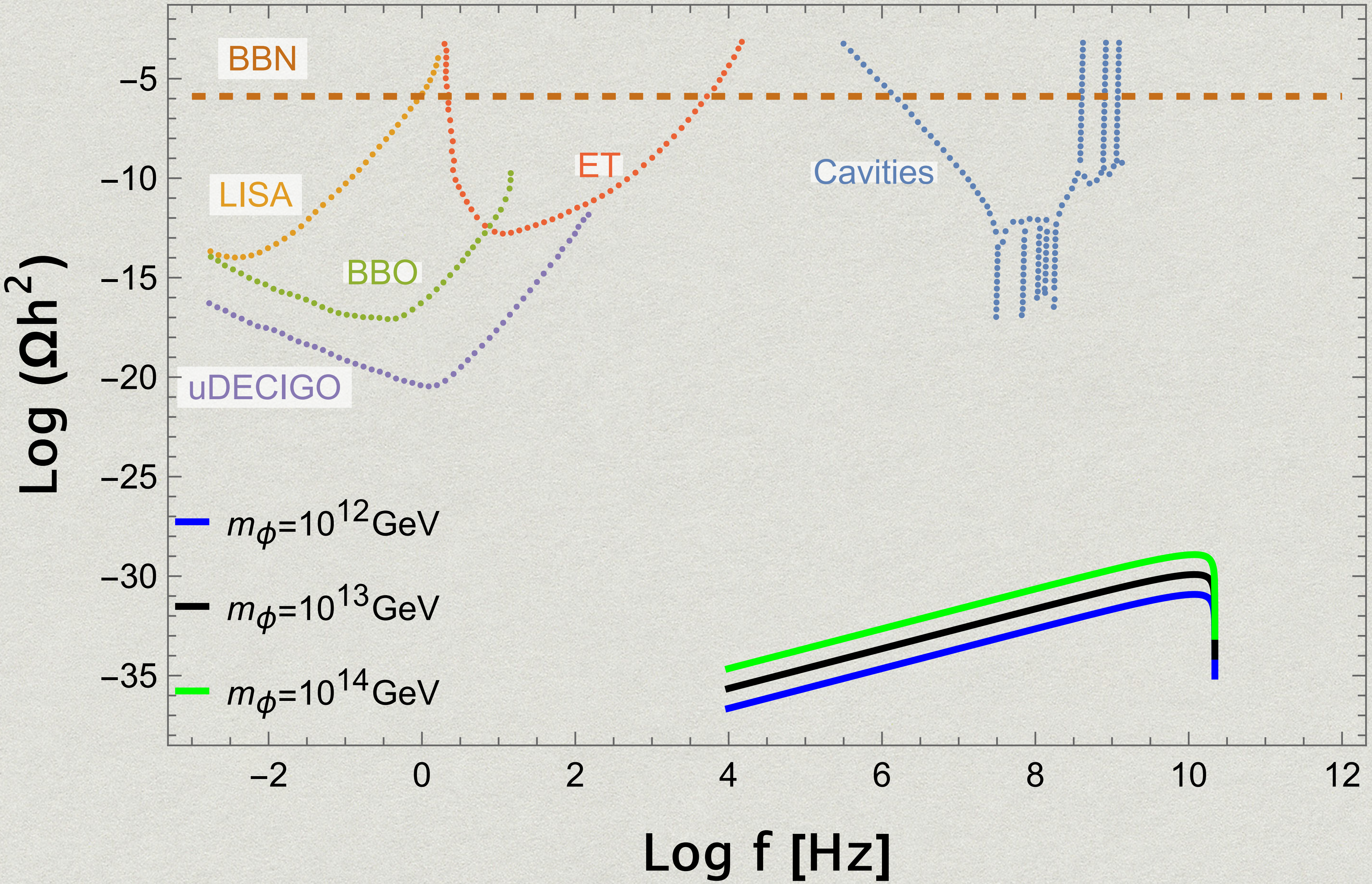
$$\Omega_{GW}(f) \equiv \frac{1}{\rho_{cr,0}} \frac{d\rho_{GW}(T_0)}{d\ln\omega_0} = \frac{1}{\rho_{cr,0}} \frac{dY_{GW}^{\infty}}{d\ln\omega_D} S^{4/3}(T_0)$$

$$= \frac{T_0^4}{\rho_{cr,0}} \frac{g_*^S(T_0)}{g_*^S(T_D)} \frac{\sqrt{90}\beta^2 T_D}{2(2\pi)^5\sqrt{g_*^\rho(T_D)}M_P} \frac{f}{T_0} \times$$

$$\left[8 - 6\pi^2 \frac{f}{T_0}\left(\frac{g_*^S(T_D)}{g_*^S(T_0)}\right)^{1/3} + 24\pi^2 \frac{f^2}{T_0^2}\left(\frac{g_*^S(T_D)}{g_*^S(T_0)}\right)^{2/3} - 8\pi^4 \frac{f^3}{T_0^3} \frac{g_*^S(T_D)}{g_*^S(T_0)}\right]$$

Graviton bremsstrahlung in the freeze-in process

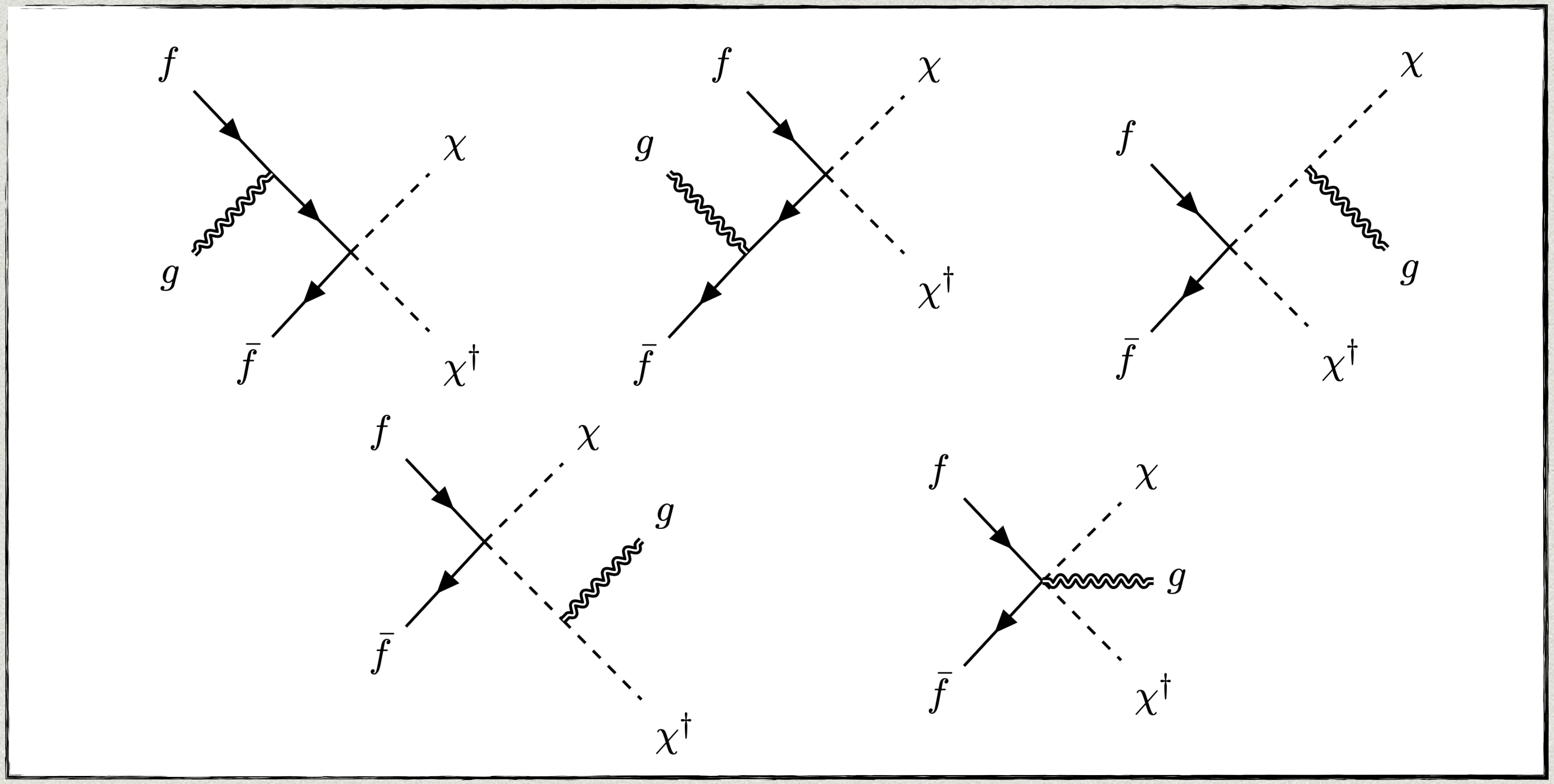
IR freeze-in case:



Max frequency: $$f_{max} = \frac{1}{2\pi}\omega_D^{max}\frac{a(T_D)}{a(T_0)} = \frac{1}{2\pi}\frac{m_\phi}{2}\left(\frac{g_*^S(T_0)}{g_*^S(T_D)}\right)^{1/3}\frac{T_0}{T_D} \simeq 2.3\times 10^{11}\mathrm{Hz}$$

Graviton bremsstrahlung in the freeze-in process

UV freeze-in case:



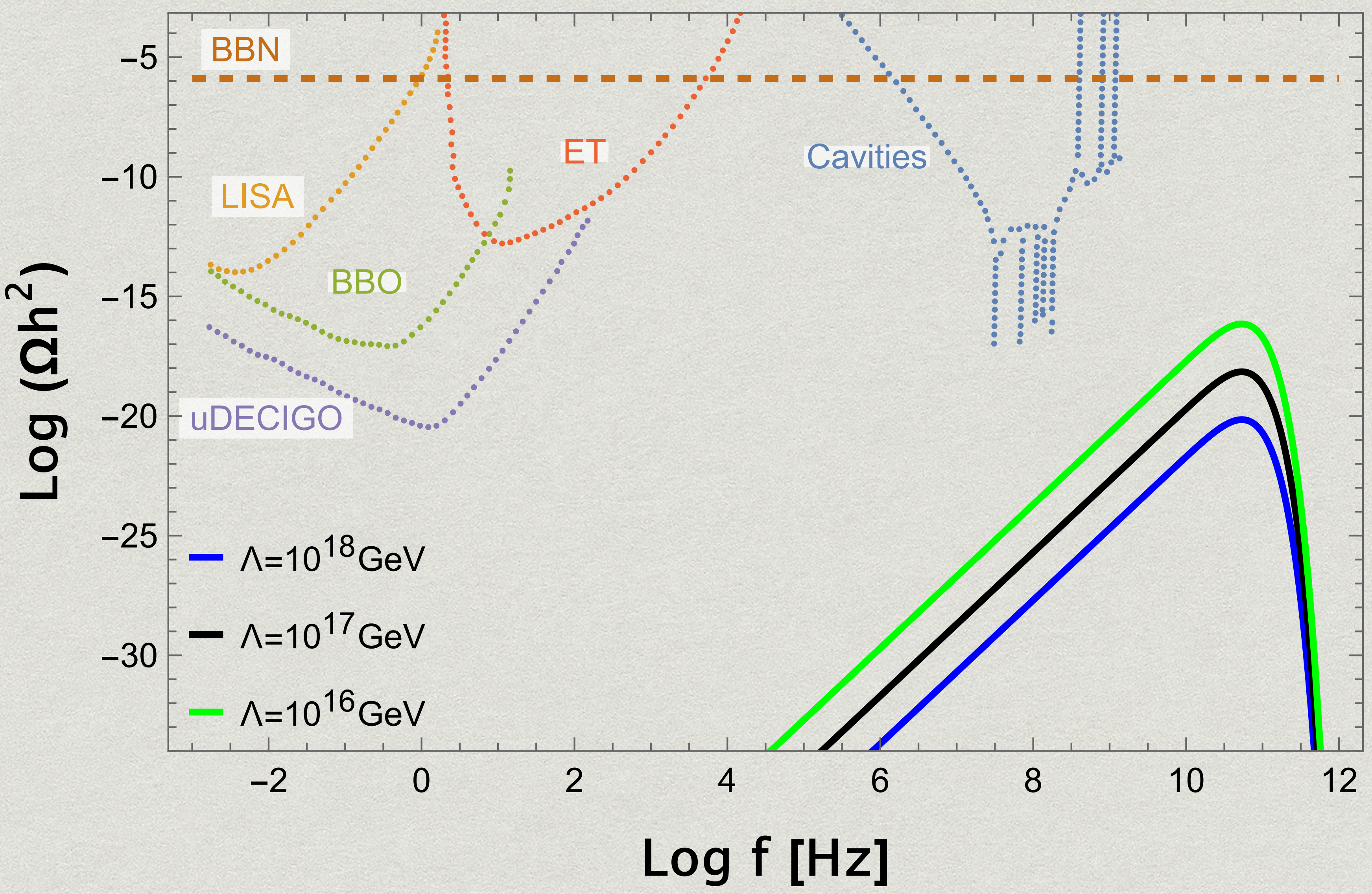
Differential collision term for GWs:

$$\frac{d}{d\omega} C_{f\bar{f}\to\chi\chi^\dagger g} = \frac{1}{(2\pi)^7}\frac{T^6\omega^2}{M_P^2\Lambda^2} G^{3,0}_{1,3}\left(\frac{\omega^2}{T^2}\middle|\begin{matrix}1\\0,2,3\end{matrix}\right)$$

$$\frac{dY^\infty_{GW}}{d\omega_E} = \left(\frac{45}{2\pi^2 g_*^S(T_E)}\right)^{4/3}\frac{\sqrt{90}}{\sqrt{g_*^\rho(T_E)}}\frac{1}{(2\pi)^8}\frac{\omega_E^2 T_R^2}{M_P\Lambda^2 T_E^2} G^{3,0}_{1,3}\left(\frac{\omega_E^2}{T_E^2}\middle|\begin{matrix}1\\0,2,3\end{matrix}\right)$$

Graviton bremsstrahlung in the freeze-in process

UV freeze-in case:



Spectrum at peak frequency $\Omega_{GW}h^2(f_{peak}) = 7.1 \times 10^{-17} \left(\frac{T_R}{10^{16}\,\mathrm{GeV}}\right)^3 \left(\frac{\Lambda^2}{10^{32}\,\mathrm{GeV}^2}\right)^{-1}$

Conclusion

We have calculated GW spectrum arising from dark matter related process:

- Nano-hertz GW can be induced from oscillation of superlight DM.

- O(1) Hz GW can be induced from WIMP triggered phase transition.

- High frequency GW can be induced from graviton of DM production process.

More systematic study is needed to build connection between dark matter and GW.

Thank you for your attention