Effects of spacetime geometry on neutrino oscillation inside a Core-Collapse Supernova

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Neutrino oscillation through curved spacetime

1. Terrestrial neutrino oscillation experiments like LBL or the study of atmospheric neutrinos do not take account of the non-flatness of spacetime geometry in the presence of matter.
2. Even when spacetime curvature can be neglected, torsional interaction can alter the neutrino oscillation parameters. [1]
3. We concentrate on neutrino oscillation inside a Core Collapse Supernova. We will focus on the torsional part of the spacetime geometry.

[1] S. Chakrabarty, A. Lahiri, Eur.Phys.J.C 79 (2019) 8, 697 *and* R. Barick et al, Eur.Phys.J.Plus 139 (2024) 6, 461

Plan of talk

1. First order theory of gravity
2. Hamiltonian of flavor dynamics
3. Oscillation in the presence of uniform density of neutrinos
4. Oscillation in the presence of varying density of neutrinos
5. Conclusions

Torsion and Neutrino oscillation

1. At tree level EW interaction gives us a quartic interaction under contact approximation. However, the ECSK theory will also produce an effective four-fermion interaction, as we will see.
2. This new interaction will change the effective mass of neutrino. It will contribute to the neutrino-matter interaction and neutrino-neutrino interaction.
3. The collapse of a dying star of 10 $M_\odot$ into a PNS of ≈ 10 km releases $\approx 10^{53}$ erg. Most of the energy is dispersed in forms of neutrinos.
4. The effect of spacetime geometry on the propagation of this intense flux of neutrinos just outside of the ultra-dense core is often ignored.

First order theory of gravity

Ingredients :

1. First order theory of gravity is an alternative formulation to gravity which contains terms upto first order derivative.
2. The tetrads (e_a^μ) and the spin connections ($A_\mu{}^{ab}$) are two independent degrees of freedom.
3. Minimal coupling with the Dirac fermions.

Conventions:

1. Latin indices $(a, b \cdots)$ are the flat indices
   Greek indices $(\alpha, \beta \cdots)$ are the curved indices.
2. $\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$.
3. We will often use γ_μ to indicate the product $e_\mu^a \gamma_a$.
4. With $e_a^\mu e_\mu^b = \delta_a^b$.

Spinor covariant derivative

Imposed a local Lorentz invariance on a curved manifold by defining

$$D'_\mu(S(\Lambda(x))\psi(x)) = S(\Lambda(x))D_\mu\psi(x). \quad (1)$$

The spin-connection follows the transformation

$$A'_\mu = SA_\mu S^{-1} + 4i\partial_\mu SS^{-1}. \quad (2)$$

Here, ${A_\mu}^{ab}\sigma_{ab} = A_\mu$. We have

$$D_\mu = \partial_\mu - \frac{i}{4}{A_\mu}^{ab}\sigma_{ab} \quad (3)$$

Einstein Cartan Sciama Kibble theory (ECSK)

Lagrangian of Dirac fermions minimally coupled to gravity is

$$\mathcal{L}_\psi = \frac{i}{2}\left(\bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\psi\gamma^\mu\psi - \frac{i}{4}A_\mu{}^{ab}\,\bar{\psi}[\gamma_c, \sigma_{ab}]_+\psi\, e^{\mu c}\right) - m\bar{\psi}\psi\,. \quad (4)$$

The Ricci scalar can be written in terms of the spin connection $A_\mu{}^{ab}$ and the tetrads e_a^μ as

$$R = F_{\mu\nu}{}^{ab}e_a^\mu e_b^\nu\,, \quad (5)$$

Where the field strength F is defined as

$$F_{\mu\nu}{}^{ab} = \partial_\mu A_\nu{}^{ab} - \partial_\nu A_\mu{}^{ab} + A_\mu{}^a{}_c A_\nu{}^{cb} - A_\nu{}^a{}_c A_\mu{}^{cb}\,. \quad (6)$$

First order theory of gravity contd.

Spin connections ${A_\mu}^{ab}$ has two parts

$$ {A_\mu}^{ab} = {\omega_\mu}^{ab} + {K_\mu}^{ab}. \tag{7} $$

1. ${\omega_\mu}^{ab}$: the Levi-Civita connection - given by the tetrads.
2. ${K_\mu}^{ab}$: the contortion usually set zero in GR.

$$ \begin{aligned} S =& \frac{1}{2\kappa} \int |e| d^4x \left(\hat{R} + e^\mu_a e^\nu_b \partial_{[\mu} {K_{\nu]}}^{ab} + e^\mu_a e^\nu_b \left[\omega_{[\mu}, {K_{\nu]}}^{ab} \right]_- \right) \\ &+ \int |e| d^4x \left(\frac{1}{2\kappa} e^\mu_a e^\nu_b \left[K_{[\mu}, {K_{\nu]}}^{ab} \right]_- + \mathcal{L}_\psi \right) . \end{aligned} \tag{8} $$

1. $\kappa = 8\pi G_N$ is the Planck Mass squared.
2. $\hat{R}$ is the Ricci scalar as calculated for the torsionless part of the connection.

First order theory of gravity contd.

Varying with respect to K we get purely algebraic and axial solution. It is

$$K_\mu{}^{ab} = \frac{\kappa}{8} e^c_\mu \bar{\psi} [\gamma_c, \sigma^{ab}]_+ \psi \,. \tag{9}$$

If we allow parity violation then, the most generic form of $\mathcal{L}_\psi$ is

$$\mathcal{L}_\psi = \sum_{i=\text{fermions}} \left(\frac{i}{2} \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{1}{8} \omega_\mu{}^{ab} e^{\mu c} \, \bar{\psi}_i \gamma_c \sigma_{ab} \psi_i - \frac{1}{2} m \bar{\psi}_i \psi_i + \text{h. c.} \right) + \sum_{i=\text{fermions}} \left(\frac{1}{8} K_\mu{}^{ab} e^{\mu c} \left(\lambda^i_L \bar{\psi}_{iL} \left[\gamma_c, \sigma_{ab} \right]_+ \psi_{iL} + \lambda^i_R \bar{\psi}_{iR} \left[\gamma_c, \sigma_{ab} \right]_+ \psi_{iR} \right) \right) \tag{10}$$

We have modified the spinor derivative only. The structure of the $\omega_\mu{}^{ab}$ is kept unchanged. Only the $K_\mu{}^{ab}$ part is generalized to accommodate the parity violation.

First order theory gravity contd.

This is not a quantum theory of gravity and hence there is no natural energy scale associated with it.
EOM of ${K_\mu}^{ab}$ with the modified spinor derivative from $\mathcal{L}_\psi$ of Eq. (10) is

$$ {K_\mu}^{ab} = \frac{\kappa}{4}\epsilon^{abcd} e_{c\mu} \sum_i \left(-\lambda_L^i \bar{\psi}_{iL}\gamma_d\psi_{iL} + \lambda_R^i \bar{\psi}_{iR}\gamma_d\psi_{iR}\right) . \qquad (11) $$

Replacing the contortion from Eq. (11) into Eq. (10) we get

$$ \mathcal{L}_\psi = \sum_i \Bigg(\frac{i}{2}\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i - \frac{i}{2}\partial_\mu\bar{\psi}_i\gamma^\mu\psi_i + \frac{1}{8}{\omega_\mu}^{ab} e^{\mu c}\bar{\psi}_i[\sigma_{ab},\gamma_c]_+\psi_i - m\bar{\psi}_i\psi_i \Bigg) - \frac{1}{\sqrt{2}}\Bigg(\sum_i \left(-\lambda_L^i \bar{\psi}_{iL}\gamma_d\psi_{iL} + \lambda_R^i \bar{\psi}_{iR}\gamma_d\psi_{iR}\right)\Bigg)^2 . \qquad (12) $$

Redefined $\lambda \rightarrow \sqrt{\frac{3\kappa}{8}}\lambda$.

First order theory of gravity contd.

The spin-torsion interaction adds a quartic interaction to the Lagrangian which is diagonal in mass basis.

$$\mathcal{L}_{\rm int} = -\frac{1}{\sqrt{2}}\left[\sum_{\substack{i=\\ \text{all fermions}}}\left(-\lambda_L^i\,\bar{\psi}_{iL}\gamma_d\psi_{iL} + \lambda_R^i\,\bar{\psi}_{iR}\gamma_d\psi_{iR}\right)\right]^2 \qquad (13)$$

1. We will include this interaction in both self-interaction (when both the summands are neutrino currents) and neutrino-non neutrino (when one of the summands is neutrino current) interaction.
2. The λ's are unknown. The κ only sets their mass dimension. Their sizes can not be fixed from theories.
3. We will assume that the new interaction is maximally chiral i.e. $\lambda_R^i = 0$.

Flavor evolution in terms of density matrix

We will work in an effective two flavor model.
Density matrix for neutrinos, $n =$ number density of neutrinos

$$\rho = \frac{n}{2}\left(\mathbb{I}_2 + \vec{P}\cdot\vec{\sigma}\right). \tag{14}$$

$\vec{P} =$ is the Bloch vector in the flavor space.
Similarly for antineutrinos

$$\bar{\rho} = \frac{\bar{n}}{2}\left(\mathbb{I}_2 + \vec{\bar{P}}\cdot\vec{\sigma}\right). \tag{15}$$

The density matrix will evolve according to the Liouville-von Neumann (LvN) equation.

$$i\frac{\partial}{\partial t}\rho = [H(\rho,\bar{\rho}),\rho]. \tag{16}$$

Neutrino flavor Hamiltonian

The Hamiltonian can be split into three parts,

$$H = H_V + H_M + H_{\nu\nu} \,. \tag{17}$$

H_V : vacuum oscillations.
H_M : interaction with the non-neutrino matter, i.e. leptons and quarks.
$H_{\nu\nu}$: term corresponds to the self-interaction of neutrinos.

$$H_V = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} = \frac{\Delta m^2}{2E}\frac{1}{2}\vec{B}\cdot\vec{\sigma} \,. \tag{18}$$

$$\begin{aligned} H_M &= \pm \frac{\Delta\lambda\lambda_f n_f}{2\sqrt{2}}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \pm \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ &= \pm \frac{\Delta\lambda\lambda_f n_f}{\sqrt{2}}\frac{1}{2}\vec{B}\cdot\vec{\sigma} \pm \sqrt{2} G_F n_e \frac{1}{2}\vec{L}\cdot\vec{\sigma} \,. \end{aligned} \tag{19}$$

Neutrino flavour Hamiltonian - self interaction

Here, $\lambda_f = \frac{\sum_d \lambda_d n_d}{\sum_d n_d}$, $\vec{B} = (\sin 2\theta, 0, -\cos 2\theta)$, $\vec{L} = (0, 0, 1)$.
$H_{\nu\nu} = H_{\nu\nu}^{\mathbf{W}\text{eak}} + H_{\nu\nu}^{\mathbf{S}\text{pin}-\mathbf{T}\text{orsion}} = H_{\nu\nu}^{W} + H_{\nu\nu}^{ST}$

$$H_{\nu\nu}^{W} = \frac{1}{2}\sqrt{2}G_F(\vec{P} - \vec{\bar{P}}) \tag{20}$$

$$H_{\nu\nu}^{ST} = \frac{\sqrt{2}}{4}\frac{1}{2}[\Delta\lambda^2 \vec{B}\cdot(n\vec{P} - \bar{n}\vec{\bar{P}})\vec{B}\cdot\vec{\sigma} + \frac{1}{4}(\lambda_{\text{tot}}^2 - \Delta\lambda^2|\vec{B}|^2)(n\vec{P} - \bar{n}\vec{\bar{P}})\cdot\vec{\sigma}]\,. \tag{21}$$

n =neutrino density and $\bar{n}$ =antineutrino density.

Self-interaction contd.

Define : $\lambda_1^2 = gG_F,\ \lambda_2 = (2r+1)\lambda_1,\ \lambda_f = a\lambda_1,\ \tau = t/(\Delta m^2/(2E))^{-1}$
Parametrizing geometrical coupling using three parameters (a, g, r) and using reduced time τ we write

$$\partial_\tau \vec{P} = \Big(\hat{\omega}\vec{B} + \sqrt{2}agrR_f\vec{B} + \sqrt{2}R_\nu gr^2\vec{B}\cdot(\vec{P}-\vec{\bar{P}})\vec{B} + \sqrt{2}R_e\vec{L} + \sqrt{2}R_\nu f_{g,r}(\vec{P}-\vec{\bar{P}})\Big)\times\vec{P} \tag{22}$$

$$\partial_\tau \vec{\bar{P}} = \Big(-\hat{\omega}\vec{B} + \sqrt{2}agrR_f\vec{B} + \sqrt{2}R_\nu gr^2\vec{B}\cdot(\vec{P}-\vec{\bar{P}})\vec{B} + \sqrt{2}R_e\vec{L} + \sqrt{2}R_\nu f_{g,r}(\vec{P}-\vec{\bar{P}})\Big)\times\vec{\bar{P}}. \tag{23}$$

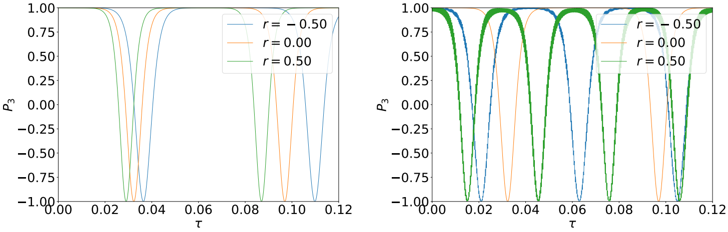
1. $R_{e,f,\nu,\bar{\nu}} = G_F n_{e,f,\nu,\bar{\nu}}/(\Delta m^2/(2E)),\ f_{g,r} = 1 + (1/4)g(2r+1)$
2. We will assume $R_\nu = R_{\bar{\nu}}$.
3. $n_p = n_n = n_e$. Hence, $R_f = 7R_e$.

Significance of the parameters

- We have calculated the background averaged Hamiltonian density.
- $\Delta\lambda = (2r+1)\sqrt{gG_F}$, r is splitting between the spin-torsion coupling constants.
- $g = \sqrt{\lambda_1^2/G_F} = \sqrt{(\lambda_2^2/G_F)}/(2r+1)$ the strength of the interaction
- a sets the scale of the neutrino interaction with non-neutrino matter through spin-torsion coupling
- By convention, $\vec{P} = (0, 0, \pm 1.0)$ is associated with $\nu_e, (\nu_x)$
- ν_x is a mixing of $\nu_{\mu,\tau}$

Oscillation pattern for uniform neutrino density (IH)

$E = 15.1$ MeV, $\theta = 8.6^\circ$, $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $\mu_0 = 1.76 \times 10^5$, $\tau = 1$ corresponds to 8.6 μs. [2].



(a) $a = 0$ and $r = 0, 0.5, -0.5$.

(b) $a = 0.1$ and $r = 0, 0.5, -0.5$.

Figure: P_3 dynamics for $g = 1, (R_\nu, R_e) = (\mu_0/10, \mu_0/10)$.

$r > 0$ hastens the flavor instability.

[2] Y.-C. Lin and H. Duan, Phys. Rev. D 107, 083034 (2023)

Oscillation pattern

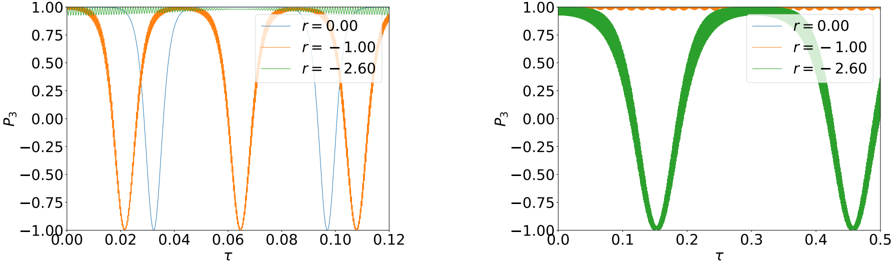
When $a = 0$

1. $r \uparrow$, quicker instability.
2. $r < 0$, slower to start oscillation and the $r > 0$ quicker to start oscillation.

When $a \neq 0$

1. $r \uparrow$, quicker instability.
2. The non-neutrino matter hastens the instability. So, $r < 0$ is quicker than the $r = 0$.
3. The hierarchy between $r \neq 0$ remains the same.

$2r + 1 < 0$ induces or suppresses flavour instability



(a) P_3 in IH for larger values of r.

(b) P_3 in NH for larger values of r.

Figure: For both of the panels $(R_\nu, R_e) = (\mu_0/10, \mu_0/10),\ g = 1,\ a = 0.1.$

$$(2r+1) < 0 \to \Delta\lambda < 0.$$

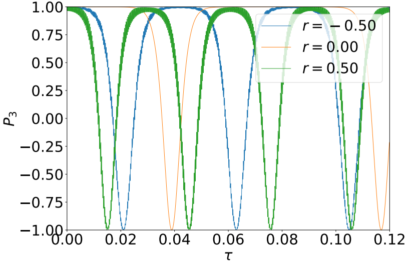
Flavor instability

$(2r + 1) < 0$ means $\Delta\lambda < 0$.

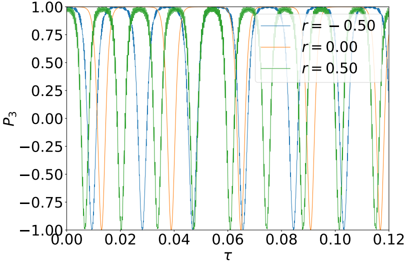
1. In IH, $r = 0 \rightarrow$ oscillation. $2r + 1 < 0 \rightarrow$ oscillation suppressed.
2. In NH, $r = 0 \rightarrow$ no oscillation. $2r + 1 < 0 \rightarrow$ oscillation.

$R_\nu > R_e$ vs $R_e > R_\nu$

Now we consider the situation when $R_\nu \neq R_e$.



(a) $(R_\nu, R_e) = (\mu_0/10, \mu_0/2)$.



(b) $(R_\nu, R_e) = (\mu_0/2, \mu_0/10)$.

Figure: Evolution of P_3 when $R_\nu \neq R_e$. We have fixed $g = 1$, $a = 0.1$ in these.

R_ν controls frequency and the R_f controls the thickness.
We see positive r accelarates the flavor instability in IH.

Oscillation for non-uniform neutrino density and uniform electron density

Neutrino number density profile [3] (d = distance from centre of core)

$$R_{\nu,\bar{\nu}}(d) = R_{\nu,\bar{\nu}}(R)\left(1-\sqrt{1-\frac{R^2}{d^2}}\right)\frac{R^2}{d^2}. \tag{24}$$

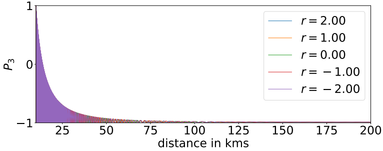
1. Neutrinos emit semi-isotropically from the surface of PNS.
2. The angluar distribution is then averaged over.

R = the radius of the core.
$\Delta m^2 = 2.5\times 10^{-3}$ eV^2 and $E = 15.1$ MeV.
The parameters are same as used before.

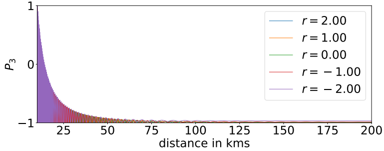
[3] H. Duan et al., 2011 J. Phys. G: Nucl. Part. Phys. 38 035201

Oscillation patterns (IH, $a = 0$)

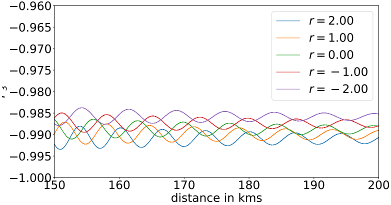
Remembering $\lambda_1^2 = gG_F,\ \lambda_2 = (2r+1)\lambda_1,\ \lambda_f = a\lambda_1$



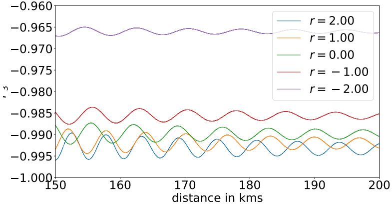
(a) P_3 for different r when $g = 0.5$.



(b) P_3 for different r when $g = 1.5$.



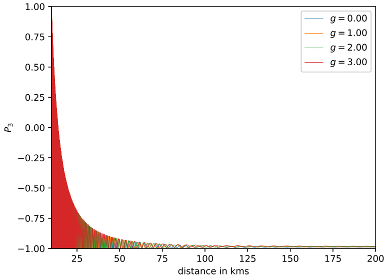
(c) Details of above figure.



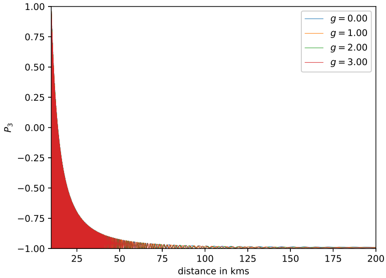
(d) Details of above figure.

Figure: In both panels $R_\nu(R) = R_{\bar{\nu}}(R) = \mu_0/10 = R_e$.

Oscillation patterns (IH, $a = 0$)



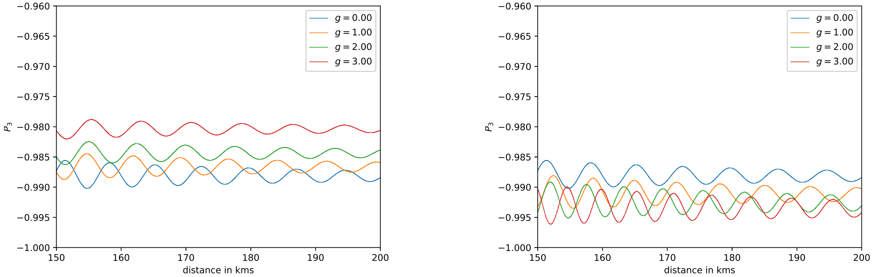
(a) P_3 for different g when $r = -1.0$.



(b) P_3 for different g when $r = +1.0$.

Figure: In both panels $R_\nu(R) = R_{\bar{\nu}}(R) = \mu_0/10 = R_e$.

Away from core of PNS again



(a) $r = -1.0$ away from core. (b) $r = 1.0$ away from core.

Figure: In both panels $R_\nu(R) = R_{\bar{\nu}}(R) = \mu_0/10 = R_e$.

Change in the detector signal (IH, $a = 0$)

The initial oscillations will not be visible at a detector. The ν_e survival probability at any point will be given by

$$\mathcal{P}_S = \frac{1}{n}\,\mathrm{Tr}\left(\rho\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\right) = \frac{1+P_3}{2} \tag{25}$$

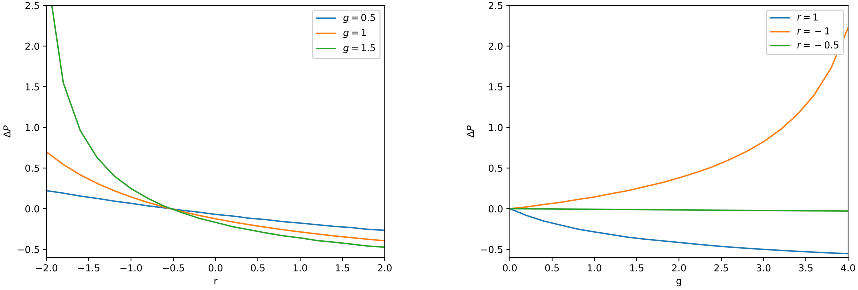
$$\text{where, } \rho = \frac{1}{2}n(\mathbb{I}_2 + \vec{P}\cdot\vec{\sigma}). \tag{26}$$

We are concerned with $\mathcal{P}_S$ far away from the core. We define

$$\Delta P(g,r) = \frac{\mathcal{P}_S(g,r) - \mathcal{P}_S(0,0)}{P_S(0,0)} = \frac{P_\infty(g,r) - P_\infty(0,0)}{1 + P_\infty(0,0)} \tag{27}$$

$P_\infty = P_3$ far away from the core.

Change in the detector signal contd. $(a = 0)$



(a) ΔP for varying r (b) ΔP for varying g

Figure: In both panels $R_\nu(R) = R_{\bar{\nu}}(R) = \mu_0/10 = R_e$, $a = 0$.

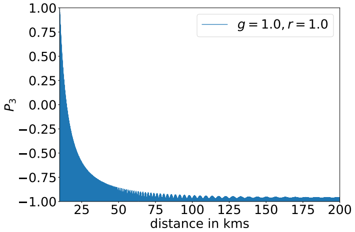
$2r + 1 = 0$ is a crossing point in left panel. $\Delta P \approx 0$ for $r = -0.5$ in the right panel.

Fractional change in survival probability

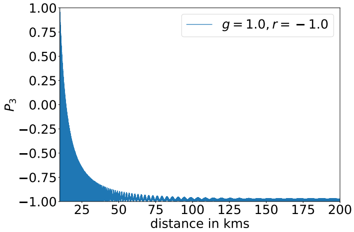
When $a = 0$,

1. Far away from the core the only term that controls the oscillation $(1 + \frac{g}{4}(2r+1))\vec{P} \times \vec{\bar{P}}$. Hence, $2r + 1 = 0$ is a fixed point of the fractional change in the probability.

Oscillation patterns (IH, $a = 0.1$)



(a) P_3 for $g = 1.0, r = 1.00$.



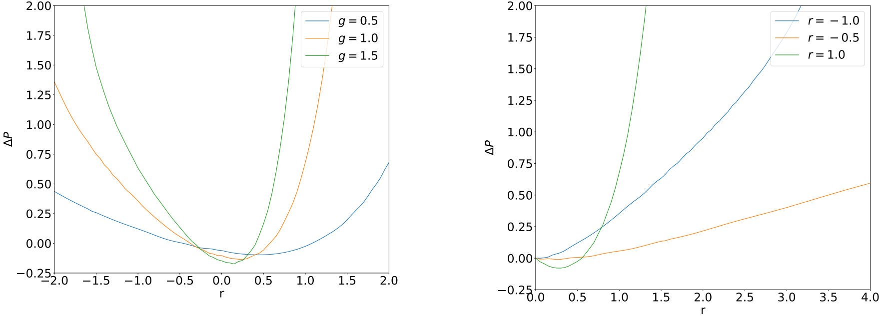
(b) P_3 for $g = 1.0, r = -1.00$.

Figure: In both panels $R_\nu(R) = R_{\bar{\nu}}(R) = \mu_0/10 = R_e$.

Change in detector signals (IH, $a = 0.1$)

$$\Delta P(g,r) = \frac{\langle \mathcal{P}_S(g,r)\rangle - \langle \mathcal{P}_S(0,0)\rangle}{\langle \mathcal{P}_S(0,0)\rangle} = \frac{P_\infty(g,r) - P_\infty(0,0)}{1 + P_\infty(0,0)} \quad (28)$$

$P_\infty = \langle P_3 \rangle$, angular bracket is average over large number of periods.



(a) ΔP when varying r.

(b) ΔP when varying g.

Figure: In both panels $R_\nu(R) = R_{\bar{\nu}}(R) = \mu_0/10 = R_e$, $a = 0.1$.

Far away from the core

When, $a \neq 0$,

1. We have an extra term $\sqrt{2}agrR_f\vec{B} \times \vec{P}$ that dictates the behavior far away from the core.
2. $2r + 1 = 0$ is not a fixed point.
3. The extra term gives an approximate $r \to -r$ symmmetry.

Conclusion

1. The effect of spin-torsion interaction affects the neutrino oscillation in CCSN.
2. In the presence of uniform neutrino density, $\lambda_2 - \lambda_1 < 0$ can alter the flavor stability in Inverted and Normal Hierarchy.
3. Uniform density induces no permanent flavour change. When the neutrino density varies with the distance from core, there is a permanent flavour change.
4. We found that the presence of spin-torsion interaction changes the survival probability by a factor of 2.
5. Data from future Megaton detectors can put a stronger constraint on the spin-torsion coupling constants. A proper event level analysis on a specific detector will be carried out elsewhere.

Acknowledgement

1. Prof. Amitabha Lahiri
2. Ms. Riya Barick
3. Part of the computation is done on the HPC facilities in SNBNCBS.

THANK YOU

Backup slides

BACKUP SLIDES

Density matrix

$$\rho = \frac{1}{2} n_\nu (\mathbb{I}_2 + \vec{P} \cdot \vec{\sigma}) \tag{29}$$

$$\bar{\rho} = \frac{1}{2} n_{\bar{\nu}} (\mathbb{I}_2 + \vec{\bar{P}} \cdot \vec{\sigma}) \tag{30}$$

$\vec{P} = (0, 0, \pm 1)$ for ν_e (ν_x).
$\vec{\bar{P}} = (0, 0, \pm 1)$ for $\hat{\nu}_e$ $(\hat{\nu}_x)$.
ν_x is a linear combination of ν_μ and ν_τ.
Survival probability of ν_e is

$$\mathcal{P}_S = \frac{1}{n} \mathrm{Tr} \left(\rho \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) = \frac{1 + P_3}{2}. \tag{31}$$