Cosmological Signatures of Neutrino Seesaw mechanism

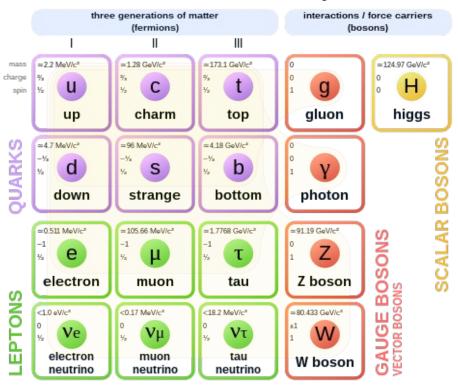
Chengcheng Han Sun Yat-sen University

With Hongjian He, Linghao Song, Jingtao You, arXiv: 2412.21045, 2412.16033(PRD)

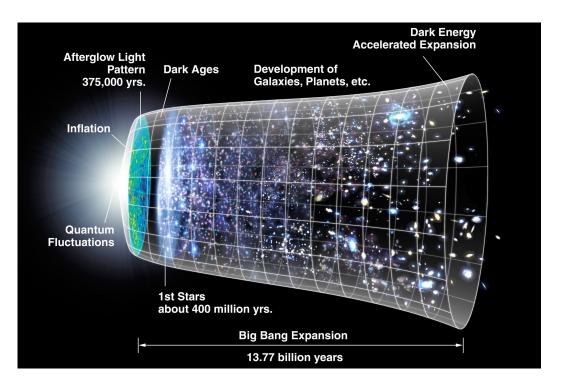
Dark Matter and Neutrino Focus Week (TDLI)

SM of particle and cosmology

Standard Model of Elementary Particles

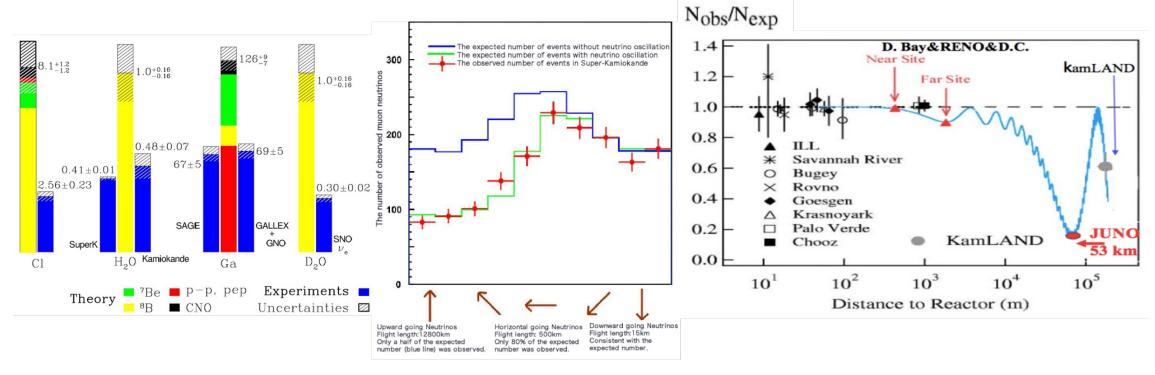


ACDM+Inflation



Neutrino masses

Neutrino oscillation indicates massive neutrinos



Solar Neutrino oscillations

Atmospheric Neutrino Oscillations

Reactor Neutrino Oscillations

 θ_{12}

 θ_{23}

 θ_{13}

$$\Delta m_{21}^2 \simeq 7.42 \times 10^{-5} \text{ eV}^2$$

$$|\Delta m_{13}^2| \approx |\Delta m_{23}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

Cosmological limit

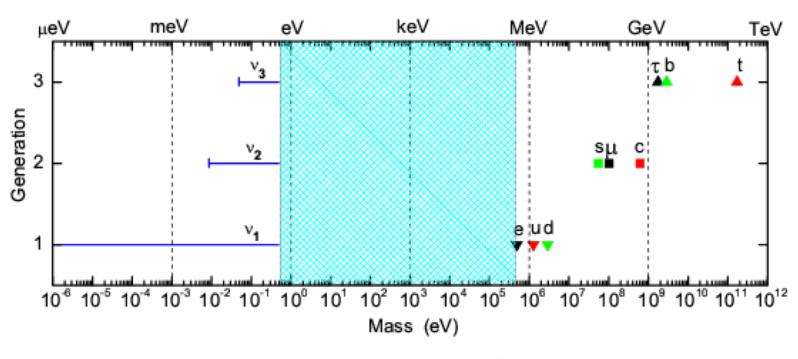
Table 26.2: Summary of	PDG		
	Model	95% CL (eV)	Ref.
CMB alone			
Pl18[TT+lowE]	$\Lambda { m CDM} + \sum m_{ u}$	< 0.54	$\overline{[24]}$
Pl18[TT,TE,EE+lowE]	$\Lambda { m CDM} + \sum m_ u$	< 0.26	[24]
CMB + probes of background evolution			
$\overline{ ext{Pl18}[ext{TT,TE,EE}+ ext{lowE}] + ext{BAO}}$	$\Lambda { m CDM} + \sum m_{ u}$	< 0.13	[49]
$Pl18[TT,TE,EE+lowE] + BAO$ ΛO	CDM+ $\sum m_{\nu}$ +5 params.	< 0.515	[25]
$\overline{ ext{CMB} + ext{LSS}}$			
Pl18[TT+lowE+lensing]	$\Lambda { m CDM} + \sum m_{ u}$	< 0.44	$\overline{[24]}$
Pl18[TT,TE,EE+lowE+lensing]	$\Lambda { m CDM} + \sum m_ u$	< 0.24	[24]
Pl18[TT,TE,EE+lowE] + ACT[lensing]	$\Lambda { m CDM} + \sum m_ u$	< 0.12	[50]
$\overline{\text{CMB}}$ + probes of background evolution + L	SS		
Pl18[TT,TE,EE+lowE] + BAO + RSD	$\Lambda { m CDM} + \sum m_{ u}$	< 0.10	[49]
Pl18[TT,TE,EE+lowE+lensing] + BAO + RSD +	Shape $\Lambda \text{CDM} + \sum m_{\nu}$	< 0.082	[51]
$Pl18[TT+lowE+lensing] + BAO + Lyman-\alpha$	$\Lambda { m CDM} + \sum m_ u$	< 0.087	[52]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DI	ES-Y1 $\Lambda \text{CDM} + \sum m_{\nu}$	< 0.12	[49]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DI	ES-Y3 $\Lambda \text{CDM} + \sum m_{\nu}$	< 0.13	[53]

DDC

The heavy neutrino mass should be around 0.05 eV(IO)-0.06eV(NO)

Neutrino masses

A large hierarchy comparing with other fermion masses



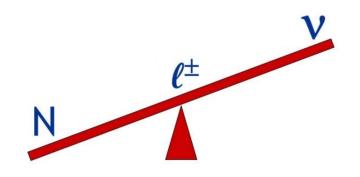
$$\mathcal{L}_{
m Yukawa} \supset -\left[y_e \bar{e}_R \Phi^{\dagger} L_L + y_e^* \bar{L}_L \Phi e_R\right]$$

Seesaw mechanism

Origin of neutrino masses: seesaw mechanism

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + y_{
u} \tilde{H} \bar{L} N_{\!\!R} - rac{1}{2} M_R \bar{N}_{\!\!R}^c N_{\!\!R} + h.c.$$
 $M = \begin{pmatrix} 0 & m_D \ m_D^T & M_R \end{pmatrix}$
 $m_{
u} \sim rac{m_D^2}{M_R} = rac{y_{
u}^2 \langle h \rangle^2}{2M_R}$

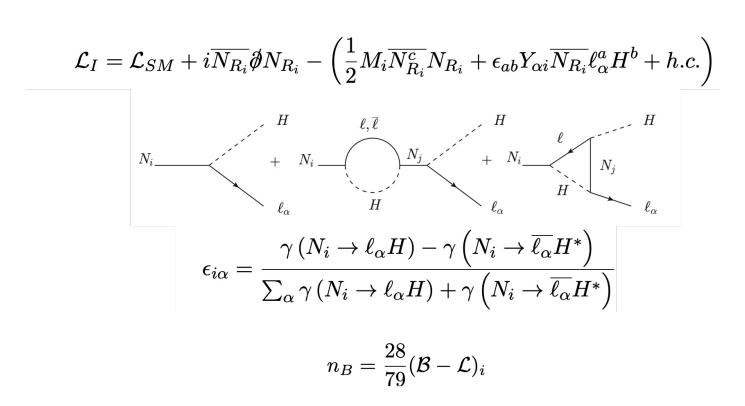
P. Minkowski; T. Yanagida; S. L. Glashow; M. Gell-Mann, P. Ramond and R. Slansky



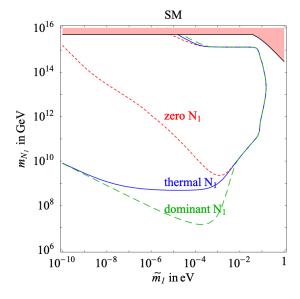
- Natural prediction of small neutrino masses
- Explain the baryon asymmetry of the universe: leptogenesis

Leptogenesis

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986'



G.F. Giudice, et al, Nucl.Phys.B 685 (2004) 89-149



Mass of the right-handed neutrino should heavier than 109 GeV

Seesaw mechanism

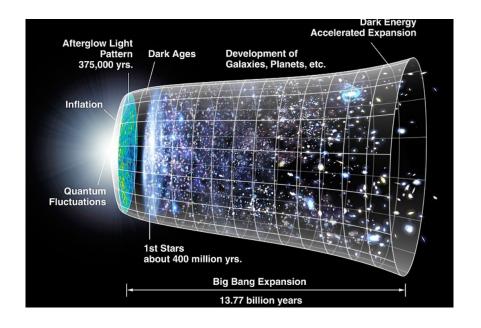
$$m_{\nu} \sim \frac{m_D^2}{M_R} = \frac{y_{\nu}^2 \langle h \rangle^2}{2M_R}$$

If the Yukawa coupling is O(1) (as predicted by the GUT), the seesaw scale M_R should be around 10^{13-14} GeV, which is much beyond the reach of particle experiments.

How to test such high scale seesaw?

Inflation

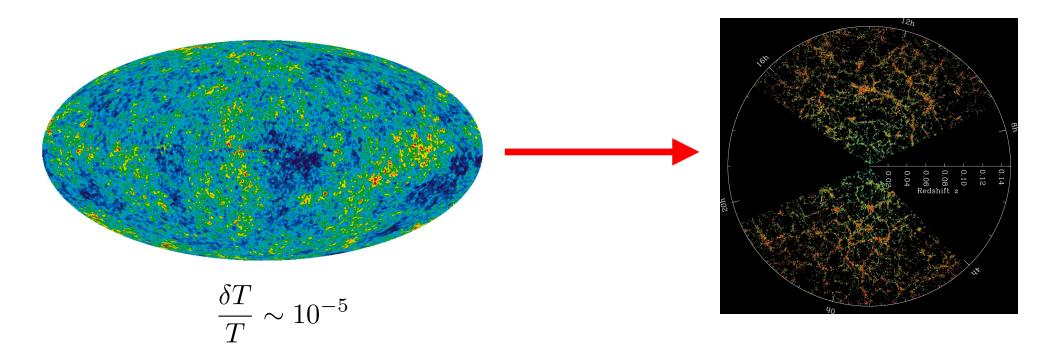
Rapid expansion of the universe in the early time



- Flatness problem
- Horizon problem
- Seeding the primordial anisotropies in CMB

Inflation

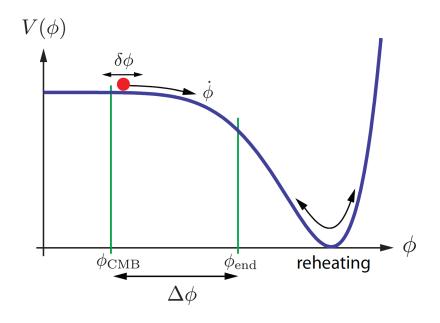
Stretching quantum fluctuations to large scale



Such small fluctuations finally develops the large structure of our universe

Slow-roll Inflation

Inflation is driven by a scalar field (inflaton)



$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$
 $H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right)$

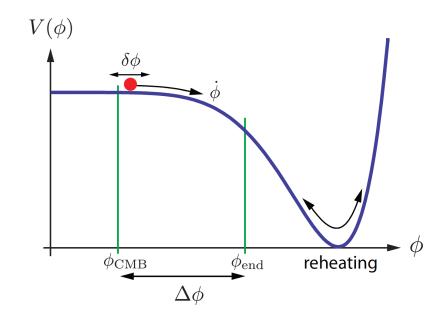
Slow roll condition

$$\dot{\phi}^2 \ll V(\phi)$$
 $|\ddot{\phi}| \ll |3H\dot{\phi}|, |V_{,\phi}|$

- Hubble parameter is nearly constant(de Sitter universe)
- After inflation, inflaton oscillates at the bottom of the potential and finally decays into SM particles, then reheats the universe(still no clear how it occurs)

Slow-roll Inflation

In a de Sitter universe, scalar fields get quantum fluctuation



$$\delta\phi(x,\tau) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \Big[u_a(\tau,\mathbf{k}) b_a(\mathbf{k}) + u_a^*(\tau,-\mathbf{k}) b_a^{\dagger}(-\mathbf{k}) \Big] e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}} \left[1 + ik\tau \right] e^{-ik\tau}$$

$$\langle \delta \phi_{\mathbf{k}} \, \delta \phi_{\mathbf{k}'} \rangle = (2\pi)^3 \, \delta(\mathbf{k} + \mathbf{k}') \, \frac{2\pi^2}{k^3} \left(\frac{H}{2\pi} \right)^2$$

$$\zeta = -\frac{H}{\dot{\phi}} \delta \phi$$

- Quantum fluctuation of inflaton induces CMB anisotropies(or curvature perturbations)
- In the single field inflation, the fluctuations should be nearly gaussian and adiabatic,
 close to scale invariant

Slow-roll Inflation

Scalar perturbations

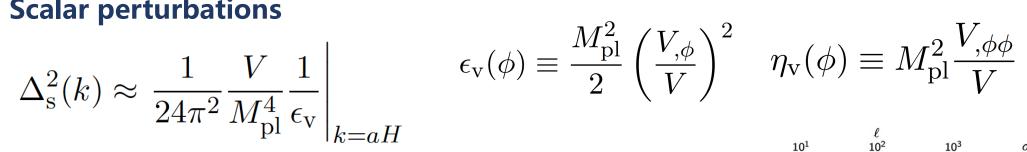
$$\Delta_{\rm s}^2(k) \approx \left. \frac{1}{24\pi^2} \frac{V}{M_{\rm pl}^4} \frac{1}{\epsilon_{\rm v}} \right|_{k=aH}$$

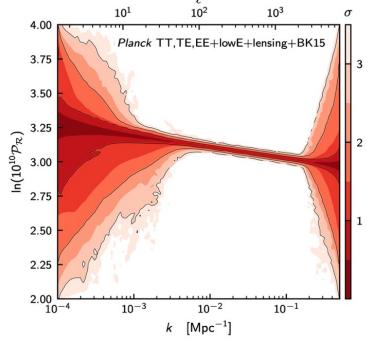
$$n_{\rm s} - 1 \equiv \frac{d \ln \Delta_{\rm s}^2}{d \ln k} = 2\eta_{\rm v} - 6\epsilon_{\rm v}$$

$$n_s \simeq 0.965$$
 n_s =1 scale invariant

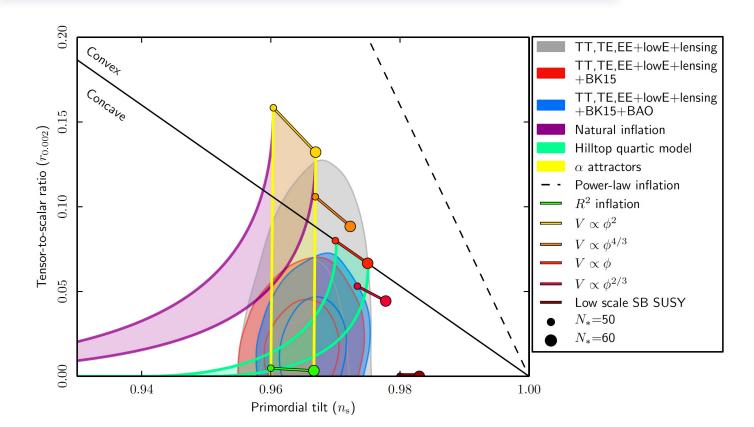
Tensor perturbations

$$\Delta_{\rm t}^2(k) \approx \left. \frac{2}{3\pi^2} \frac{V}{M_{\rm pl}^4} \right|_{k=aH} \quad r \equiv \frac{\Delta_{\rm t}^2}{\Delta_{\rm s}^2} = 16\epsilon_{\rm v}$$





Inflation



$$\epsilon_{
m v}(\phi) \equiv rac{M_{
m pl}^2}{2} \left(rac{V_{,\phi}}{V}
ight)^2
onumber \ \eta_{
m v}(\phi) \equiv M_{
m pl}^2 rac{V_{,\phi\phi}}{V}$$

$$\eta_{
m v}(\phi) \equiv M_{
m pl}^2 rac{V_{,\phi\phi}}{V}$$

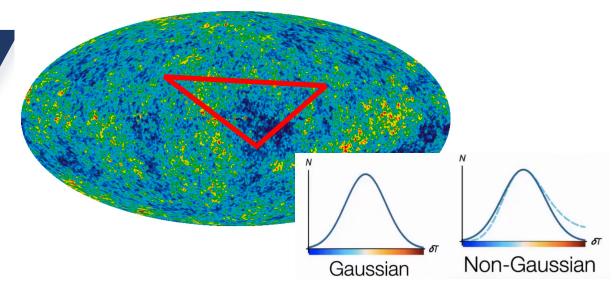
$$\epsilon_{V} < 0.0097$$

$$\eta_V = -0.010^{+0.007}_{-0.011}$$

$$\frac{H_*}{M_{\rm Pl}} < 2.5 \times 10^{-5}$$

- Inflaton potential should be flat enough(shift-symmetry?)
- Hubble scale could be as high as 6*10¹³ GeV(close to seesaw scale), providing access to the high scale physics

Non-Gaussianity



Non-Gaussianity is sensitive to new physics

- New physics could induce large non-Gaussianity : multi-field inflation models, modulated reheating, curvaton scenario...
- Current limit from Planck on local type f_{NL}~ O(10), future CMB observations, CMB S4, large scale structure observations DESI O(1), 21 cm tomography O(0.01-0.1)
- Non-Gaussianity could provide information to the new particle mass, spin, interactions:
 cosmological collider signals
 Nima Arkani-Hamed, Juan Maldacena, arXiv:1503.08043

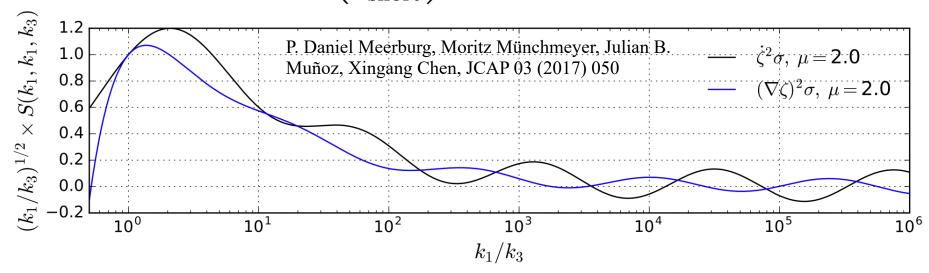
Xingang Chen, Yi Wang, JCAP 04 (2010) 027

Cosmological collider signals

$$\langle \zeta^3 \rangle \equiv (2\pi)^3 \delta_D(\mathbf{k}_{123}) \frac{A^2}{(k_1 k_2 k_3)^2} S(k_1, k_2, k_3) \quad P_{\zeta}(k) = A/k^3$$

Massive particle coupling to the inflaton could induce

$$S_{
m squeezed} \propto \left(rac{k_{
m long}}{k_{
m short}}
ight)^{1/2\pm i\mu} \quad \mu = \sqrt{rac{m^2}{H^2} - rac{9}{4}}$$



- Probing new particles with mass around Hubble scale
- Signature would be suppressed when mass is large(e-pi m/H)

A minimal model

Minimal model incorporates inflation and seesaw

$$\Delta \mathcal{L} = \sqrt{-g} \left[-\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) + \overline{N}_{R} i \partial N_{R} + \frac{1}{\Lambda} \partial_{\mu} \phi \, \overline{N}_{R} \gamma^{\mu} \gamma^{5} N_{R} \right]$$
$$+ \left(-\frac{1}{2} M \overline{N_{R}^{c}} N_{R} - y_{\nu} \, \overline{L}_{L} \tilde{\mathbb{H}} N_{R} + \text{H.c.} \right)$$

- V(phi) is the potential for inflation is unknown but denominated by the mass term after inflation
- Derivative coupling to keep the flatness of the inflaton potential(shift-symmetry, dim-4 coupling should be suppressed, otherwise the induced phi⁴ potential would destroy the flatness of the potential)
- Lambda > 60 Hubble to keep perturbative unitarity
- After inflation, inflaton oscillates at the bottom of the potential until decays into heavy neutrinos (mphi > 2 mN). The heavy neutrinos quickly decay into SM particles and reheat the universe

Seesaw mechanism

Consequence of the seesaw mechanism

$$\mathcal{L} \supset rac{1}{2}ar{\psi}_L\mathbf{M}_
u\psi_R + \mathrm{h.c.}, \qquad \mathbf{M}_
u = egin{pmatrix} 0 & rac{y_
u h}{\sqrt{2}} \ rac{y_
u h}{\sqrt{2}} & M \end{pmatrix}$$
 $m_
u \simeq -rac{y_
u^2 h^2}{2M}, \qquad M_N \simeq M + rac{y_
u^2 h^2}{2M}$

- Light neutrino gets a mass
- Heavy neutrino mass are get lifted (h dependent)

Heavy neutrino mass is h dependent, then decay rate of the inflaton is h dependent:

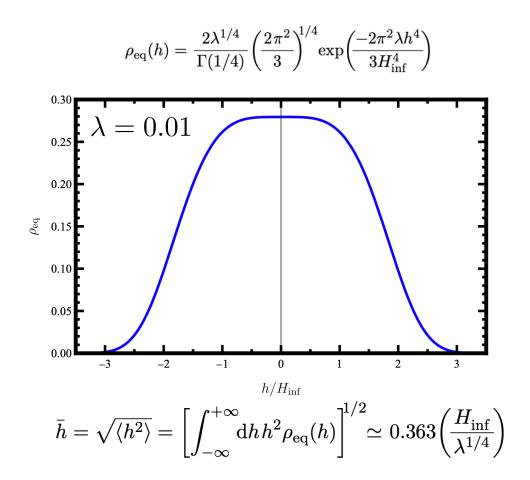
$$\Gamma \simeq rac{m_\phi M^2}{4\pi\Lambda^2} \Biggl[1 + rac{1}{4} \Biggl(rac{y_
u h}{M} \Biggr)^2 \Biggr]$$

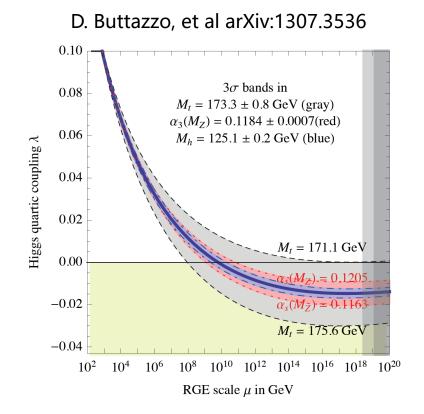
What happens to h in the early universe?

Higgs during inflation

Alexei A. Starobinsky, Jun'ichi Yokoyama, Phys.Rev.D 50 (1994) 6357-6368

- During inflation(de-Sitter universe), Higgs also gets quantum fluctuations
- The fluctuations reach a equilibrium state(solution of F-P equation)

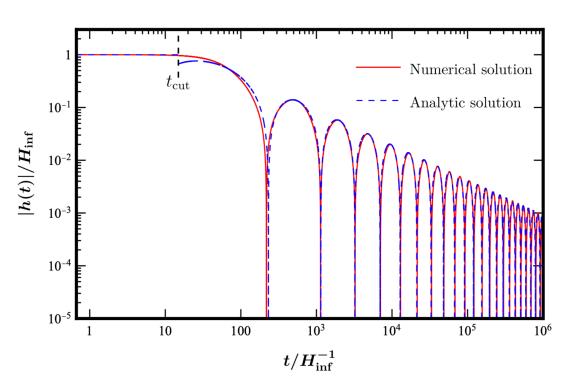




Higgs after inflation

Inflaton oscillates at the bottom potential. If the inflaton potential is dominated by the mass term, the Universe is matter-dominated

$$\ddot{h}(t) + \frac{2}{t}\dot{h}(t) + \lambda h^{3}(t) = 0$$



Numerical solution Analytic solution
$$h(t) = \begin{cases} h_{\inf}, & t \leq t_{\text{cut}} \\ AH_{\inf}(\frac{h_{\inf}}{H_{\inf}\lambda})^{\frac{1}{3}} (H_{\inf}t)^{-\frac{2}{3}} \cos\left(\lambda^{\frac{1}{6}}|h_{\inf}|^{\frac{1}{3}}\omega t^{\frac{1}{3}} + \theta\right), & t > t_{\text{cut}} \end{cases}$$

$$t_{\rm cut} = \frac{\sqrt{2}}{3\sqrt{\lambda} h_{\rm inf}} \qquad A \qquad = 2^{1/3} 3^{-2/3} 5^{1/4} \simeq 0.9$$

$$\omega \qquad = \frac{\Gamma^2(3/4)}{\sqrt{\pi}} 2^{1/3} 3^{1/3} 5^{1/4} \simeq 2.3$$

$$\theta \qquad = -3^{-1/3} 2^{1/6} \omega - \arctan 2 \simeq -2.9$$

Higgs value would oscillate and decrease

Considering decay rate of the inflaton is h dependent

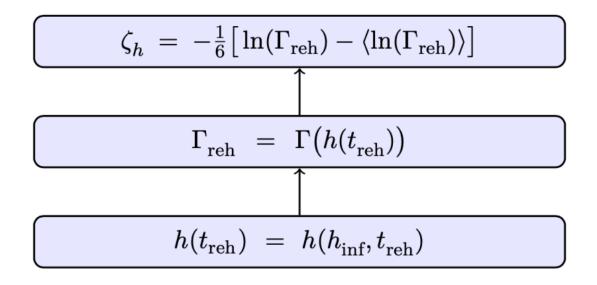
$$\Gamma \simeq rac{m_\phi M^2}{4\pi\Lambda^2} \Biggl[1 + rac{1}{4} \Biggl(rac{y_
u h}{M} \Biggr)^2 \Biggr]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga, Phys.Rev. D69 (2004) 023505

- Different patches of the universe reheat differently (modulated reheating)
- The curvature perturbation is generated by Higgs field
- Delta N formalism (zeta=delta N~ N- <N>)

$$\zeta_h(t > t_{\rm reh}, \mathbf{x}) = \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle
= -\frac{1}{6} \left[\ln(\Gamma_{\rm reh}) - \langle \ln(\Gamma_{\rm reh}) \rangle \right]$$

$$\zeta = \zeta_{\phi} + \zeta_h$$



$$h(t) = A \left(\frac{h_{\mathrm{inf}}}{\lambda}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} \cos\left(\lambda^{\frac{1}{6}} h_{\mathrm{inf}}^{\frac{1}{3}} \omega t^{\frac{1}{3}} + \theta\right)$$

n-point correlation function of zeta changes into n-point correlation function of hinf

Curvature perturbation contains two parts

$$\zeta = \zeta_{\phi} + \zeta_{h}$$
 $\mathcal{P}_{\zeta}^{(\phi)} = \left(\frac{H}{\dot{\phi}}\right)^{2} \mathcal{P}_{\phi} = \left(\frac{H}{\dot{\phi}}\right)^{2} \frac{H^{2}}{4\pi^{2}}$

Taylor expansion of the curvature perturbation

$$\zeta_h(\mathbf{x}) = -\frac{1}{6} \left[\frac{\Gamma_0'}{\Gamma_0} \delta h_{\text{inf}}(\mathbf{x}) + \frac{\Gamma_0 \Gamma_0'' - \Gamma_0' \Gamma_0'}{2\Gamma_0^2} \delta h_{\text{inf}}^2(\mathbf{x}) \right] \equiv z_1 \delta h_{\text{inf}}(\mathbf{x}) + \frac{1}{2} z_2 \delta h_{\text{inf}}^2(\mathbf{x})$$

$$\mathcal{P}_{\zeta}^{(h)} = z_1^2 \mathcal{P}_{\delta h} = \frac{z_1^2 H^2}{4\pi^2}$$
 $R = \left(\frac{\mathcal{P}_{\zeta}^{(h)}}{\mathcal{P}_{\zeta}^{(o)}}\right)^{1/2} = |z_1| \left(\frac{\mathcal{P}_{\delta h}}{\mathcal{P}_{\zeta}^{(o)}}\right)^{1/2}$

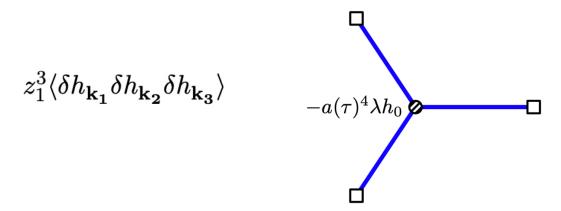
R should be less than 1

Bispectrum

Considering the three point correlation function

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle_h = z_1^3 \langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle + z_1^2 z_2 \langle \delta h^4 \rangle_{2nd}(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$$

First term is from Higgs self-coupling



Calculated by in-in formalism/Schwinger-Keldysh formalism

Steven Weinberg, Phys.Rev.D 72 (2005) 043514, Phys.Rev.D 74 (2006) 023508

Xingang Chen, Yi Wang, Zhong-Zhi Xianyu, JCAP 1712 (2017) 006

Bispectrum

$$\langle \delta h_{\mathbf{k}_{1}} \delta h_{\mathbf{k}_{2}} \delta h_{\mathbf{k}_{3}} \rangle' = 12 \lambda \bar{h} \operatorname{Im} \left(\int_{-\infty}^{\tau_{f}} a^{4} \prod_{i=1}^{3} G_{+} \left(\mathbf{k}_{i}, \tau \right) d\tau \right)$$

$$\operatorname{Im} \left(\int_{-\infty}^{\tau_{f}} a^{4} \prod_{i=1}^{3} G_{+} \left(\mathbf{k}_{i}, \tau \right) d\tau \right)$$

$$N_{e} = \log \left(\frac{a_{\operatorname{end}}}{a_{k}} \right) = \log \left(\frac{-\frac{1}{H\tau_{f}}}{\frac{k_{t}}{H}} \right) = -\log \left(k_{t} | \tau_{f} | \right) \sim 60$$

$$= \operatorname{Im} \int_{-\infty}^{\tau_{f}} \frac{d\tau}{(H\tau)^{4}} \cdot \frac{H^{6}}{8k_{1}^{3}k_{2}^{3}k_{3}^{3}} \left(\prod_{i=1}^{3} (1 - ik_{i}\tau) \right) e^{i(k_{1} + k_{2} + k_{3})\tau}$$

$$= \frac{H^{2}}{24k_{1}^{3}k_{2}^{3}k_{3}^{3}} \cdot \left\{ (k_{1}^{3} + k_{2}^{3} + k_{3}^{3}) [\log(k_{t} | \tau_{f} |) + \gamma - \frac{4}{3}] + k_{1}k_{2}k_{3} - \sum_{a \neq b} k_{a}^{2}k_{b} \right\}$$

Bispectrum

Second term is from non-linear evolution of the Higgs

$$\begin{split} z_1^2 z_2 \langle \delta h^4 \rangle (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= \frac{z_1^2 z_2}{2} \int \frac{\mathrm{d}^3 \mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1) \delta h(\mathbf{k}_2) \delta h(\mathbf{k}_0) \delta h(\mathbf{k}_3 - \mathbf{k}_0) \rangle + (2 \, \mathrm{perm.}) \\ &= \frac{z_1^2 z_2}{2} \bigg[\int \frac{\mathrm{d}^3 \mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1) \delta h(\mathbf{k}_0) \rangle \langle \delta h(\mathbf{k}_2) \delta h(\mathbf{k}_3 - \mathbf{k}_0) \rangle + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2) \bigg] + (2 \, \mathrm{perm.}) \\ &= \frac{z_1^2 z_2}{2} \bigg[\int \mathrm{d}^3 \mathbf{k}_0 \, (2\pi)^3 \delta^3 (\mathbf{k}_1 + \mathbf{k}_0) \delta^3 (\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_0) \frac{H^4}{4k_1^3 k_2^3} + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2) \bigg] + (2 \, \mathrm{perm.}) \\ &= (2\pi)^3 \delta^3 (\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) z_1^2 z_2 \bigg[\frac{H^4}{4k_1^3 k_2^3} + (2 \, \mathrm{perm.}) \bigg]. \end{split}$$

Local type non-gaussianity

The local type non-gaussianity which is defined by Bardeen Potential $\Phi \equiv {3\over 5} \phi$

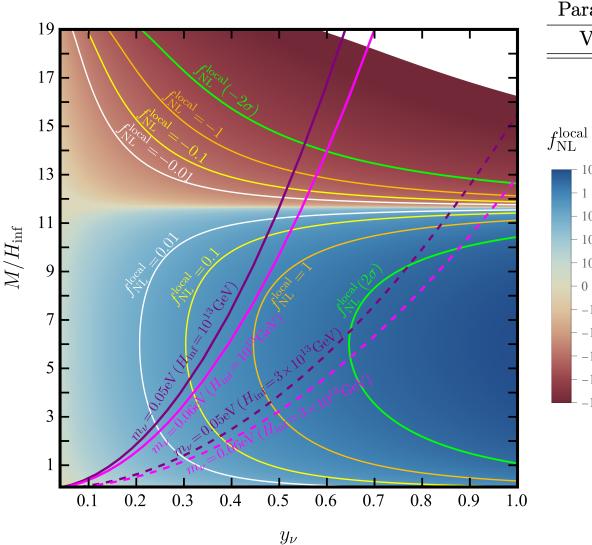
$$\langle \Phi_{\mathbf{k_1}} \Phi_{\mathbf{k_2}} \Phi_{\mathbf{k_3}} \rangle_{\text{local}}' = 2A^2 f_{\text{NL}}^{\text{local}} \left\{ \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right\}$$

In the limit $k_1 \sim k_2 >> k_3$, we find

$$f_{
m NL}^{
m local} \simeq -rac{10}{3} rac{z_1^3 H^3}{(2\pi)^4 \mathcal{P}_{\zeta}^2} igg(rac{\lambda ar{h}}{2H} N_e - rac{z_2 H}{4z_1} igg)$$

$$f_{
m NL}^{
m local} = -0.9 \pm 5.1 \quad (68\% \ {
m C.L., \ Planck \ 2018})$$

Local type non-gaussianity



Parameters	\mathcal{P}_{ζ}	N_e	$H_{ m inf}$	m_{ϕ}	Λ	λ
Values	2.1×10^{-9}	60	$(1,3) \times 10^{13} \text{GeV}$	$40H_{ m inf}$	$60H_{ m inf}$	0.01

Colored curves indicating future searches

 -10^{-2}

 -10^{-4} -10^{-6}

- ()

 -10^{-3}

-10

- Parameter space with Yukawa O(1) could be probed by future observations
- The contribution from self-interaction and non-linear term are both important
- Interplaying with neutrino experiments(JUNO, **DUNE for neutrino ordering)**

Summary

- We propose a minimal model incorporating inflation and seesaw
- Seesaw generated non-Gaussianity could be probed in near future
 CMB or large-scale structure observations
- Cosmological collider signals(in progress)

Thanks!

S-K formalism

$$Q(\tau) \equiv \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N)$$

$$\langle Q(\tau) \rangle = \langle \Omega | \overline{F}(\tau, \tau_0) Q_I(\tau) F(\tau, \tau_0) | \Omega \rangle$$

$$F(\tau, \tau_0) = \operatorname{T} \exp \left(-i \int_{\tau_0}^{\tau} d\tau_1 H_I(\tau_1) \right),$$

$$\overline{F}(\tau, \tau_0) = \overline{T} \exp \left(i \int_{\tau_0}^{\tau} d\tau_1 H_I(\tau_1) \right),$$

S-K formalism

$$\begin{cases} G_{++}\left(\mathbf{k};\tau_{1},\tau_{2}\right) &\equiv G_{>}\left(\mathbf{k};\tau_{1},\tau_{2}\right)\theta(\tau_{1}-\tau_{2})+G_{<}\left(\mathbf{k};\tau_{1},\tau_{2}\right)\theta(\tau_{2}-\tau_{1}) \\ G_{+-}\left(\mathbf{k};\tau_{1},\tau_{2}\right) &\equiv G_{<}\left(\mathbf{k};\tau_{1},\tau_{2}\right) \\ G_{-+}\left(\mathbf{k};\tau_{1},\tau_{2}\right) &\equiv G_{>}\left(\mathbf{k};\tau_{1},\tau_{2}\right) \\ G_{--}\left(\mathbf{k};\tau_{1},\tau_{2}\right) &\equiv G_{<}\left(\mathbf{k};\tau_{1},\tau_{2}\right)\theta(\tau_{1}-\tau_{2})+G_{>}\left(\mathbf{k};\tau_{1},\tau_{2}\right)\theta(\tau_{2}-\tau_{1}) \end{cases} \qquad \begin{matrix} \tau_{1} & \tau_{2} \\ \tau_{1} & \tau_{2} \\ \tau_{1} & \tau_{2} \\ \tau_{2} & \tau_{3} \\ \tau_{1} & \tau_{2} \\ \tau_{2} & \tau_{3} \\ \tau_{3} & \tau_{4} \\ \tau_{3} & \tau_{4} \\ \tau_{5} & \tau_{5} \\ \tau_{7} & \tau_{7} \\ \tau_$$

$$G_{>}(k; \tau_1, \tau_2) \equiv u(\tau_1, k)u^*(\tau_2, k)$$

 $G_{<}(k; \tau_1, \tau_2) \equiv u^*(\tau_1, k)u(\tau_2, k)$

$$\Box u_{\mathbf{k}} = \ddot{u}_{\mathbf{k}} + 3H\dot{u}_{\mathbf{k}} + \frac{\mathbf{k}^2}{a^2(t)}u_{\mathbf{k}} = 0$$

$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}} \left[1 + ik\tau \right] e^{-ik\tau}$$

$$\begin{array}{ccc}
\tau \\
\bullet & = G_{-}(k;\tau) \equiv G_{-+}(k;\tau,\tau_f)
\end{array}$$

S-K formalism

Bulk-to-Boundary propagator

$$G_{\pm}\left(\mathbf{k}, au
ight)\equiv G_{\pm+}\left(\mathbf{k}; au, au_{f}
ight)$$
 $au=G_{+}\left(\mathbf{k}, au
ight)$
 $au=G_{-}\left(\mathbf{k}, au
ight)$
 $au=G_{+}\left(\mathbf{k}, au
ight)$
 $au=G_{+}\left(\mathbf{k}, au
ight)+G_{-}\left(\mathbf{k}, au
ight)$

$$G_{+}(\mathbf{k},\tau) = \frac{H^{2}}{2k^{3}} \left[1 - ik(\tau - \tau_{f}) + k^{2}\tau\tau_{f} \right] e^{ik(\tau - \tau_{f})} \qquad G_{-}(\mathbf{k},\tau) \simeq \frac{H^{2}}{2k^{3}} \left[1 + ik\tau \right] e^{-ik\tau}$$

$$\simeq \frac{H^{2}}{2k^{3}} \left[1 - ik\tau \right] e^{ik\tau}$$

Higgs during inflation

During inflation(de-Sitter universe), Higgs also gets quantum fluctuations

暴胀期间,希格斯场可以分为长波和短波两部分

Alexei A. Starobinsky, Jun'ichi Yokoyama, Phys.Rev.D 50 (1994) 6357-6368

$$\begin{split} h(\mathbf{x},t) &= h_L(\mathbf{x},t) + \int \!\! \frac{d^3k}{(2\pi)^3} \theta \big(k - \epsilon a(t) H_{\mathrm{inf}} \big) \Big[a_\mathbf{k} h_\mathbf{k}(t) e^{-\mathrm{i}\,\mathbf{k}\cdot\mathbf{x}} + a_\mathbf{k}^\dagger h_\mathbf{k}^*(t) e^{\mathrm{i}\,\mathbf{k}\cdot\mathbf{x}} \Big] \\ h_\mathbf{k} &= \frac{H_{\mathrm{inf}}}{\sqrt{2k^3}} \left(1 + \mathrm{i}k\tau\right) e^{-\mathrm{i}k\tau} \end{split}$$

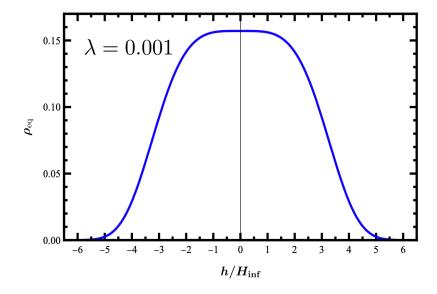
长波部分可以用郎之万方程描述

$$\dot{h}_L(\mathbf{x},t) = -\frac{1}{3H_{\rm inf}} \frac{\partial V}{\partial h_L} + f(\mathbf{x},t)$$

$$\langle f(\mathbf{x}_1, t_1) f(\mathbf{x}_2, t_2) \rangle = \frac{H_{\text{inf}}^3}{4\pi^2} \delta(t_1 - t_2) j_0 \left(\epsilon a(t_1) H_{\text{inf}} |\mathbf{x}_1 - \mathbf{x}_2| \right)$$

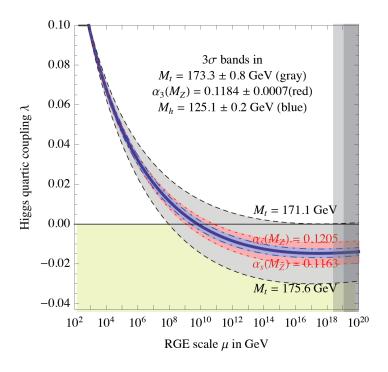
Higgs during inflation

- If inflation lasts long enough, these fluctuations reach a equilibrium state
- Different part of universe Higgs field takes different value



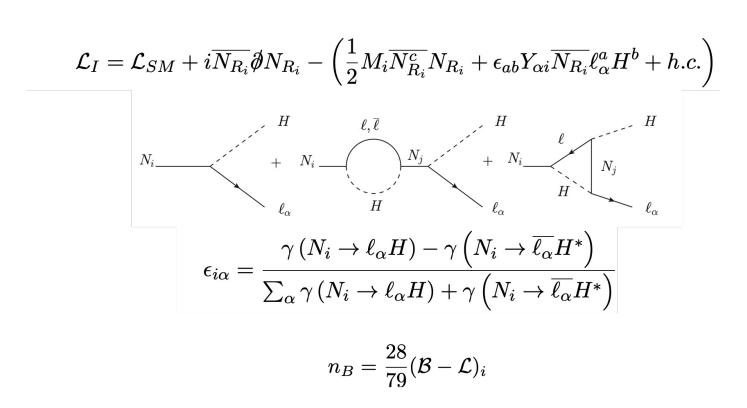
$$ar{h} = \sqrt{\langle h^2 \rangle} = \left[\int_{-\infty}^{+\infty} \! \mathrm{d}h \, h^2
ho_{\mathrm{eq}}(h) \right]^{\!\!1/2} \simeq 0.363 \! \left(\frac{H_{\mathrm{inf}}}{\lambda^{1/4}} \right)$$

D. Buttazzo, et al arXiv:1307.3536

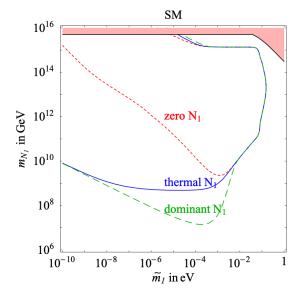


Leptogenesis

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986'



G.F. Giudice, et al, Nucl.Phys.B 685 (2004) 89-149



Mass of the right-handed neutrino should heavier than 109 GeV

Equation of state:
$$\dot{\rho} + 3H(1+\omega)\rho = 0$$

From matter-dominated universe to radiation dominated universe

$$N(\mathbf{x}) = -\frac{1}{3} \ln \frac{\rho_{\text{reh}}(h(\mathbf{x}))}{\rho_{\text{inf}}} - \frac{1}{4} \ln \frac{\rho_f}{\rho_{\text{reh}}(h(\mathbf{x}))}$$

Reheating occurs $H_{\rm reh} = \Gamma_{\rm reh} \quad 3H^2M_n^2 = \rho$

Curvature perturbation in terms of the decay rate

$$\zeta_h(t > t_{\rm reh}, \mathbf{x}) = \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle$$
$$= -\frac{1}{6} \left[\ln(\Gamma_{\rm reh}) - \langle \ln(\Gamma_{\rm reh}) \rangle \right]$$