Dynamical Evolutions in Globular Clusters and Dwarf Galaxies: Conduction Fluid Simulations

Yi-Ming Zhong

香港城市大學
City University of Hong Kong

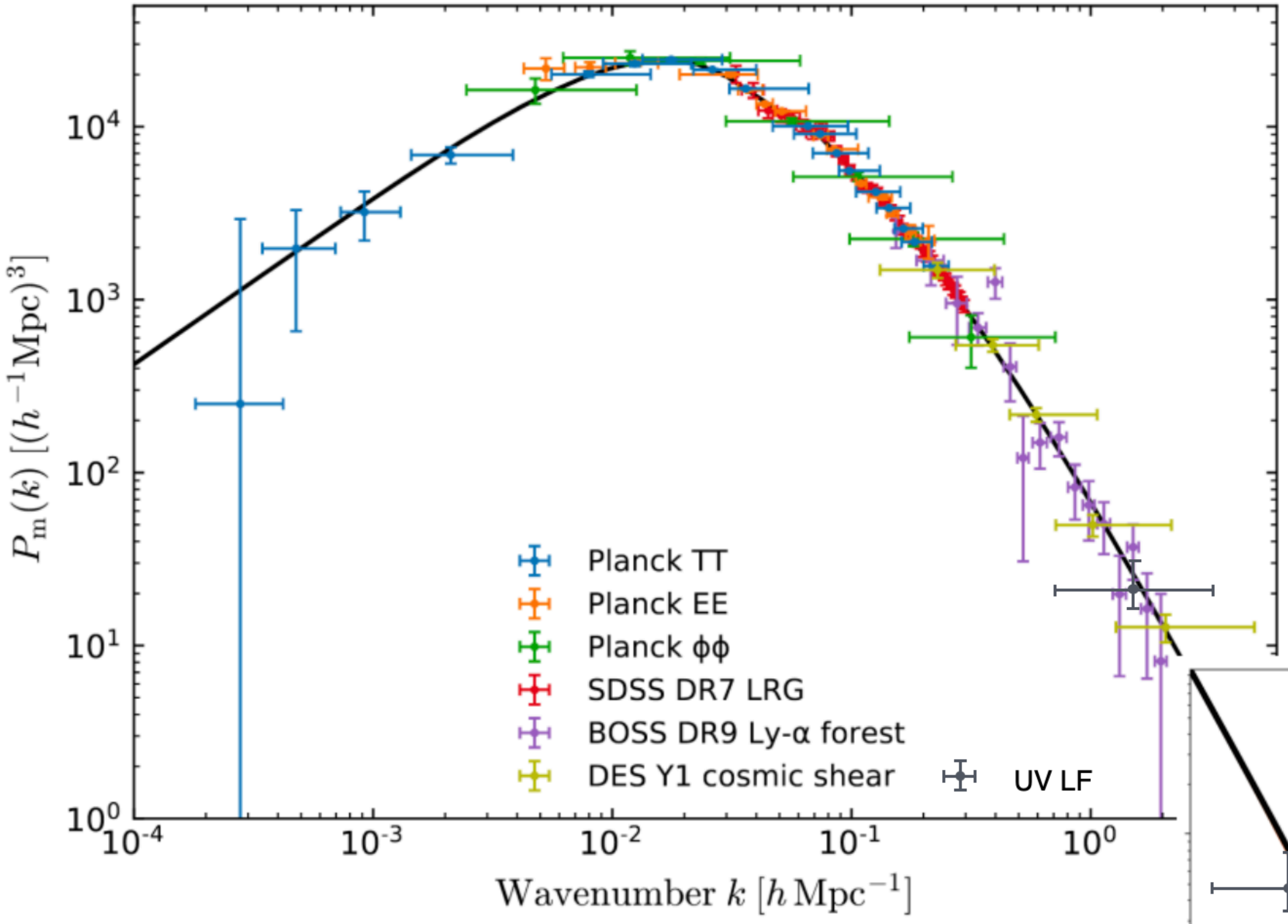
w/ Stuart L. Shapiro, PRD 112 (2025), arXiv: 2505.18251

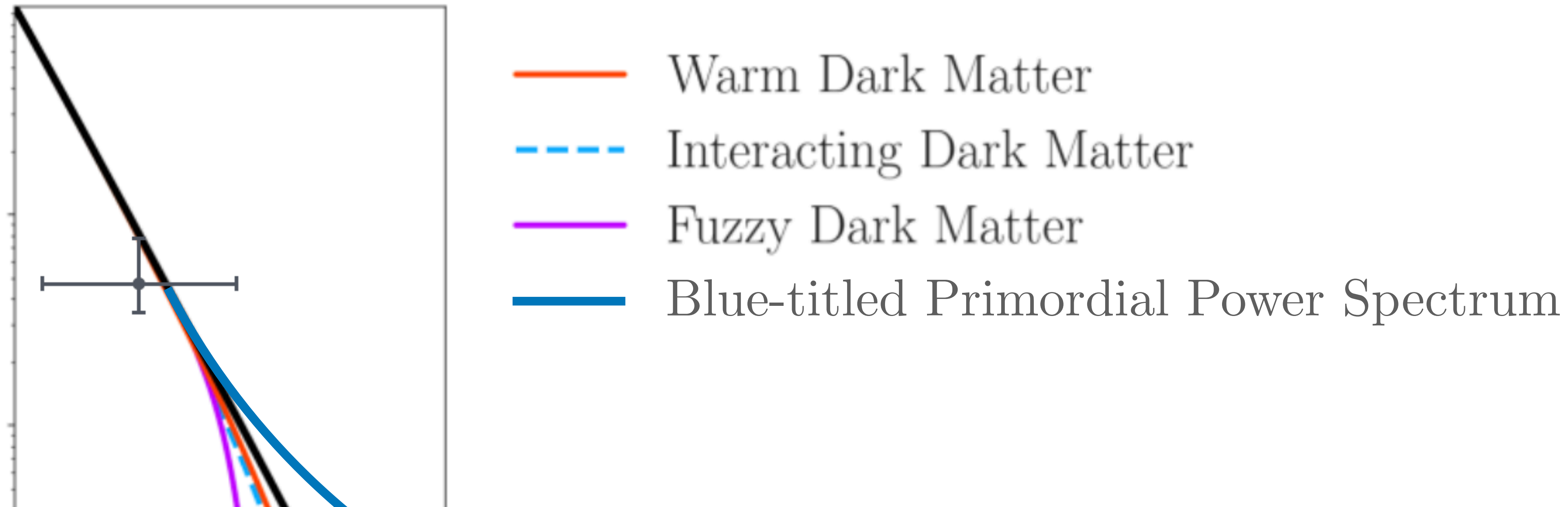
Dark Matter and Neutrino Focus Week, TDLI, 19 August 2025

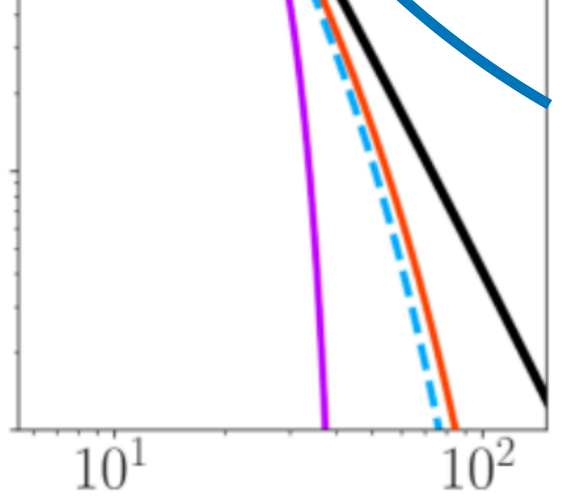
Outline

- Introduction
- Method: conduction fluid simulations
- Results and applications to local globular clusters (GCs) and dwarfs
- Conclusion

Opportunities at the small scales

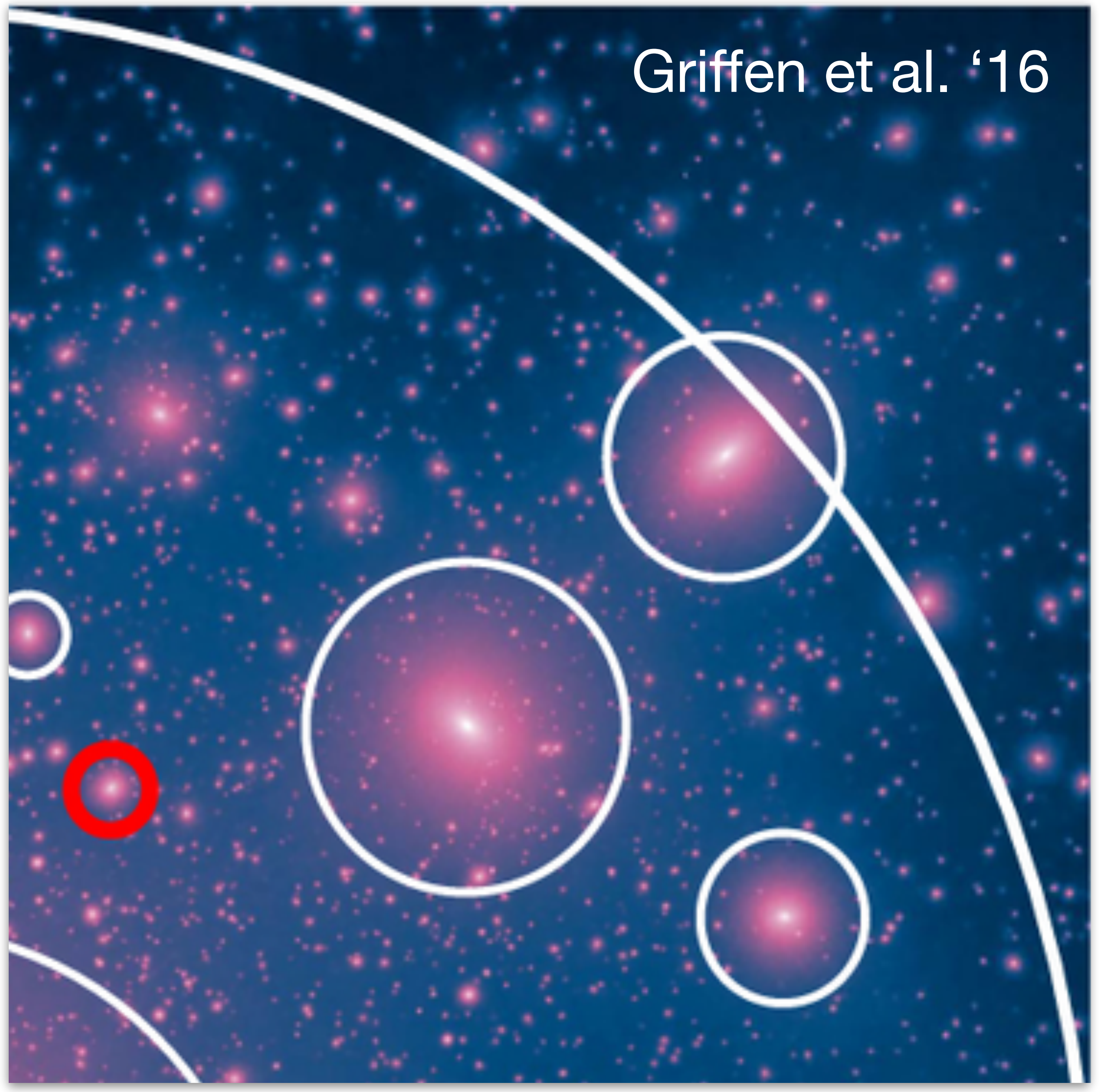


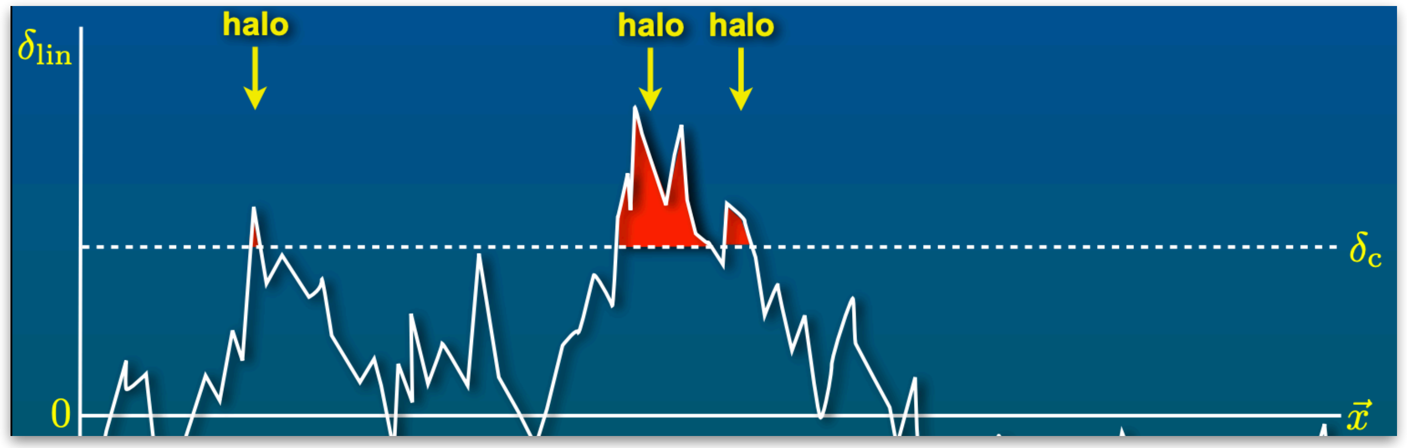




Adapted from E. Nadler

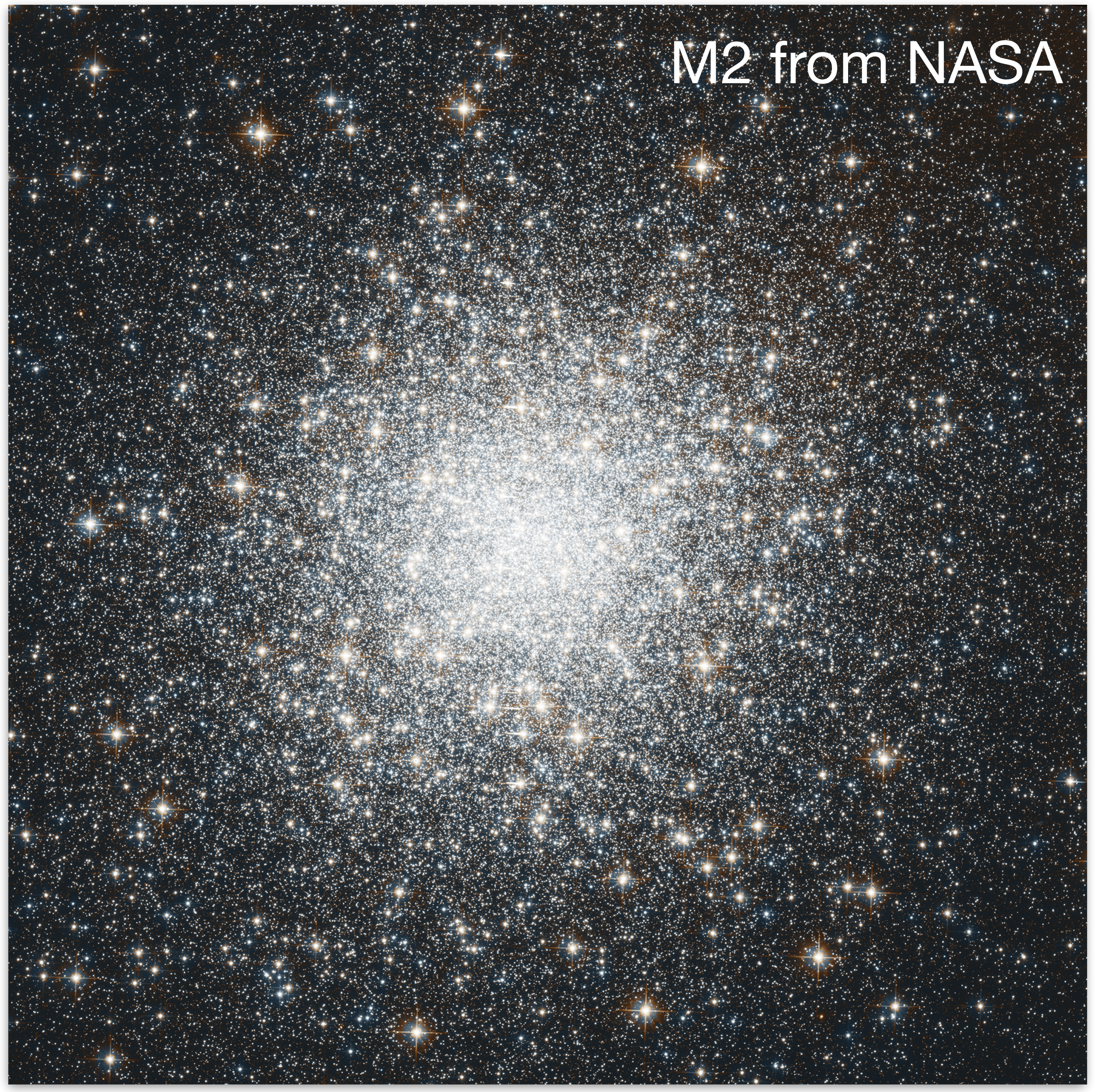
Dwarfs

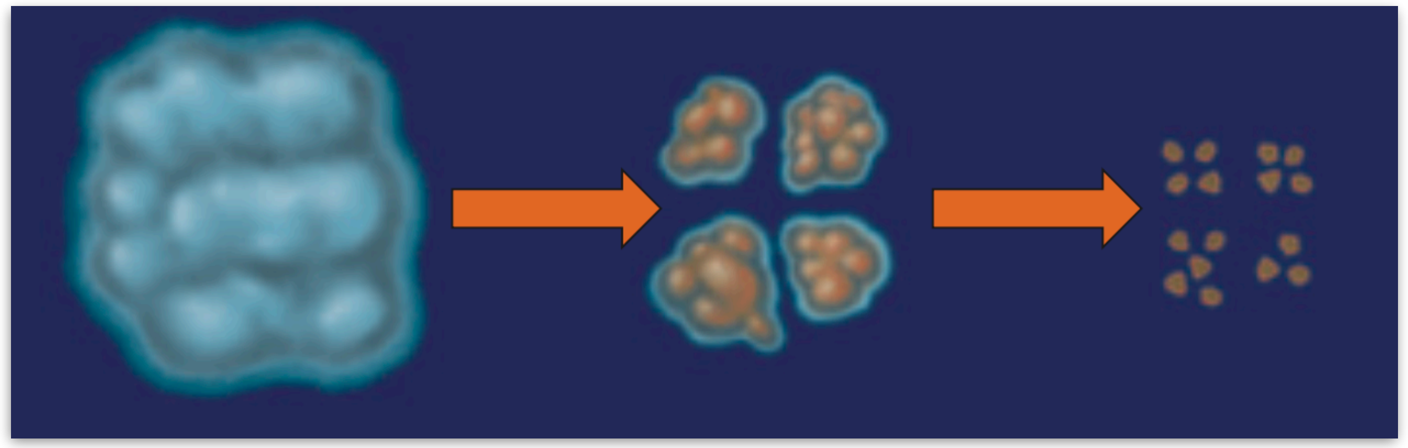




van den Bosch' lecture note

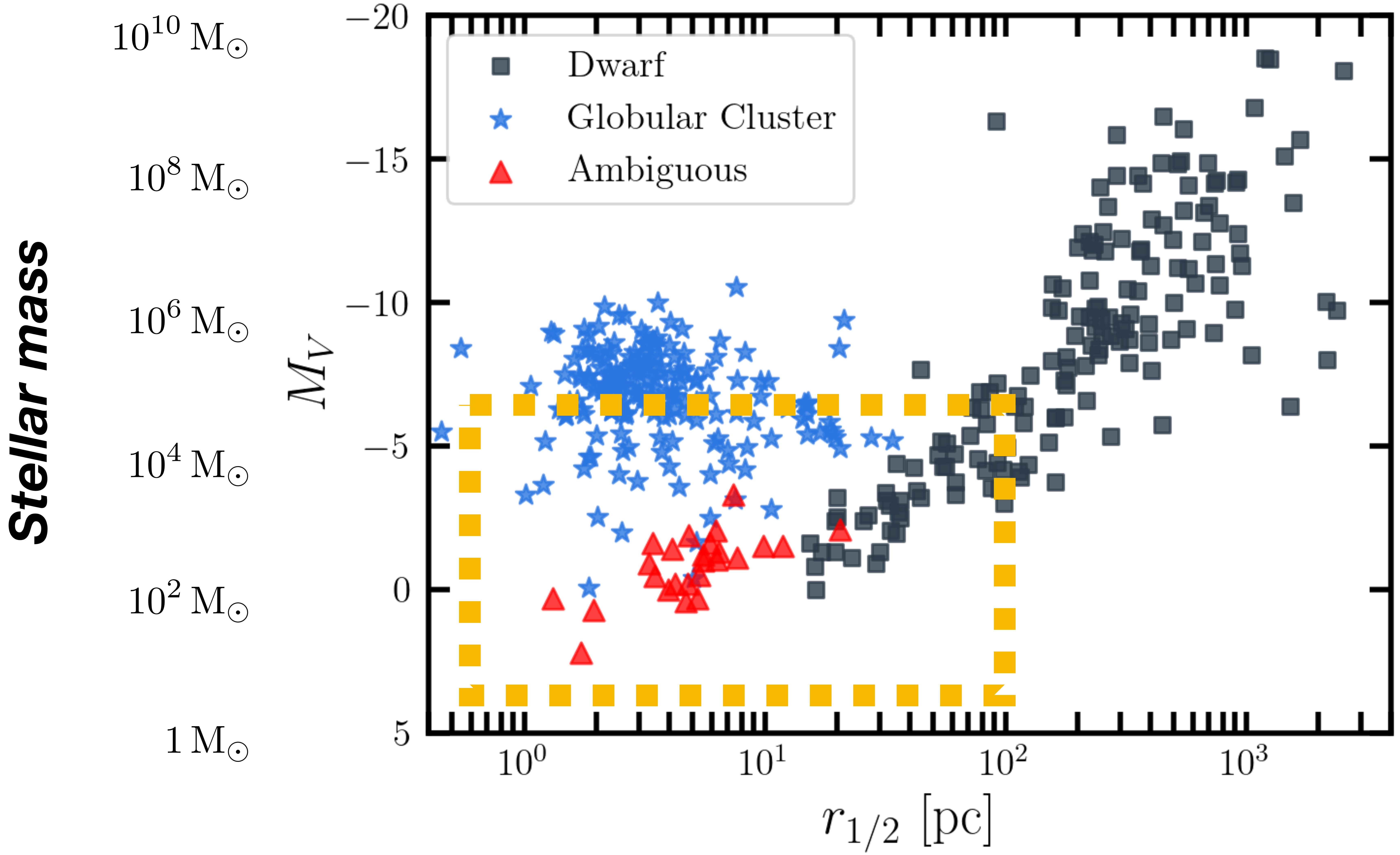
Globular clusters





Astronomy today

GCs & dwarf galaxies



Similar size, luminosity, chemical decomposition

……

They are difficult to distinguish observationally.

Based on Local Volume Database
Pace '24

Motivation

- GCs and ultra-faint-compact dwarfs may come from same type of progenitor. 🤔

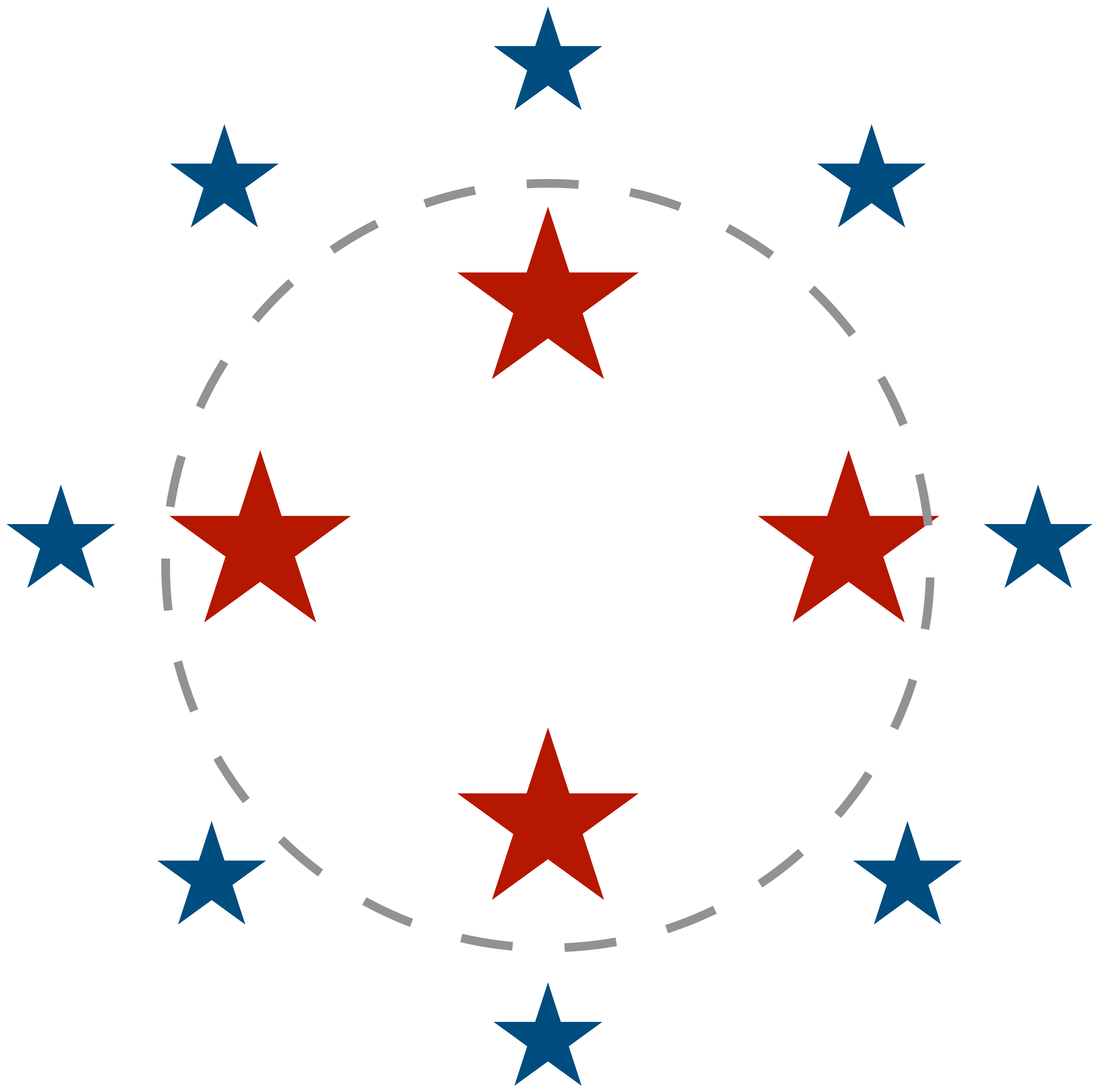
  Baumgardt & Mieske ’08, Ardi & Baumgardt ’20

  - More test is needed. → Simulate stellar-dark matter systems.

- More generally, it is interesting to understand the evolution of a two-component system.

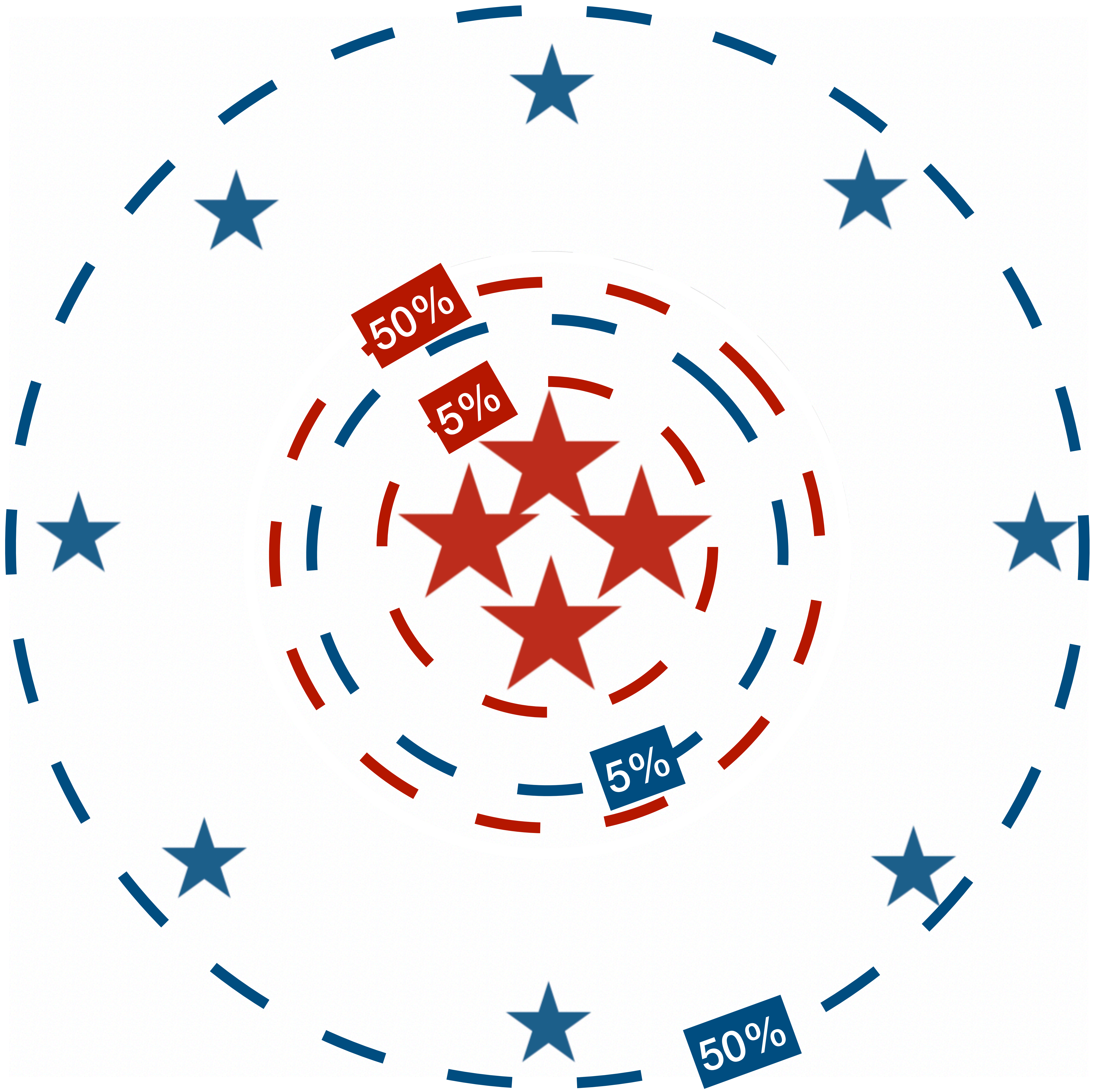
Stars 🤝 **Dark matter**

Dynamical evolution



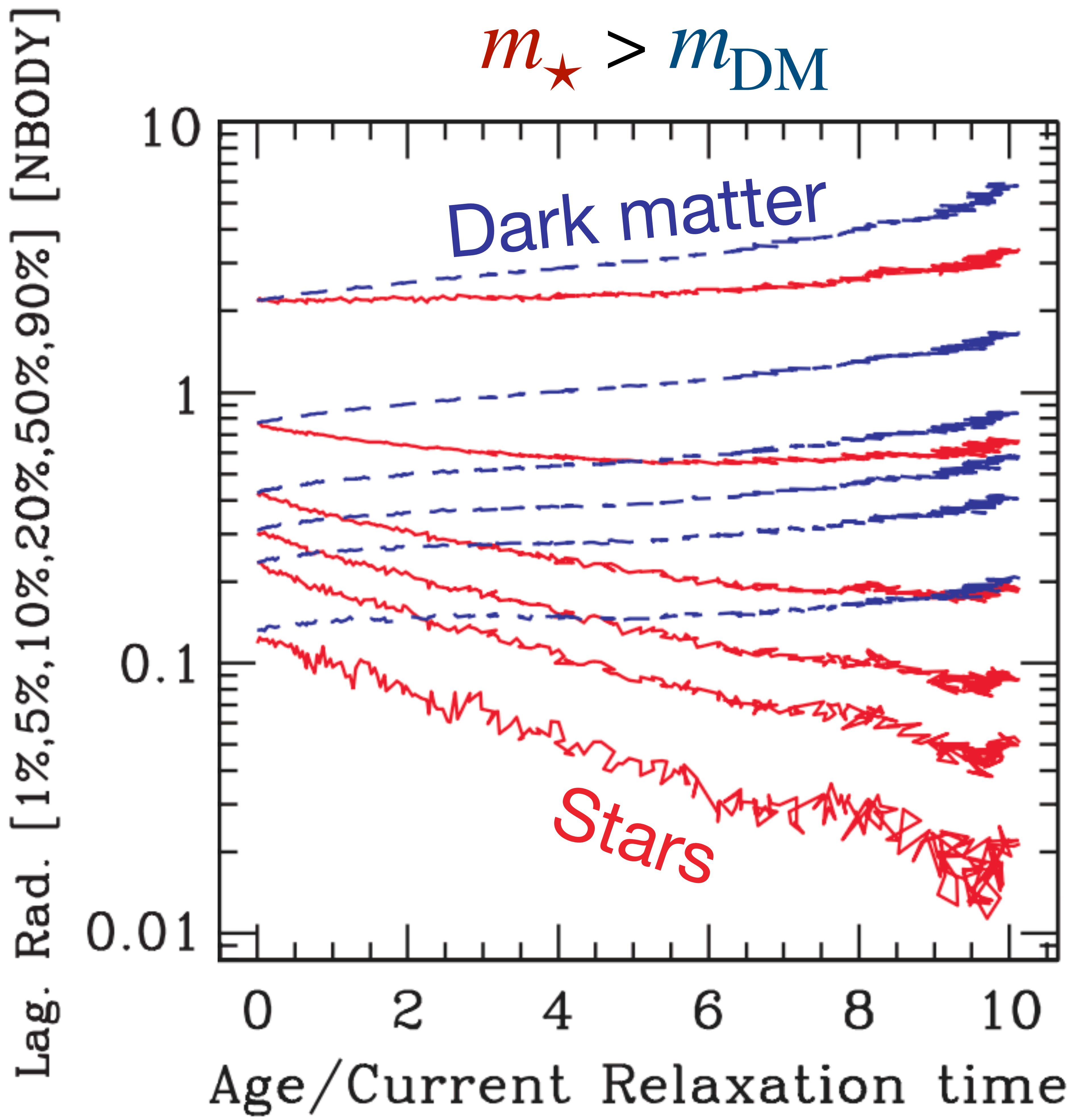
- Dynamical evolution:
  Core collapse & **mass segregation**
- First studied for GCs with stars with different masses.
- Later on other systems.

Dynamical evolution

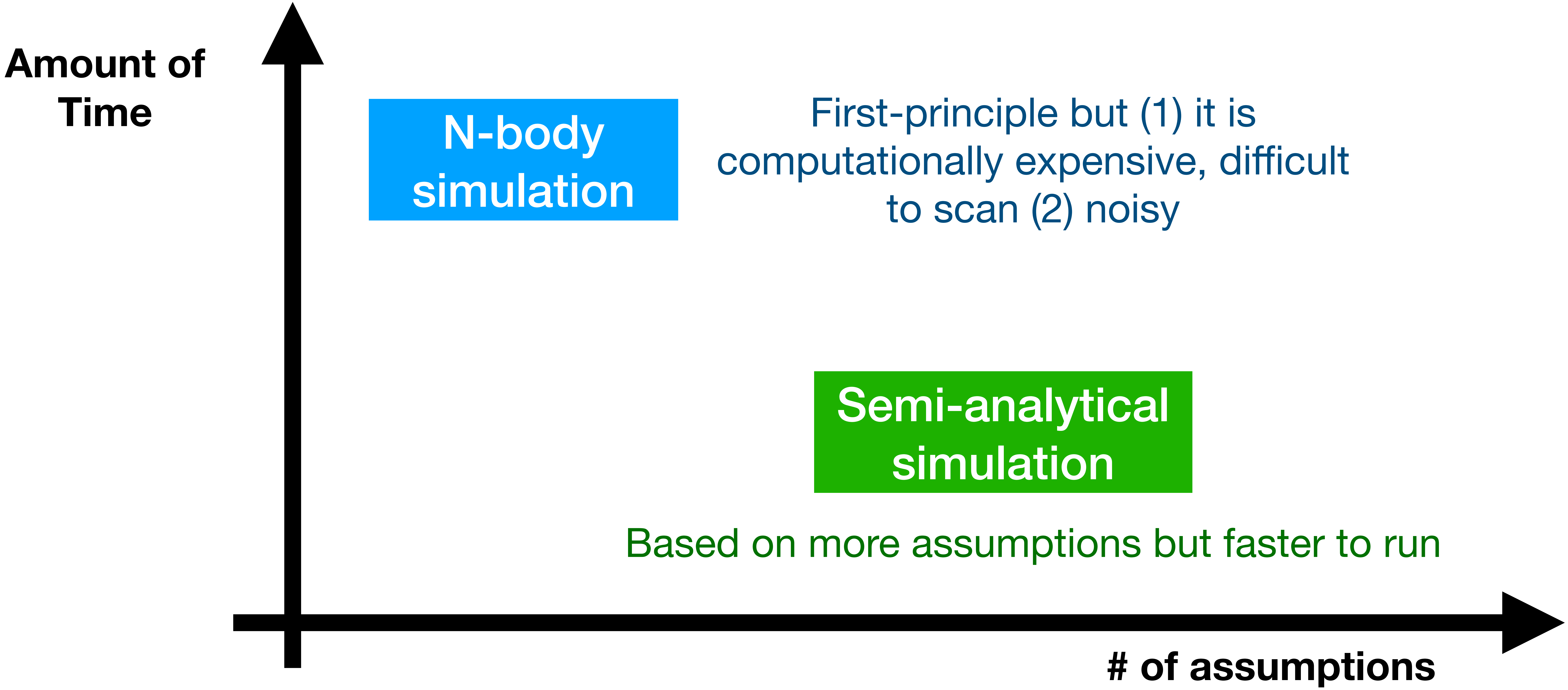


Baumgardt & Mieske '08 (BM08)

$m_\star > m_{\rm DM}$



Simulation methods



Simulation methods

Amount of Time

Model self-gravitating components as hydrostatic-equilibrium fluids;
Use kinetic theory to evolve the system

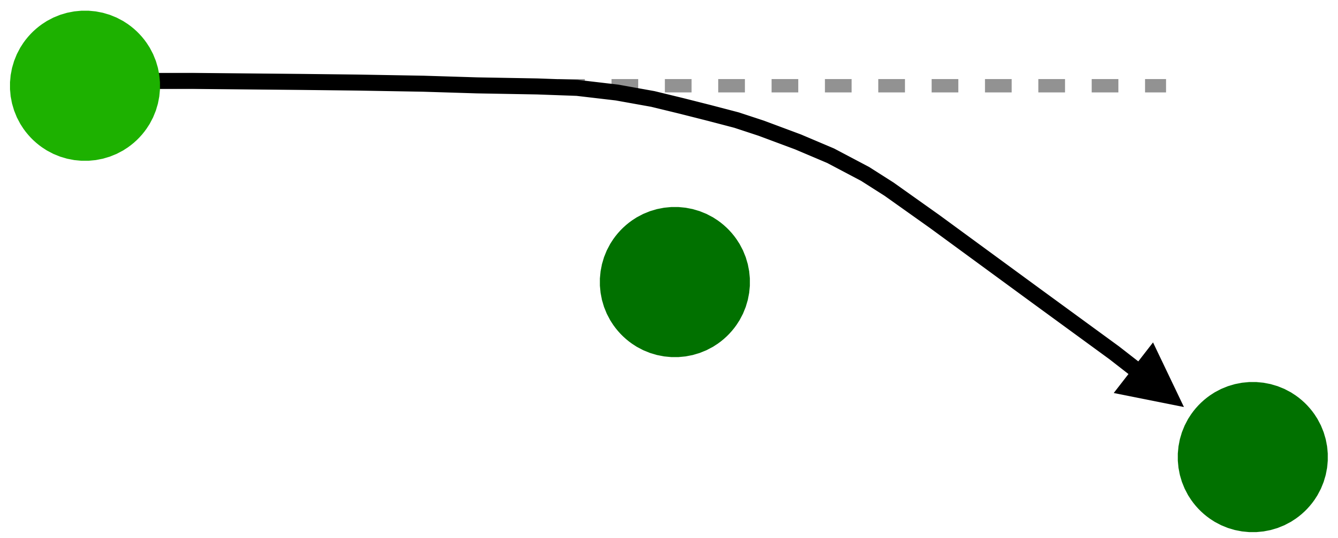
Conduction fluid model

# of assumptions

Method

Model star/dark matter as fluid

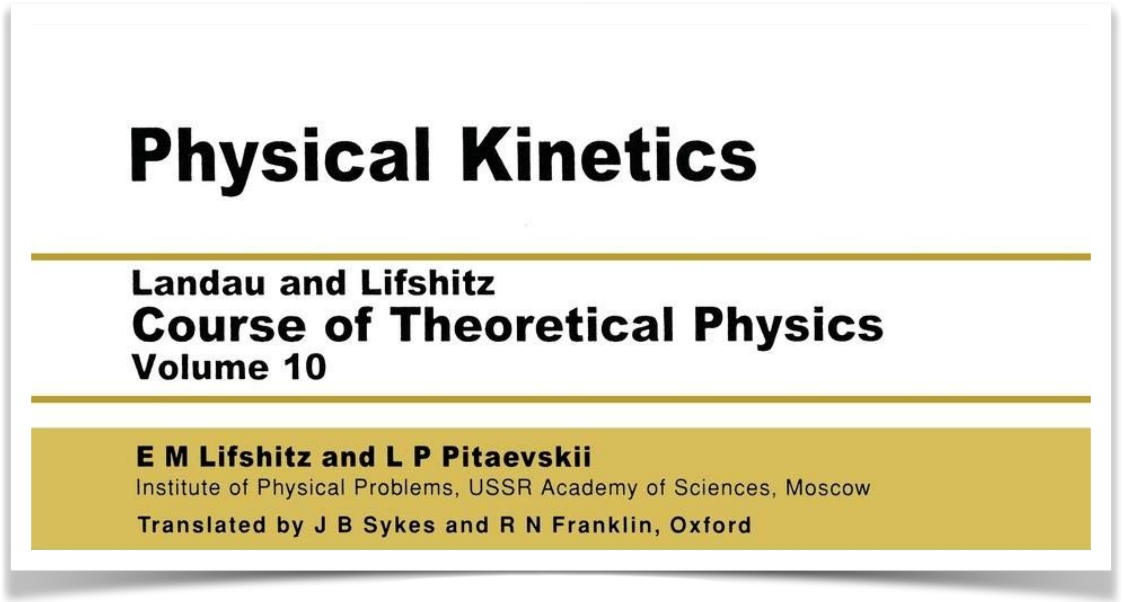
- Does cold dark matter behave as a conducting fluid?
  - No. The relaxation takes too long.

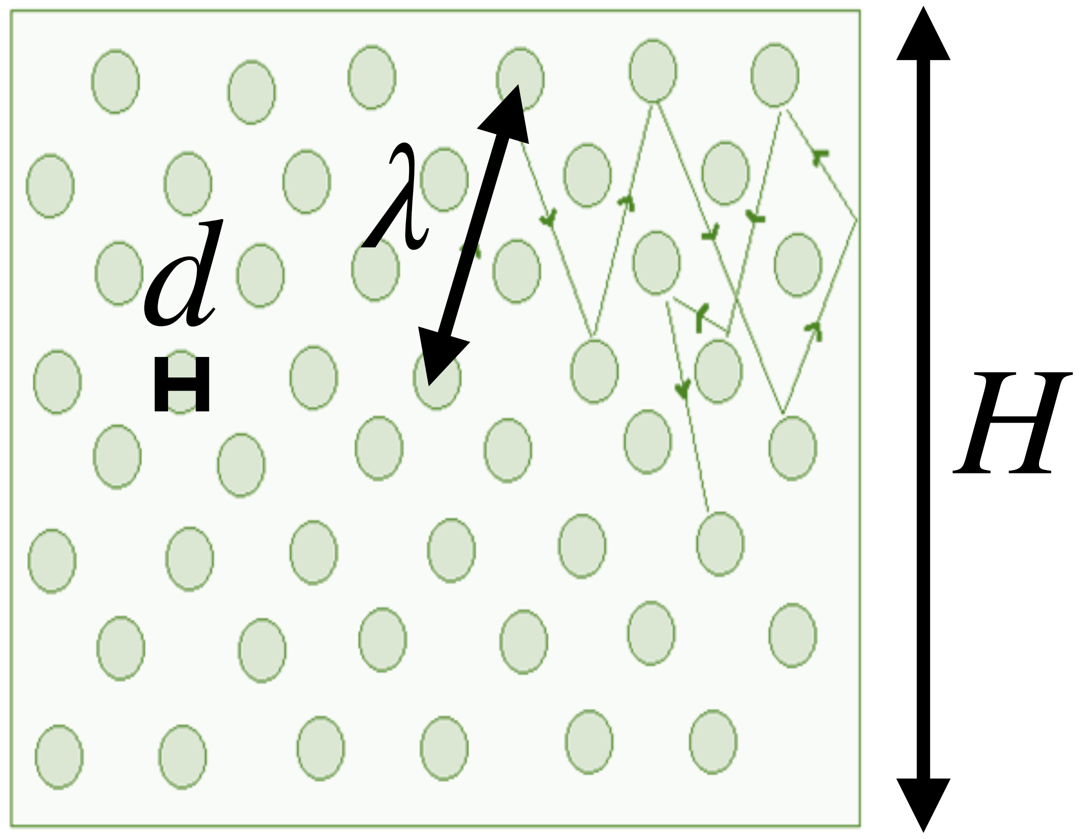


$$t_{2-\mathrm{body}} = \mathcal{O}\left(\frac{N}{\ln N}\right) t_{\mathrm{dyn}} \quad \text{or} \quad \lambda = \mathcal{O}\left(\frac{N}{\ln N}\right) H$$

Mean free path (λ) — Scale height (H)

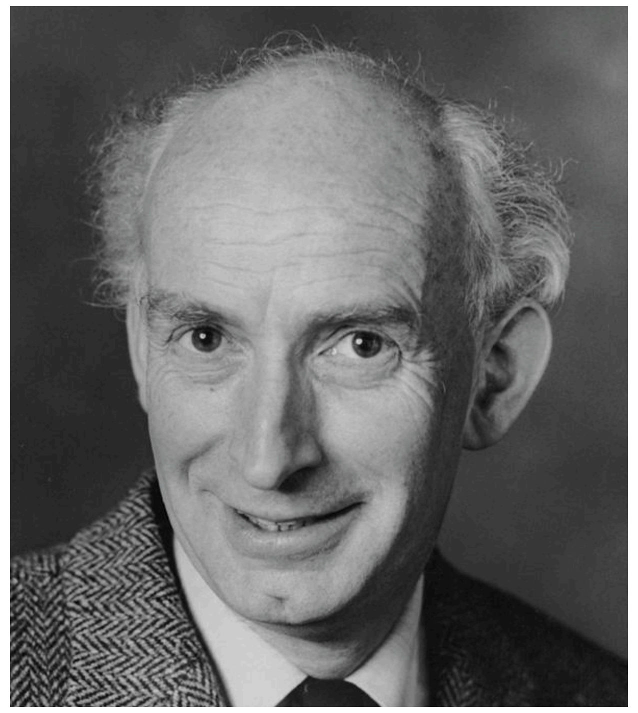
- To apply kinetic theory, fluid needs to satisfy $d \ll \lambda \ll H$.



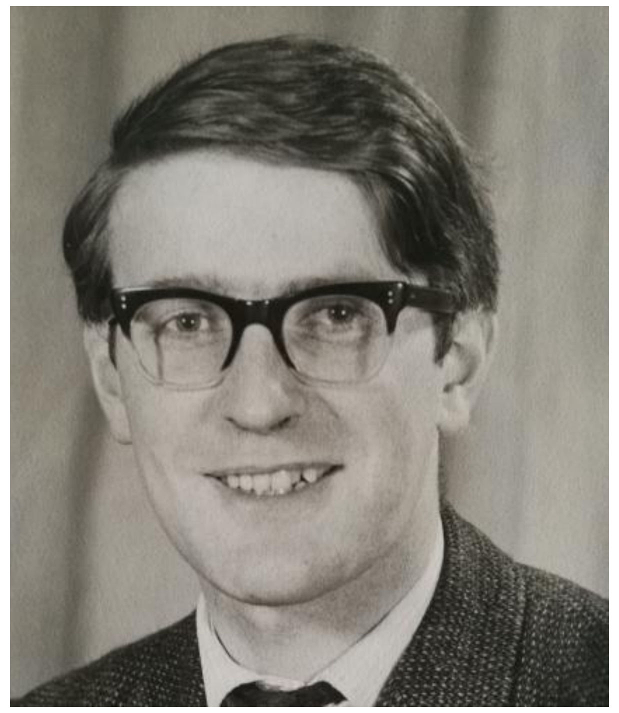


Model star/dark matter as fluid

- Lynden-Bell & Eggleton ’80 built an ***ad-hoc*** conductivity for core collapse of GCs.
- The conduction fluid model yields results in good agreement with N-body / Fokker-Planck simulations.
- Let us treat cold dark matter as a fluid.



D. Lynden-Bell



P. P. Eggleton

$$\kappa = \kappa(\lambda) \sim \lambda^2/\tau$$

$$\kappa = \kappa(H) \sim H^2/\tau$$

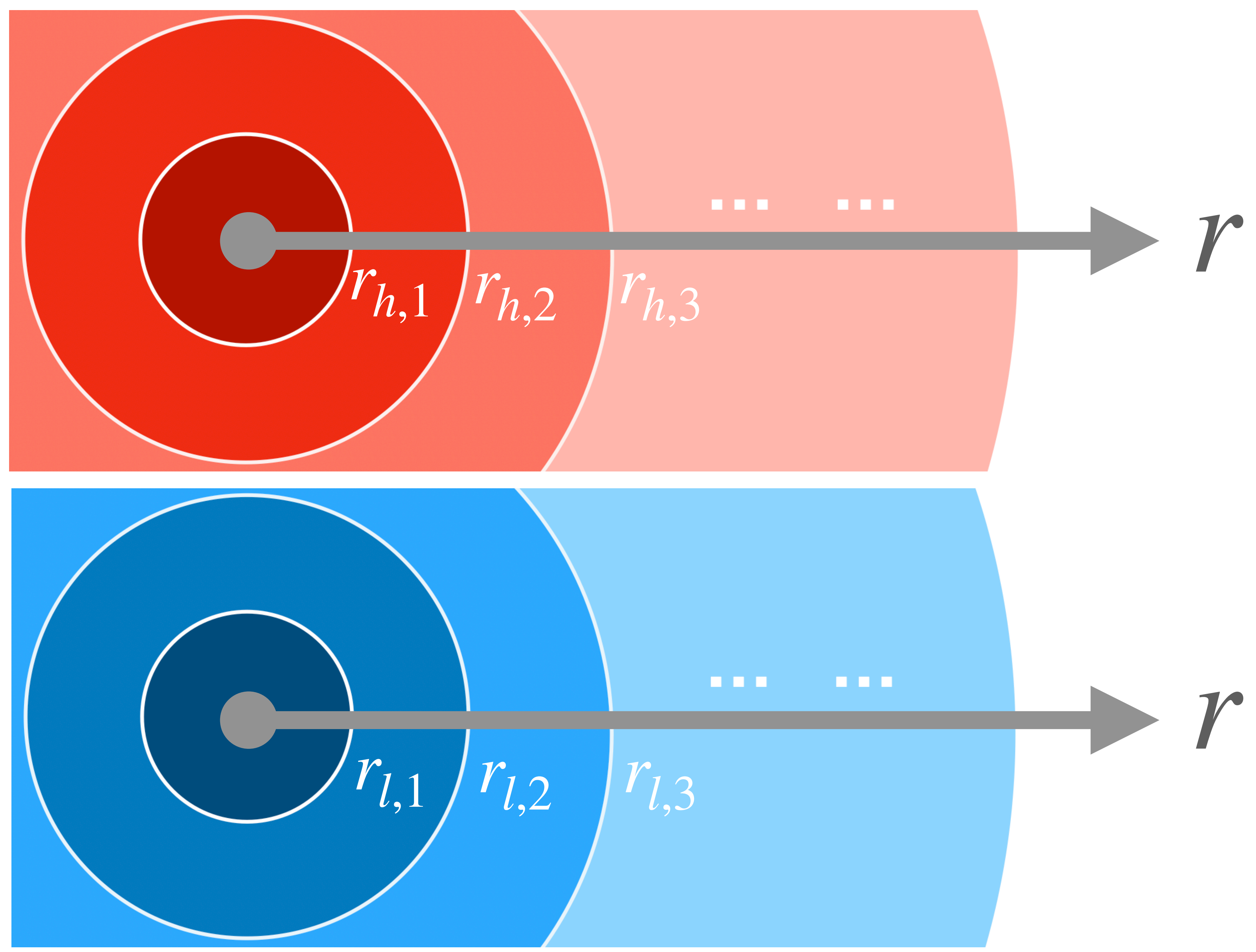
Conductivity

Two-fluid setup

- Consider a progenitor galaxy consisting of a heavier fluid (h) and a lighter fluid (l). Both are spherical, conducting, non-rotating fluids.

Heavier fluid

Lighter fluid



One-fluid equations

- ✦ Mass conservation $$\frac{\partial M}{\partial r} = 4\pi r^2 \rho$$
- ✦ Linear momentum conservation $$\frac{\nabla P}{\rho} = -\nabla \Phi$$
- ✦ Energy conservation $$dw = T\,ds + V\,dp$$
- ✦ Conduction (closing condition) $$\frac{L}{4\pi r^2} = -\kappa \frac{\partial T}{\partial r}$$

Two-fluid equations

- Mass conservation

$$\frac{\partial M_h}{\partial r} = 4\pi r^2 \rho_h, \quad \frac{\partial M_l}{\partial r} = 4\pi r^2 \rho_l.$$

- Linear momentum conservation

$$\frac{1}{\rho_h}\frac{\partial\left(\rho_h\sigma_h^2\right)}{\partial r} = \frac{1}{\rho_l}\frac{\partial\left(\rho_l\sigma_l^2\right)}{\partial r} = -\frac{G(M_l + M_h)}{r^2}.$$

- Energy conservation

$$\begin{cases} \rho_h\sigma_h^2\left(\frac{D}{Dt}\right)\ln\frac{\sigma_h^3}{\rho_h} = -\frac{1}{4\pi r^2}\frac{\partial L_h}{\partial r} - R \\ \rho_l\sigma_l^2\left(\frac{D}{Dt}\right)\ln\frac{\sigma_l^3}{\rho_l} = -\frac{1}{4\pi r^2}\frac{\partial L_l}{\partial r} + R \end{cases}$$

σ_h, σ_l: 1D velocity dispersion

- Conduction

$$\frac{L_i}{4\pi r^2} = -\beta_i\frac{\alpha b G\rho_i m_i \ln\Lambda_i}{\sqrt{3}\sigma_i}\frac{\partial\sigma_i^2}{\partial r} \quad (i = h, l)$$

Dynamical interaction (local)

Change of velocity in a single collision

$$\Delta \boldsymbol{v}_h = \frac{m_l}{m_l + m_h} |\boldsymbol{v}_h - \boldsymbol{v}_l| \left(\hat{n} - \frac{\boldsymbol{v}_h - \boldsymbol{v}_l}{|\boldsymbol{v}_h - \boldsymbol{v}_l|} \right)$$

Change of energy in a single collision

$$\Delta E_l = m_l \boldsymbol{v}_{\rm CM} \cdot \Delta \boldsymbol{v}_l = -m_h \boldsymbol{v}_{\rm CM} \cdot \Delta \boldsymbol{v}_h$$

Average rate

$$\frac{d\langle E_l \rangle}{dt} = m_l n_h \int d^3 v_l f_l \int d^3 v_h f_h |\boldsymbol{v}_l - \boldsymbol{v}_h| \int d\hat{n} \frac{d\sigma}{d\hat{n}} \boldsymbol{v}_{\rm CM} \cdot \Delta \boldsymbol{v}_l$$

Gravitational interaction

$$R = n_l \frac{d\langle E_l \rangle}{dt} = \frac{4(2\pi)^{1/2} G^2 \rho_h \rho_l \ln \Lambda_{hl}}{(\sigma_l^2 + \sigma_h^2)^{3/2}} \left(m_h \sigma_h^2 - m_l \sigma_l^2 \right)$$

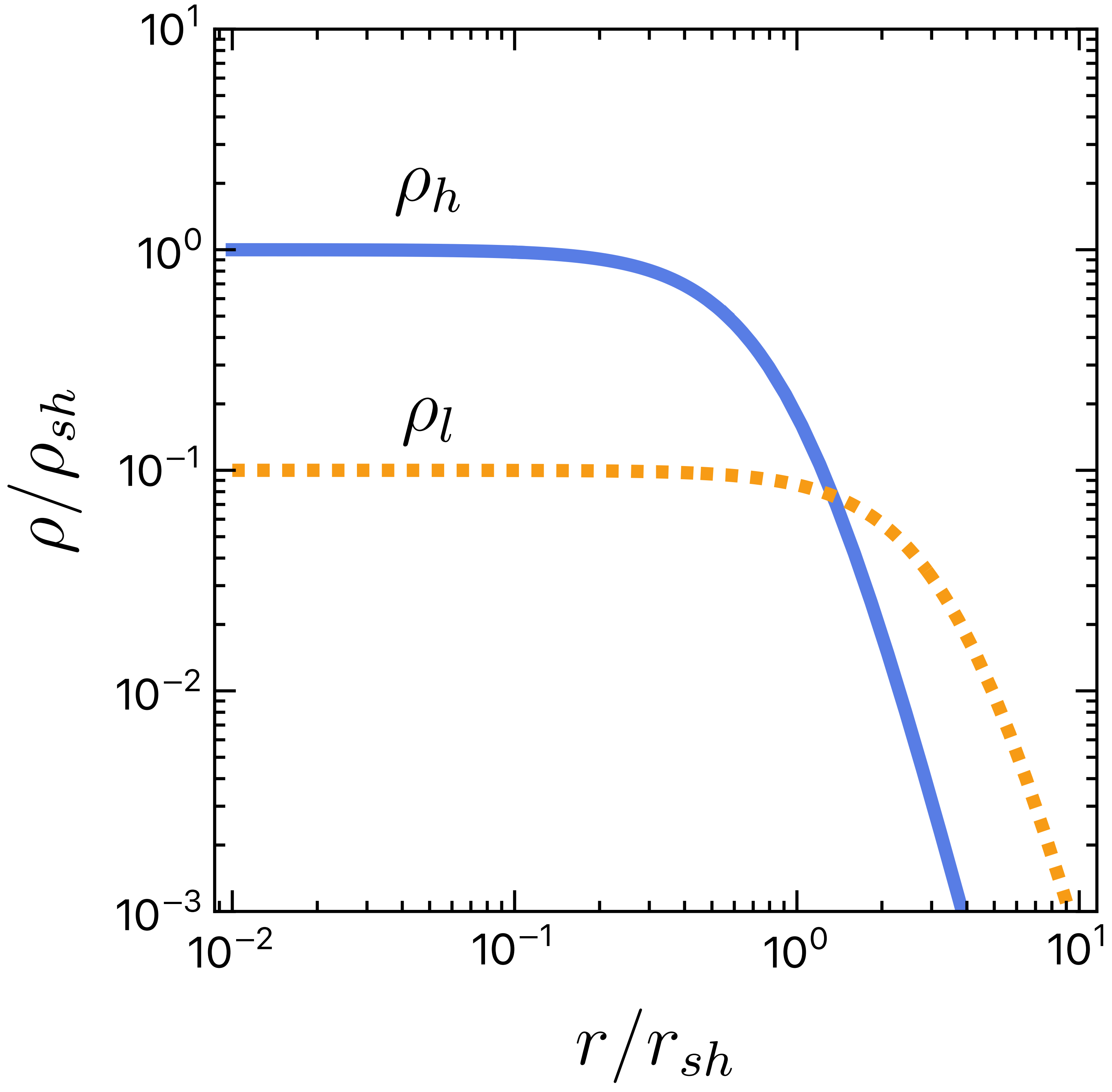
See e.g. Graham & Ramani ‘23

Initial condition

✦ Initial profiles: Plummer

$$\rho_h(r) = \frac{\rho_{sh}}{(1 + r^2/r_{sh}^2)^{5/2}}, \rho_l(r) = \frac{\rho_{sl}}{(1 + r^2/r_{sl}^2)^{5/2}}.$$

$$\begin{cases} \xi \equiv \rho_{sl}/\rho_{sh} \\ \zeta \equiv r_{sl}/r_{sh} \end{cases} \Rightarrow \frac{M_{\mathrm{tot},l}}{M_{\mathrm{tot},h}} = \xi\zeta^3$$



Two-fluid simulation procedure

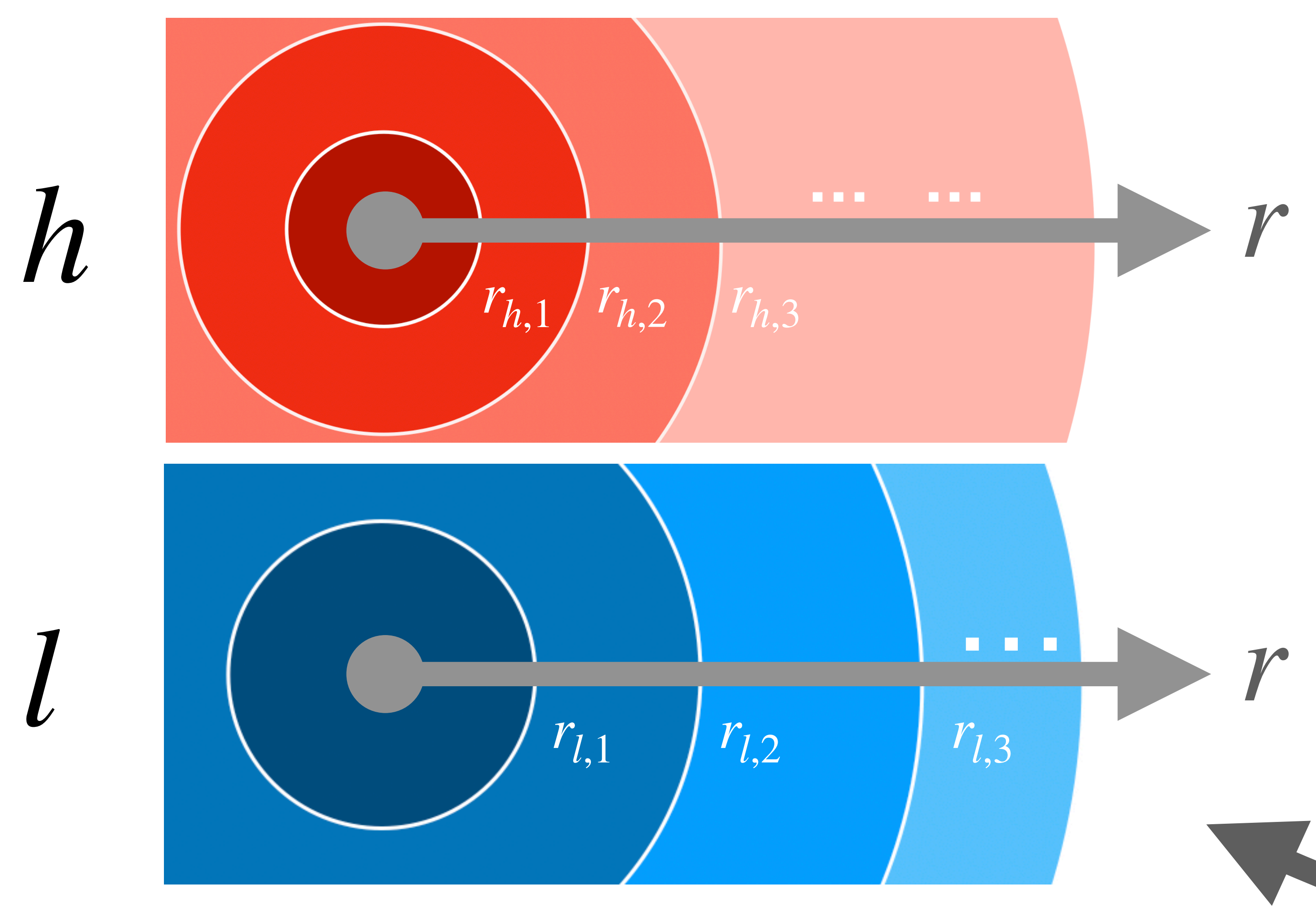
$t \to t + \Delta t$

Conduction & interaction

Solve a $2N \times 2N$ sparse matrix

Semi-implicit method

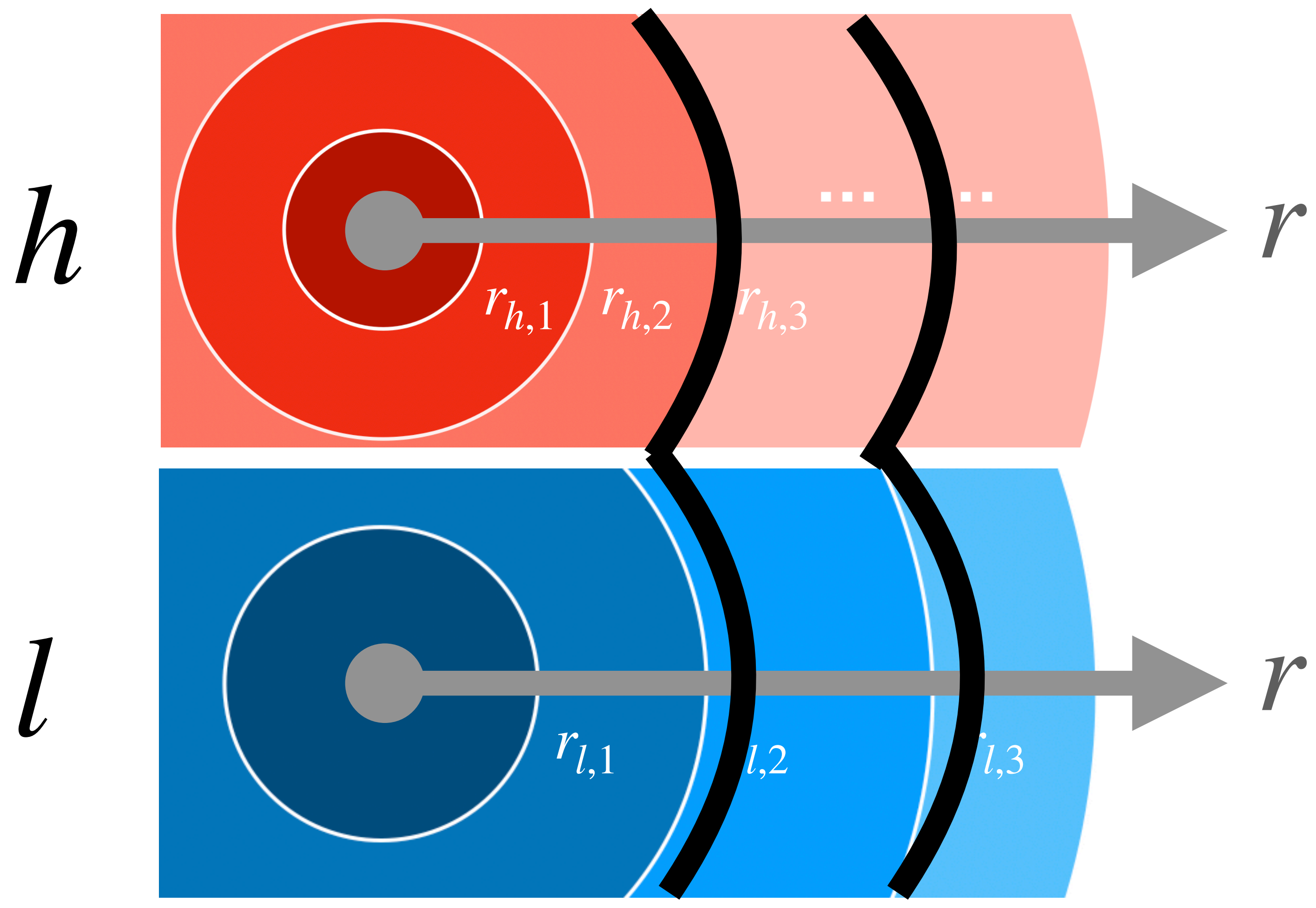
Hydrostatic relaxation



Solve a $2N \times 2N$ tridiagonal matrix

Two-fluid simulation procedure

$t \to t + \Delta t$



Conduction & interaction

Hydrostatic relaxation

Realignment

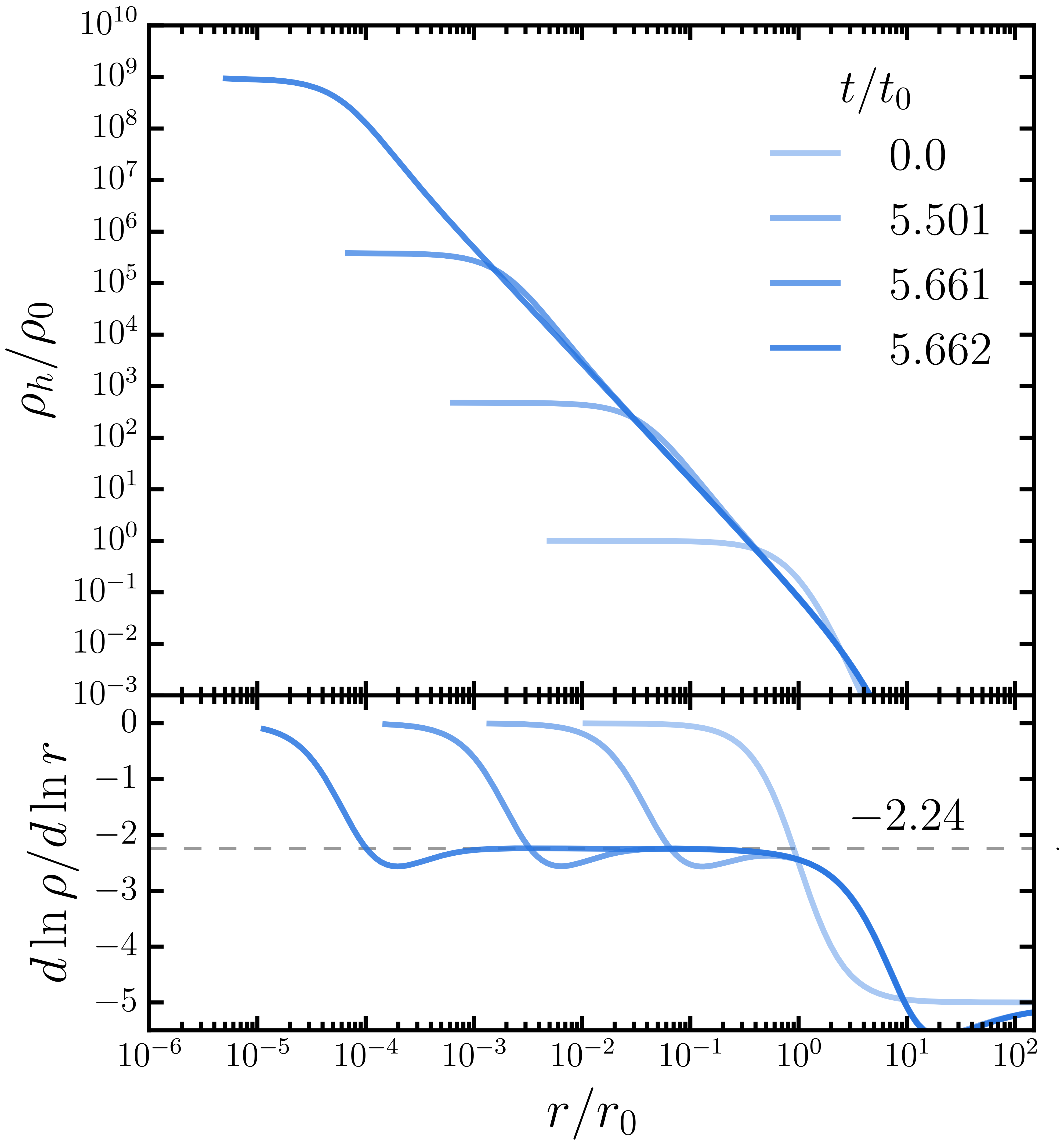
Solve a $2N \times 2N$ sparse matrix
Semi-implicit method

Solve a $2N \times 2N$ tridiagonal matrix

Result & Application

Self-consistency checks

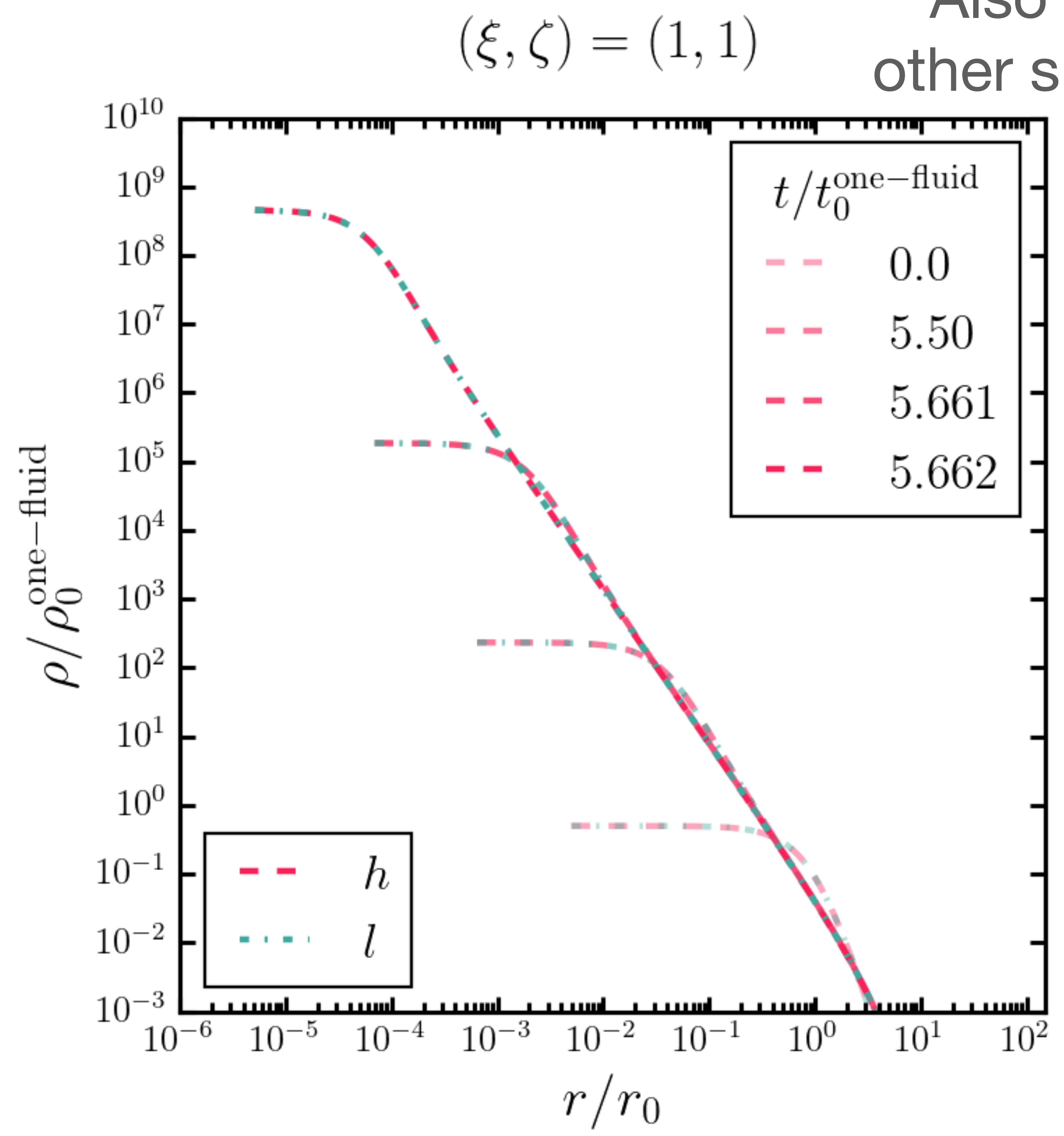
- ✦ One-fluid limit. Agree w/ Shapiro '18 ✅



Self-consistency checks

- ✦ One fluid splitting in two:
  - ✦ Two fluids evolve according to the splitting ratio.✅

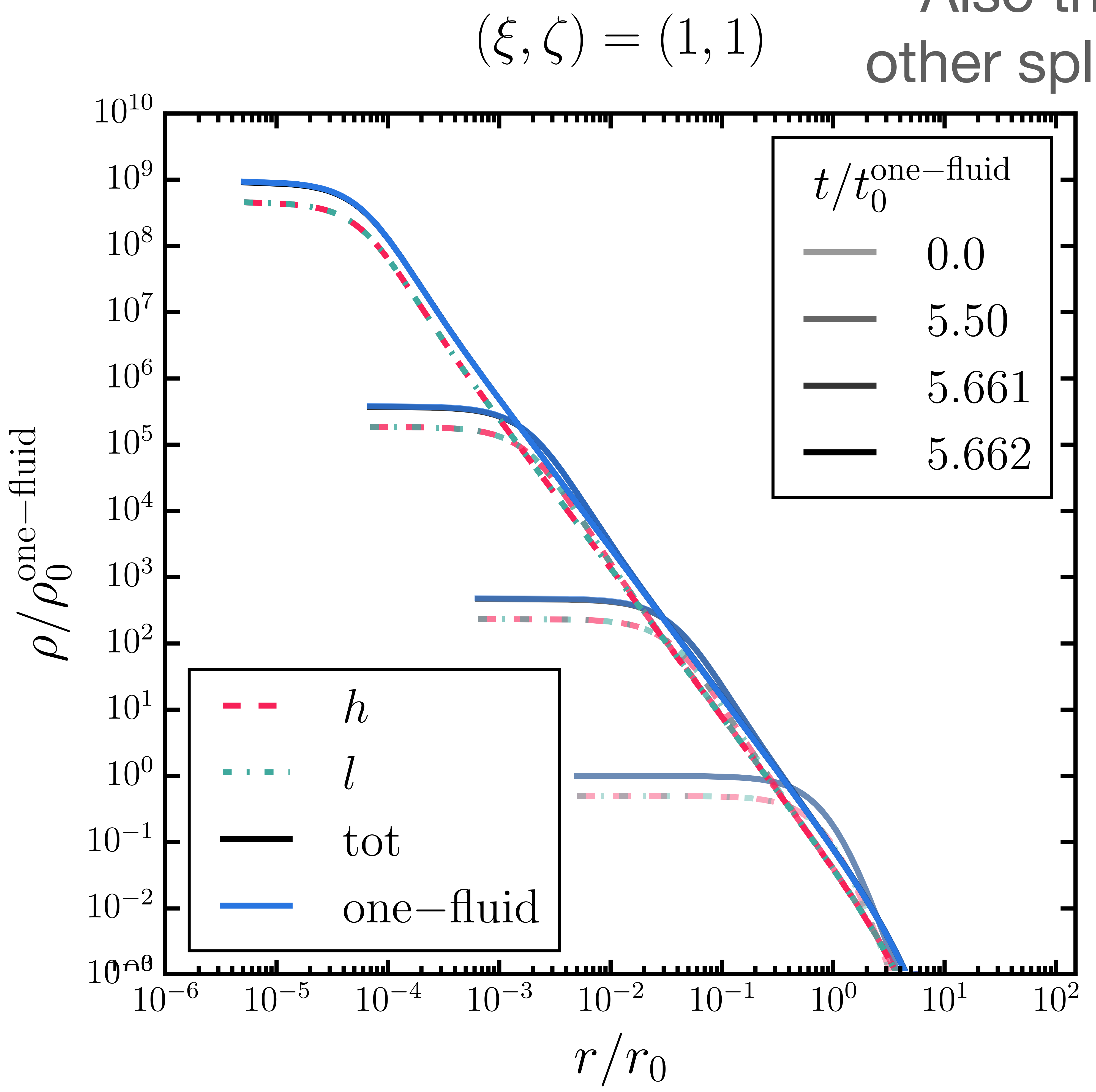
Also tried other splitting



Self-consistency checks

- One fluid splitting in two:
  - The sum of the two fluids agrees w/ one fluid. ✅

Also tried other splitting



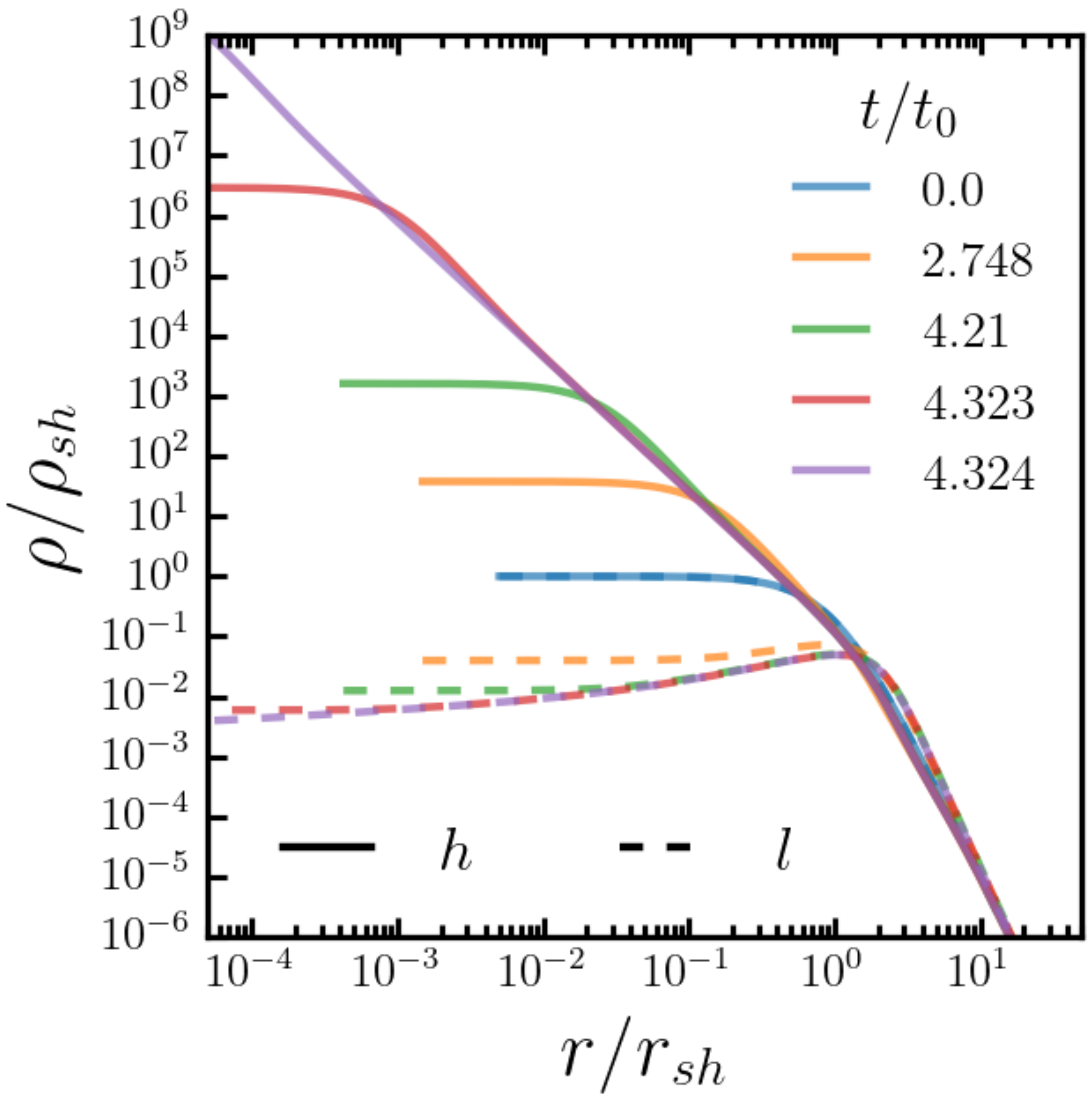
Black lines are under blue lines

Fluid vs. N-body

Central relaxation time

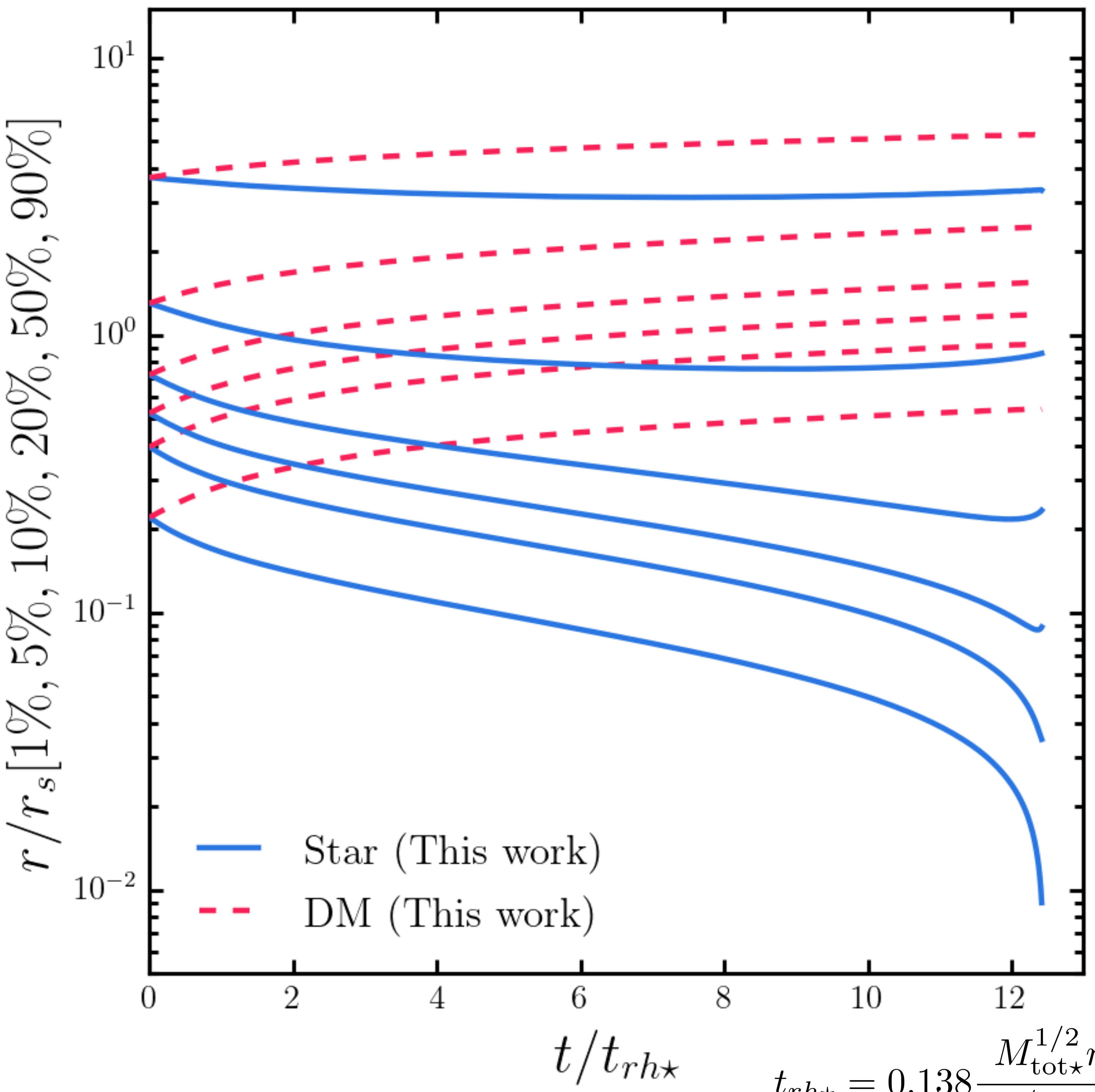
$$t_0 = \frac{\sigma_{sh}^3}{12\pi G^2 m_h \rho_{sh} \ln \Lambda_{hl}} = \frac{(4\pi)^{1/2}\rho_{sh}^{1/2} r_{sh}^3}{3G^{1/2} m_h \ln \Lambda_{hl}}$$

- Simulate benchmark I of BM08: $< m_\star > = 0.34\,\mathrm{M}_\odot$ and $m_\chi = 0.03\,\mathrm{M}_\odot$ and identical initial Plummer profiles.
- Observe core collapse of heavier fluid and suppression of lighter fluid.



Fluid vs. N-body

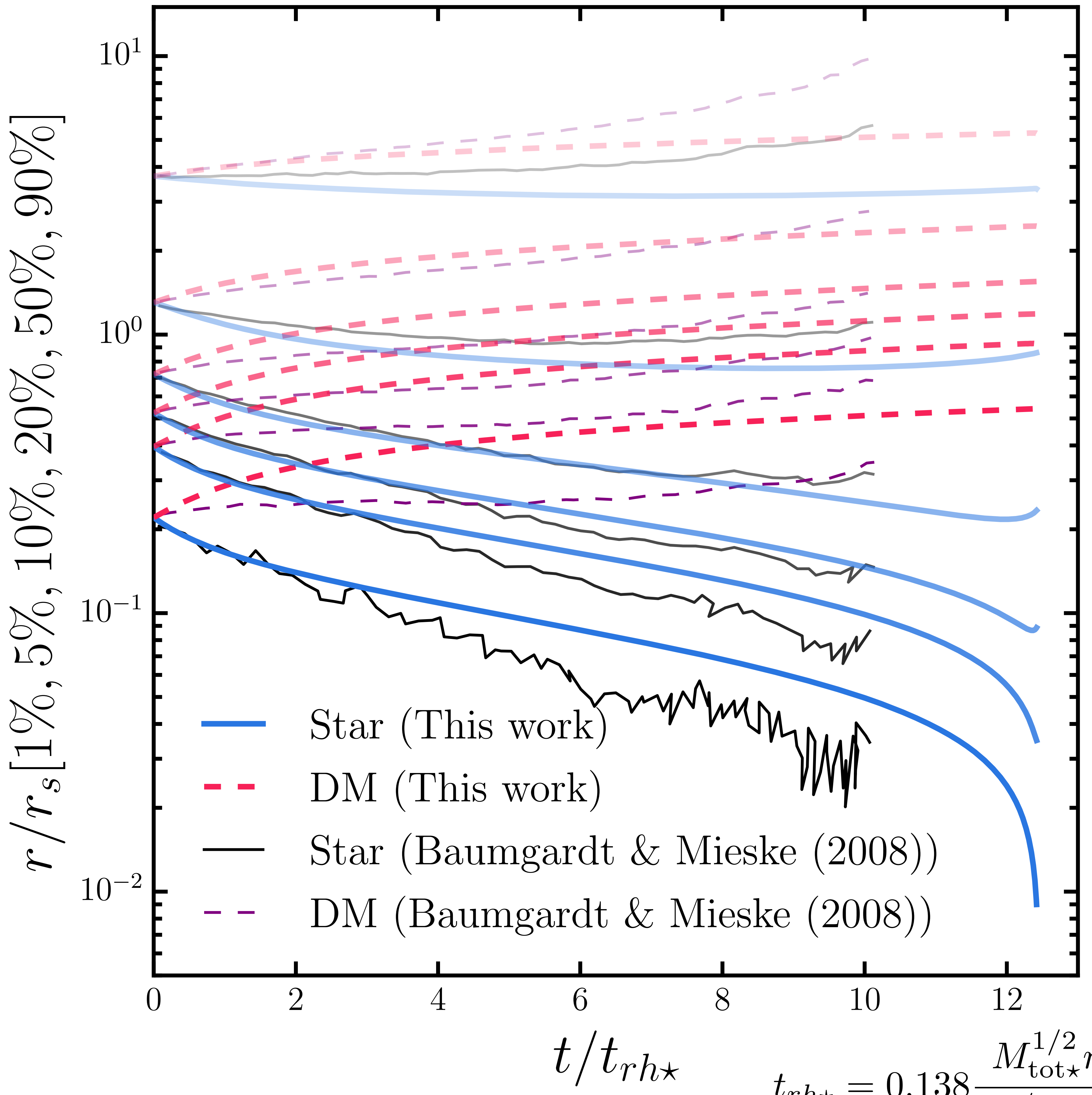
- ✦ Observe mass segregation in the evolution of Lagrangian radii.



$$t_{rh\star} = 0.138 \frac{M_{\mathrm{tot}\star}^{1/2} r_{50\%\star}^{3/2}}{G^{1/2} m_\star \ln \Lambda_\star}$$

Fluid vs. N-body

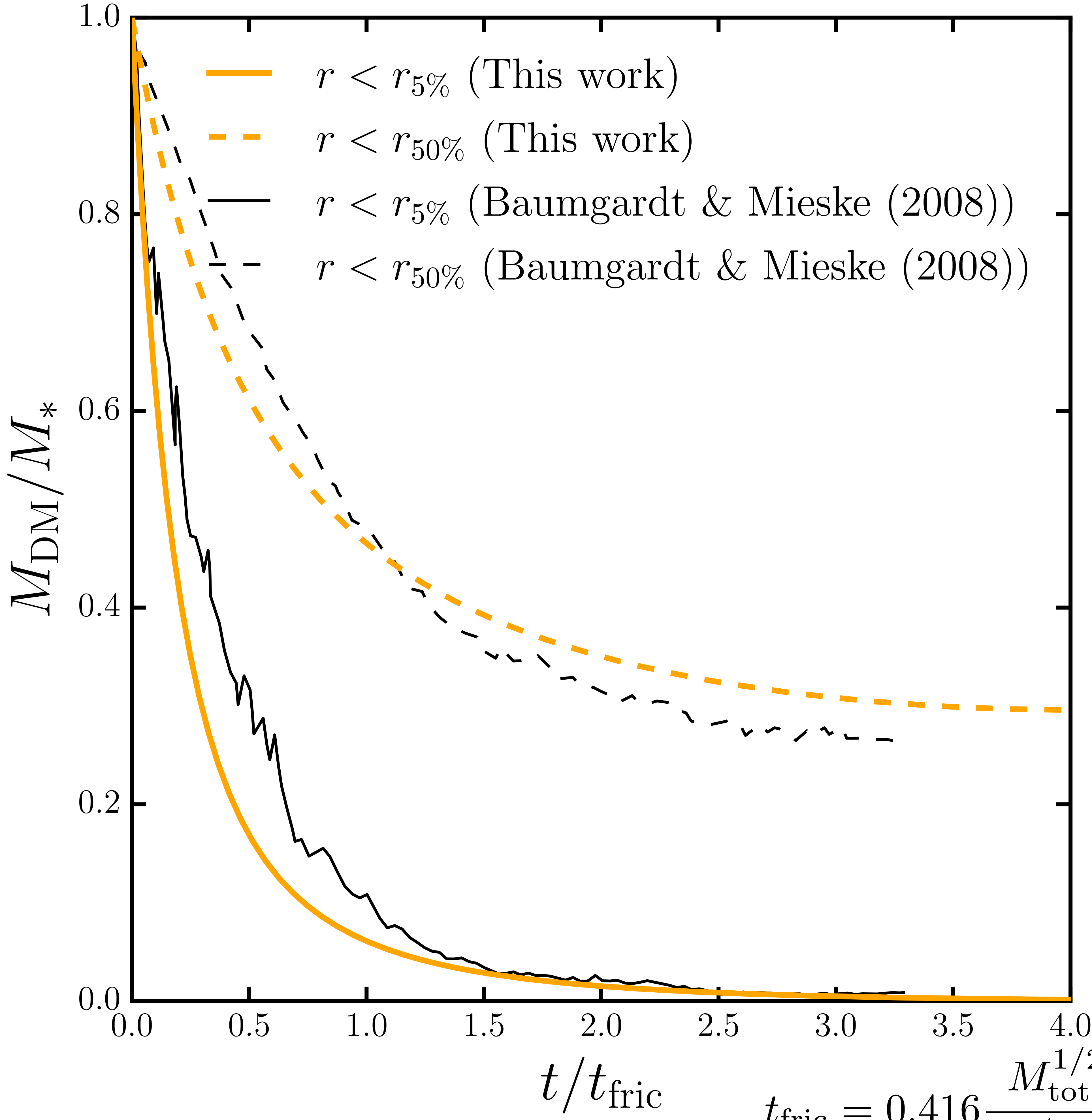
- ✦ Reasonable agreement with BM08.
- ✦ Go deeper in the core-collapse region.



$$t_{rh\star} = 0.138 \frac{M_{\mathrm{tot}\star}^{1/2} r_{50\%\star}^{3/2}}{G^{1/2} m_\star \ln \Lambda_\star}$$

Fluid vs. N-body

- ✦ Observe dark matter mass depletion in the inner halo in the enclosed mass ratio $M_\chi/M_\star$ evolution.
- ✦ Reasonable agreement with BM08.
- ✦ Similar agreements for other benchmarks of BM08.

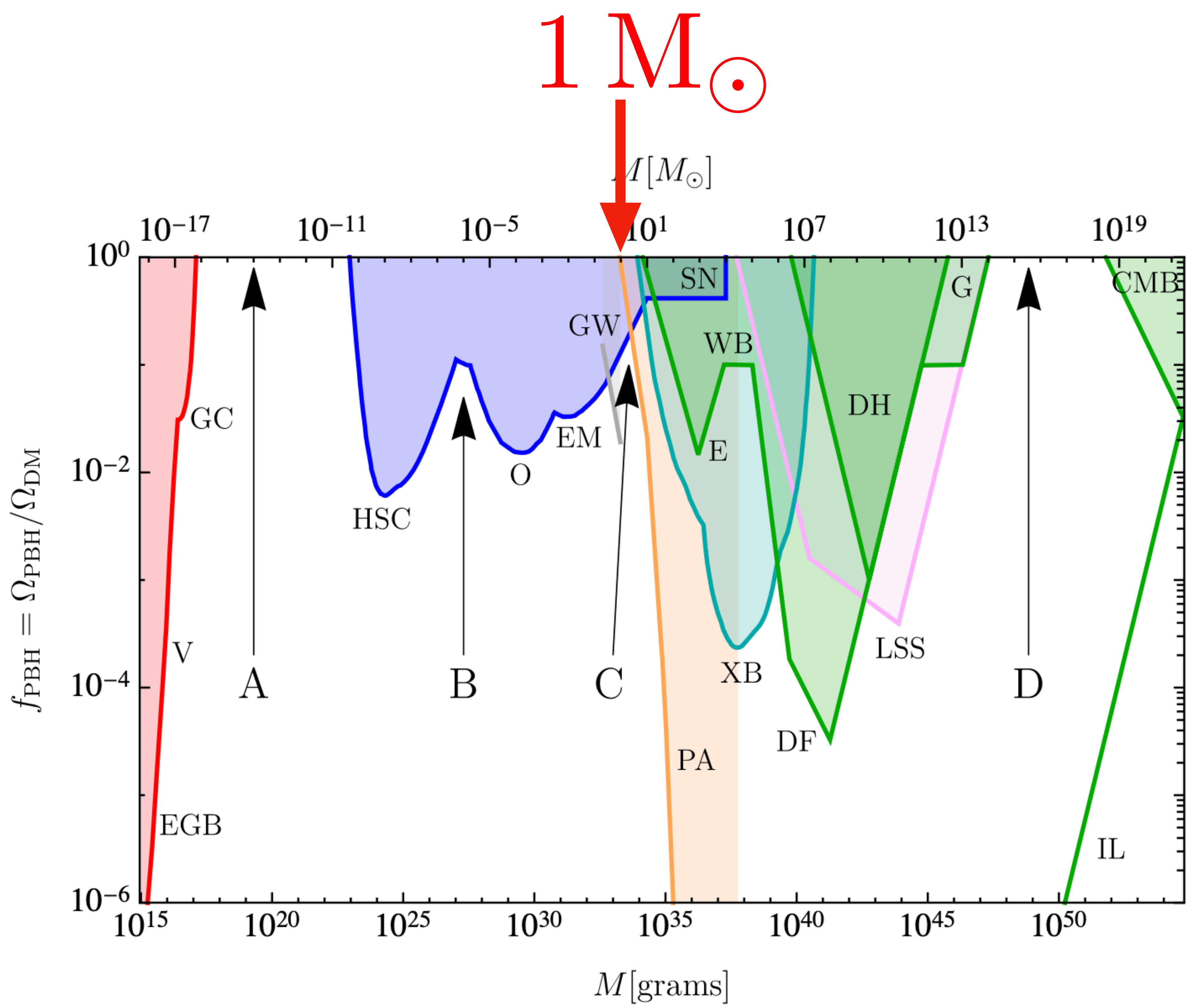


$$t_{\rm fric} = 0.416\frac{M_{\rm tot}^{1/2} r_{50\%\star}^{3/2}}{G^{1/2} m_\star \ln \Lambda_\star}$$

Scan initial profiles

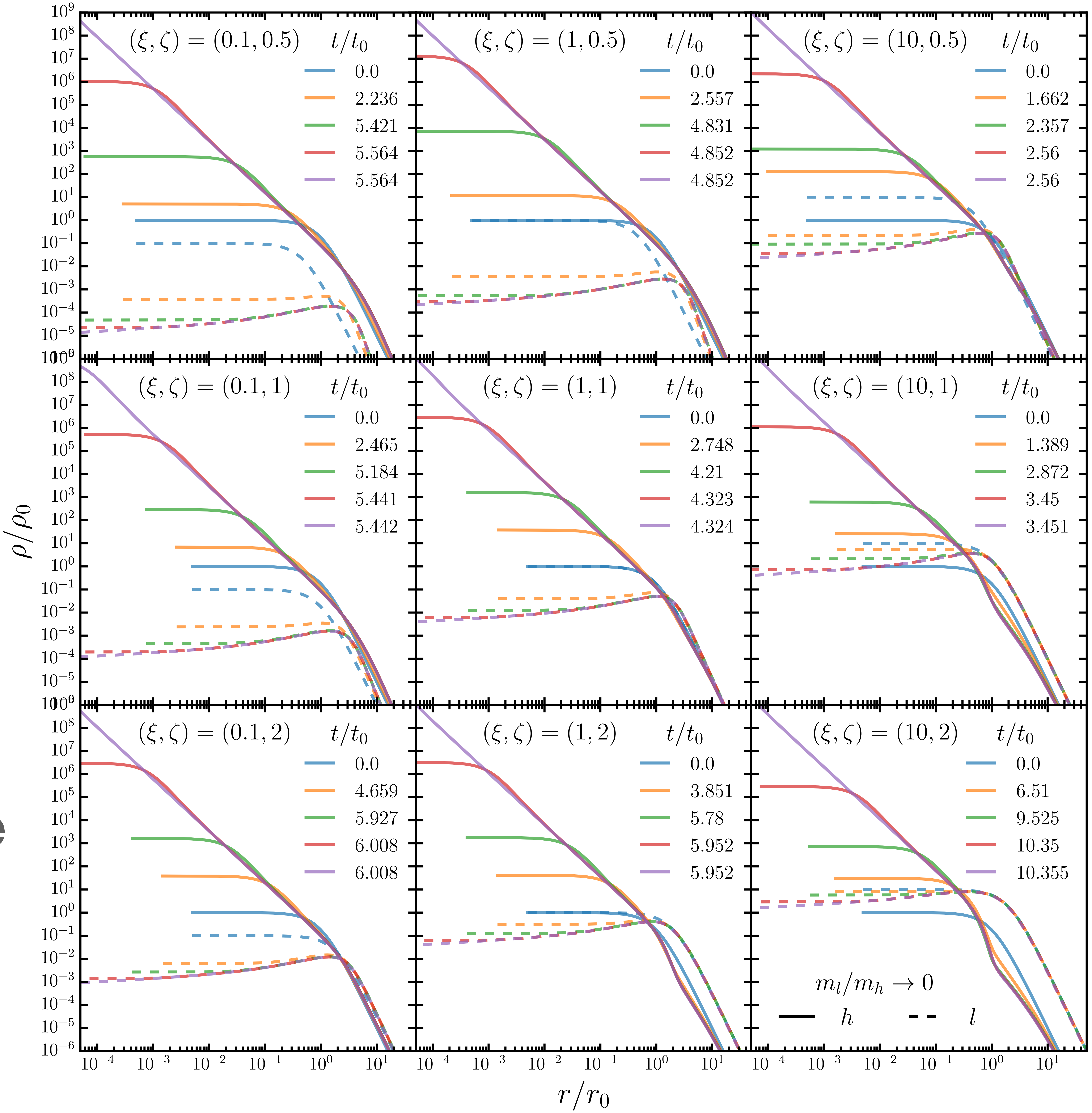
- Consider the mass hierarchy $m_l \ll m_h$. Motivated by mass windows of primordial black holes (PBHs).

- Scan initial Plummer profiles w/ $\rho_{sl}/\rho_{sh} = (0.1,\ 1,\ 10)$ and $r_{sl}/r_{sh} = (0.5,\ 1,\ 2)$.



PBH constraints

Carr & Kuhnel, '22



Core-collapse at

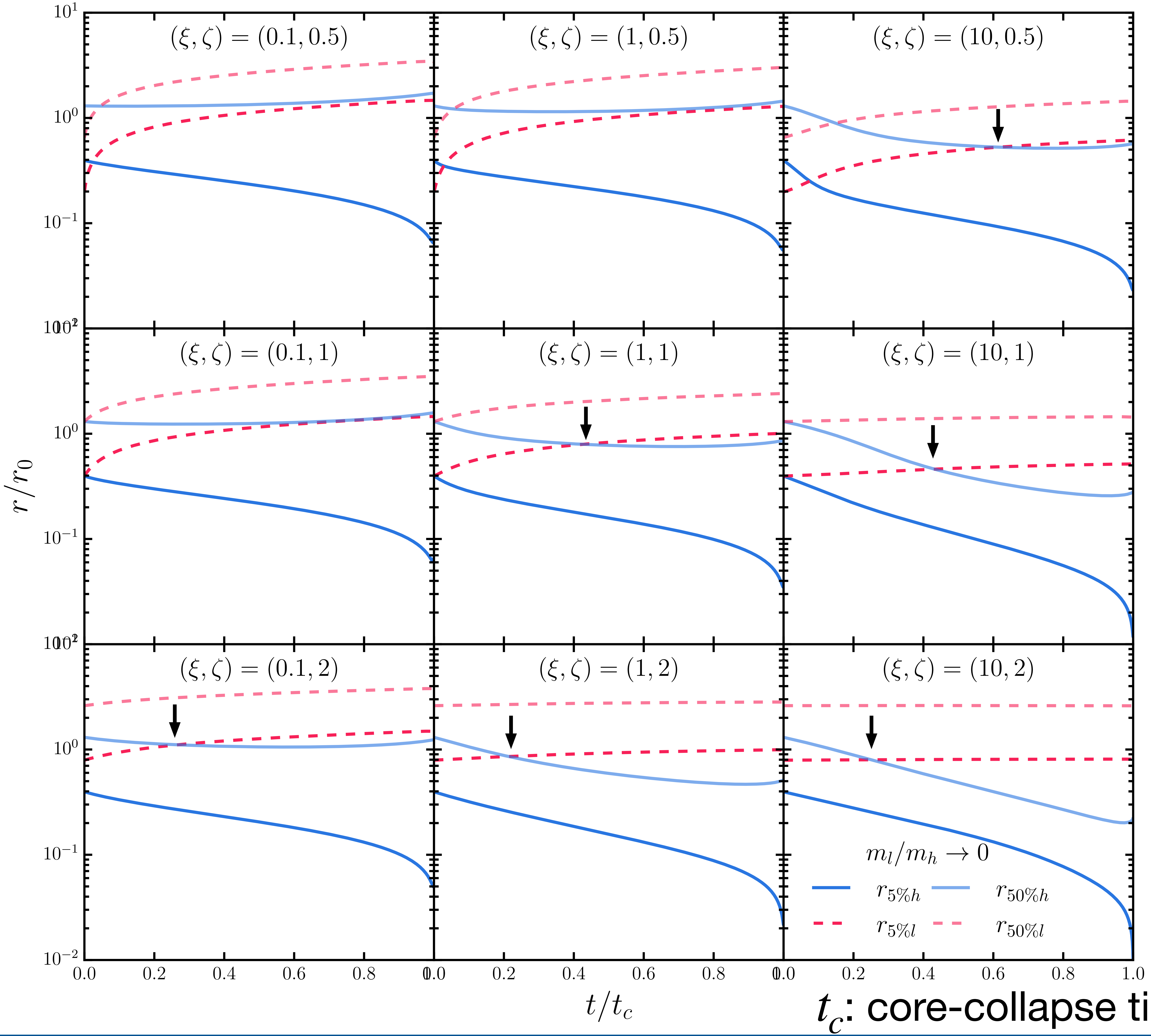
$t_c = \text{few} \times t_0.$

$$t_0 = \frac{(4\pi)^{1/2}\rho_{sh}^{1/2} r_{sh}^3}{3G^{1/2} m_h \ln \Lambda_{hl}}$$

Central relaxation time of the heavier fluid

Significant Mass segregation happens at $t \sim t_0$.

(when $r_{5\%l}$ cross $r_{50\%h}$)



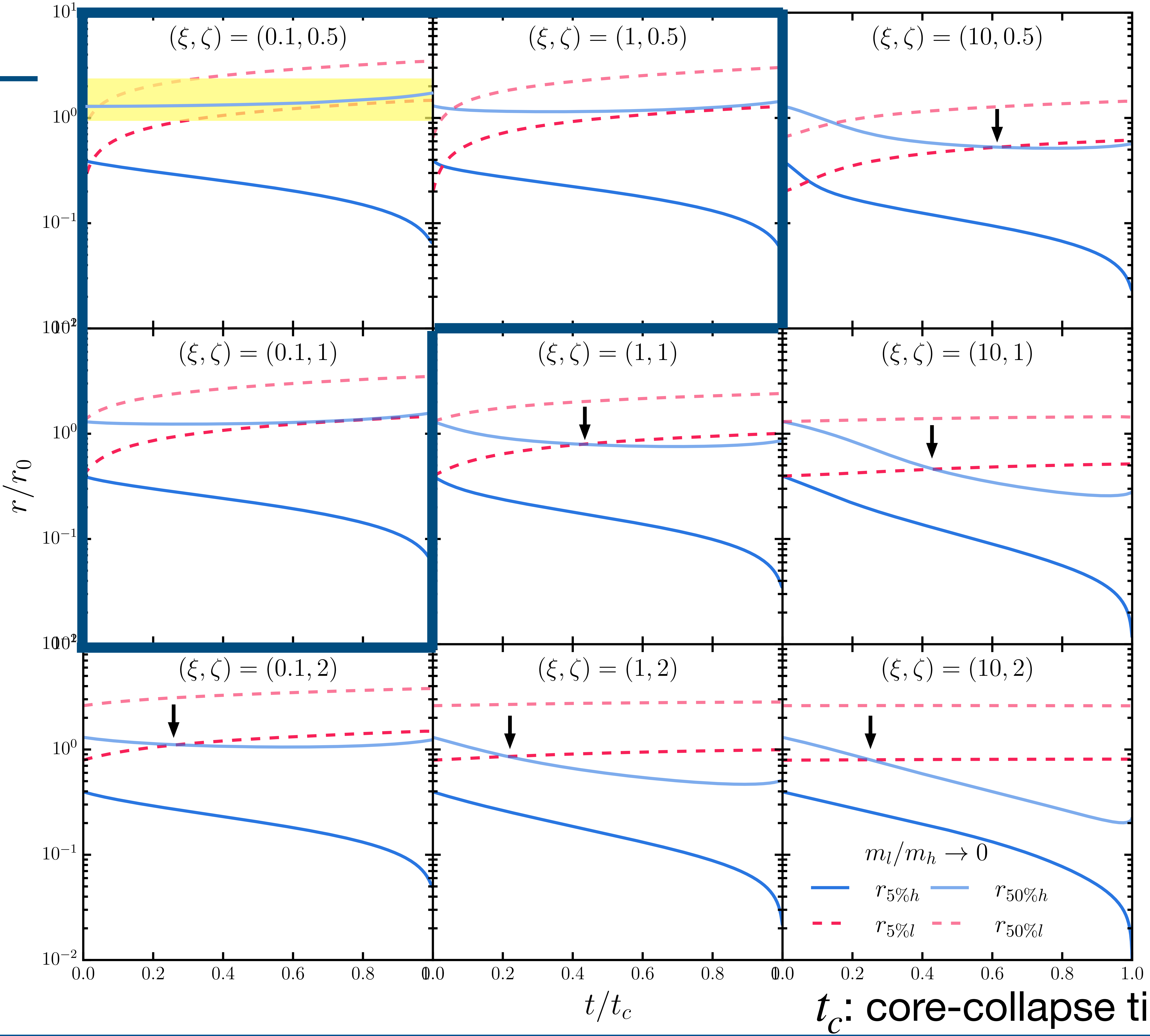
t_c: core-collapse time

$\xi \equiv \rho_{sl}/\rho_{sh}$

$\zeta \equiv r_{sl}/r_{sh}$

$$\frac{M_{\mathrm{tot},l}}{M_{\mathrm{tot},h}} = \xi\zeta^3$$

Heavier fluid at large radii barely move if $M_{\mathrm{tot},l}/M_{\mathrm{tot},h} \ll 1$

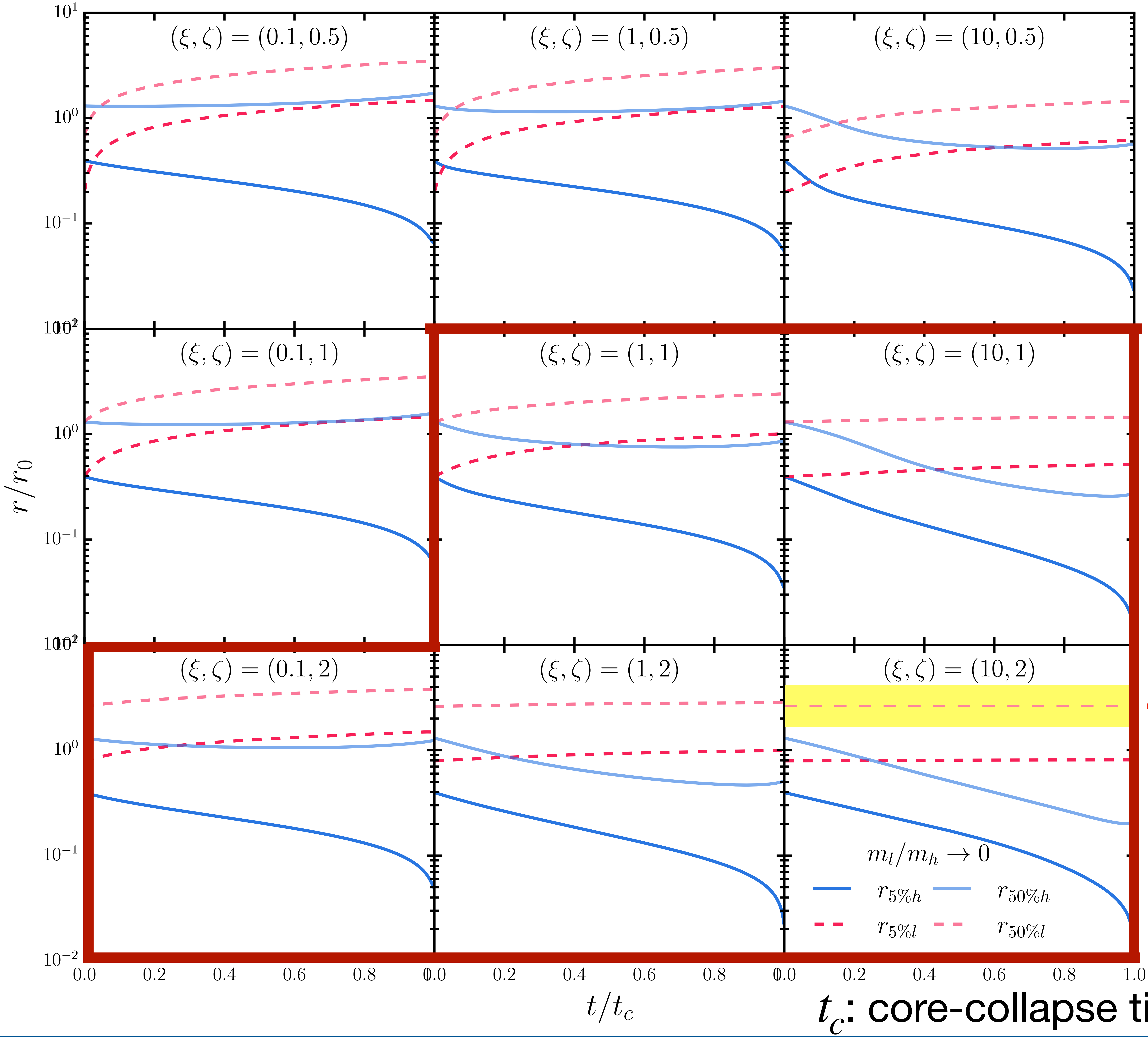


t_c: core-collapse time

$\xi \equiv \rho_{sl}/\rho_{sh}$

$\zeta \equiv r_{sl}/r_{sh}$

$$\frac{M_{\mathrm{tot},l}}{M_{\mathrm{tot},h}} = \xi\zeta^3$$



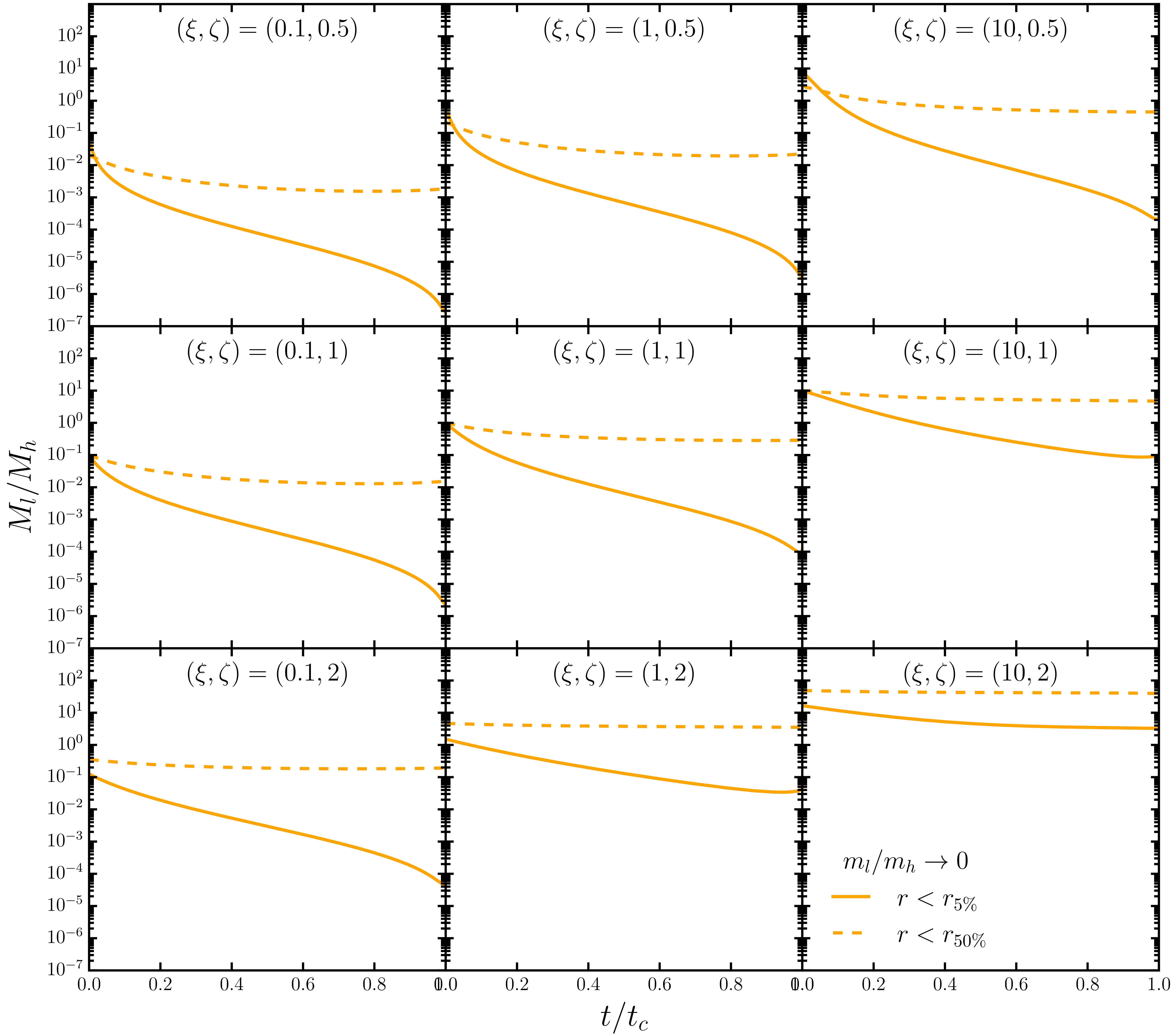
$$\xi \equiv \rho_{sl}/\rho_{sh}$$

$$\zeta \equiv r_{sl}/r_{sh}$$

$$\frac{M_{\mathrm{tot},l}}{M_{\mathrm{tot},h}} = \xi\zeta^3$$

→ No significant depletion for lighter fluid if $M_{\mathrm{tot},l}/M_{\mathrm{tot},h} \gg 1$

t_c: core-collapse time



$\xi \equiv \rho_{sl}/\rho_{sh}$

$\zeta \equiv r_{sl}/r_{sh}$

$$\frac{M_{\mathrm{tot},l}}{M_{\mathrm{tot},h}} = \xi\zeta^3$$

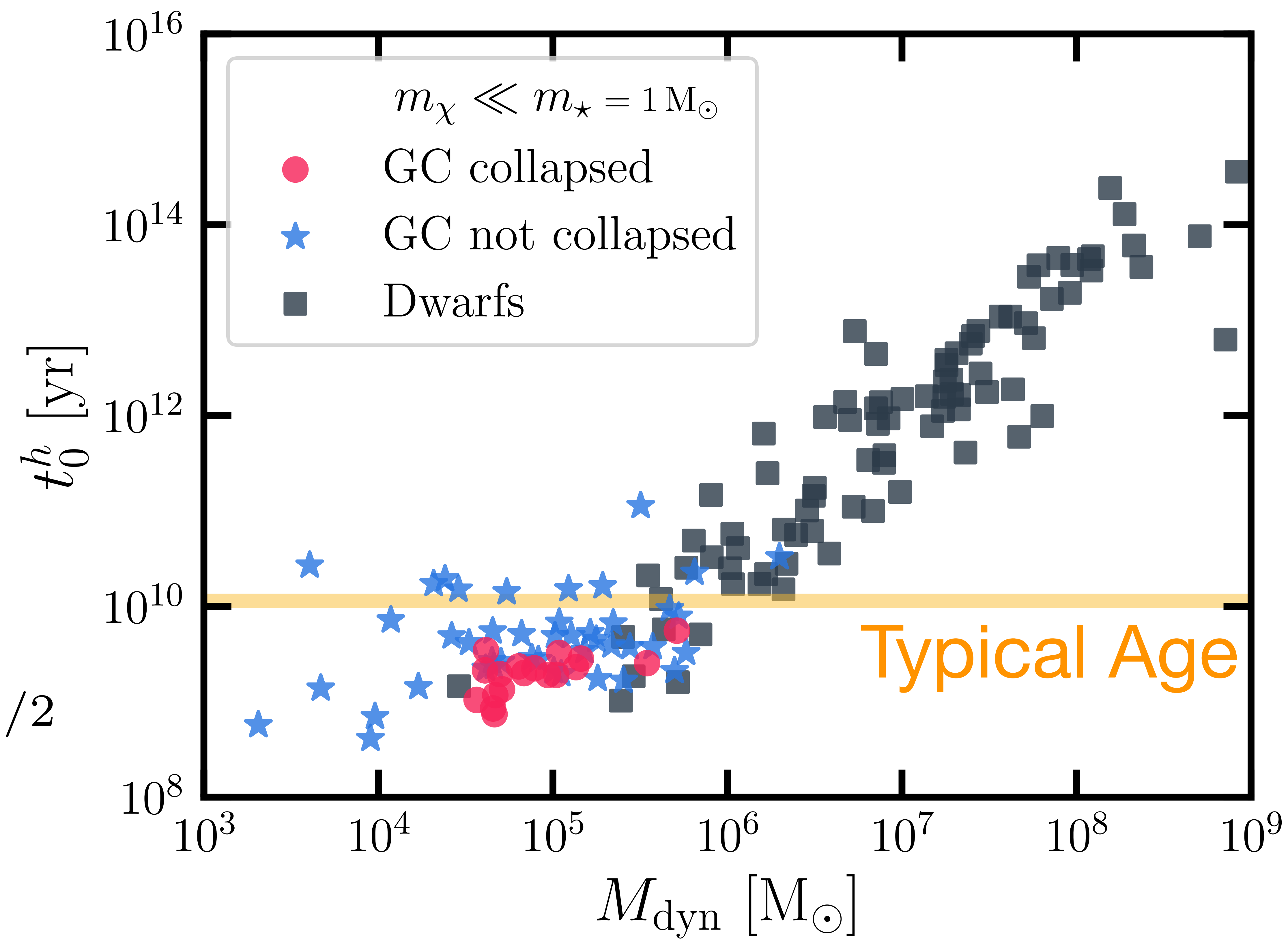
t_c: core-collapse time for each system

Sub-$\mathrm{M}_\odot$ dark matter

- Time for mass segregation and core collapse ≈ **central relaxation time of the heavier component.**

- If $m_\chi \ll m_\star$, the dynamical evolution timescale is

$$t_0^\star = \frac{5.8 \times 10^6\,\mathrm{yr}}{\ln(2\gamma M_{\rm tot}/m_\star)} \left(\frac{\mathrm{M}_\odot}{m_\star}\right) \left(\frac{M_{\rm tot\star}}{\mathrm{M}_\odot}\right)^{1/2} \left(\frac{r_{50\%\star}}{\mathrm{pc}}\right)^{3/2}$$

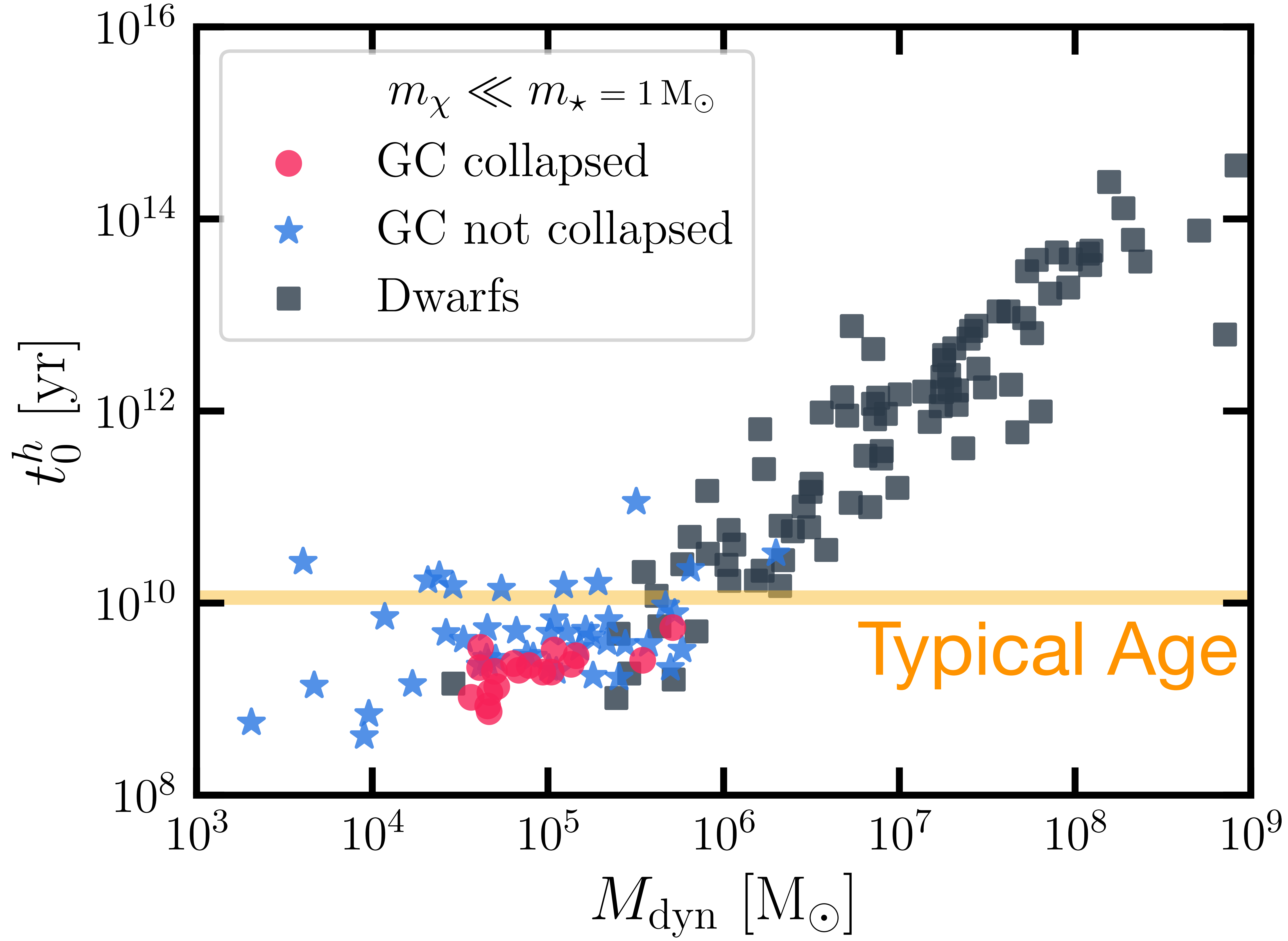


Selected from Harris GCs & Local Volume Database

Sub-$M_\odot$ dark matter

- ✦ The majority of the GCs should show dynamical evolution.

- ✦ The majority of dwarfs show no stellar core-collapse or mass segregation. (except the ones with least stars.)

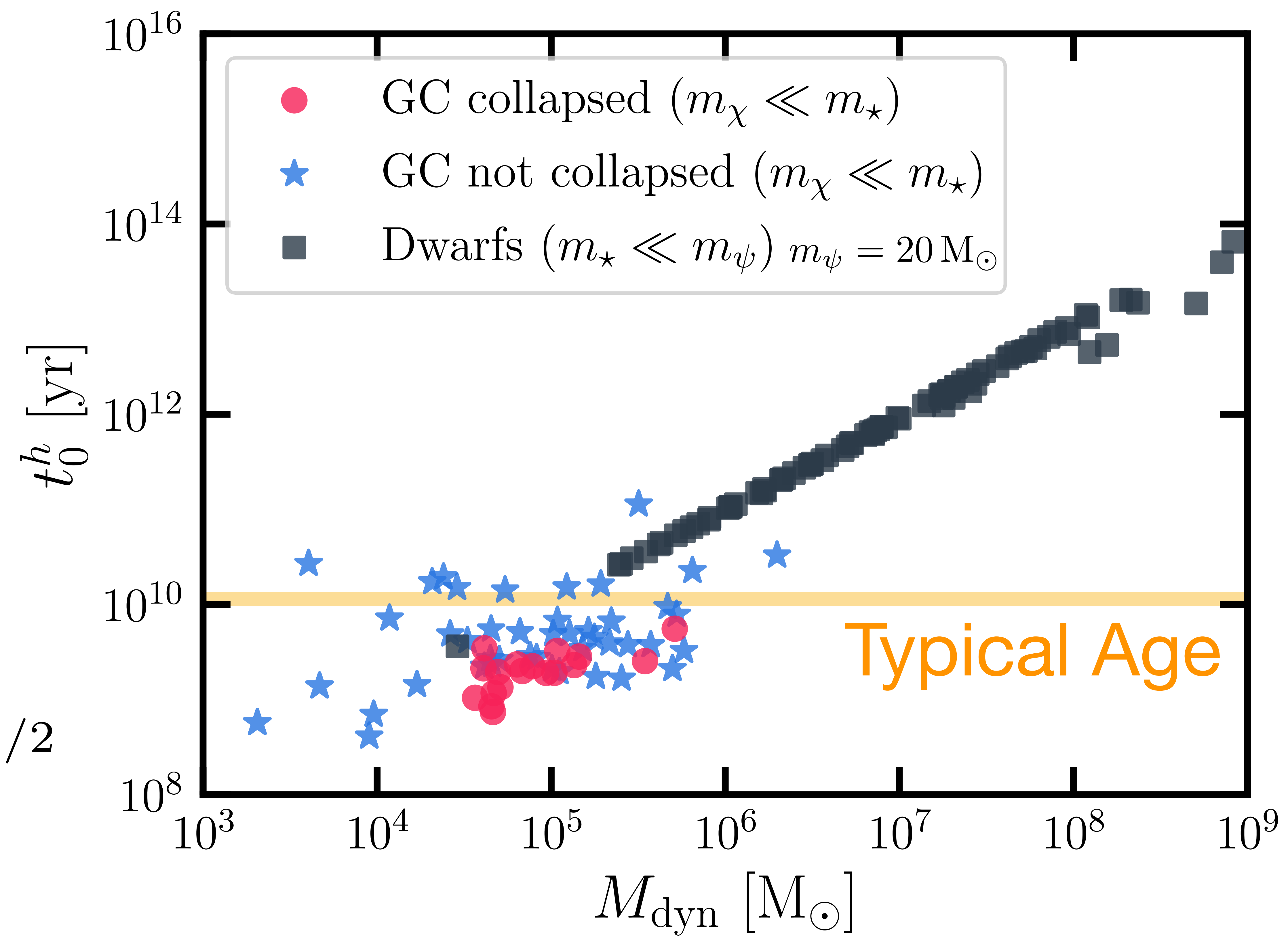


Super-$M_\odot$ dark matter

- Consider an extreme case, $m_\chi \ll m_\star \ll m_\psi$ and dwarfs (GCs) are mostly made of ψ (χ),

- The dynamical evolution timescale for the dwarfs is

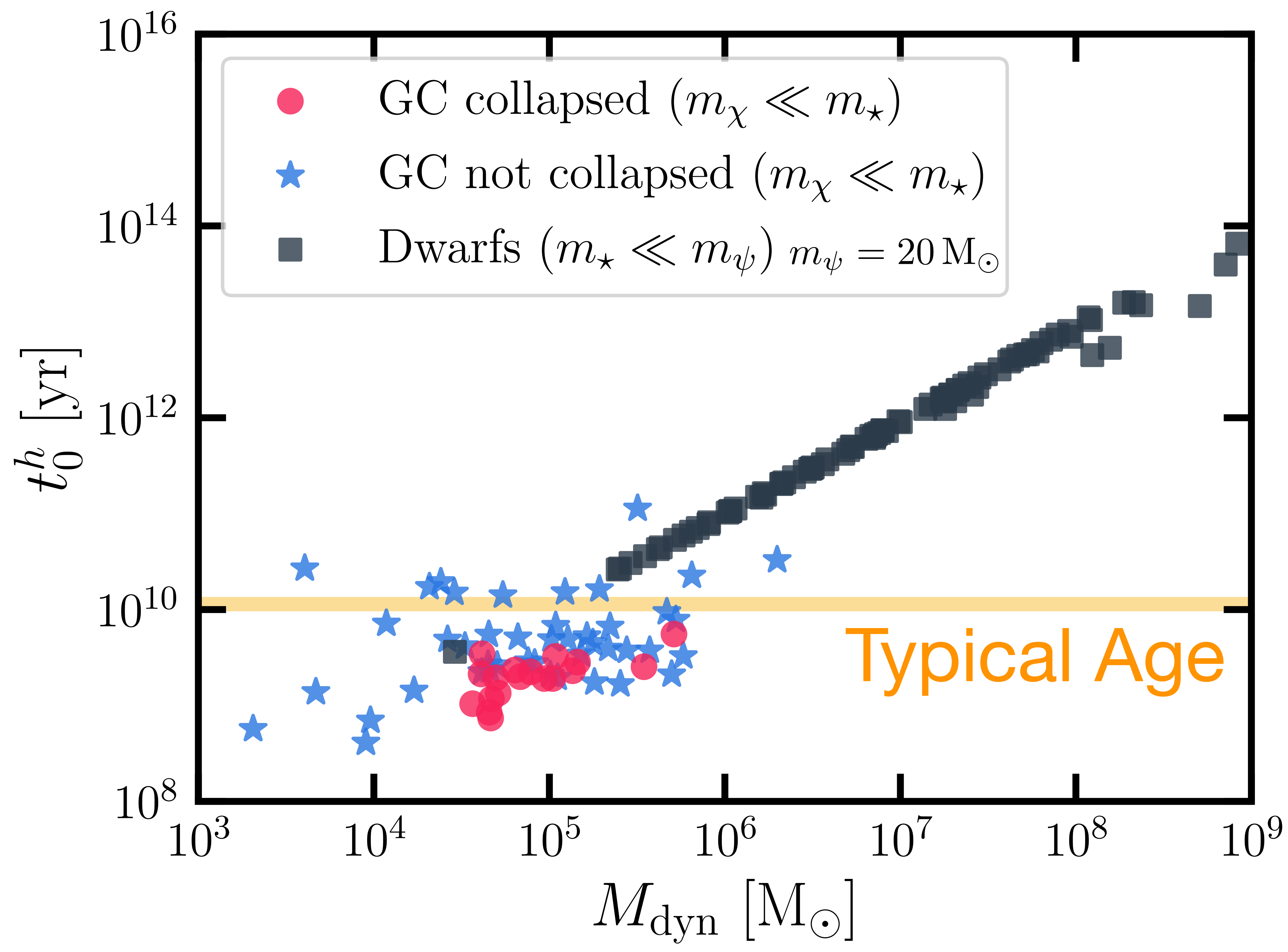
$$t_0^\psi = \frac{4.3 \times 10^5\,\mathrm{yr}}{\ln(2\gamma M_{\rm tot}/m_\psi)} \left(\frac{20\,\mathrm{M}_\odot}{m_\psi}\right) \left(\frac{M_{{\rm tot}\psi}}{\mathrm{M}_\odot}\right)^{1/2} \left(\frac{r_{s\psi}}{\mathrm{pc}}\right)^{3/2}$$



Super-$M_\odot$ dark matter

- Assuming $m_\psi = 20\,M_\odot$, dark matter core-collapse only occurs in dwarf galaxies with smallest dynamical mass.

  (Estimate $r_{s\psi}$ using concentration-mass-redshift Ludlow et al. '16)



Summary

- ✦ Understanding the origin of ultra-faint-compact dwarfs is interesting and important.
- ✦ The conduction fluid model can effectively simulate the dynamical evolution of stellar-dark matter systems.
- ✦ The timescale is around the central relaxation time of the heavier component.
- ✦ Mass segregation alone is not sufficient to deplete the lighter component, especially at large radii.
- ✦ Caveats/future work: tidal interactions, binary, central black hole…
  CDM → SIDM.