



Search for Dark Matter in 2HDMS at LHC and Future Lepton Colliders

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Shanghai, Aug 22



Based on [arXiv:2308.05653]
[arXiv:2504.14529]

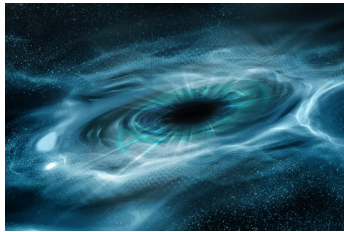
Why 2HDM+complex singlet?



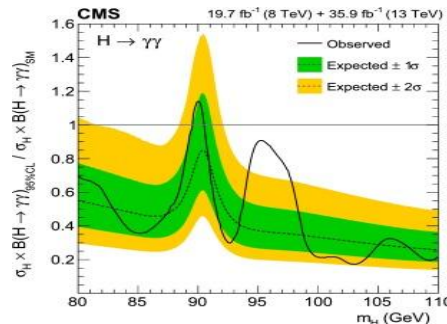
$$V_{SM} = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Explain:

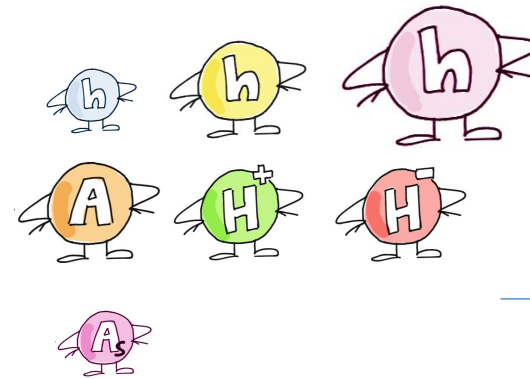
- cold dark matter (DM)



- 95 GeV excess
(at LHC and LEP)



* $\gamma\gamma$ channel at CMS ($\sim 2.9 \sigma$),
bb channel at LEP ($\sim 2 \sigma$),
investigated in S. Heinemeyer,
C. Li, et al, 2021,
arxiv:2112.11958



$$V_{2HDM S} = V_{2HDM} + V_S$$

Reference model to
obtain different **DM mass
scenarios**, which lead to
various **signal topologies**
at future colliders

2HDMS Type II, Higgs Sector Potential

[Notation as in: Baum and Shah, arXiv: 1808.02667]

Type II, Couplings to Fermions

Down-type quarks	Leptons	Up-type quarks
Φ_1	Φ_1	Φ_2

$$V = V_{2HDM} + V_S$$

$$V_{2HDM} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.] + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 \\ + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + h.c. \right]$$

$$V_S = m_S^2 S^\dagger S + \left[\frac{m_S'^2}{2} S^2 + h.c. \right] + \left[\frac{\lambda_1''}{24} S^4 + h.c. \right] + \left[\frac{\lambda_2''}{6} (S^2 S^\dagger S) + h.c. \right] + \frac{\lambda_3''}{4} (S^\dagger S)^2 \\ + S^\dagger S [\lambda_1' \Phi_1^\dagger \Phi_1 + \lambda_2' \Phi_2^\dagger \Phi_2] + [S^2 (\lambda_4' \Phi_1^\dagger \Phi_1 + \lambda_5' \Phi_2^\dagger \Phi_2) + h.c.]$$

for this study:
 $\lambda_2'' = \lambda_1''$

V_{2HDM} Symmetries

$\Phi_j \xrightarrow{U(1)} e^{i\theta} \Phi_j$ $\Phi_j^\dagger \xrightarrow{U(1)} e^{-i\theta} \Phi_j^\dagger$	avoids charge-parity violation
$\Phi_1 \xrightarrow{Z_2} -\Phi_1$ $\Phi_2 \xrightarrow{Z_2} \Phi_2$ (softly broken by m_{12}^2)	avoids flavour changing neutral currents
$\Phi_j \xrightarrow{Z_2'} \Phi_j$ $S \xrightarrow{Z_2'} -S$	stabilization of DM

2HDMS Type II, Higgs Sector Potential

[Notation as in: Baum and Shah, arXiv: 1808.02667]

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\eta_i) \end{pmatrix}$$

$$S = \frac{1}{\sqrt{2}}(v_S + \rho_S + \textcircled{iA_S})$$

$$\langle \Phi_i \rangle = \begin{pmatrix} 0 \\ \frac{v_i}{\sqrt{2}} \end{pmatrix}$$

$$\langle S \rangle = \frac{v_S}{\sqrt{2}}$$

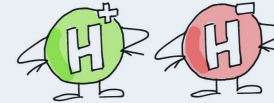
DM
Candidate



DM Candidate Properties:
massive
electrically neutral
colourless
stable

Higgs Sector
particles:

2 charged: H_{\pm} ,



1 charged GB: G_{\pm} ,

3 scalars: $h_1, \textcircled{h_2}, h_3$,

SM-like

1 pseudo scalar: A ,

1 pseudo scalar GB: G_0 ,

1 DM: A_s

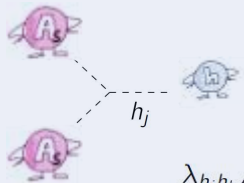


This model is different from 2HDMa, where the pseudoscalar is the DM

2HDMS Type II, Higgs Sector Potential

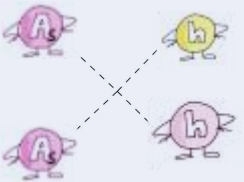
[Notation as in: Baum and Shah, arXiv: 1808.02667]

DM to Higgs portal couplings



$$\lambda_{h_j h_k A_S A_S} = \frac{\partial^4 V}{\partial h_j \partial h_k \partial A_S \partial A_S}$$

$$= -i[(\lambda'_1 - 2\lambda'_4)R_{j1}R_{k1} + (\lambda'_2 - 2\lambda'_5)R_{j2}R_{k2} - \frac{1}{2}(\lambda''_1 - \lambda''_3)R_{j3}R_{k3}]$$



$$\lambda_{h_j h_k A_S A_S} = \frac{\partial^4 V}{\partial h_j \partial h_k \partial A_S \partial A_S}$$

$$= -i[(\lambda'_1 - 2\lambda'_4)R_{j1}R_{k1} + (\lambda'_2 - 2\lambda'_5)R_{j2}R_{k2} - \frac{1}{2}(\lambda''_1 - \lambda''_3)R_{j3}R_{k3}]$$

DM mass:

$$m_{A_S}^2 = \frac{\partial^2 V}{\partial A_S^\dagger \partial A_S} \Big|_{\Phi_1 = \langle \Phi_1 \rangle, \Phi_2 = \langle \Phi_2 \rangle, S = \langle S \rangle}$$

$$= -(2m_S'^2 + v_S^2(\frac{\lambda''_1}{3} + \frac{\lambda''_3}{3}) + 2(\lambda'_4 v_1^2 + \lambda'_5 v_2^2))$$

2HDMS Basis Change

Interaction Basis Parameters:

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, m_{12}^2, \tan\beta, v_S, m_S^{2'},$$

$$\lambda'_1, \lambda'_2, \lambda'_4, \lambda'_5, \lambda''_1 = \lambda''_2, \lambda''_3$$



Mass Basis Parameters:

$$m_{h_1}, m_{h_2}, m_{h_3}, m_A, m_{A_S}, m_{H^\pm}, \alpha_1, \alpha_2, \alpha_3,$$

$$\tan\beta, v_S, \tilde{\mu}^2, \lambda'_{14}, \lambda'_{25}, \lambda''_{13}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R(\alpha_{1,2,3}) \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}$$

$$v = \sqrt{v_1^2 + v_2^2}$$

$$\tan(\beta) = \frac{v_2}{v_1}$$

$$\tilde{\mu}^2 = \frac{m_{12}^2}{\sin\beta \cos\beta}$$

$$\lambda'_{14} = \lambda'_1 - 2\lambda'_4$$

$$\lambda'_{25} = \lambda'_2 - 2\lambda'_5$$

$$\lambda''_{13} = \lambda''_1 - \lambda''_3$$

Benchmark scenarios

- Light $10 \text{ GeV} < m_{\text{DM}} < 100 \text{ GeV}$
- Intermediate $100 \text{ GeV} < m_{\text{DM}} < 1 \text{ TeV}$
- Heavy $m_{\text{DM}} > 1 \text{ TeV}$

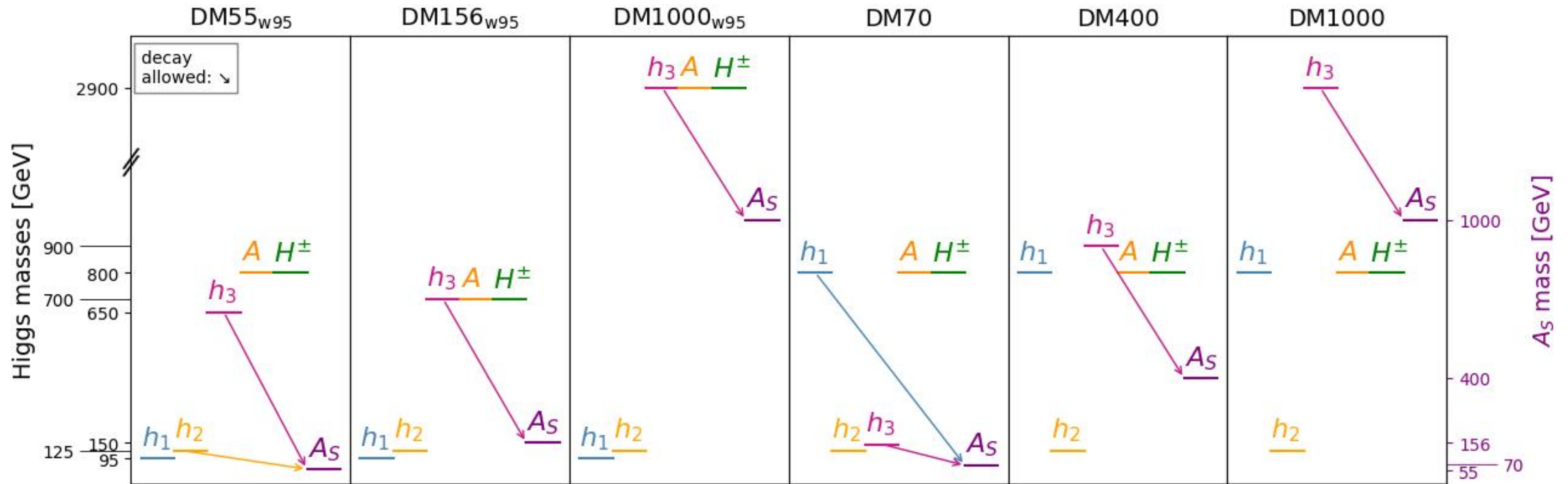
Dark Matter (DM) Phenomenology
(Relic Density,
Indirect Detection,
Direct Detection)

SARAH
SPheno
micrOMEGAs

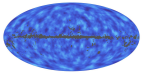
Collider
Phenomenology
(HL-LHC,
Future Lepton
Colliders)

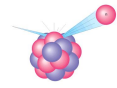
Pythia
Delphes
MadAnalysis
WHIZARD
Madgraph

Constraints:
- Theoretical: bfb,
unitarity
- Experimental:
HiggsBounds, Planck,
LUX-ZEPLIN, Fermi-LAT
*including 95 GeV excess

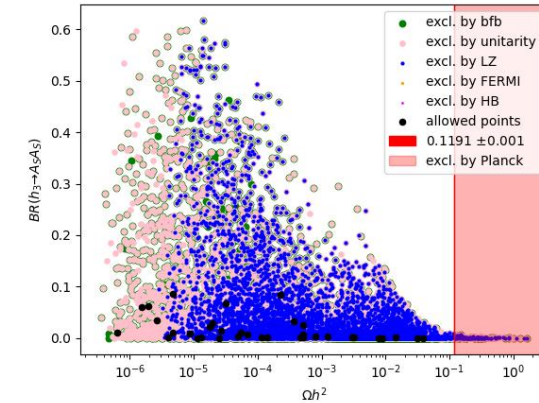


DM Phenomenology

Relic Density 
(=amount of DM left in universe today),
constraints from Planck :
 $\Omega h^2 \approx 0.12$

Direct Detection CS 
(=elastic scattering of DM on nucleon),
constraints from LUX-ZEPLIN (LZ)

Indirect Detection CS 
(=annihilation of two DM particles),
constraints from Fermi-LAT



Benchmark	Ωh^2	σ_{pA_S}/pb	σ_{nA_S}/pb	$\sigma_{A_SA_S \rightarrow XX}/\frac{\text{cm}^3}{\text{s}}$	$BR(h_3 \rightarrow A_SA_S)$	$BR(h_2 \rightarrow A_SA_S)$	$BR(h_1 \rightarrow A_SA_S)$
DM55 _{w95}	0.11	4.21×10^{-12}	4.08×10^{-12}	1.98×10^{-28}	3.81×10^{-9}	0.0199	-
DM156 _{w95}	1.61×10^{-4}	3.903×10^{-11}	4.160×10^{-11}	3.875×10^{-29}	0.69	-	-
DM1000 _{w95}	0.111	3.323×10^{-11}	3.369×10^{-11}	2.045×10^{-26}	0.0359	-	-
DM70	0.113	8.938×10^{-16}	2.651×10^{-13}	2.13×10^{-28}	0.99934	-	1.80×10^{-4}
DM400	0.106	4.960×10^{-11}	5.101×10^{-11}	3.67×10^{-26}	0.82203	-	-
DM1000	0.117	8.263×10^{-11}	8.464×10^{-11}	2.018×10^{-26}	0.005	-	-

- The larger DM relic density, the lower interaction strength between DM and other particles

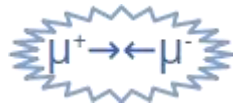
Collider Phenomenology



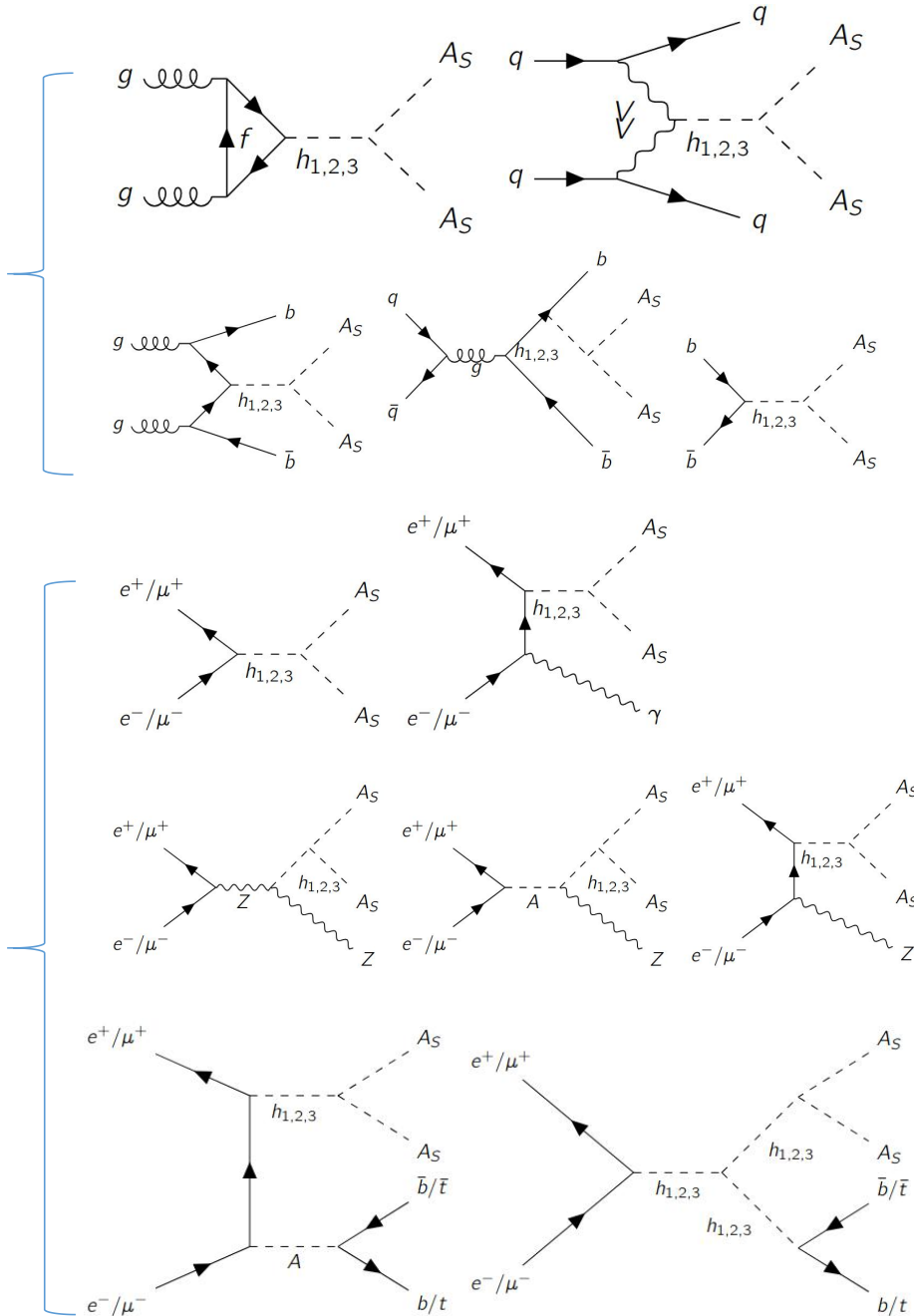
**Hadron Collider
(HL-LHC)**



**e^+e^- Colliders
(ILC, FCC-ee, CEPC, CLIC)**



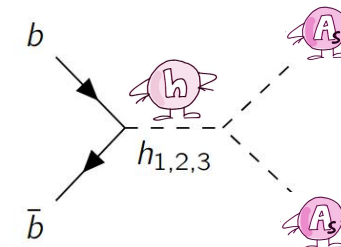
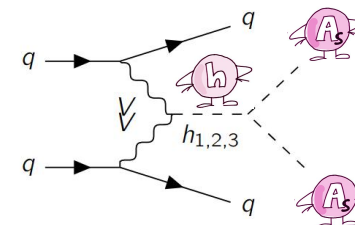
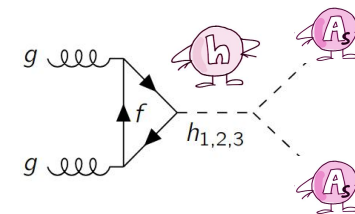
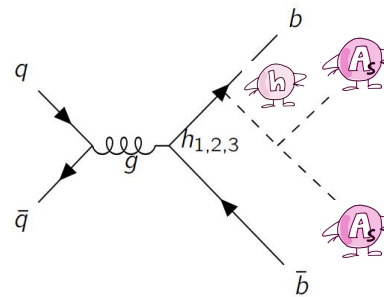
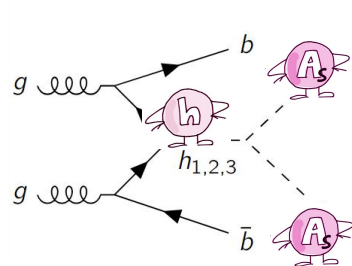
$\mu^+\mu^-$ Collider



Collider Phenomenology, HL-LHC

Process	Production cross-section (fb) at $\sqrt{s} = 14$ TeV		
	DM55 _{w95}	DM156 _{w95}	DM70
GGF($h_2 \rightarrow A_S A_S$)	533.9	-	19.29×10^3
GGF($h_3 \rightarrow A_S A_S$)	-	0.015	-
VBF($h_2 \rightarrow A_S A_S$)	54.33	-	2.72×10^3
VBF($h_3 \rightarrow A_S A_S$)	-	0.134	0.0022
BBH ($(b\bar{b}h_2 \rightarrow A_S A_S)$)	21.6	-	0.137
BBH ($(b\bar{b}h_3 \rightarrow A_S A_S)$)	-	47.24	-

Process	Production cross-section (fb) at $\sqrt{s} = 14$ TeV		
	DM400	DM1000	DM1000 _{w95}
GGF($h_3 \rightarrow A_S A_S$)	0.013	6.35×10^{-7}	4.5×10^{-6}
VBF($h_3 \rightarrow A_S A_S$)	0.0008	-	-
BBH($h_3 \rightarrow A_S A_S$)	0.007	-	-



GGF + Mono jet

Benchmark	Significance
DM55 _{w95}	0.30σ
DM70	0.55σ

VBF

Benchmark	Significance
DM70	1.94σ

BBH

Benchmark	Significance
DM156 _{w95}	1.95σ

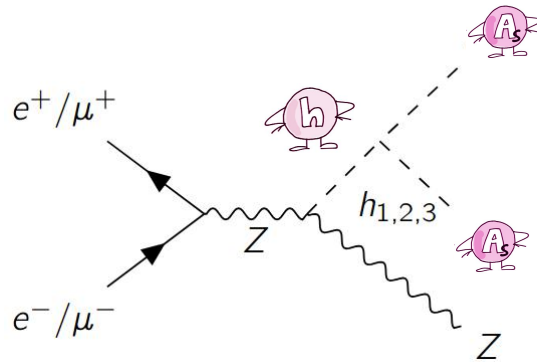
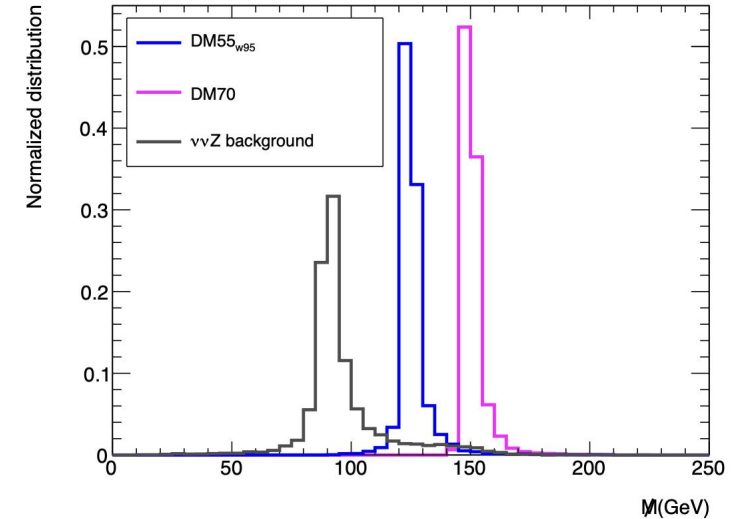
- DM55 has low significance at hadron collider
- DM70 has large cross-section via offshell h_{125}

Collider Phenomenology, e^+e^- Colliders



Z+MET

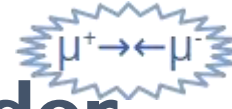
Benchmark	Production cross-section (fb)		
	at $\sqrt{s} = 250$ GeV	at $\sqrt{s} = 500$ GeV	at $\sqrt{s} = 1$ TeV
DM55_{w95}	4.42	1.1	0.24
DM70	0.33	0.15	0.035
$\nu\bar{\nu}Z$ background	503	491	950



Benchmark	\sqrt{s}	Cut	Significance
DM55_{w95}	250 GeV	$M > 100$ GeV	11σ ($1ab^{-1}$)
DM70	250 GeV	$M > 130$ GeV	3σ ($3ab^{-1}$)
DM55_{w95}	500 GeV	$M > 100$ GeV and $M < 150$ GeV	3.6σ ($1ab^{-1}$)
DM70	500 GeV	$M > 140$ GeV and $M < 190$ GeV	1.5σ ($3ab^{-1}$)
DM55_{w95}	1 TeV	$M > 120$ GeV and $M < 250$ GeV	2.4σ ($3ab^{-1}$)
DM70	1 TeV	$M > 120$ GeV and $M < 250$ GeV	0.36σ ($3ab^{-1}$)

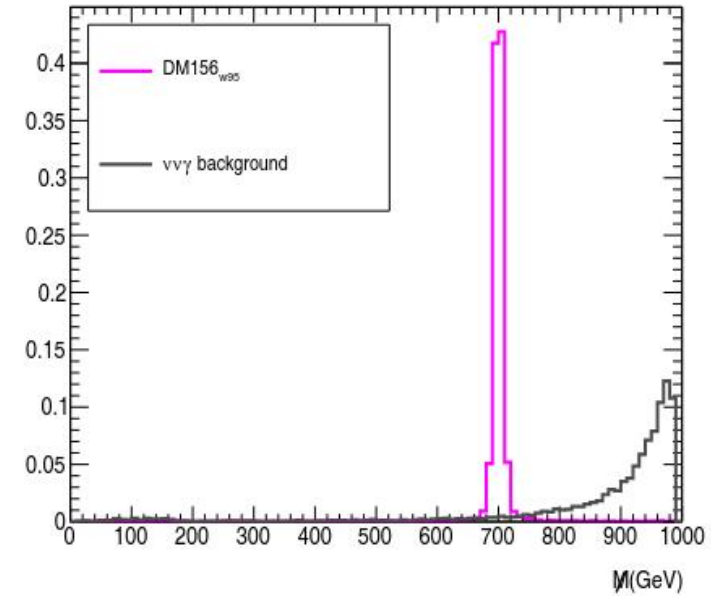
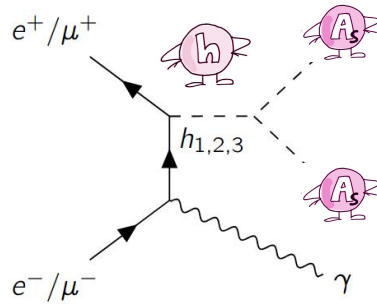
- e^+e^- colliders can be sensitive to the light DM scenarios with h_{125} as mediator

Collider Phenomenology, $\mu^+\mu^-$ Collider



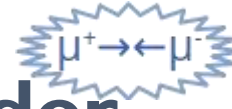
Mono photon

Benchmark	Production cross-section (fb) at $\sqrt{s} = 1$ TeV
DM156_{w95}	0.23
$\nu\nu\gamma$ background	2.45



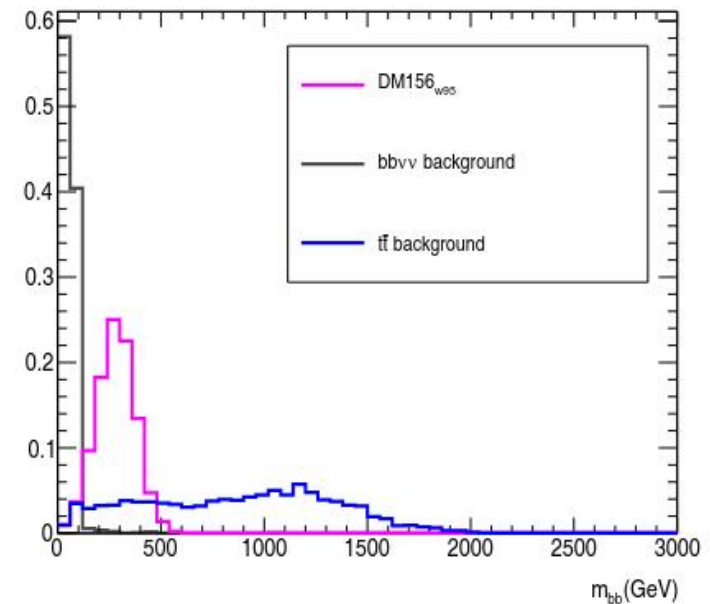
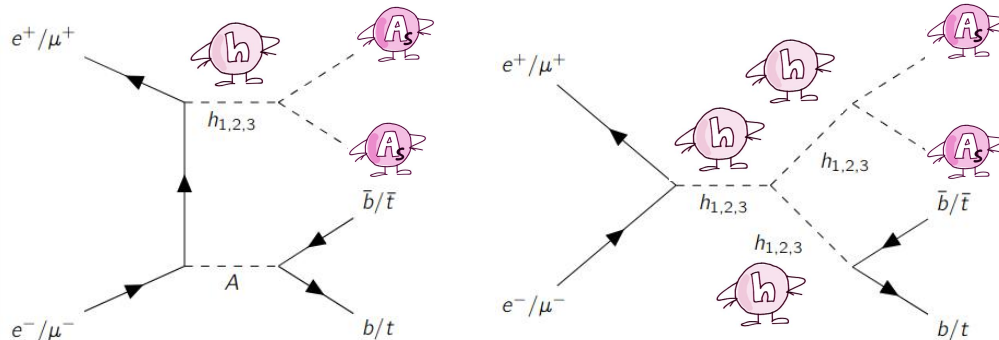
Benchmark	Cut	Significance
DM156_{w95}	$690 \text{ GeV} < M < 710 \text{ GeV}$	3σ ($3ab^{-1}$), 5.3σ ($10ab^{-1}$)

Collider Phenomenology, $\mu^+\mu^-$ Collider



bb+MET

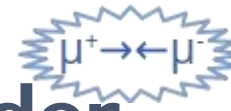
Benchmark	Production cross-section (fb)	
	at $\sqrt{s} = 3$ TeV	at $\sqrt{s} = 10$ TeV
DM156_{w95}	0.48	0.063
$b\bar{b}\nu\nu$ background	758	1.3
$t\bar{t}$ background	20	1.7



Benchmark	Cut	Significance
DM156_{w95}	$100 \text{ GeV} < m_{bb} < 500 \text{ GeV}$	6.3σ ($3ab^{-1}$)

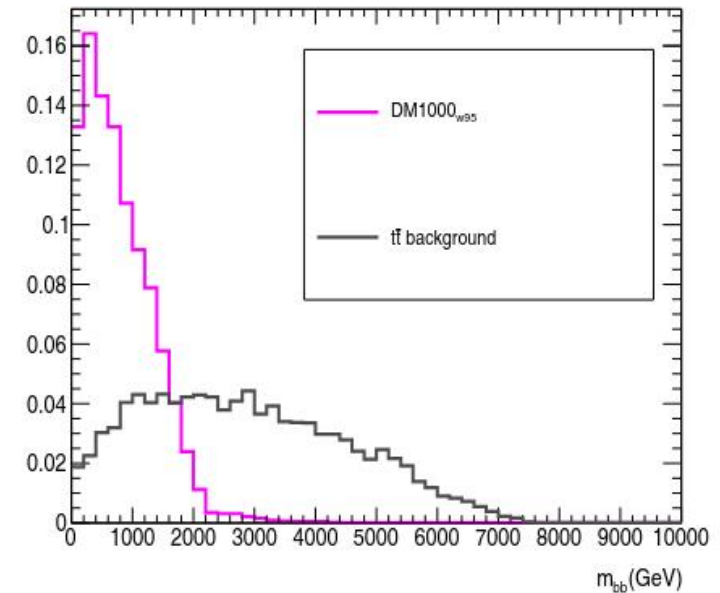
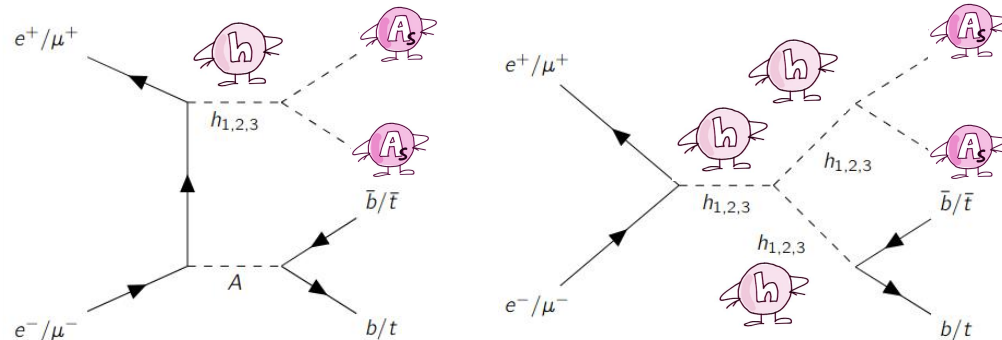
- Muon collider can be sensitive to the intermediate DM scenarios

Collider Phenomenology, $\mu^+\mu^-$ Collider



tt+MET

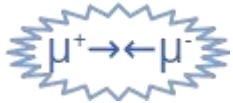
Benchmark	Production cross-section (fb) at $\sqrt{s} = 10$ TeV
DM1000_{w95}	0.027
$t\bar{t}$ +MET background	1.66



Benchmark	Cut	Significance
DM1000_{w95}	$m_{bb} < 2$ TeV	2.9σ ($10ab^{-1}$)

- Muon collider can be sensitive to the heavy DM scenarios, while the DM mediator is at 3 TeV

Collider Phenomenology, Challenging Scenarios



DM400

Final state	Production cross-section (fb) at muon collider	
	at $\sqrt{s} = 3$ TeV	at $\sqrt{s} = 10$ TeV
γ +MET	5.3×10^{-7}	4.9×10^{-8}
Z +MET	1.1×10^{-5}	1.5×10^{-6}
$b\bar{b}$ +MET	2.7×10^{-3}	4.5×10^{-3}
$t\bar{t}$ +MET	3.7×10^{-3}	8.9×10^{-3}

DM1000

Final state	Production cross-section (fb) at muon collider	
	at $\sqrt{s} = 3$ TeV	at $\sqrt{s} = 10$ TeV
γ +MET	3.5×10^{-9}	1.3×10^{-10}
Z +MET	4.4×10^{-8}	2.2×10^{-6}
$b\bar{b}$ +MET	3.7×10^{-8}	2.0×10^{-5}
$t\bar{t}$ +MET	7.8×10^{-9}	3.7×10^{-5}

LHC

Process	Production cross-section (fb) at $\sqrt{s} = 14$ TeV	
	DM400	DM1000
GGF	0.016	1.27×10^{-4}
VBF	0.001	4.7×10^{-6}
BBH	0.008	1.96×10^{-6}

FCC-hh

Process	Production cross-section (fb) at $\sqrt{s} = 100$ TeV	
	DM400	DM1000
GGF	1.456	0.117
VBF	0.039	1.182
BBH	0.264	0.029

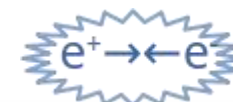
- The scenario with the singlet Higgs as the DM mediator would be difficult for lepton colliders

Conclusions

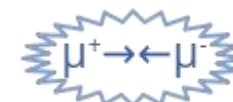
- DM search at future colliders:
 - 6 BPs (light, intermediate, heavy)
 - Hardon, electron, muon colliders
- Best prospects for light BPs at e^+e^- colliders
- Best prospects for intermediate and heavy BPs at $\mu^+\mu^-$ colliders
- Challenging scenarios probably require much higher energy (e.g. FCC-hh)

Thank you! 

Process	Benchmark	Significance
GGF	DM55 _{w95}	0.30 σ
GGF	DM70	0.55 σ
VBF	DM70	1.94 σ
BBH	DM156 _{w95}	1.95 σ



Final State	Benchmark	\sqrt{s}	Significance
Z+MET	DM55 _{w95}	250 GeV	11 σ ($1 ab^{-1}$)
Z+MET	DM70	250 GeV	3 σ ($3 ab^{-1}$)
Z+MET	DM55 _{w95}	500 GeV	3.6 σ ($1 ab^{-1}$)
Z+MET	DM70	500 GeV	1.5 σ ($3 ab^{-1}$)
Z+MET	DM55 _{w95}	1 TeV	2.4 σ ($3 ab^{-1}$)
Z+MET	DM70	1 TeV	0.36 σ ($3 ab^{-1}$)



Final State	Benchmark	\sqrt{s}	Significance
$b\bar{b}$ +MET	DM156 _{w95}	3 TeV	6.3 σ ($3 ab^{-1}$)
γ +MET	DM156 _{w95}	1 TeV	3 σ ($3 ab^{-1}$), 5.3 σ ($10 ab^{-1}$)
$t\bar{t}$ +MET	DM1000 _{w95}	10 TeV	2.9 σ ($10 ab^{-1}$)

Backup

m_{h_1}	m_{h_2}	m_{h_3}	m_A	m_{H^\pm}	χ^2
95.4 GeV	125.09 GeV	650 GeV	800 GeV	800 GeV	1.26
m_{A_S}	$\lambda'_1 - 2\lambda'_4$	$\lambda'_2 - 2\lambda'_5$	$\lambda''_1 - \lambda''_3$	$\tan \beta$	
55.596 GeV	0.0020912	0.00074611	-0.025735	2	
v_S	$\tilde{\mu}$	α_1	α_2	α_3	
300 GeV	650 GeV	-1.932	1.272	1.484	

Table 23: The benchmark point **DM55_{w95}** in the mass basis.

m_{h_1}	m_{h_2}	m_{h_3}	m_A	m_{H^\pm}	χ^2
95.4 GeV	125.09 GeV	700 GeV	700 GeV	700 GeV	0.422
m_{A_S}	$\lambda'_1 - 2\lambda'_4$	$\lambda'_2 - 2\lambda'_5$	$\lambda''_1 - \lambda''_3$	$\tan \beta$	
156 GeV	12.753	-0.31351	-2.6747	6.6	
v_S	$\tilde{\mu}$	α_1	α_2	α_3	
239.86 GeV	700 GeV	1.4661	1.1920	-1.5989	

Table 24: The benchmark point **DM156_{w95}** in the mass basis.

m_{h_1}	m_{h_2}	m_{h_3}	m_A	m_{H^\pm}	χ^2
95.4 GeV	125.09 GeV	2950 GeV	2950 GeV	2950 GeV	2.13
m_{A_S}	$\lambda'_1 - 2\lambda'_4$	$\lambda'_2 - 2\lambda'_5$	$\lambda''_1 - \lambda''_3$	$\tan \beta$	
1000 GeV	21.231	0	-1.4153	5	
v_S	$\tilde{\mu}$	α_1	α_2	α_3	
10005 GeV	2949.29 GeV	-1.769	1.250	1.569	

Table 25: The benchmark point **DM1000_{w95}** in the mass basis.

m_{h_1}	m_{h_2}	m_{h_3}	m_A	m_{H^\pm}
800 GeV	125.09 GeV	150 GeV	800 GeV	800 GeV
m_{A_S}	$\lambda'_1 - 2\lambda'_4$	$\lambda'_2 - 2\lambda'_5$	$\lambda''_1 - \lambda''_3$	$\tan \beta$
70 GeV	-0.10783	0.063127	-0.47818	1.3728
v_S	$\tilde{\mu}$	α_1	α_2	α_3
219.05 GeV	751.54 GeV	-0.60016	0.042445	-0.054807

Table 26: The benchmark point **DM70** in the mass basis.

m_{h_1}	m_{h_2}	m_{h_3}	m_A	m_{H^\pm}
800 GeV	125.09 GeV	900 GeV	800 GeV	800 GeV
m_{A_S}	$\lambda'_1 - 2\lambda'_4$	$\lambda'_2 - 2\lambda'_5$	$\lambda''_1 - \lambda''_3$	$\tan \beta$
400 GeV	0.077784	0.036923	-0.42725	2.1309
v_S	$\tilde{\mu}$	α_1	α_2	α_3
587.17 GeV	755.39 GeV	-0.41245	-0.0086501	-0.0055431

Table 27: The benchmark point **DM400** in the mass basis.

m_{h_1}	m_{h_2}	m_{h_3}	m_A	m_{H^\pm}
800 GeV	125.09 GeV	2900 GeV	800 GeV	800 GeV
m_{A_S}	$\lambda'_1 - 2\lambda'_4$	$\lambda'_2 - 2\lambda'_5$	$\lambda''_1 - \lambda''_3$	$\tan \beta$
1000 GeV	0.32873	0.21320	-0.41541	1.3414
v_S	$\tilde{\mu}$	α_1	α_2	α_3
2271.3 GeV	768.14 GeV	-0.54917	0.036530	-0.056095

Table 28: The benchmark point **DM1000** in the mass basis.

Backup, Collider Pheno, HL-LHC



$$M^2 = (p_{in} - p_{out})^2$$

$$\sigma^{w_i} = \frac{\sigma^{4f} + w_i \sigma^{5f}}{1 + w_i}$$

$$w_i = \ln\left(\frac{m_{h_i}}{m_b}\right) - 2$$

Benchmark	Cross-section after cuts (fb)
DM156_{w95}	0.357
SM Background	
$b\bar{b}Z$	18.3
$b\bar{b}\nu\bar{\nu}$	13.46
$t\bar{t}$	66.46
$Z + j$	2.04
hZ	0.012
Total Background	100.27

Table 8: The cross-sections for the signal and backgrounds after applying the cuts **E1-E4** as discussed in the text for signal-background distinction for BBH for HL-LHC at an integrated luminosity of 3000 fb⁻¹.

Gluon Fusion

We consider the final state mono-jet + MET from the gluon fusion production channel. For the collider analyses, we use the following cuts [70]:

- **C1:** The final state consists of up to four jets with $p_T > 30$ GeV and $|\eta| < 2.8$.
- **C2:** We demand a large $\cancel{E}_T > 250$ GeV.
- **C3:** The hardest leading jet has $p_T > 250$ GeV with $|\eta| < 2.4$.
- **C4:** We demand $\Delta\Phi(j, \cancel{E}_T) > 0.4$ for all jets and $\Delta\Phi(j, \cancel{E}_T) > 0.6$ for the leading jet.
- **C5:** A lepton-veto is imposed for electrons with $p_T > 20$ GeV and $|\eta| < 2.47$ and muons with $p_T > 10$ GeV and $|\eta| < 2.5$.

The SM background of 7.07 pb is obtained from the mono-jet + \cancel{E}_T search studied in Ref. [71].

Vector Boson Fusion

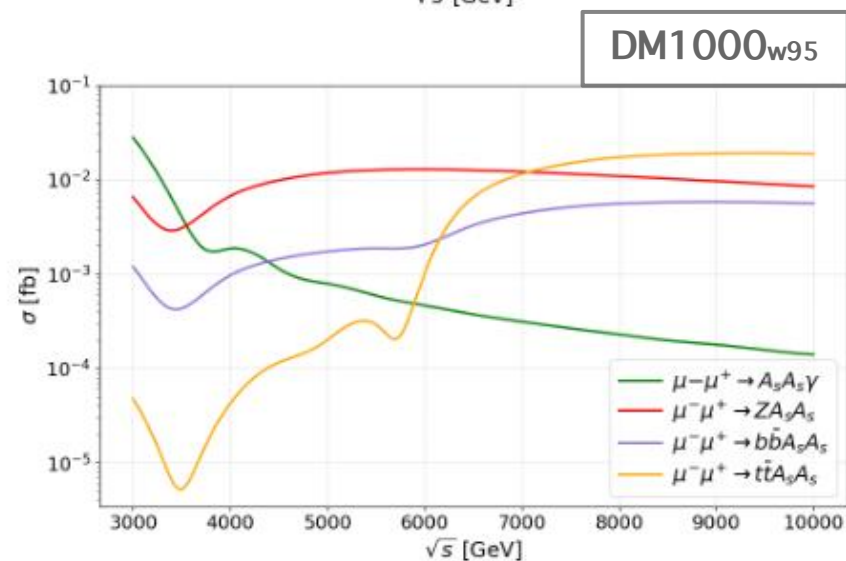
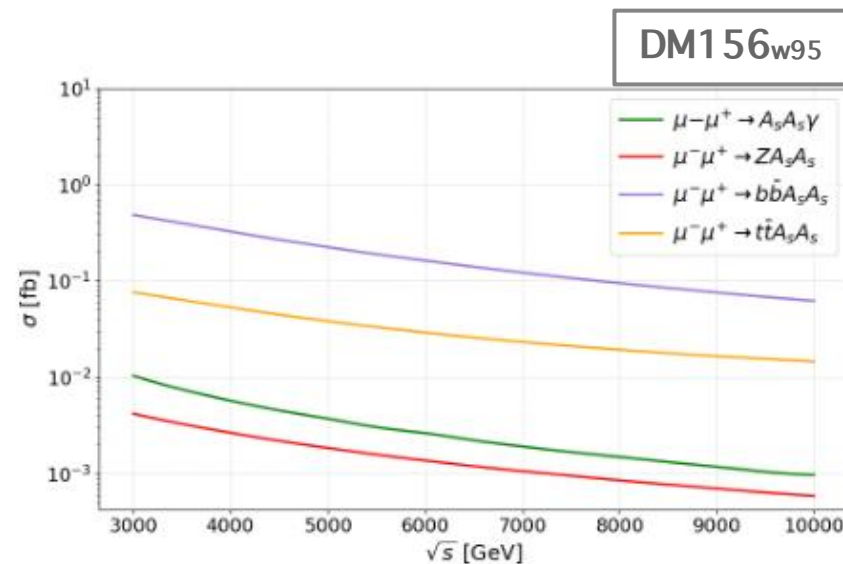
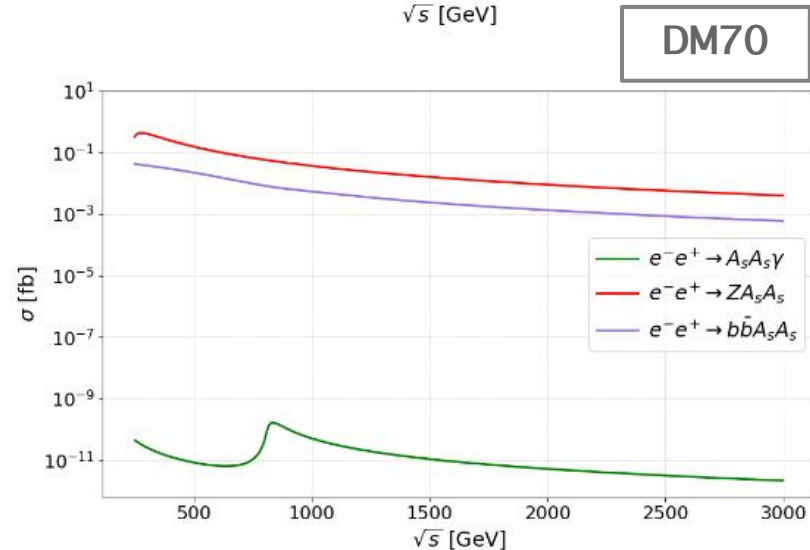
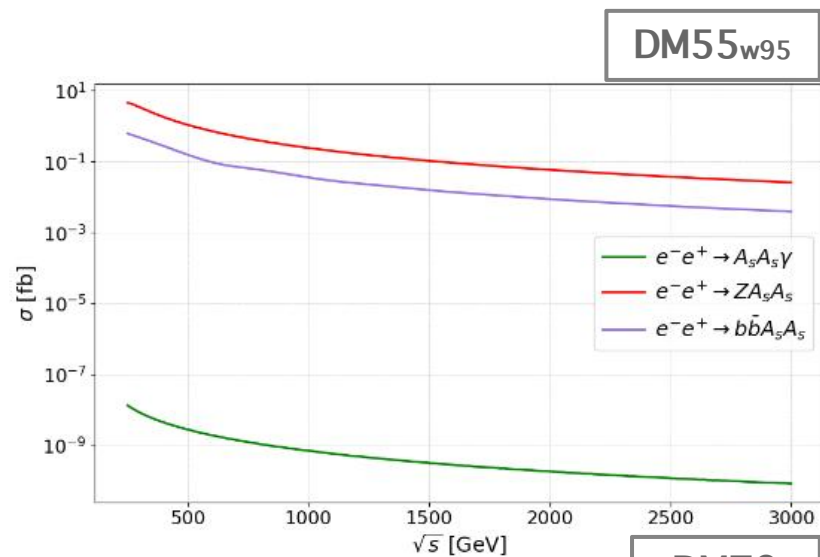
We consider the final state two forward-jets + MET from the vector boson fusion production channel. For the collider analyses, we use the following cuts [72]:

- **D1:** The final state consists of at least two jets with $p_T(j_1) > 80$ and $p_T(j_2) > 40$ GeV and $\Delta\Phi(j_i, \cancel{E}_T) > 0.5$.
- **D2:** We demand $\eta(j_1 j_2) < 0$ and $\Delta\Phi j_1 j_2 < 1.5$.
- **D3:** We demand $|\Delta\eta|_{jj} > 3.0$.
- **D4:** The invariant mass of the two forward jets is required to be large, i.e, $M_{jj} > 600$ GeV.
- **D5:** We demand $\cancel{E}_T > 200$ GeV.
- **D6:** Furthermore, a lepton veto is imposed for electrons with $p_T > 20$ GeV or muons with $p_T > 10$ GeV.

$b\bar{b}$ Higgs associated production

- **E1:** The final state consists of two b jets and no photons or leptons. We demand $\Delta R(b_1, b_2) > 0.4$, $p_T(b_1) > 150$ GeV and $p_T(b_2) > 100$ GeV.
- **E2:** We demand a large missing transverse momenta (MET) $\cancel{E}_T > 200$ GeV to reduce SM background.
- **E3:** We demand the invariant mass of the $b\bar{b}$ pair (as seen in Fig. 5) is outside the Z (76 GeV $< M(b\bar{b}) < 105$ GeV) or SM Higgs mass window (115 GeV $< M(b\bar{b}) < 135$ GeV) to remove background contributions from on-shell Z or Higgs bosons.
- **E4:** Further, we demand $M(b\bar{b}) > 200$ GeV to reduce SM background contributions.

Backup, Collider Pheno, Lepton colliders



Backup

$$0 = \frac{\partial V}{\partial \Phi_1} \Big|_{\substack{\Phi_1=\langle\Phi_1\rangle \\ \Phi_2=\langle\Phi_2\rangle \\ S=\langle S\rangle}} = \frac{1}{\sqrt{2}} [m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{\lambda_1}{2} v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 + (\frac{\lambda'_1}{2} v_1 + \lambda'_4 v_1) v_S^2] \quad (\text{A.1a})$$

$$0 = \frac{\partial V}{\partial \Phi_2} \Big|_{\substack{\Phi_1=\langle\Phi_1\rangle \\ \Phi_2=\langle\Phi_2\rangle \\ S=\langle S\rangle}} = \frac{1}{\sqrt{2}} [m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{\lambda_2}{2} v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 + (\frac{\lambda'_2}{2} v_2 + \lambda'_5 v_2) v_S^2] \quad (\text{A.1b})$$

$$0 = \frac{\partial V}{\partial S} \Big|_{\substack{\Phi_1=\langle\Phi_1\rangle \\ \Phi_2=\langle\Phi_2\rangle \\ S=\langle S\rangle}} = \frac{1}{\sqrt{2}} [m_S^2 v_S + m_S'^2 v_S + \frac{\lambda_1''}{12} v_S^3 + \frac{\lambda_2''}{3} v_S^3 + \frac{\lambda_3''}{4} v_S^3 + \frac{v_S}{2} (\lambda'_1 v_1^2 + \lambda'_2 v_2^2) + v_S (\lambda'_4 v_1^2 + \lambda'_5 v_2^2)]. \quad (\text{A.1c})$$

$$R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -s_{\alpha_1} c_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ s_{\alpha_1} s_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & -c_{\alpha_1} s_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

$$m_{12}^2 = \tilde{\mu}^2 \cdot \sin \beta \cos \beta$$

$$\lambda_1 = \frac{1}{v^2 \cos^2 \beta} (\Sigma_{i=1}^3 m_i^2 R_{i1}^2 - \tilde{\mu}^2 \sin^2 \beta),$$

$$\lambda_2 = \frac{1}{v^2 \sin^2 \beta} (\Sigma_{i=1}^3 m_i^2 R_{i2}^2 - \tilde{\mu}^2 \cos^2 \beta),$$

$$\lambda_3 = \frac{1}{v^2} \left(\frac{1}{\sin \beta \cos \beta} \Sigma_{i=1}^3 m_i^2 R_{i1} R_{i2} - \tilde{\mu}^2 + 2m_{H^\pm}^2 \right),$$

$$\lambda_4 = \frac{1}{v^2} (m_A^2 + \tilde{\mu}^2 - 2m_{H^\pm}^2),$$

$$\lambda_5 = \frac{1}{v^2} (-m_A^2 + \tilde{\mu}^2),$$

$$\lambda'_1 = \frac{1}{2} \left(\frac{1}{vv_S \cos \beta} \Sigma_{i=1}^3 m_i^2 R_{i1} R_{i3} + \lambda'_{14} \right),$$

$$\lambda'_2 = \frac{1}{2} \left(\frac{1}{vv_S \sin \beta} \Sigma_{i=1}^3 m_i^2 R_{i2} R_{i3} + \lambda'_{25} \right),$$

$$\lambda'_4 = \frac{1}{4} \left(\frac{1}{vv_S \cos \beta} \Sigma_{i=1}^3 m_i^2 R_{i1} R_{i3} - \lambda'_{14} \right),$$

$$\lambda'_5 = \frac{1}{4} \left(\frac{1}{vv_S \sin \beta} \Sigma_{i=1}^3 m_i^2 R_{i2} R_{i3} - \lambda'_{25} \right),$$

$$\lambda''_1 = \frac{3}{4v_S^2} (\Sigma_{i=1}^3 m_i^2 R_{i3}^2 + \frac{v_S^2}{2} \lambda''_{13}),$$

$$\lambda''_2 = \lambda''_1,$$

$$\lambda''_3 = \frac{3}{4v_S^2} (\Sigma_{i=1}^3 m_i^2 R_{i3}^2 + \frac{5v_S^2}{6} \lambda''_{13}),$$

$$m_S'^2 = -\left(\frac{1}{2} m_{A_S}^2 + \frac{1}{4} \Sigma_{i=1}^3 m_i^2 (R_{i3}^2 + R_{i1} R_{i3} \frac{v \cos \beta}{v_S} + R_{i2} R_{i3} \frac{v \sin \beta}{v_S}) - \frac{v^2}{4} (\lambda'_{14} \cos^2 \beta + \lambda'_{25} \sin^2 \beta) + \frac{v_S^2}{8} \lambda''_{13} \right)$$