Sterile Neutrino Oscillations with a Crossing-width Term

Yi-Lei Tang

School of Physics
Sun Yat-sen University

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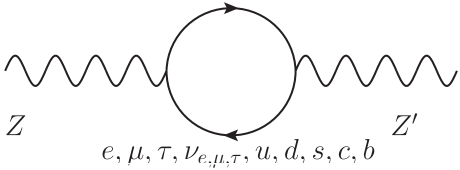
Contents

Problem 0, What is a "Particle"?

- For stable particles, "trajectories" in the detectors corresponding to "asymptotic states".
- For unstable particles, rigorous asymptotic states no longer exist. It is the decay products as well as the "secondary vertiex" that are reconstructed from the trajectories of the stable particles, which are the "real" final states.
- For unstable particles, Breit-Wigner propagators are utilized to describe their contribution in the Feynmann Diagram.
- For the nearly degenerate fields with the overlapping Breit-Wigner resonances, if regarded as one oscillating system, how can we calculate the phenomenology in a complete QFT framework?

Problem 1, Crossing-width Term in the Fermionic Case

- In our JHEP05(2024)167, we discussed a nearly degenerate Z-Z' system, in which the so-called Z-resonance observed at a collider is actually formulated by the superposition of two nearly-degenerate Z_0-Z' propagators.
- When two fields are nearly degenerate, the "crossing-width term" might become non-negligible. Such a term should also be resummed as well as the usual width terms.



Problem 1, Crossing-width Term in the Fermionic Case

$$\begin{pmatrix} m_{\hat{Z}'}^2 + ic_{\hat{Z}'\hat{Z}'} & \hat{g}'(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) & -\hat{g}(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) \\ \hat{g}'(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) & \frac{\hat{g}'^2}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) & -\frac{\hat{g}'\hat{g}}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) \\ -\hat{g}(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) & -\frac{\hat{g}'\hat{g}}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) & \frac{\hat{g}^2}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) \end{pmatrix}.$$

Although "fields" are diagonalized as a mathematical techniques, actually the resummed propagators are "diagonalized".

- Traditional algorithm: Diagonalization → Compute the width.
- Our proposal: Compute the (crossing-)width → Diagonalization.

Problem 1, Crossing-width Term in the Fermionic Case

Fermionic case?

- Pseudo-Dirac sterile neutrino. (Discussed in this work)
- neutron oscillation in the framework of some GUTs.

Complexity: Imaginary elements can arise in the original mass matrix. How can we distinct the phase of the mass matrix elements with the (crossing-)width terms?

Problem 1, Crossing-width Term in the Fermionic Case

Pseudo-Dirac sterile neutrino can arise from the "inverse", "linear", or "radiation-inverse" seesaw models

$$\mathcal{M}_{N\nu} = \begin{pmatrix} 0_{3\times 3} & m_{D3\times 1} & m'_{D3\times 1} \\ m^T_{D3\times 1} & \mu_1 & m_R \\ m'^T_{D3\times 1} & m_R & \mu_2 \end{pmatrix}.$$

Diagonalizing the $\mathcal{M}_{N\nu 2} = \mathcal{M}_{N\nu}\mathcal{M}^\dagger_{N\nu} = \mathcal{M}^T_{N\nu}\mathcal{M}^*_{N\nu}$ gives

$$U^T \mathcal{M}^T_{N\nu 2} U^* = U^\dagger \mathcal{M}_{N\nu 2} U = \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & \hat{m}_3^2 \end{pmatrix},$$

and the corresponding fields are reorganized into three "mass eigen-states". Traditionally, the (diagonal-)width terms are then computed and added in the particle propagators.

Problem 1, Crossing-width Term in the Fermionic Case

Straightforwardly including the crossing-width terms in $\mathcal{M}_{N\nu}$ cause problems: propagators for the Majorana 2-spinors are too complicated, and it is difficult to calculate the resummation processes. Therefore we have to follow the following steps:

- Diagonalize the $\mathcal{M}_{N\nu}$, and rotate the Weyl spinors χ by the U matrix to acquire the mass eigen-states.
- Combine the 2-spinors χ into the 4-spinors $\mathcal{N}_i = \begin{pmatrix} \chi_i \\ i\sigma^2 \chi_i^* \end{pmatrix}$, satisfying the $\mathcal{N}_i^C = \mathcal{N}_i$, and then calculate all the diagonal- and crossing-width terms.
- Diagonalize the mass matrix with the widths considered.

$$\begin{pmatrix} \hat{m}_1 & 0 & 0 \\ 0 & \hat{m}_2 - i\frac{\Gamma_{22}}{2} & -i\frac{\Gamma_{23}}{2} \\ 0 & -i\frac{\Gamma_{23}}{2} & \hat{m}_3 - i\frac{\Gamma_{33}}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \hat{m}_1 & 0 & 0 \\ 0 & m_{\mathcal{N}_2'} - \frac{i}{2}\Gamma_{\mathcal{N}_2'} & 0 \\ 0 & 0 & m_{\mathcal{N}_3'} - \frac{i}{2}\Gamma_{\mathcal{N}_3'} \end{pmatrix}.$$

Problem 1, Crossing-width Term in the Fermionic Case

- Traditional algorithm: Diagonalization in the 2-spinor basis $\rightarrow$ Compute the width.
- Our proposal: Diagonalization in the 2-spinor basis $\rightarrow$ Compute the (corssing-)width $\rightarrow$ Re-diagonalization in the 4-spinor bases.

The first-step diagonalization process deals with the phase in the complex mass matrix, the second-step diagonalization process focus on the width, thus separate the two imaginary ingredients. Dubbed Re-diagonalization algorithm?

Problem 1, Crossing-width Term in the Fermionic Case

Since the diagonalization matrix V is not unitary, the "diagonalized" fields $\mathcal{N}'_{1,2,3}$ no longer satisfy the "self-conjugate" conditions $\mathcal{N}'^{C}_{1,2,3} = \mathcal{N}'_{1,2,3}$. Again, this is still only a mathematical technique to resum the crossing-width term contributions.

- However, these $\mathcal{N}'_{1,2,3}$ can be straightforwardly input into the simulators like MadGraph, **without modifying** the code.
- No asymptotic external $N'_{2,3}$ should arise, and they can only be regarded as propagators

the Model

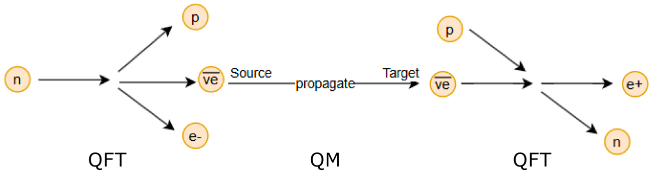
In the standard inverse/linear seesaw models, the crossing-width terms Γ_{23} disappear due to the approximate chiral symmetry rotating each 2-spinor element of the $\mathcal{N}_D$. However, if the left-hand and right-hand part of the $\mathcal{N}_D$ couple with the dark matter in a different way,

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\overline{\chi}(i\gamma^\mu\partial_\mu - m_\chi)\chi + \overline{N_D}(i\gamma^\mu\partial_\mu - m_{N_D})N_D + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_\phi^2\phi^2) \\
&+ (\mu_1\overline{N_D^C}P_L N_D + \mu_2\overline{N_D^C}P_R N_D + \text{h.c.}) + \frac{\lambda}{4}\phi^4 + \lambda_{h\phi}\phi^2 H^\dagger H \\
&+ (y_{\chi D}\overline{\chi}N_D\phi + i y_{\chi D5}\overline{\chi}\gamma^5 N_D\phi + y_{Ni}\overline{N}P_L l_i\cdot H + y_{NCi}\overline{N^C}P_L l_i\cdot H \\
&+ \text{h.c.}) + \mathcal{L}_{\text{SM}},
\end{aligned}$$

such a term can arise.

Problem 2, Oscillation in the QFT Framework

Traditionally, oscillation is understood from the aspect of the quantum mechanics.



Simulating the oscillation at a collider usually involves modifying the simulator's code, computing the generation and decay processes by QFT, while computing the propagation of the neutrino by QM.

Problem 2, Oscillation in the QFT Framework

Regarding the propagating sterile neutrinos as propagator rather than on-shell particles is not a new trick. However, how can we simulate the particle's oscillation while regarding them as the intermediate propagators?

Notice in the S-matrix

$$S = \cdots \int \frac{d^4 p}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 + \cdots - p)$$
$$\sum_i A_i \Delta_{(F)i}(p) (2\pi)^4 \delta^{(4)}(p - p_1' - p_2' - \dots) \dots,$$

each δ-function originates from the integration over the full space-time,

$$\int d^4 x_1 e^{i(p_1 + p_2 + \cdots - p)\cdot x_1} \to (2\pi)^4 \delta^{(4)}(p_1 + p_2 + \cdots - p),$$
$$\int d^4 x_2 e^{i(p - p_1' - p_2' - \dots)\cdot x_2} \to (2\pi)^4 \delta^{(4)}(p_1 + p_2 + \cdots - p).$$

Problem 2, Oscillation in the QFT Framework

The full space-time integration conceals the space dependence of the S-matrix. Keeping the distance of the "flying" propagator while integrating out all other space-time parameters gives

$$S \approx \cdots \int d|\Delta \vec{x}| \frac{(2\pi)^4}{2} \delta^4(p_1 + p_2 + \cdots - p_1' - p_2' - \dots) e^{-i|\vec{p}_1+\vec{p}_2+\dots||\Delta\vec{x}|}$$

$$\frac{-2\pi}{i|\vec{p}|} \times \sum_i \frac{i}{4\pi} A_i \left(\not{p}+m_i\right) \left.\frac{e^{i\sqrt{(p^0)^2-m_i^2+im_i\Gamma_i}|\Delta\vec{x}|}}{2}\right|_{p^0=p_1^0+p_2^0+\dots} \cdots .$$

Therefore, the oscillation probability

$$P_{|\Delta\vec{x}|} \propto \left| \cdots \sum_i A_i i(p' \cdot \gamma + m_i) e^{i\sqrt{(p^0)^2-m_i^2+im_i\Gamma_i}|\Delta\vec{x}|} \dots \right|^2 .$$

Problem 2, Oscillation in the QFT Framework

- Follow the normal way to calculate the S-matrix for the total probability when the intermediate $\mathcal{N}'_{2,3}$ are nearly on-shell.
- Replace all the $\frac{i}{p^2-m_i^2+im_i\Gamma_i}$ with the corresponding $e^{i\sqrt{(p^0)^2-m_i^2+im_i\Gamma_i}}$ factors to formulate the probability function $P_{|\Delta\vec{x}|}$.
- Generate the $|\Delta\vec{x}|$ randomly with the probability distribution function $P_{|\Delta\vec{x}|}$ for the displaced vertex distance value.

Problem: at a collider simulator, usually the analytic expression of the S-matrix is hidden. We want to avoid hacking into the event generator's codes.

Problem 2, Oscillation in the QFT Framework

However, if we change into the "flavor eigenstates" $\mathcal{N}_D$ and its anti-particle $\overline{\mathcal{N}_D}$, it is easier to compute the four full propagators,

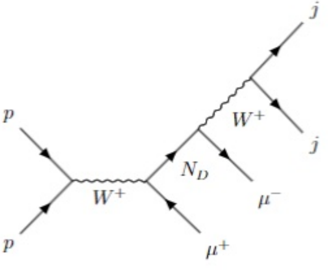
$$\langle 0|T[N_D\overline{N_D}]|0\rangle, \langle 0|T[N_D\overline{N_D^c}]|0\rangle,$$
$$\langle 0|T[N_D^c\overline{N_D}]|0\rangle, \langle 0|T[N_D^c\overline{N_D^c}]|0\rangle.$$

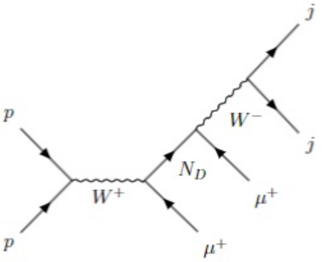
One can judge which propagator to apply by analyzing the by-products in each event. Replacing the poles with the oscillation terms one can acquire

$$\begin{aligned}
P_{N_D,\rho\to N_D,\lambda} &\propto \delta_{\lambda\rho}[c_1c_1^*e^{-\frac{m\Gamma_{\mathcal{N}_2'}}{E}t}+c_2c_2^*e^{-\frac{m\Gamma_{\mathcal{N}_3'}}{E}t}+2\mathrm{Re}(c_1c_2^*e^{i(E_2-E_1)t})e^{-\frac{m(\Gamma_{\mathcal{N}_2'}+\Gamma_{\mathcal{N}_3'})}{2E}t}],\\
P_{N_D,\lambda_1\to\overline{N}_D,\lambda_2}(t) &\propto \delta_{\lambda\rho}[c_3c_3^*e^{-\frac{m\Gamma_{\mathcal{N}_2'}}{E}t}+c_4c_4^*e^{-\frac{m\Gamma_{\mathcal{N}_3'}}{E}t}+2\mathrm{Re}(c_3c_4^*e^{i(E_2-E_1)t})e^{-\frac{m(\Gamma_{\mathcal{N}_2'}+\Gamma_{\mathcal{N}_3'})}{2E}t}],\\
P_{\overline{N}_D,\lambda_1\to\overline{N}_D\lambda_2}(t) &\propto \delta_{\lambda\rho}[c_5c_5^*e^{-\frac{m\Gamma_{\mathcal{N}_2'}}{E}t}+c_6c_6^*e^{-\frac{m\Gamma_{\mathcal{N}_3'}}{E}t}+2\mathrm{Re}(c_5c_6^*e^{i(E_2-E_1)t})e^{-\frac{m(\Gamma_{\mathcal{N}_2'}+\Gamma_{\mathcal{N}_3'})}{2E}t}],\\
P_{\overline{N}_D,\lambda_1\to N_D,\lambda_2}(t) &\propto \delta_{\lambda\rho}[c_7c_7^*e^{-\frac{m\Gamma_{\mathcal{N}_2'}}{E}t}+c_8c_8^*e^{-\frac{m\Gamma_{\mathcal{N}_3'}}{E}t}+2\mathrm{Re}(c_7c_8^*e^{i(E_2-E_1)t})e^{-\frac{m(\Gamma_{\mathcal{N}_2'}+\Gamma_{\mathcal{N}_3'})}{2E}t}],
\end{aligned}$$

Parts of the Numerical Results

Selected observable: $R_{ll} = \frac{\sigma_{\text{LNV}}}{\sigma_{\text{LNC}}}$. When more and more same-sign leptons are created, more "majorana-like" the sterile neutrino becomes.

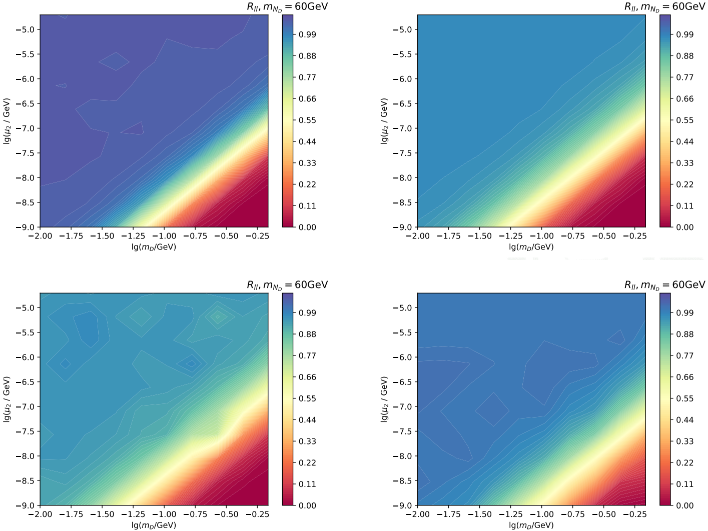




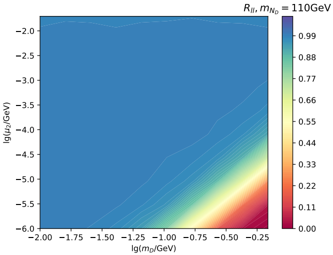
Parts of the Numerical Results

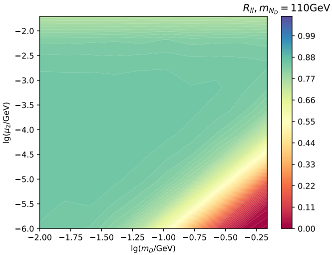
$\frac{\Gamma_{23}}{\Delta M} = 0.1, 0.5, 1, 5$ respectively, where $\Delta M = |m_{\mathcal{N}_2} - m_{\mathcal{N}_3}|$.
$m_{N_D} = 60, 110$ GeV.

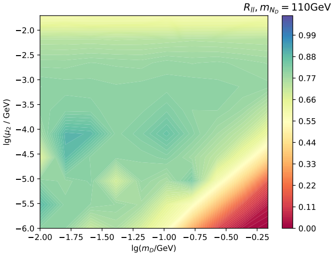
Parts of the Numerical Results

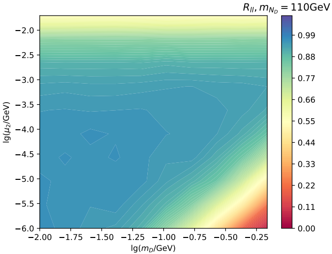


Parts of the Numerical Results



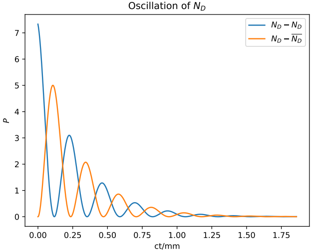


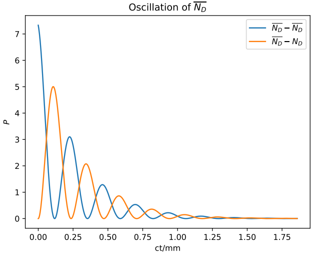




Parts of the Numerical Results

Here are the simulations of the sterile neutrino's oscillation between the $\mathcal{N}_D$-$\overline{\mathcal{N}_D}$ states. Here $m_{N_D} < m_W$ so that the lifetime arises, leaving us the possibility to observe the macroscopic displaced vertices.





Summary

- By our two-step diagonaliztion processes, the crossing-width term can be considered in the Fermionic cases.
- Oscillation can be understood and calculated in the QFT framework in which the space-time information should be retrieved to compute the distance of the displaced vertex.

Thanks!