Induced Domain Walls of QCD Axion, and Gravitational Waves

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Building for Tomorrow

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collaboration with **Kai Murai, Fuminobu Takahashi, and Wen Yin**

QCD Axion

QCD axion is an axion that dynamically solves the strong CP problem through its shift symmetry,

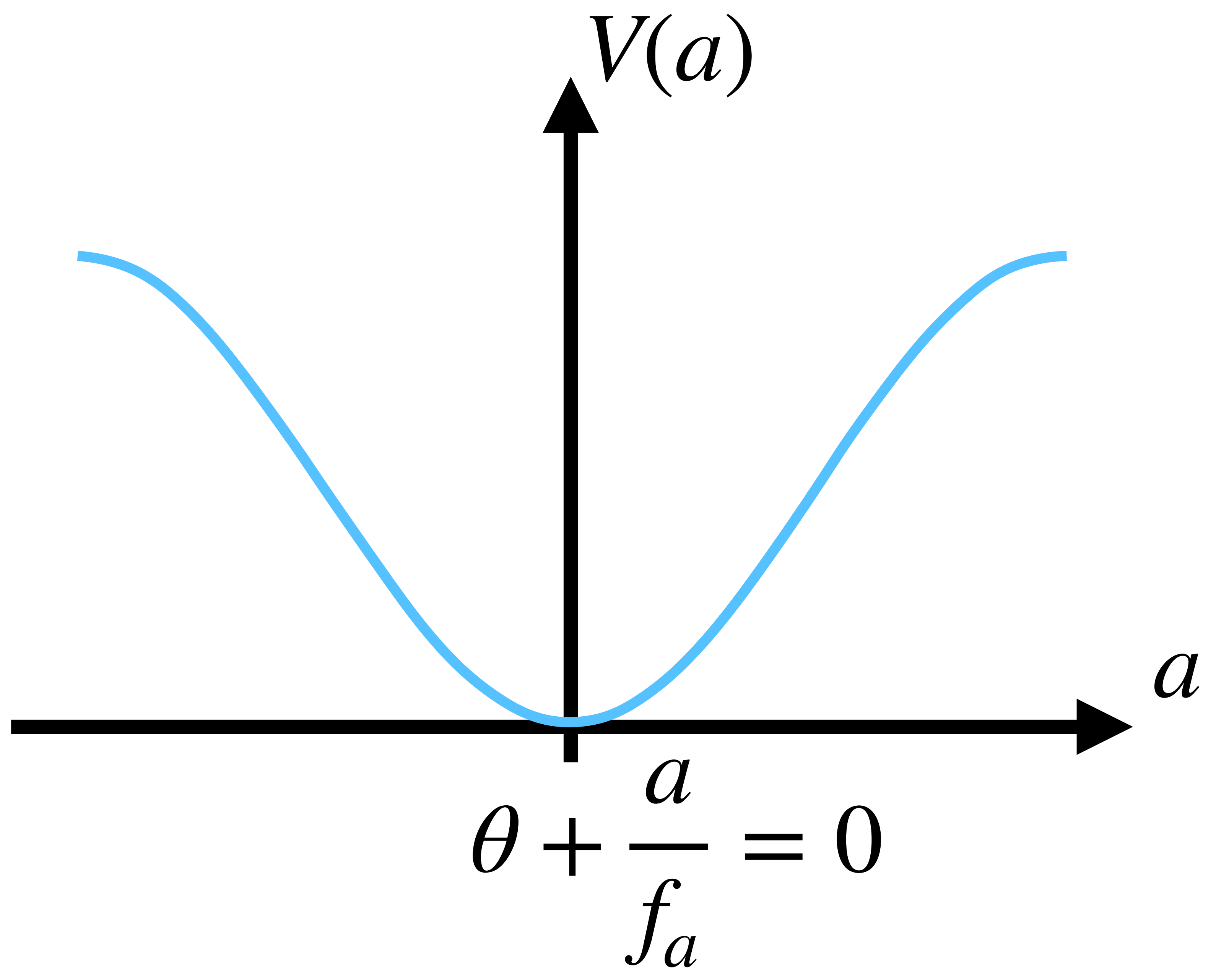
$$\phi \to \phi + \mathrm{constant},$$

and the coupling,

$$\mathscr{L} \supset -\frac{\alpha_s}{8\pi}\left(\theta + \frac{a}{f_a}\right) G_{\mu\nu}\tilde{G}^{\mu\nu}\,.$$

Peccei, Quinn `77, Weinberg `78, Wilczek `78

QCD instanton generates the potential of QCD axion.



Misalignment Mechanism

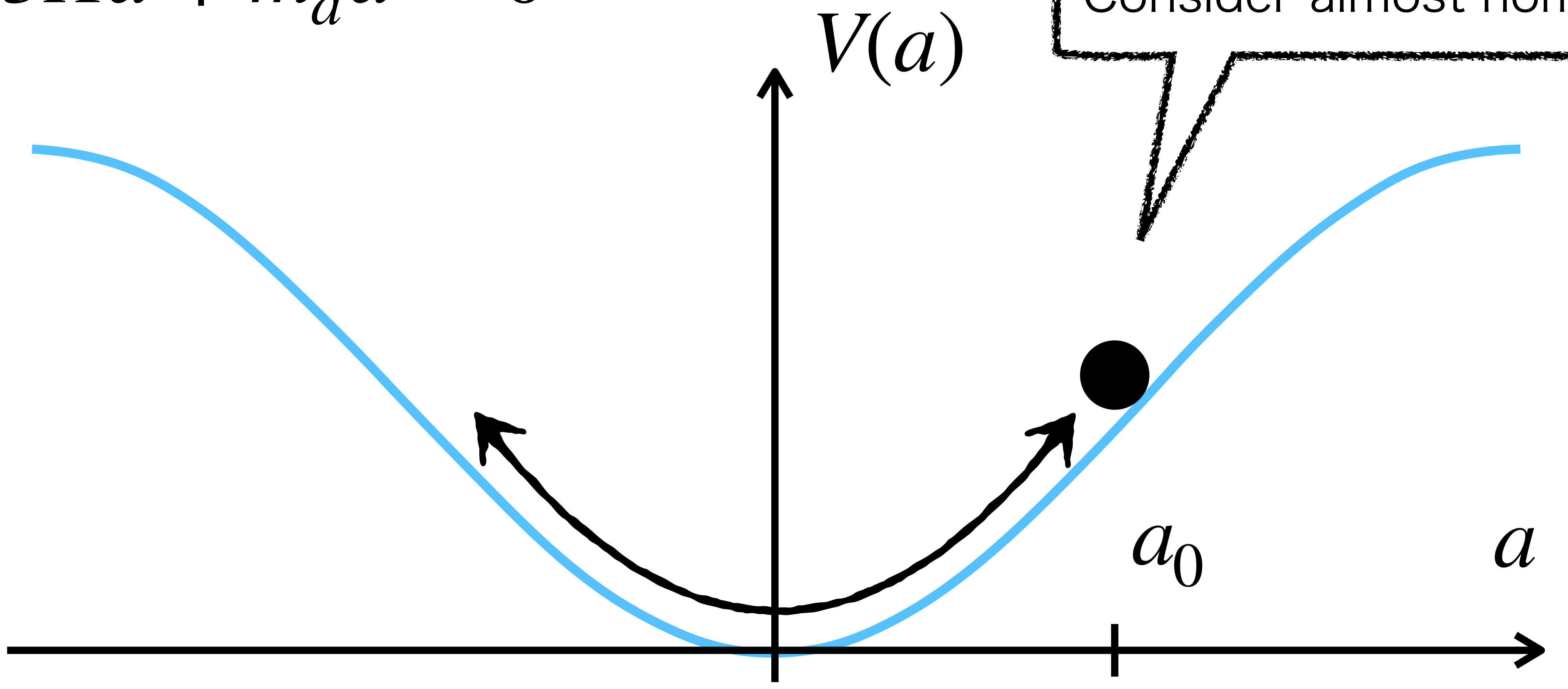
If Peccei-Quinn symmetry is broken before/during inflation (pre-inflationary), the axion will have a nearly uniform value after inflation.

Coherent oscillation of the axion acts as dark matter.

Preskill, Wise, Wilczek `83, Abbott, Sikivie `83, Dine, Fischler `83

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

Consider almost homogeneous initial condition $a = a_0$



$$\frac{\rho_a}{s} \simeq \frac{m_a^2 a_0^2}{(M_{\rm p} m_a)^{3/2}}$$

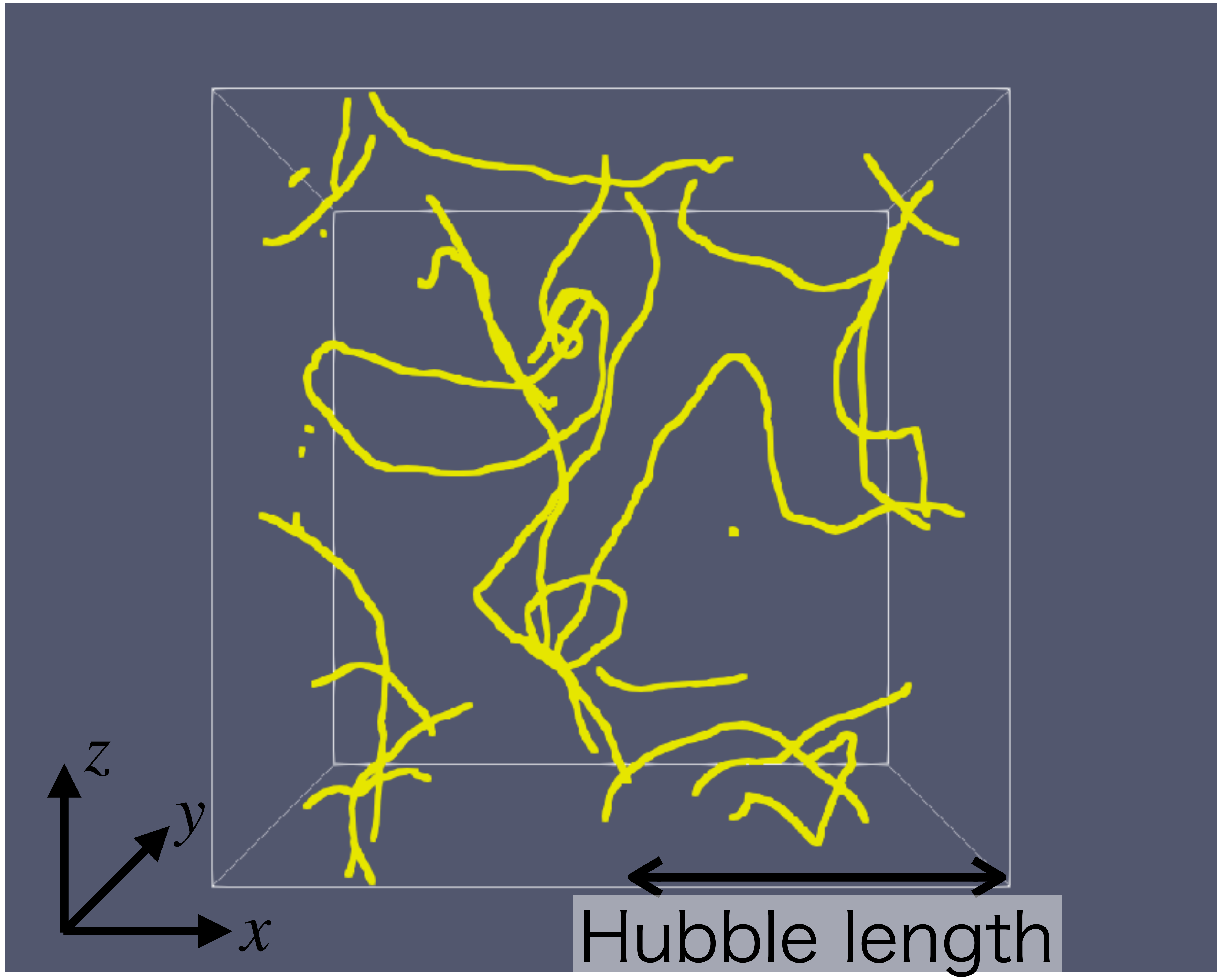
If $f_a \sim 10^{12}\,\mathrm{GeV}$, the QCD axion accounts for whole dark matter without fine-tuning of the initial condition.

Network of defects

If the PQ symmetry is restored and spontaneously broken after inflation (post-inflationary), the network of cosmic strings are formed.

$\mathcal{O}(1)$ strings in each Hubble patch

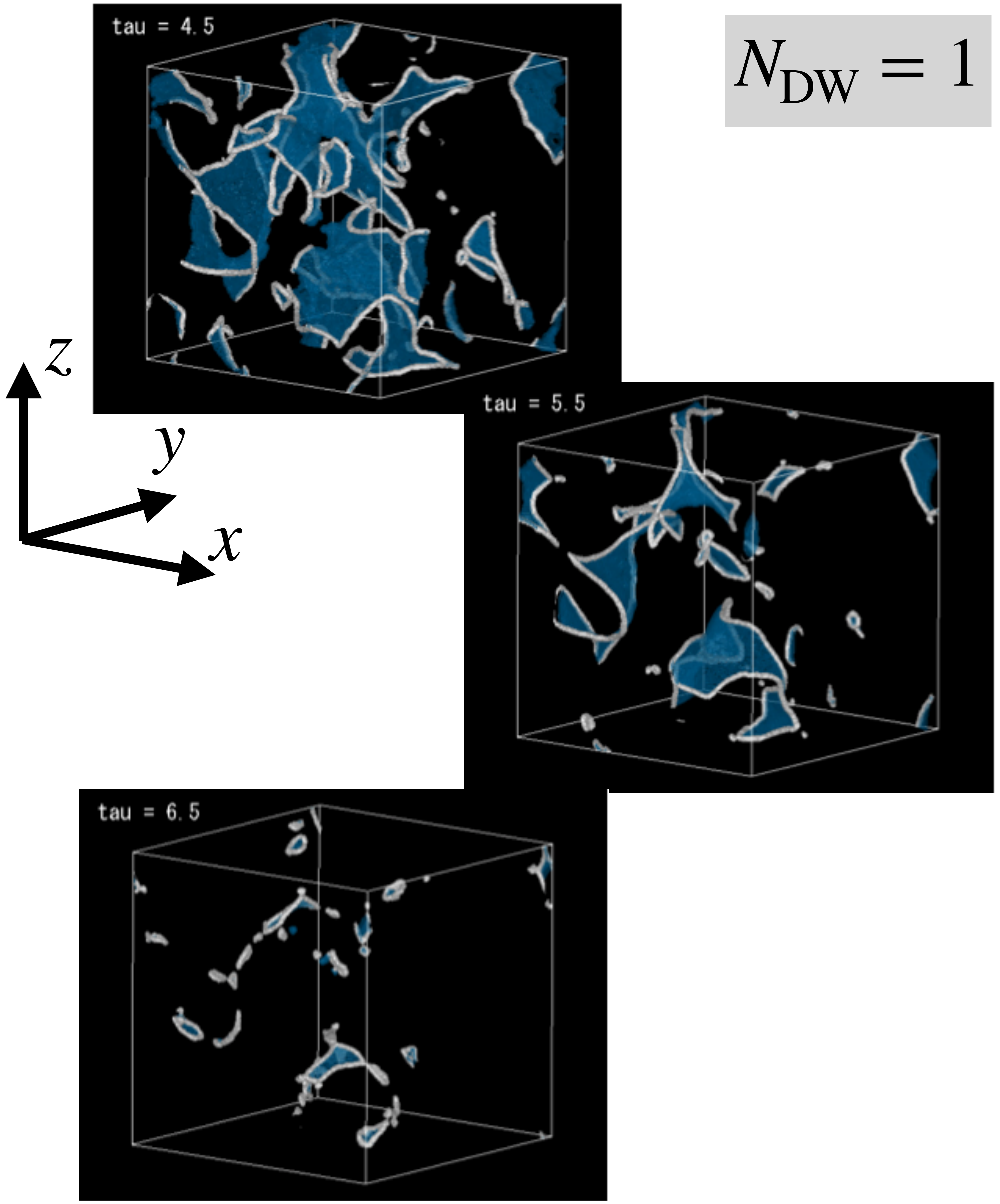
scaling solution



Short-lived strings

Taken from Hiramatsu, Kawasaki, Saikawa, Sekiguchi `12

$N_{\rm DW} = 1$



Domain walls are attached to strings through the explicit breaking of the symmetry.

If $N_{\rm DW} = 1$, the network of cosmic strings rapidly decays due to the domain wall tension and emits axion particles.

The emitted axions contribute to dark matter. (generally more axions than in the misalignment mechanism)

Case of long-lived domain walls

If the domain wall number is not unity, the network of cosmic strings and domain walls is long-lived. The potential bias is required to collapse the network, but it spoils the solution of the strong CP problem.

relative phase

$$V_{\rm QCD}(a) + V_{\rm other}(a) = \chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}\right)\right] + \Lambda_{\rm other}^4\left[1-\cos\left(N_{\rm other}\frac{a}{f_a}+\theta\right)\right]$$

Case of long-lived domain walls

If the domain wall number is not unity, the network of cosmic strings and domain walls is long-lived. The potential bias is required to collapse the network, but it spoils the solution of the strong CP problem.

What if multiple axions following distinct scenarios are present and mixed in the potential?

$$V_{\rm QCD}(a) + V_{\rm other}(a) = \chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}\right)\right] + \Lambda^4_{\rm other}\left[1-\cos\left(N_{\rm other}\frac{a}{f_a}+\theta\right)\right]$$

Model

QCD axion a and heavy axion(-like particle) ϕ mix in the potential.

$$\begin{cases} a \text{ is initially homogeneous } (a = a_{\rm ini}). \\ \phi \text{ forms cosmic strings.} \end{cases}$$

$$\begin{aligned} V(a,\phi) &= V_{\rm DW}(\phi) + V_{\rm mix}(a,\phi) \\ &= \Lambda^4 \left[1 - \cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right] + \chi(T)\left[1 - \cos\left(N_a \frac{a}{f_a} + N_\phi \frac{\phi}{f_\phi}\right)\right] , \end{aligned}$$

$$\Lambda^4 \equiv \frac{m_\phi^2 f_\phi^2}{{N_{\rm DW}}^2}, \quad \chi(T) \equiv \frac{m_a^2(T) f_a^2}{N_a^2} .$$

Model

$N_{\rm DW} \geq 2$: stable string-wall network of ϕ

$|N_a|, |N_\phi| \geq 1$: a and ϕ mix in the potential

$m_\phi \gg m_{a0} \equiv m_a(T=0)$: ϕ-domain wall first

$\frac{m_\phi f_\phi^2}{N_{\rm DW}^2} \gg \frac{m_{a0} f_a^2}{N_a^2}$: negligible backreaction from a to ϕ

$$V(a,\phi) = V_{\rm DW}(\phi) + V_{\rm mix}(a,\phi)$$
$$= \Lambda^4 \left[1 - \cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right] + \chi(T)\left[1 - \cos\left(N_a \frac{a}{f_a} + N_\phi \frac{\phi}{f_\phi}\right)\right] \, ,$$

$$\Lambda^4 \equiv \frac{m_\phi^2 f_\phi^2}{{N_{\rm DW}}^2}\, , \quad \chi(T) \equiv \frac{m_a^2(T) f_a^2}{N_a^2}\, .$$

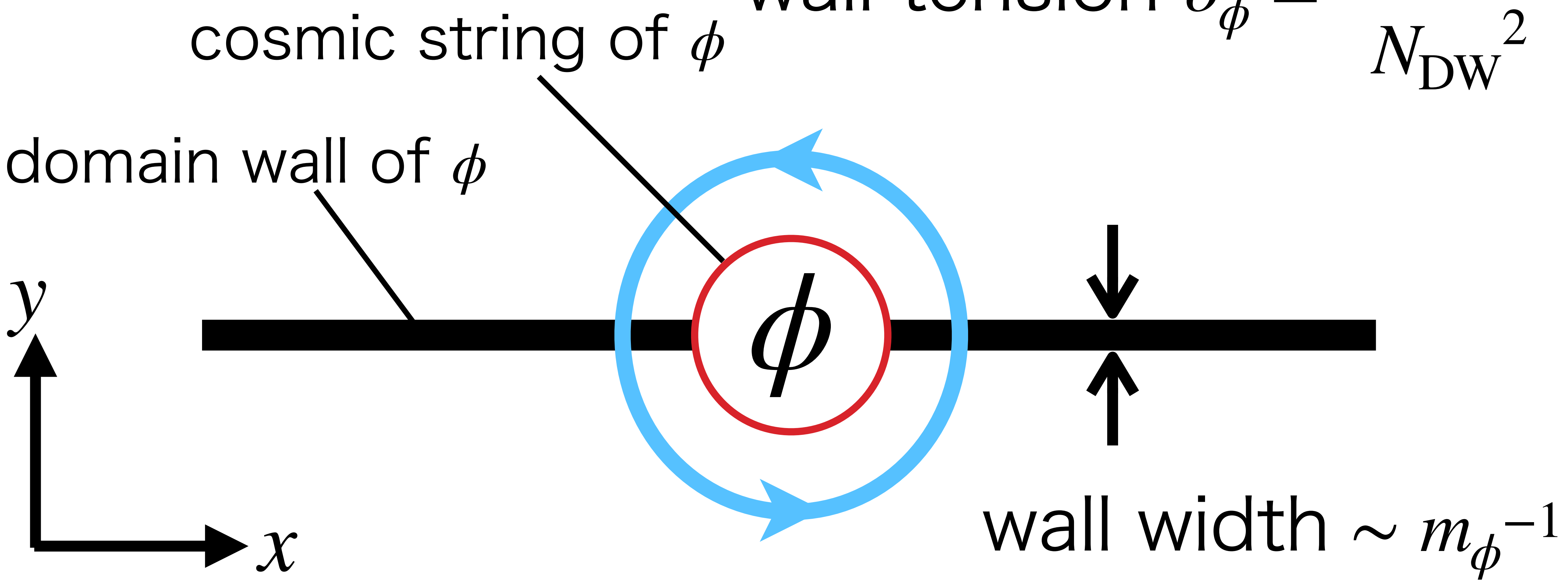
Domain walls of ϕ

$$V_{\rm DW} = \Lambda^4 \left[1 - \cos\left(N_{\rm DW} \frac{\phi}{f_\phi} \right) \right]$$

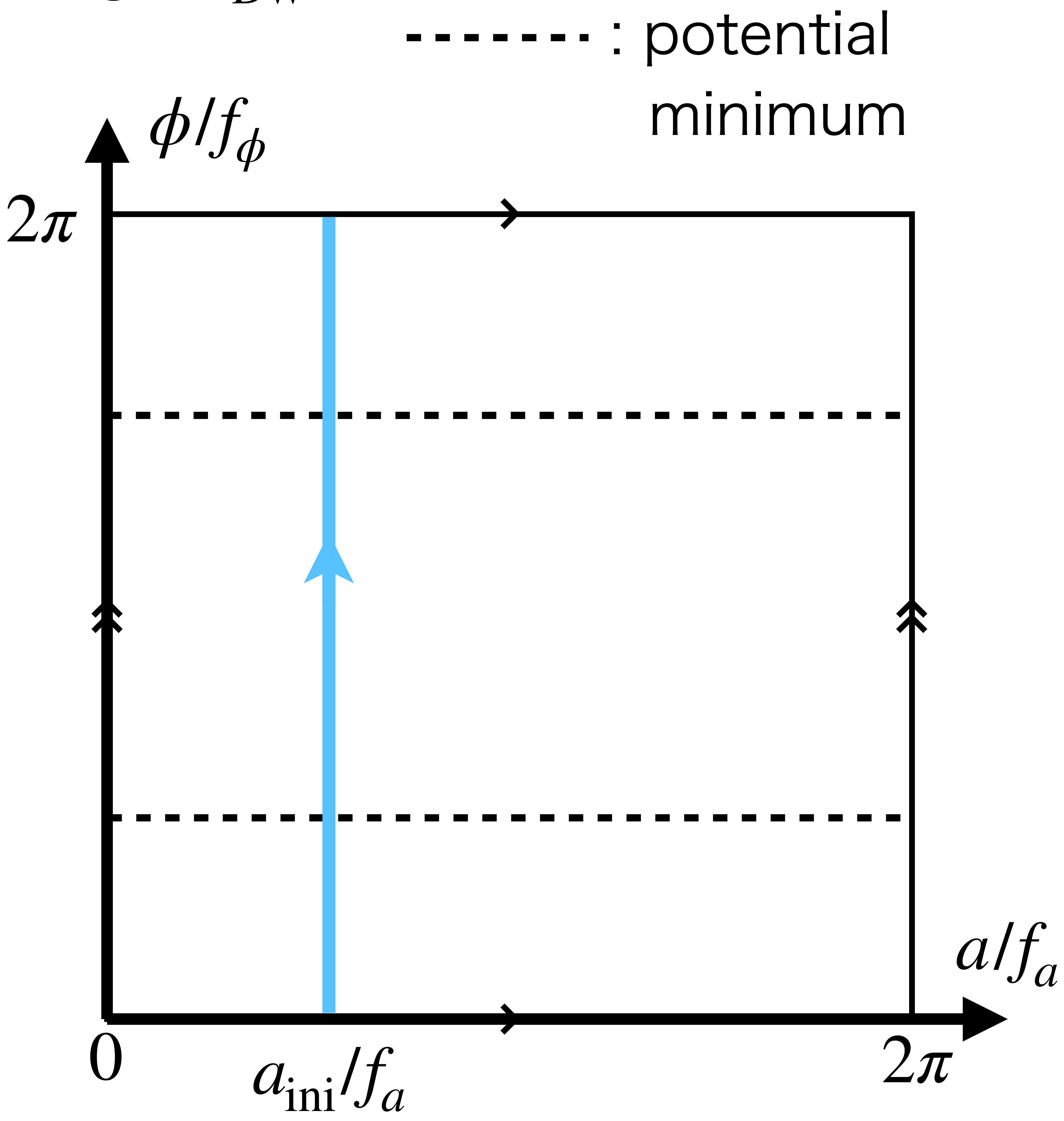
In each bulk, ϕ takes $\phi_k = 2\pi k f_\phi / N_{\rm DW}$

$(k = 0, \cdots, N_{\rm DW} - 1)$.

wall tension $\sigma_\phi = \dfrac{8 m_\phi {f_\phi}^2}{{N_{\rm DW}}^2}$



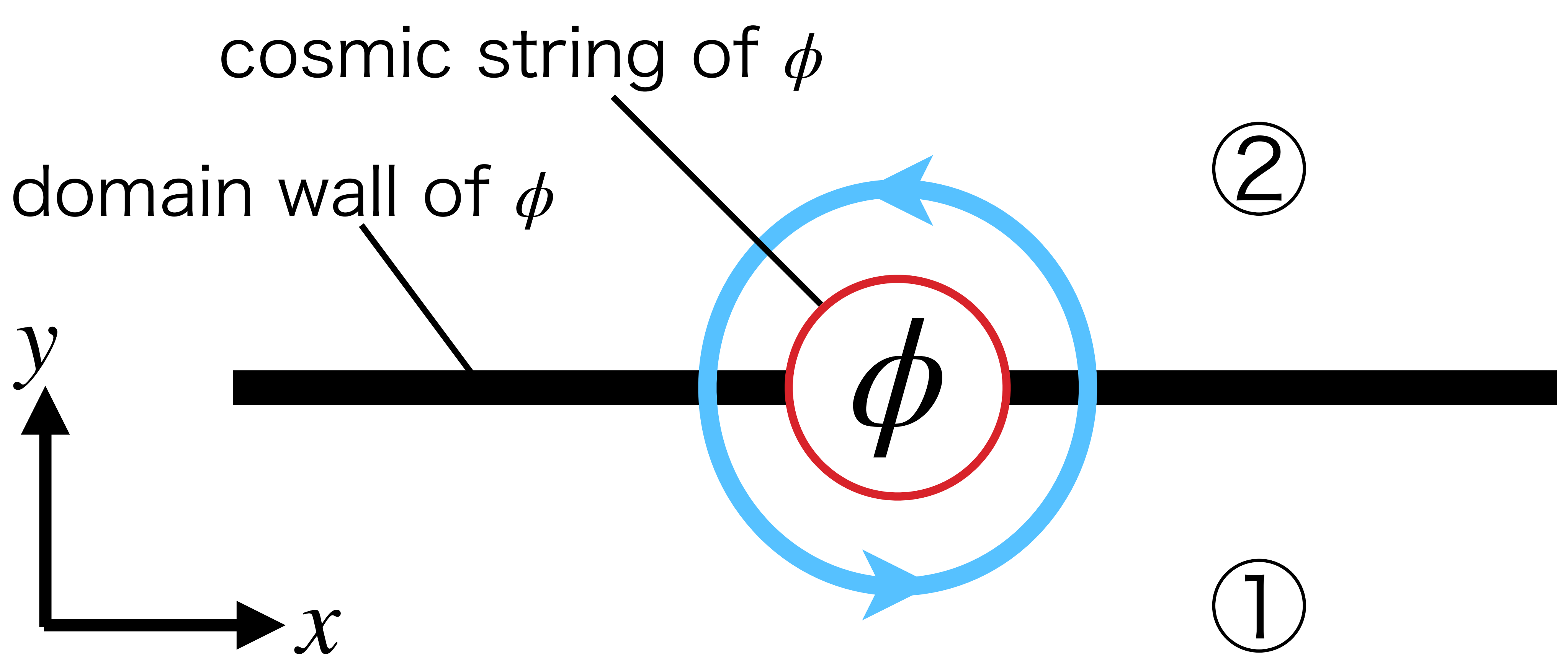
e.g., $N_{\rm DW} = 2$

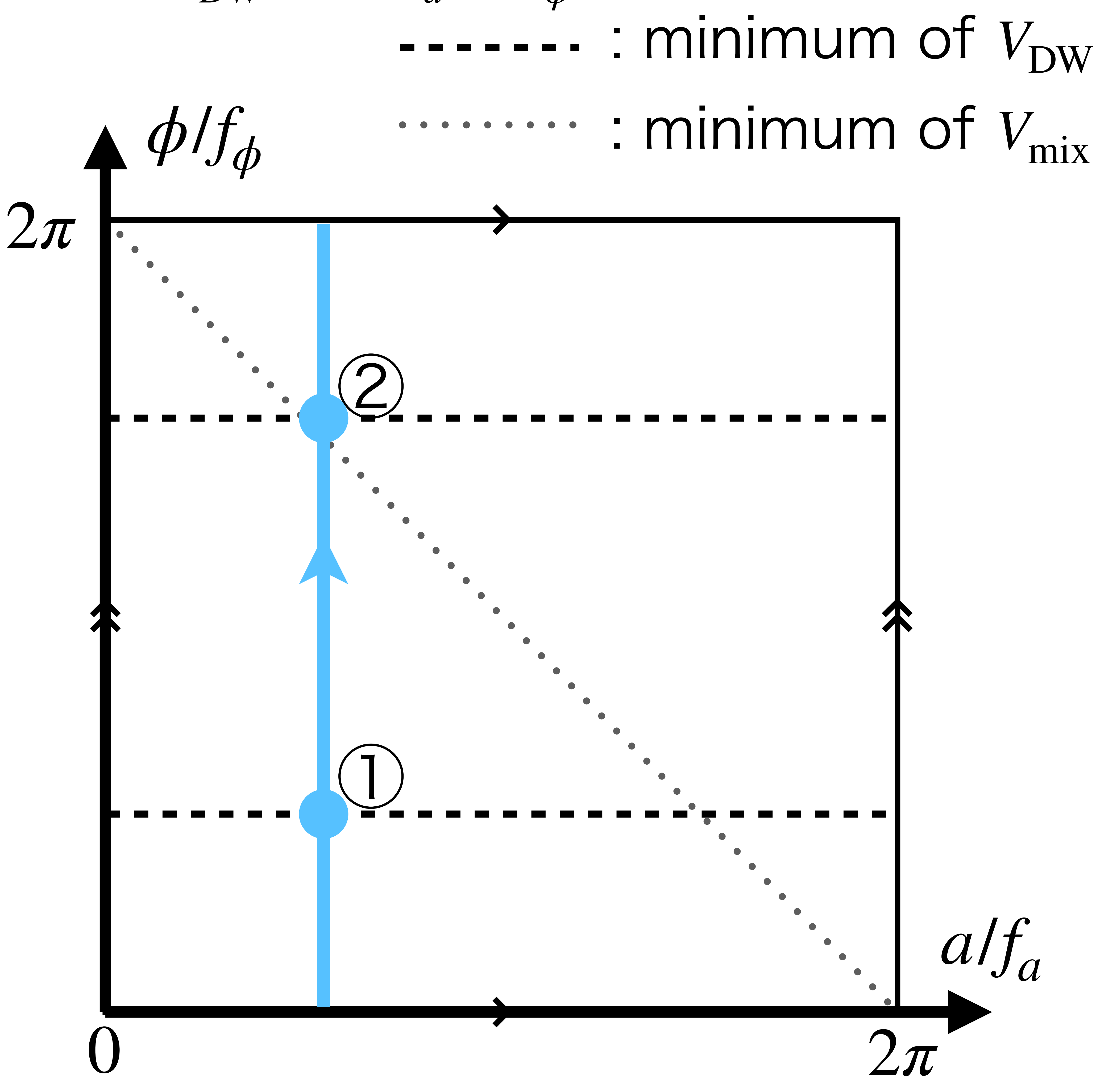


Potential from QCD

$$\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]+\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]$$

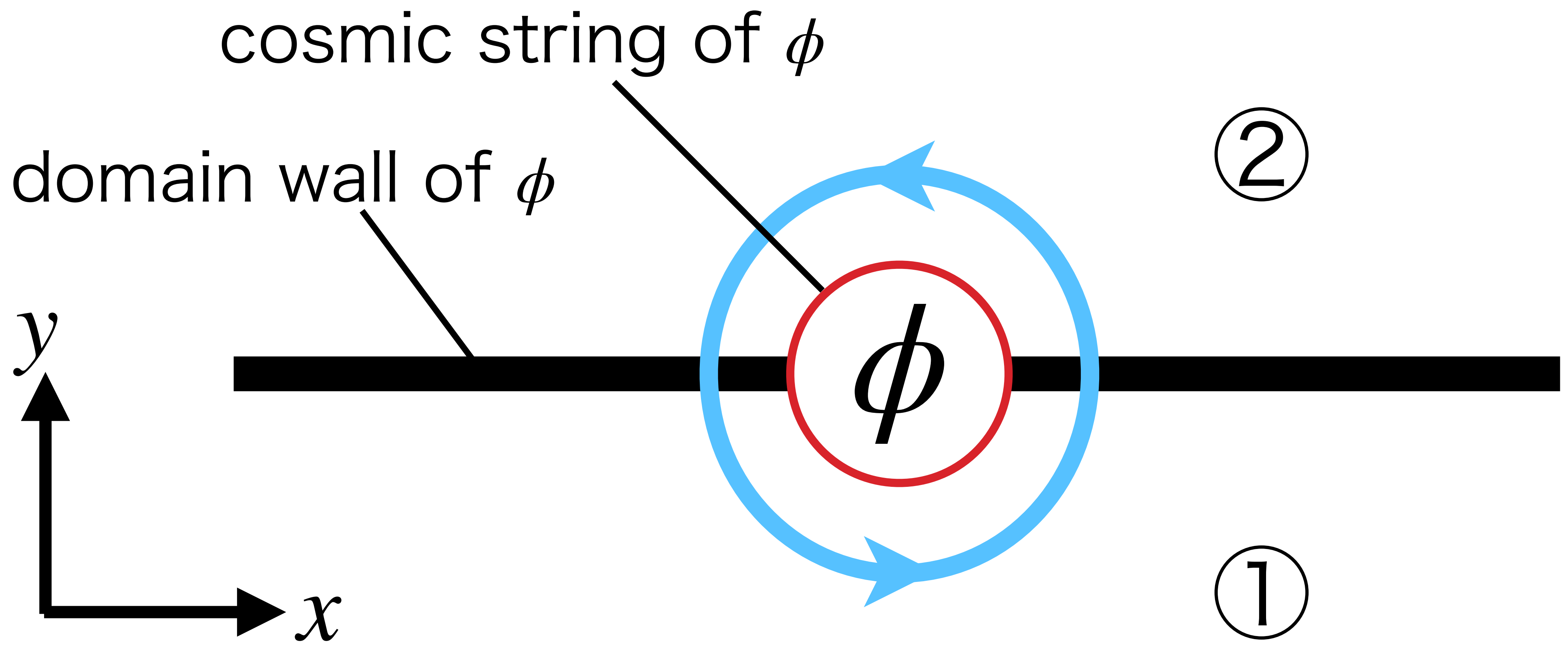
e.g., $N_{\rm DW}=2\,,N_a=N_\phi=1$

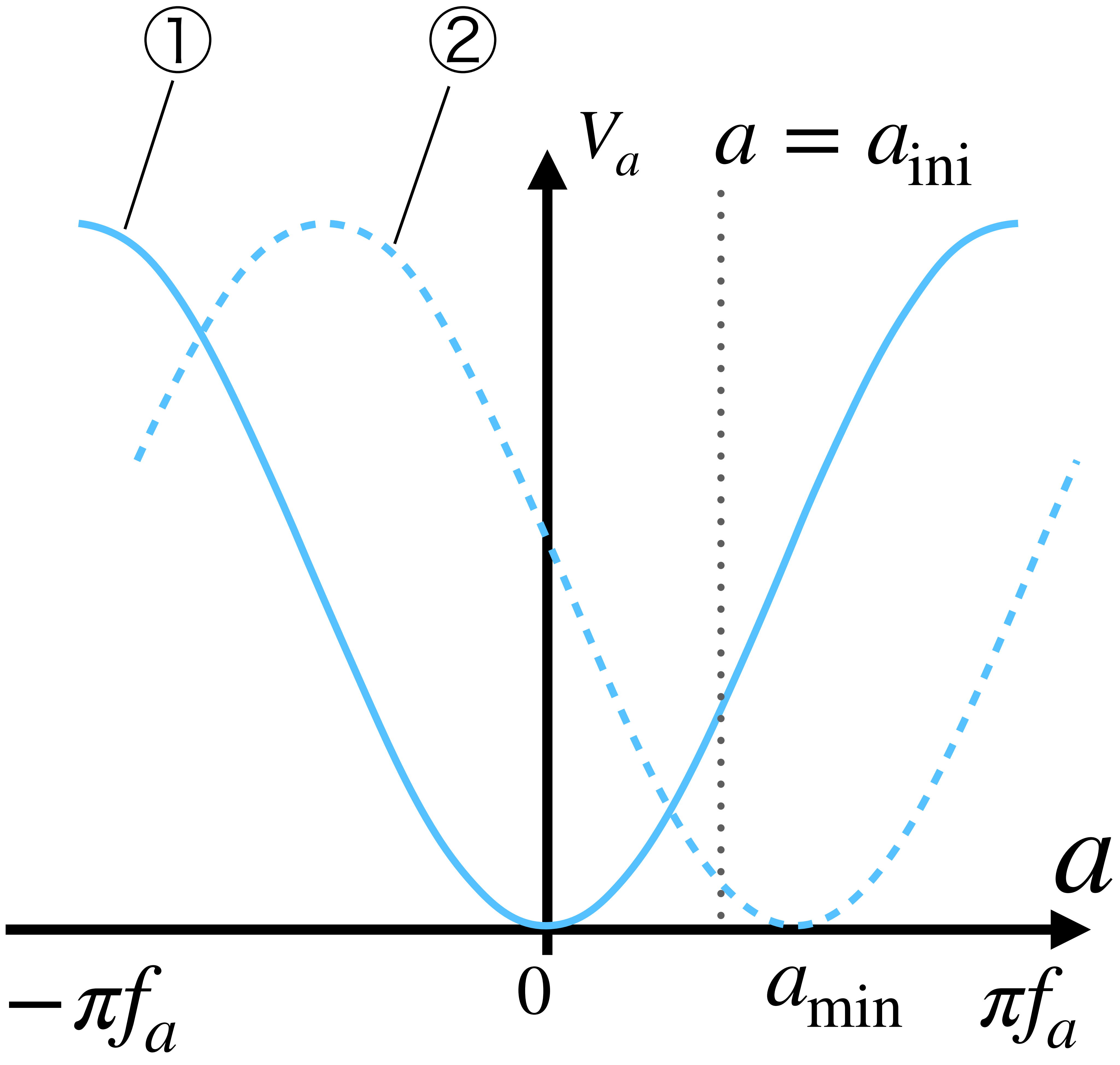




Potential from QCD

$$\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]+\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]$$



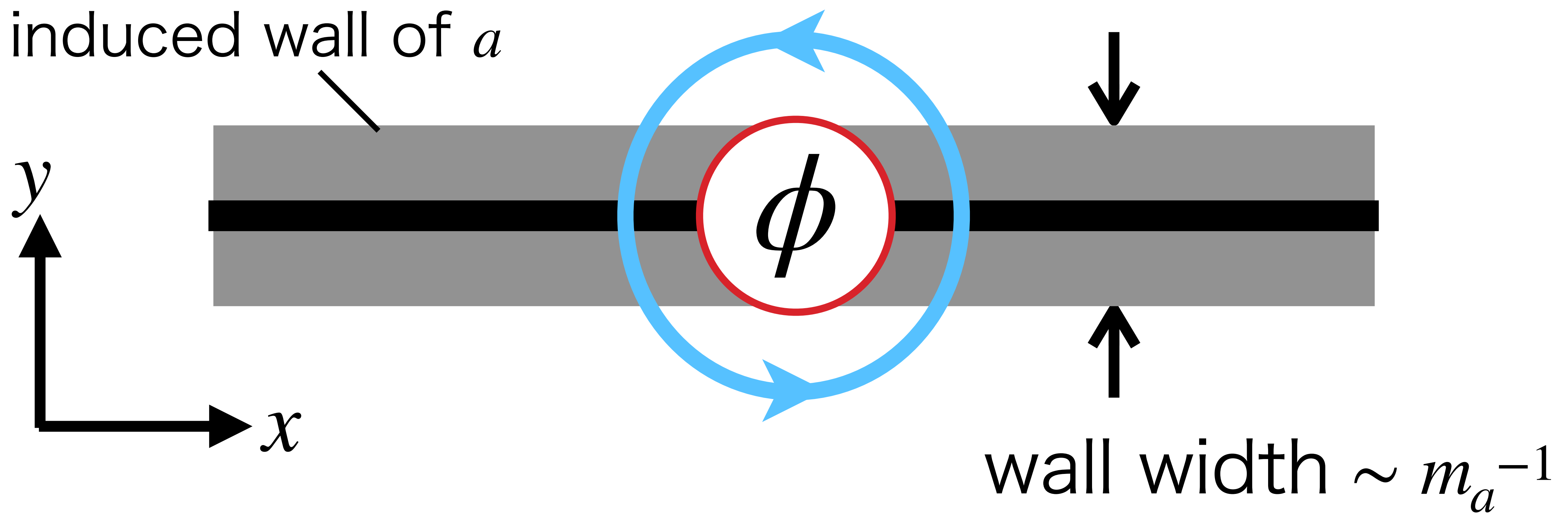


Induced domain wall

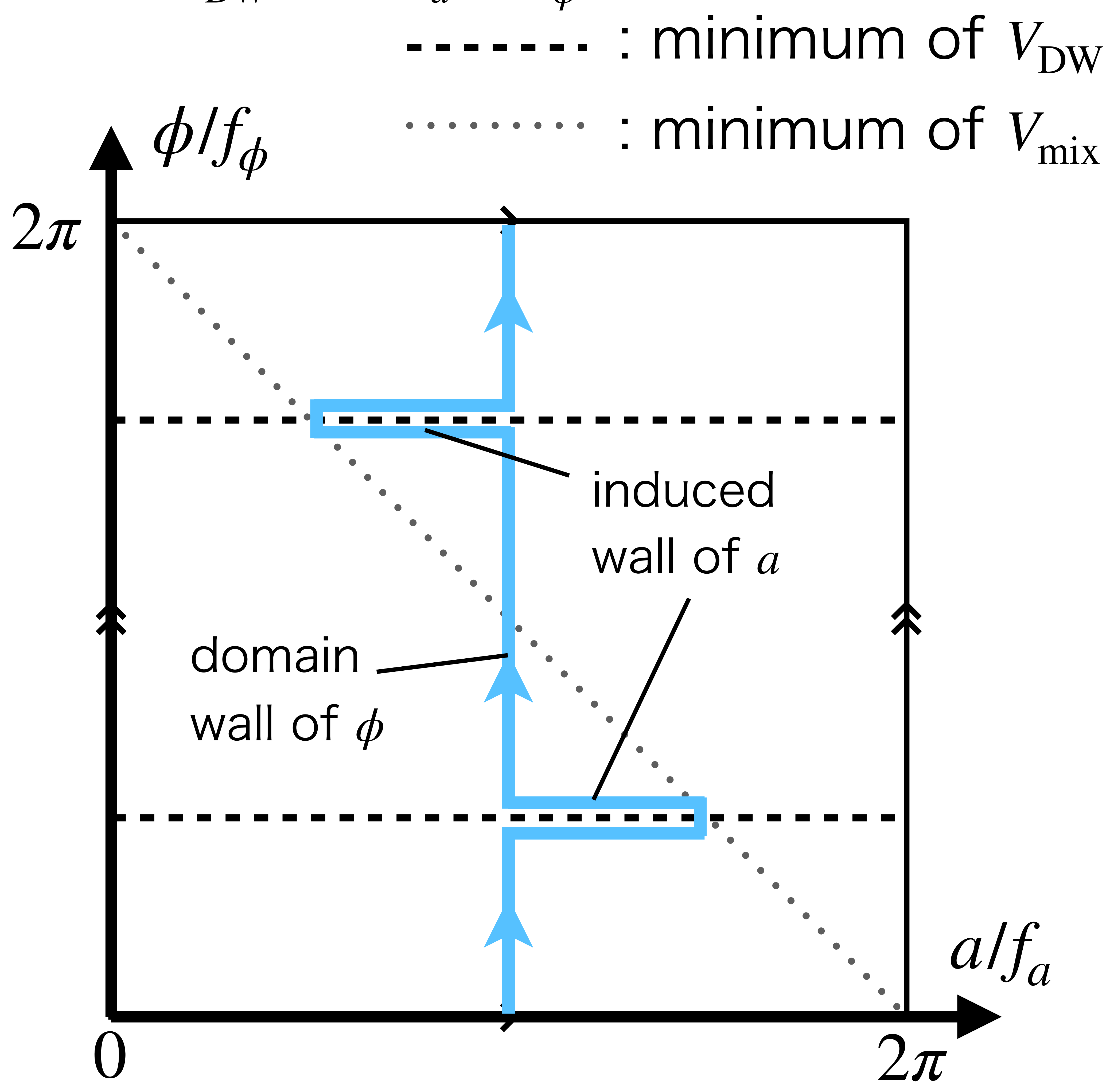
$$\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]+\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]$$

For a, the effective θ parameter varies across the heavy axion domain walls shifting the potential minimum of a, in different bulks.

$$a_{\min,k}=\frac{2\pi f_a}{N_a}\left(-\frac{kN_\phi}{N_{\rm DW}}+\mathbb{Z}\right)$$ at domain of ϕ_k



e.g., $N_{\rm DW}=2\,, N_a=N_\phi=1$



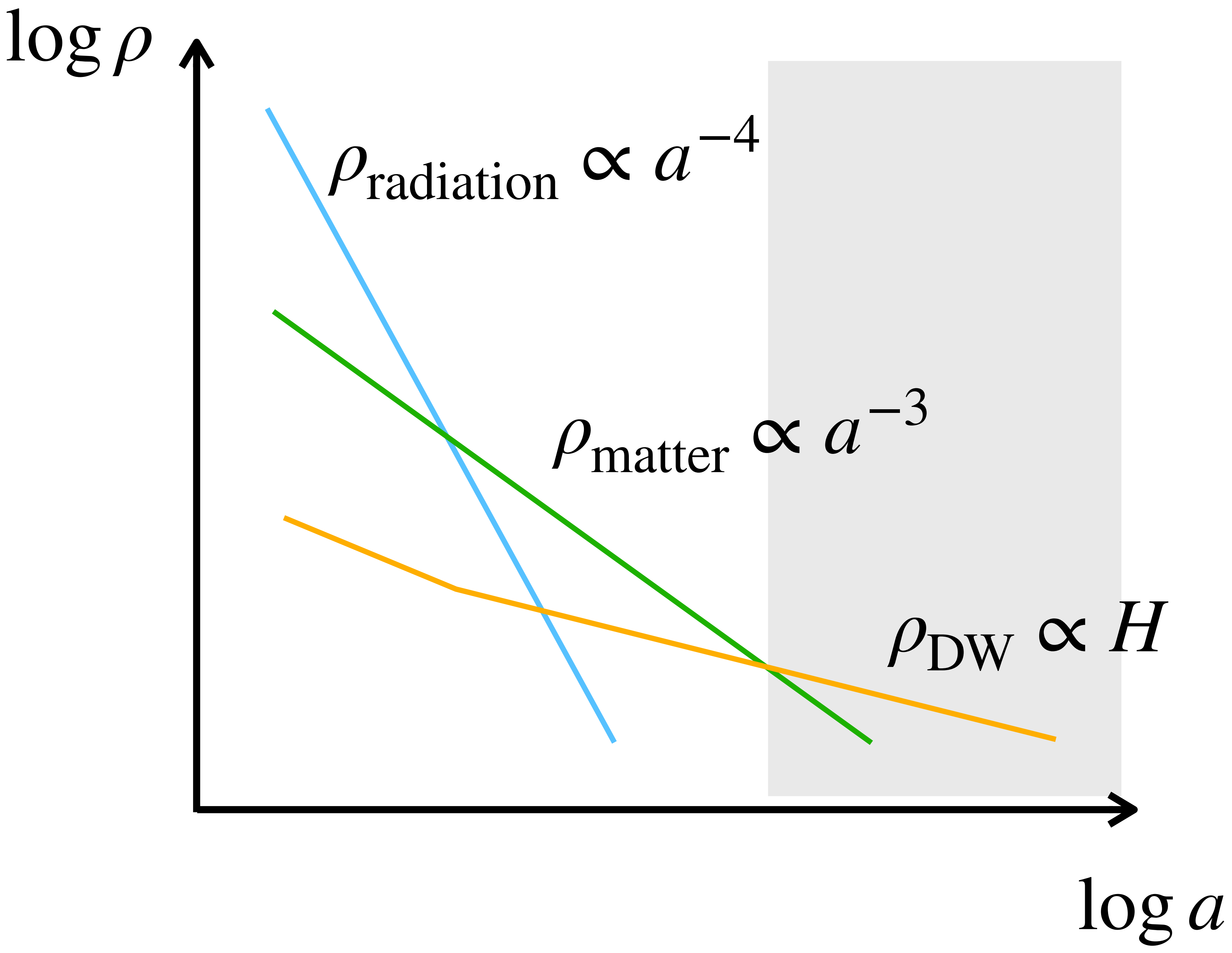
Domain walls are problematic

The energy density of domain walls in the scaling regime decreases as

$$\rho_{\rm DW} \approx \sigma H,$$

which is slower than radiation and non-relativistic matter, causing the overclosure of the universe.

Zel'dovich, Kobzarev, Okun `74



Substantial instability of domain walls is required to avoid anisotropy.

Good news:
Induced walls collapse as well when heavy axion domain walls collapse.

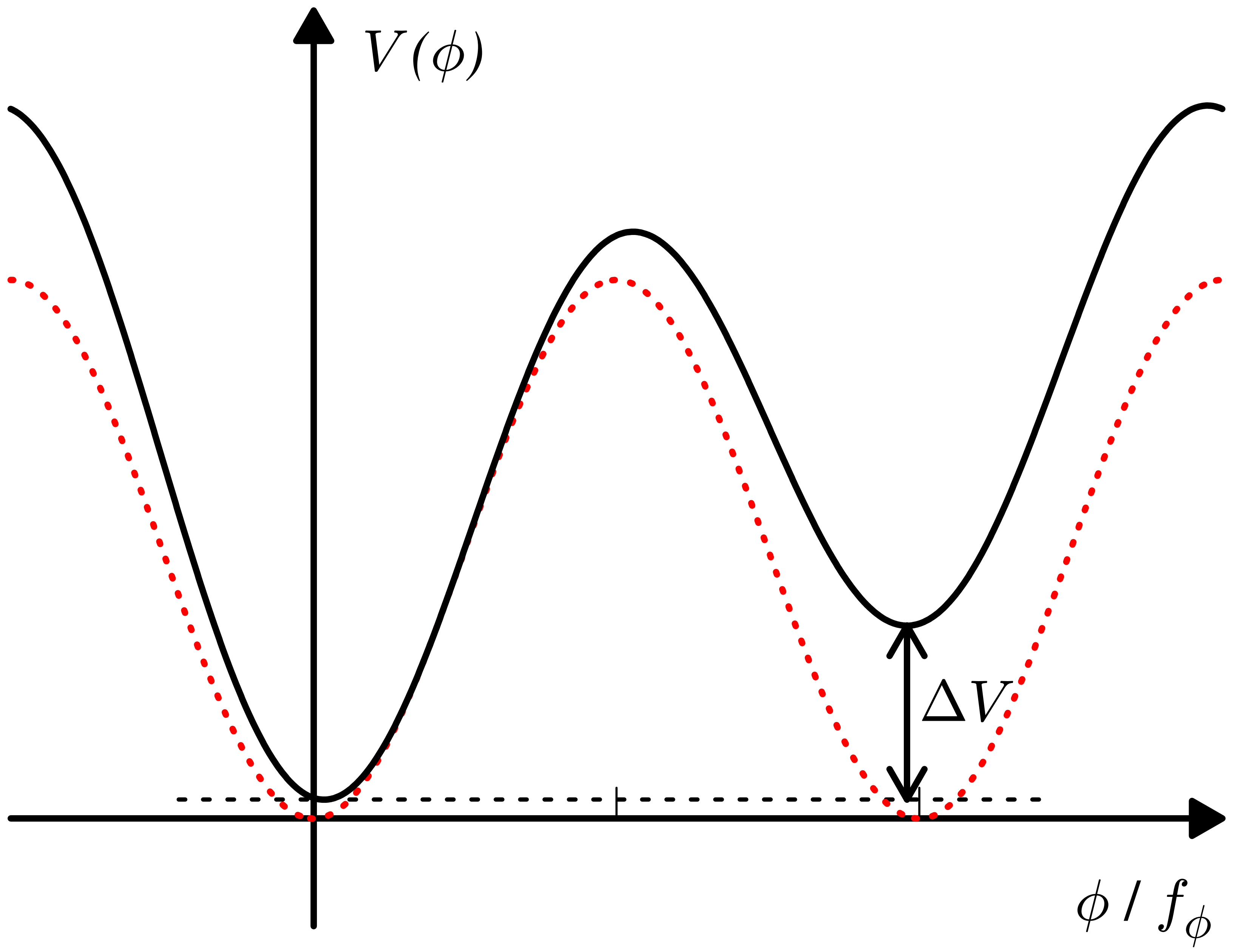
Additional potential

Additional potential $V_{\rm bias}$ to avoid the overclosure of the universe

$$V(a,\phi) = V_{\rm DW}(\phi) + V_{\rm mix}(a,\phi) + V_{\rm bias}(\phi)$$

$$V_{\rm bias}(\phi) = \epsilon\Lambda^4\left[1-\cos\left(N_{\rm b}\frac{\phi}{f_\phi}+\theta\right)\right]$$

$\epsilon \ll 1$



Additional potential

Additional potential $V_{\rm bias}$ to avoid the overclosure of the universe

from coupling to gluons

$$V(a,\phi) = V_{\rm DW}(\phi) + V_{\rm mix}(a,\phi) + V_{\rm bias}(\phi)$$

$$V_{\rm bias}(\phi) = \epsilon\Lambda^4\left[1 - \cos\left(N_{\rm b}\frac{\phi}{f_\phi} + \theta\right)\right]$$

$$\epsilon \ll 1$$

This bias term **does not spoil the solution to the strong CP problem**.

QCD axion production

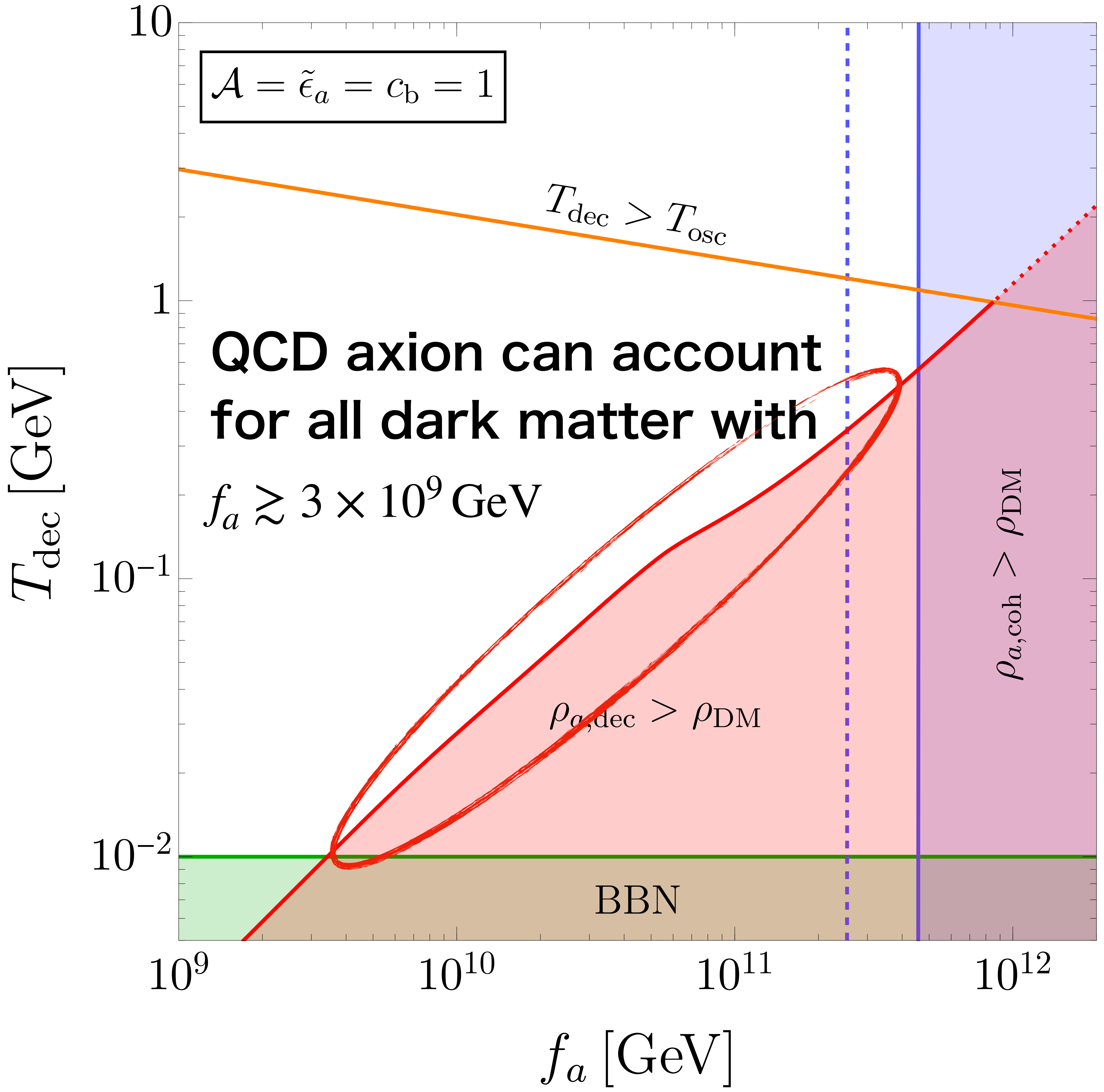
The number density of a emitted from the induced domain wall collapse is

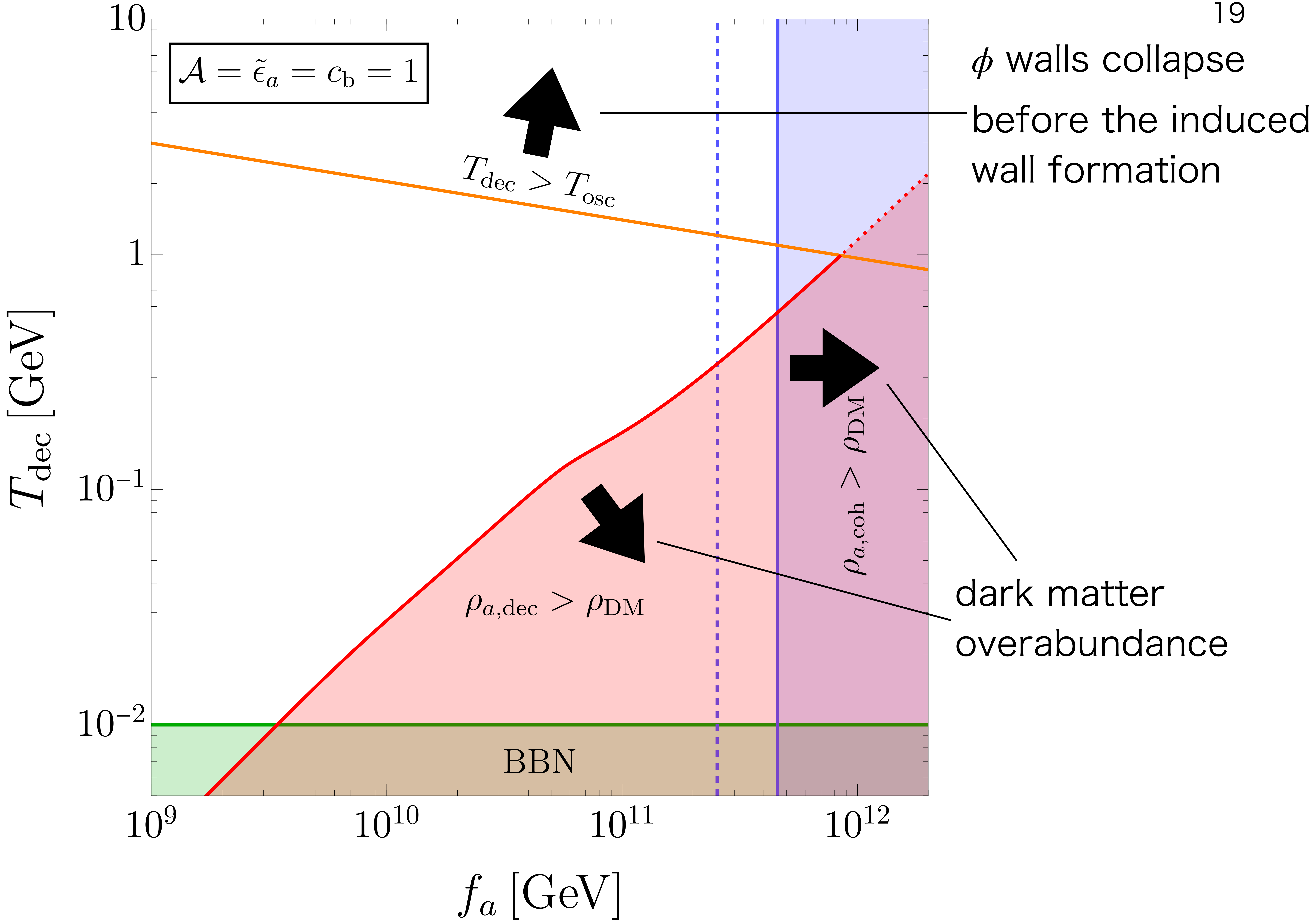
$$n_{a,\mathrm{dec}} \sim \frac{\sigma_a H_{\mathrm{dec}}}{m_a} \, .$$

$$\frac{\rho_{a,\mathrm{dec}}}{s} \simeq 0.43\,\mathrm{eV} \times \left(\frac{g_{*,\mathrm{dec}}}{10.75}\right)^{1/2} \left(\frac{g_{*s,\mathrm{dec}}}{10.75}\right)^{-1} \left(\frac{f_a}{4\times 10^9\,\mathrm{GeV}}\right) \left(\frac{T_{\mathrm{dec}}}{12\,\mathrm{MeV}}\right)^{-1}$$

(for $T_{\mathrm{dec}} < T_{\mathrm{QCD}}$)

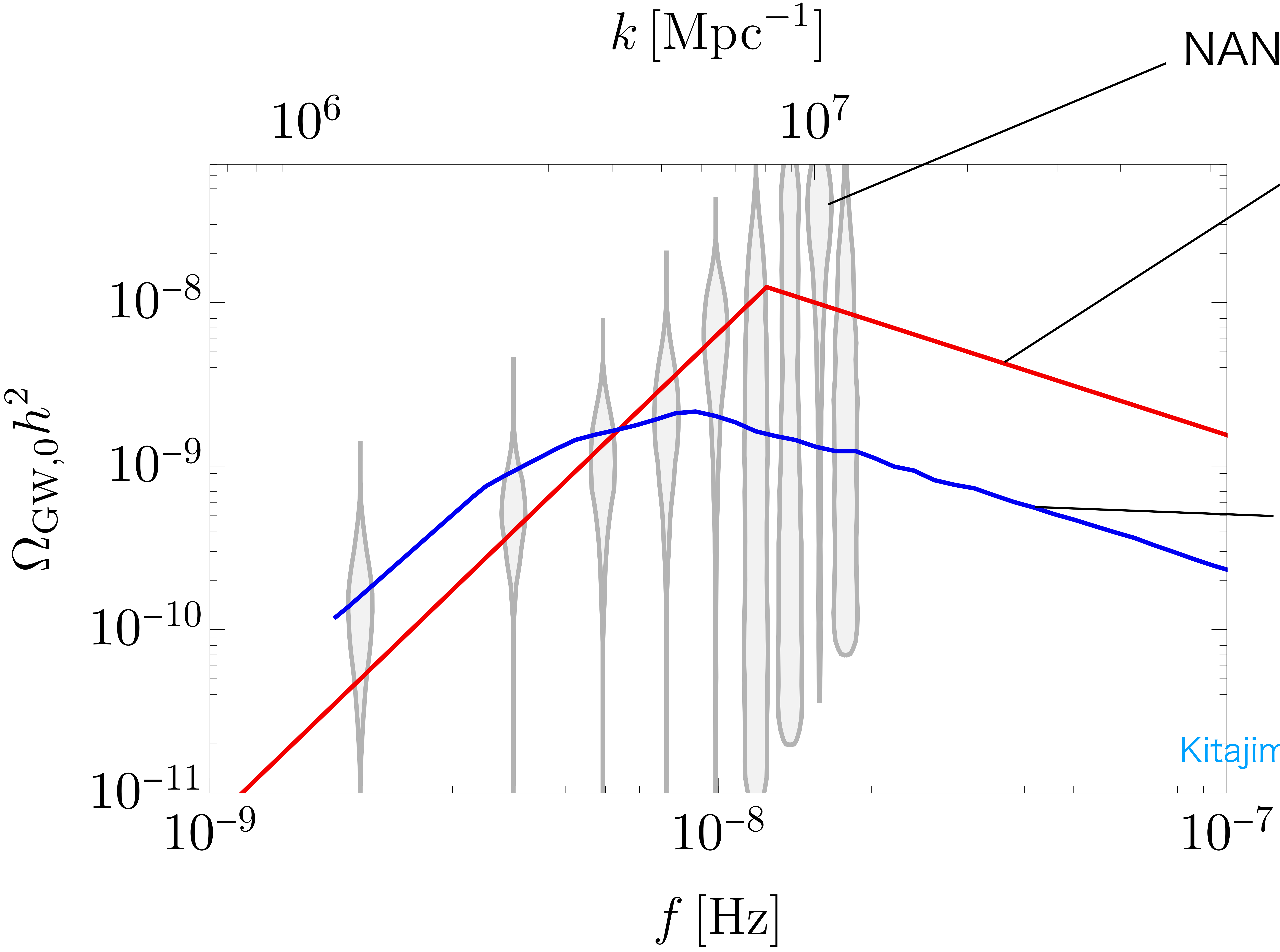
We assume that the emitted ϕ immediately decays into standard model particles and a is stable and contributes to dark matter.


$\mathcal{A} = \tilde{\epsilon}_a = c_b = 1$
$T_{dec} > T_{osc}$
QCD axion can account
for all dark matter with
$f_a \gtrsim 3 \times 10^9$ GeV
$\rho_{a,dec} > \rho_{DM}$
$\rho_{a,coh} > \rho_{DM}$
BBN
T_{dec} [GeV]
f_a [GeV]
10
1
10^{-1}
10^{-2}
10^9
10^{10}
10^{11}
10^{12}



$\mathcal{A} = \tilde{\epsilon}_a = c_\mathrm{b} = 1$
$T_\mathrm{dec} > T_\mathrm{osc}$
$\rho_{a,\mathrm{dec}} > \rho_\mathrm{DM}$
$\rho_{a,\mathrm{coh}} > \rho_\mathrm{DM}$
BBN
T_dec [GeV]
f_a [GeV]
10
1
10^{-1}
10^{-2}
10^{9}
10^{10}
10^{11}
10^{12}
ϕ walls collapse
before the induced
wall formation
dark matter
overabundance

Gravitational wave spectrum



NANOGrav dataset

analytic estimate with

$\sigma_\phi = 6 \times 10^{15}\,\mathrm{GeV}^3$

$T_{\mathrm{dec}} = 100\,\mathrm{MeV}$

numerical result with

$\sigma_\phi = 1.3 \times 10^{15}\,\mathrm{GeV}^3$

$T_{\mathrm{dec}} = 150\,\mathrm{MeV}$

Kitajima, JL, Murai, Takahashi, Yin `23

Summary

- Multiple axions significantly alter the evolution of topological defects.
- Heavy axion domain walls can induce domain walls of the lighter axion even with a spatially homogeneous initial condition.
- The bias potential term for the heavy axion does not spoil the solution of the strong CP problem, collapsing the domain walls.
  - See 2407.09478 for another mechanism, “transient bias”.

See also Ibe, Kobayashi, Suzuki, Yanagida `19

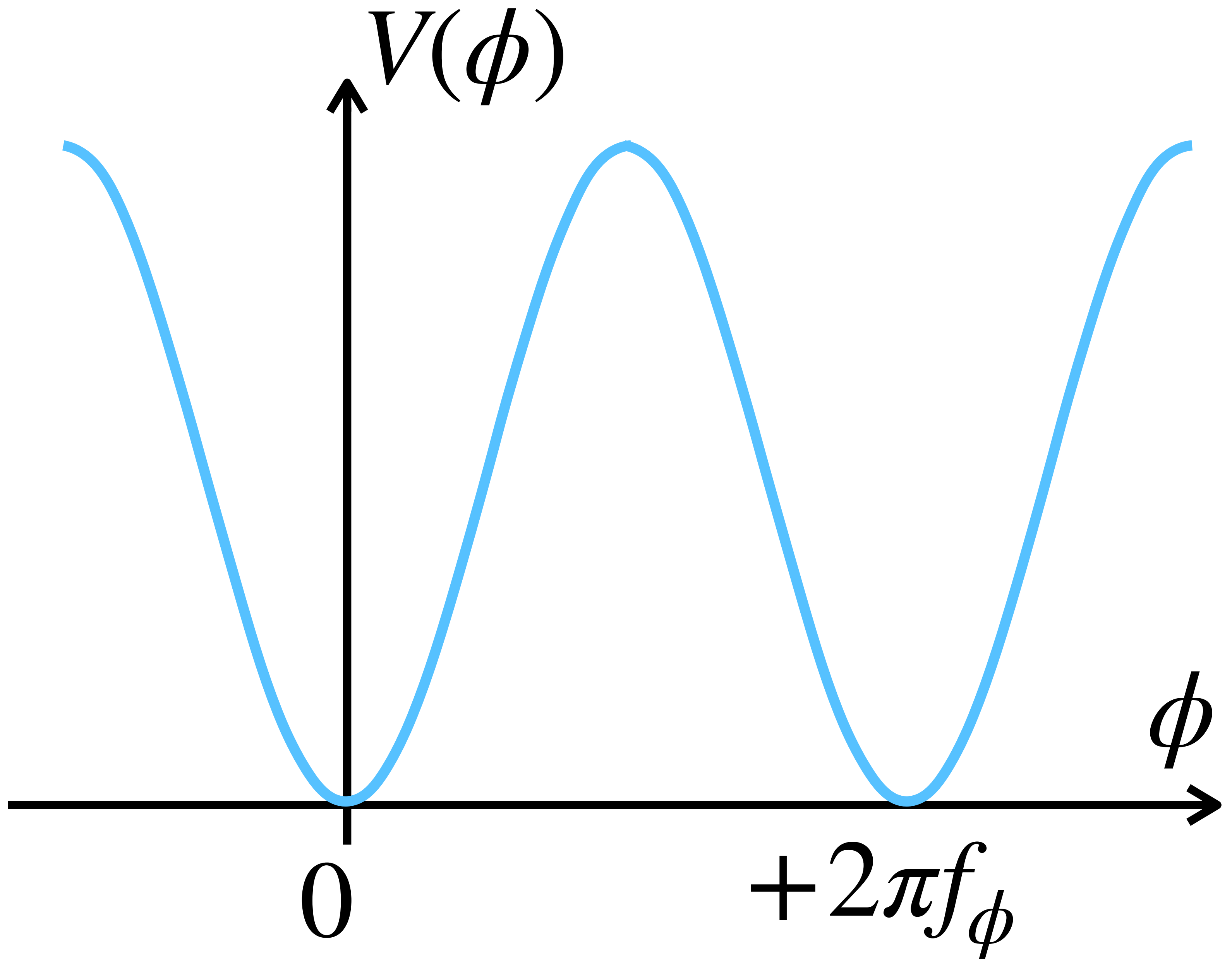
- The collapse of domain walls produces the QCD axion, which can account for all dark matter with $f_a \gtrsim 3 \times 10^9\,\mathrm{GeV}$.
- Gravitational waves from collapsing domain walls can account for the Pulsar Timing Array results.

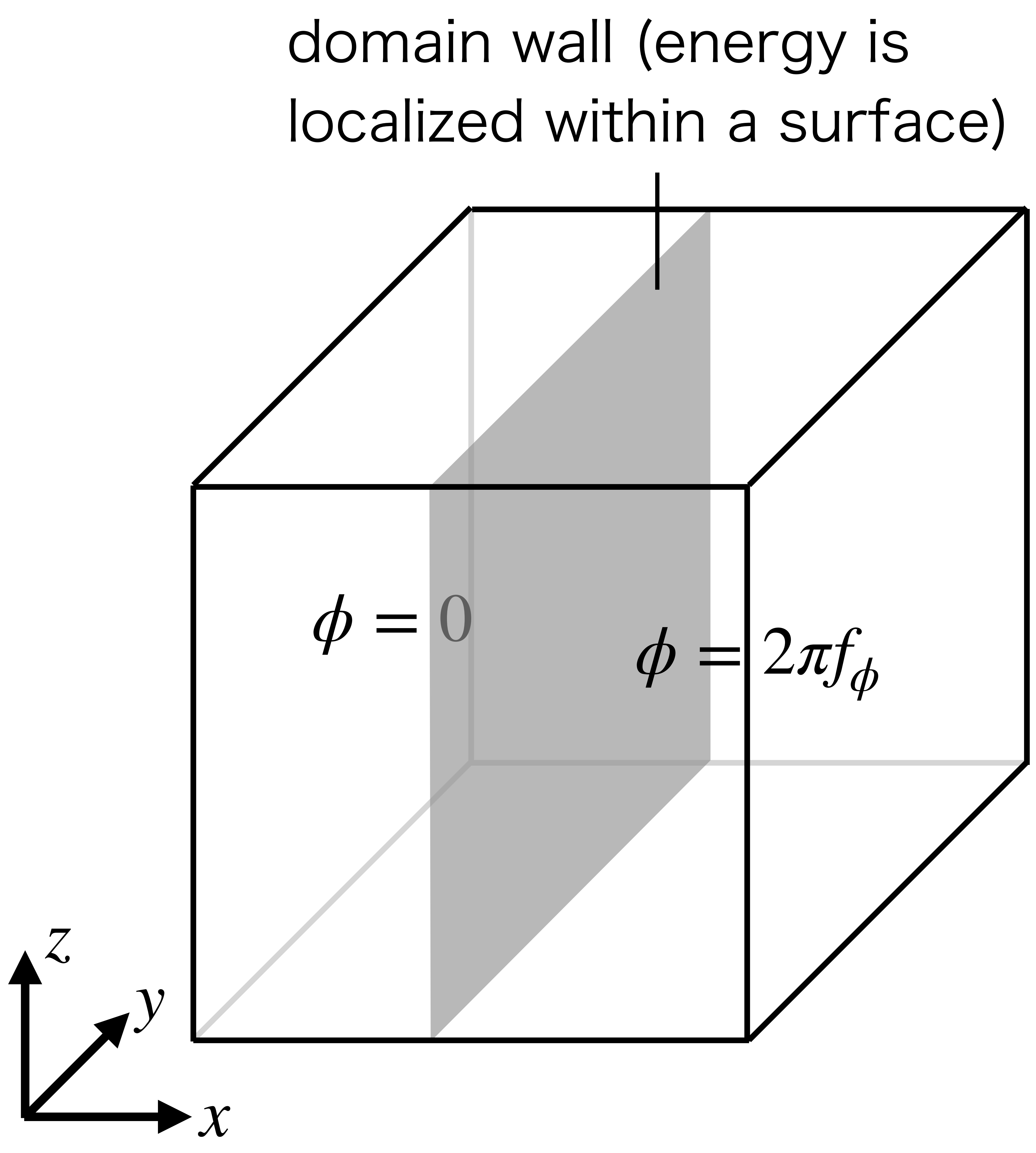
Back up

Domain wall

Spontaneous breaking of discrete symmetry leads to the domain wall formation in the early universe.

Zel'dovich, Kobzarev, Okun `74, Kibble `76, ⋯

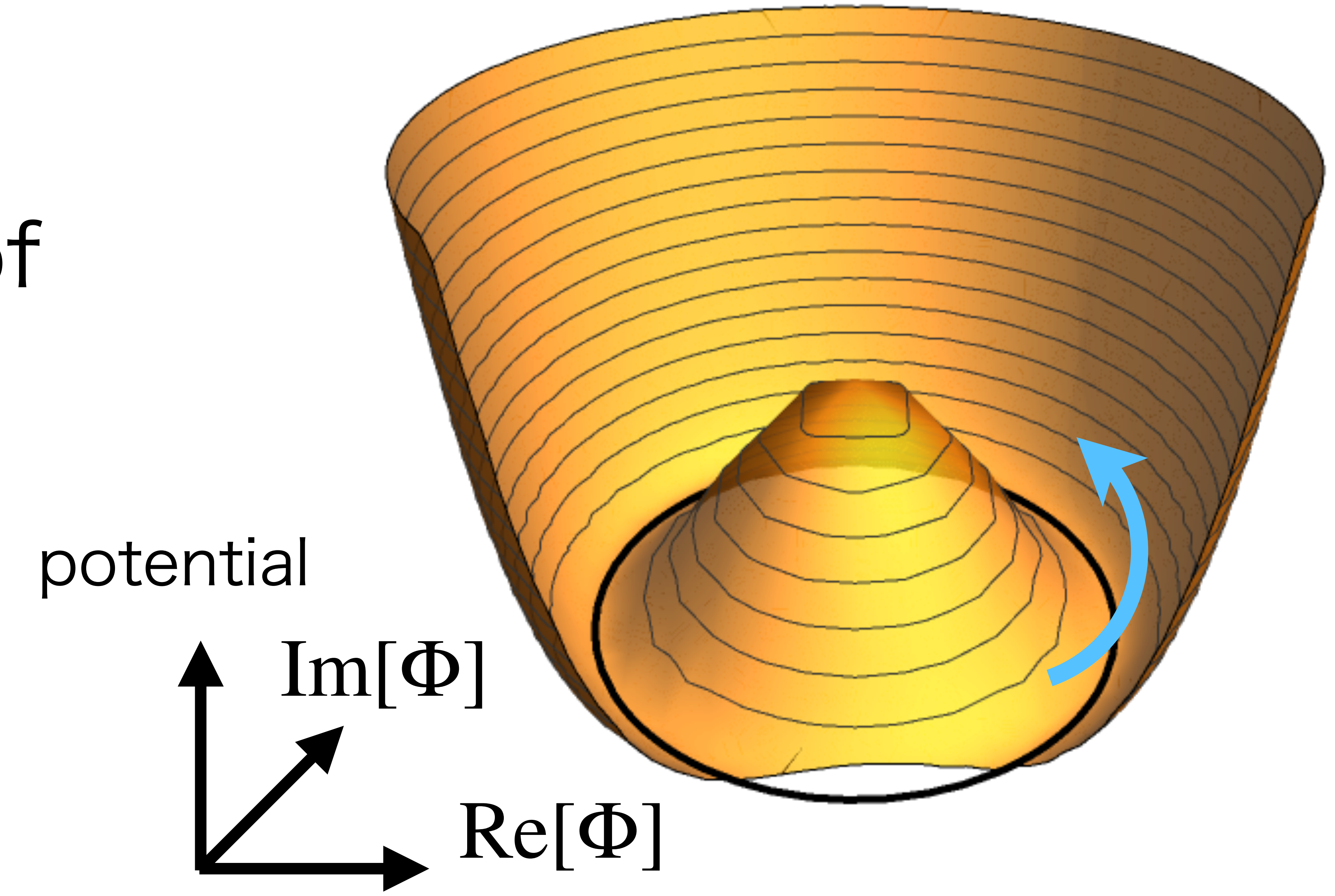




Cosmic string

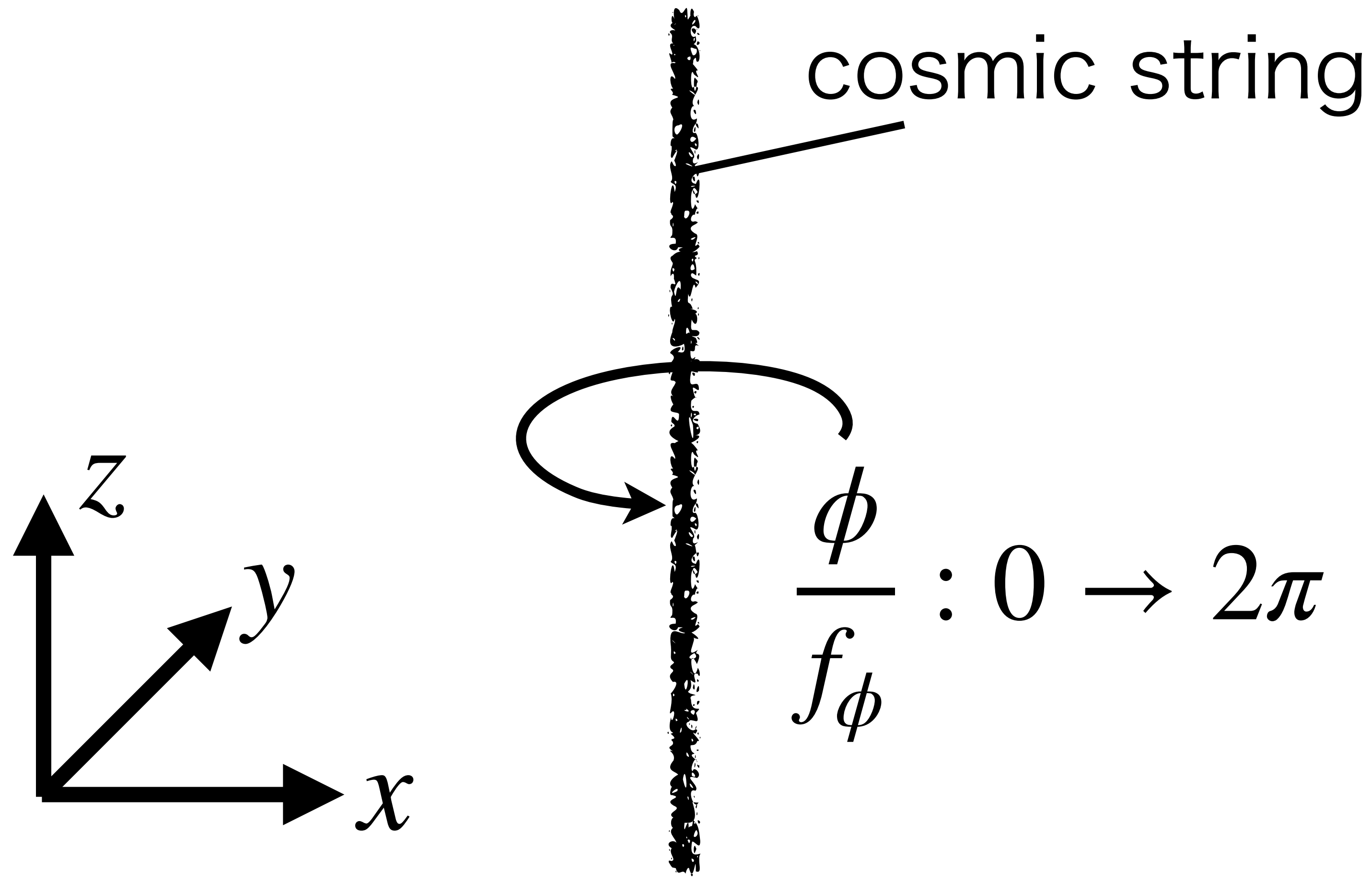
When the axion ϕ is embedded in a phase of a complex scalar field Φ with a wine-bottle potential as

$$\Phi = \frac{f_\phi}{\sqrt{2}} e^{i\frac{\phi}{f_\phi}},$$



the axion is the Nambu-Goldstone boson that arises from the spontaneous symmetry breaking of global $U(1)$ symmetry.

There is a cosmic string configuration winding the phase of Φ.



Strings and domain walls

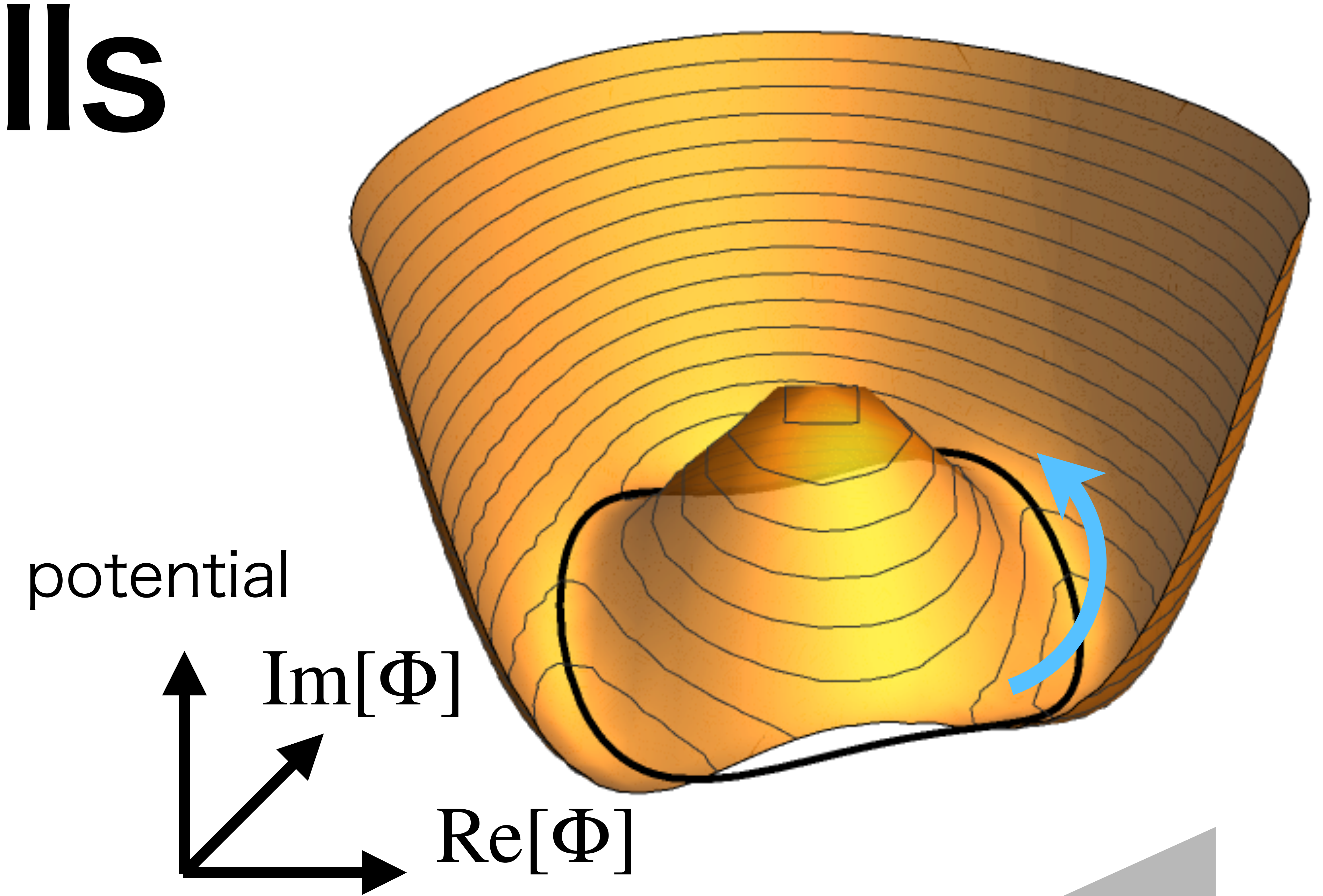
With explicit breaking of $U(1)$ symmetry,

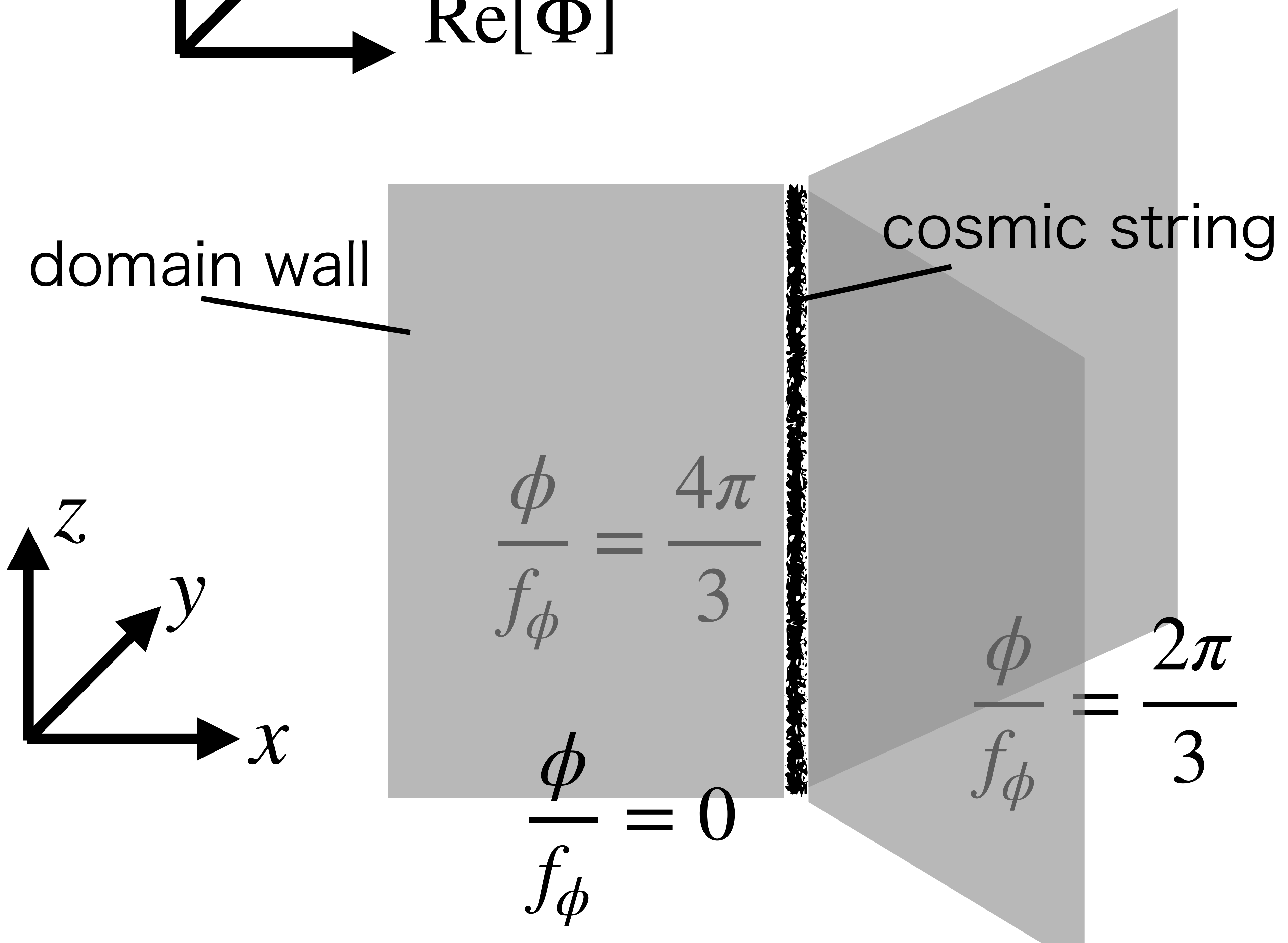
$$U(1) \to \mathbb{Z}_{N_{\rm DW}} ,$$

the axion has periodic potential and obtains mass m_ϕ.

When $H < m_\phi$, the axion rolls down the potential and domain walls are formed.

Domain walls are attached to axion strings.





Scaling solution of domain walls

Once formed, domain walls follow
the scaling solution:

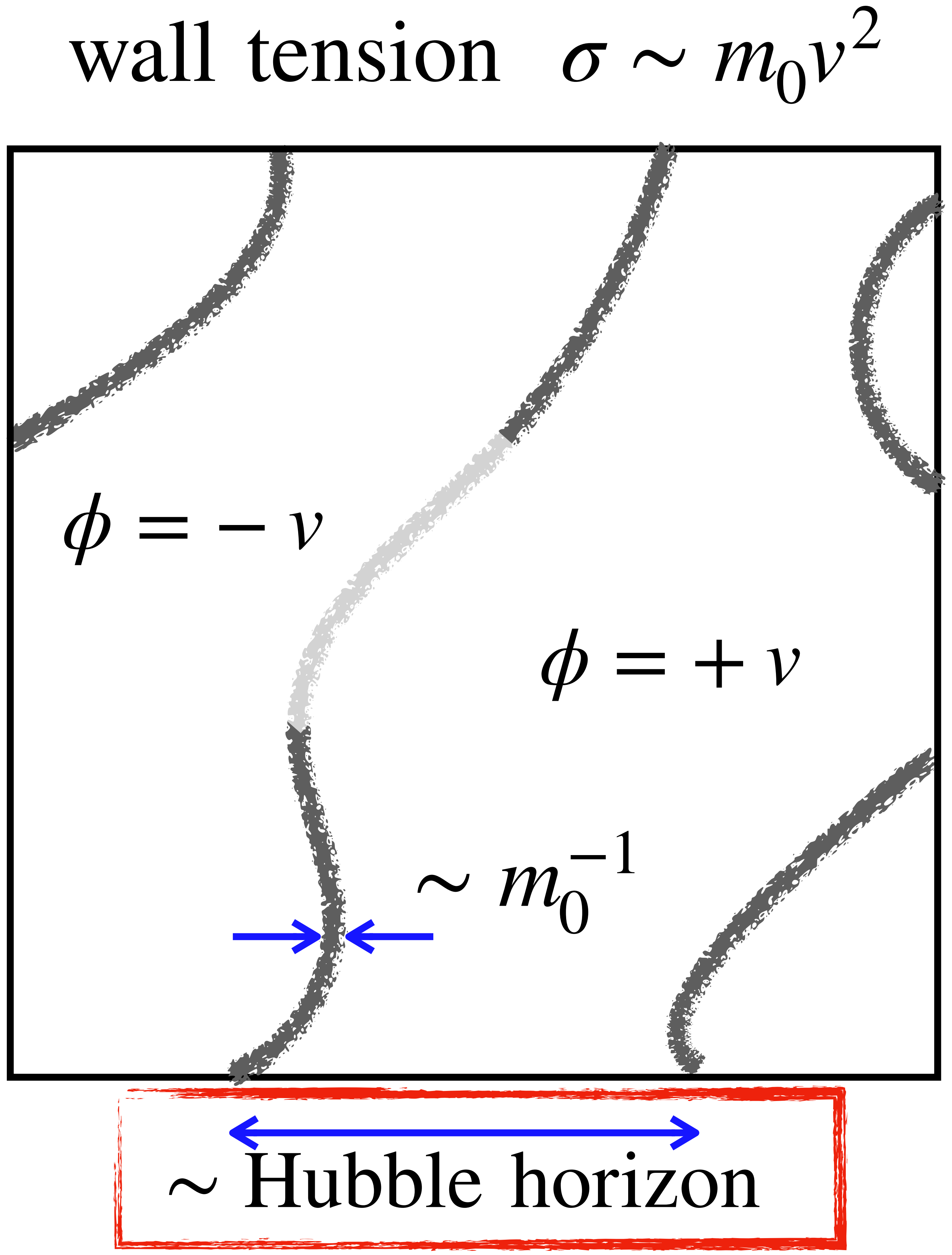
Each Hubble horizon (H^{-1}) contains

$\mathcal{O}(1)$ domain walls Sikivie `82, Vilenkin `85, Press, Ryden, Spergel `89, ⋯

$$\rho_{\rm DW} = \frac{\mathscr{A}\sigma H^{-2}}{H^{-3}} = \mathscr{A}\sigma H$$

$\mathscr{A}$: the area parameter, $\sim \mathcal{O}(1)$ @scaling solution
(How many domain walls in the Hubble volume)

Scaling solution causes **the cosmological domain wall problem** Zel'dovich, Kobzarev, Okun `74



Additional potential provide bias?

$H_{\rm decay} \simeq \dfrac{\Lambda'^4}{\Lambda^4/m_{\rm heavy}}$: Hubble parameter when domain walls decay with potential bias Λ'^4

$H_{\rm osc} \simeq m_{\rm light}$: Hubble parameter when the light axion starts to oscillate

$$\frac{H_{\rm decay}}{H_{\rm osc}} \simeq \left(\frac{\Lambda'}{\Lambda}\right)^4 \frac{m_{\rm heavy}}{m_{\rm light}} \simeq \left(\frac{\Lambda'}{\Lambda}\right)^2 \frac{n_1 f_2/f_1 + n_2 f_1/f_2}{|n_1 n_2' - n_2 n_1'|}$$

Basically, the light axion starts to oscillate before V_2 provides the bias.

(without large hierarchy between decay constants)

Domain wall tension

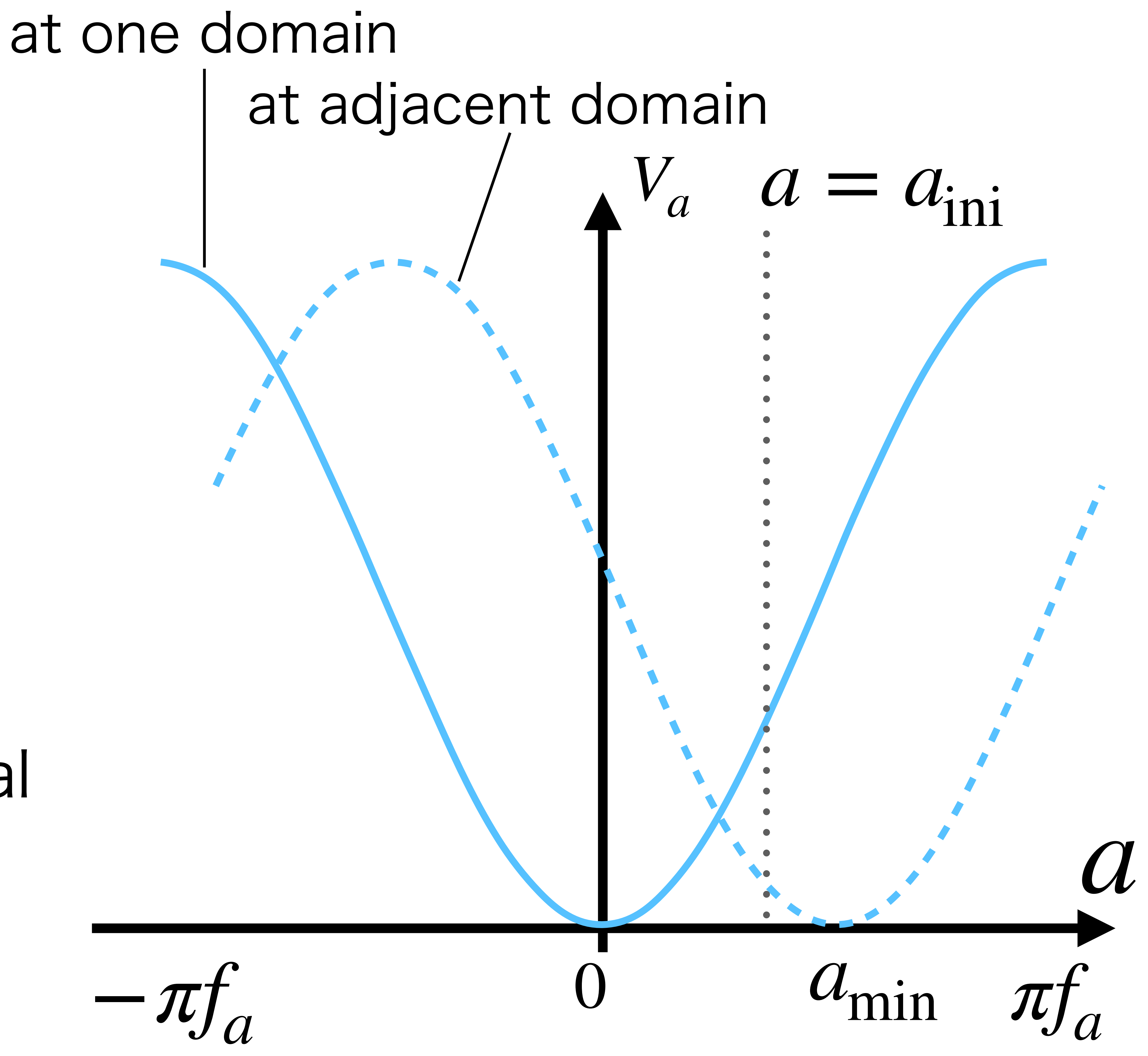
The tension of induced wall is estimated to be

$$\sigma_a = \kappa \left(\frac{a_{\rm min}}{m_a^{-1}} \right)^2 m_a^{-1} = \kappa m_a a_{\rm min}{}^2 \,.$$

($\kappa \simeq 2.34$ when $N_{\rm DW} = 2\,, N_a = N_\phi = 1$)

The tension of the induced wall is generally smaller than in the conventional case,

$$\sigma_{a,{\rm max}} = \frac{8 m_a f_a^2}{N_a^2} \,.$$



Bias potential and domain wall collapse

introduce additional potential $V_{\rm bias}$ to avoid the overclosure of the universe

$$V(a,\phi) = V_{\rm DW}(\phi) + V_{\rm mix}(a,\phi) + V_{\rm bias}(\phi)$$

This bias term **does not spoil the solution to the strong CP problem**.

$$V_{\rm bias}(\phi) = \epsilon\Lambda^4 \left[1 - \cos\left(N_{\rm b}\frac{\phi}{f_\phi} + \theta\right)\right]$$

Induced walls collapse as well when heavy axion domain walls collapse.

$$\epsilon \ll 1$$

potential difference: $\Delta V \sim \epsilon\Lambda^4$

Domain walls collapse when the bias overcomes the tension: $H_{\rm dec} \sim \Delta V/\sigma_\phi \sim \epsilon m_\phi/8$

Temperature $T_{\rm dec} \sim \sqrt{\epsilon M_{\rm Pl} m_\phi/8} \gtrsim 10\,{\rm MeV}$ for domain walls to collapse before BBN.

GWs from DW collapse assuming scaling regime

Hiramatsu, Kawasaki, Saikawa `13

Before DWs collapse, the DWs follow the scaling solution.

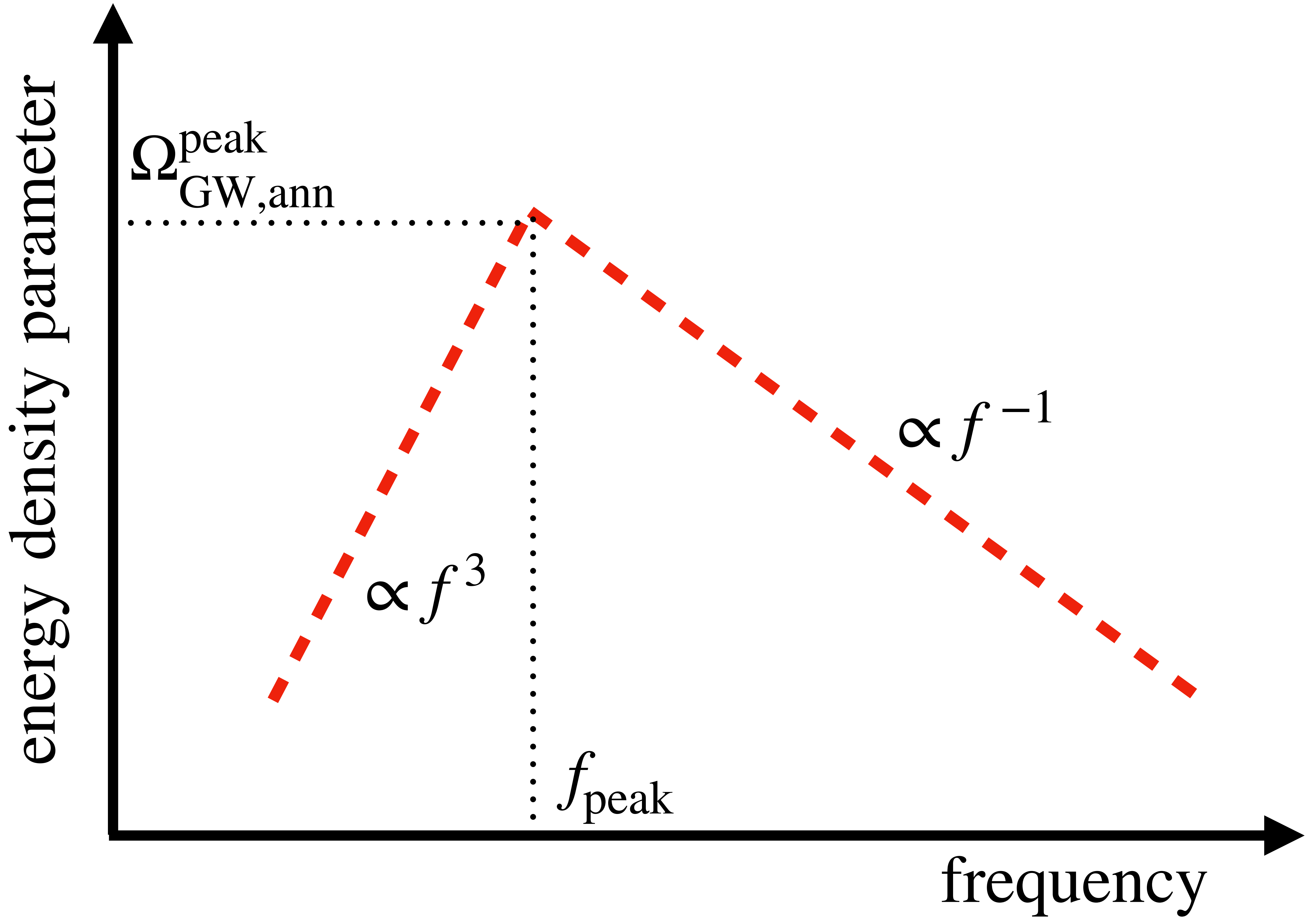
Sources are distributed over the Hubble horizon scale.

$$f_{\rm peak} \sim H_{\rm dec}$$

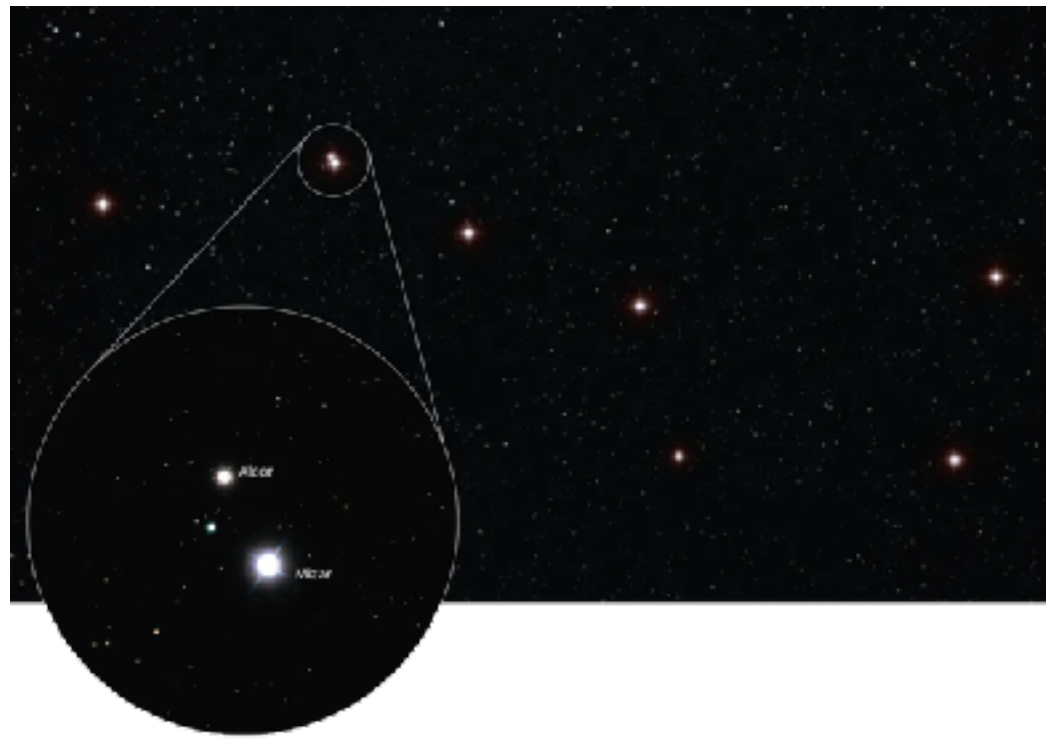
Stored gravitational energy

$\propto$ (DW area × tension $)^2$

$$\Omega_{\rm GW,dec}^{\rm peak} \sim \frac{\sigma^2}{24\pi M_{\rm pl}^4 H_{\rm dec}^2}$$



When there are multiple axions


Credit: Christophe Lehenaff / Getty Images

If there are $N(>1)$ axions and $N-1$ potentials as constraints, there remains one light axion in a low energy theory.

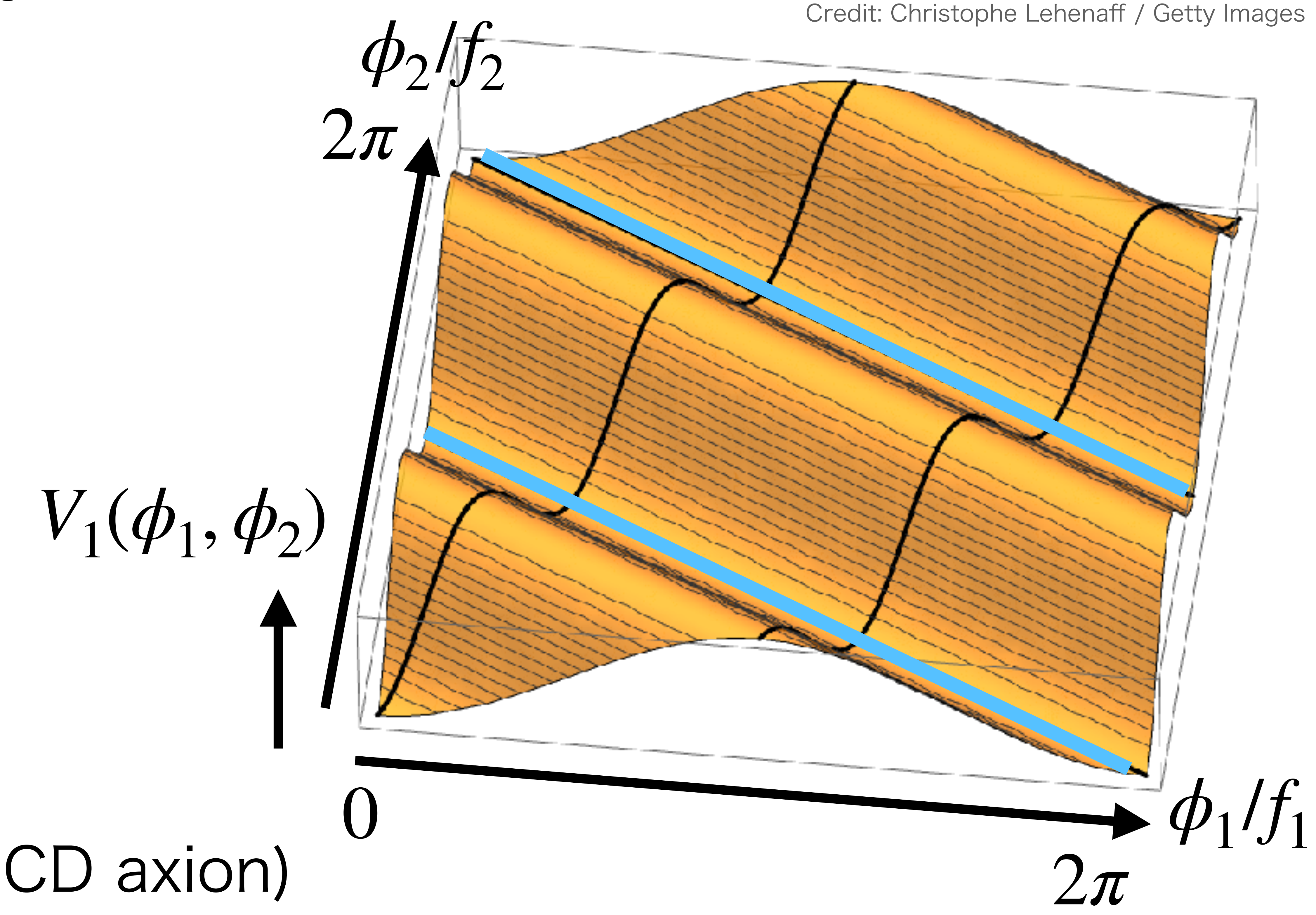
e.g.,

$$\Phi_1 = \frac{f_1}{\sqrt{2}} e^{i\frac{\phi_1}{f_1}}, \quad \Phi_2 = \frac{f_2}{\sqrt{2}} e^{i\frac{\phi_2}{f_2}}$$

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(\frac{\phi_1}{f_1} + 2\frac{\phi_2}{f_2}\right)\right]$$

$$U(1)_{\phi_1} \times U(1)_{\phi_2} \xrightarrow{V_1} U(1)_{\text{light}}$$

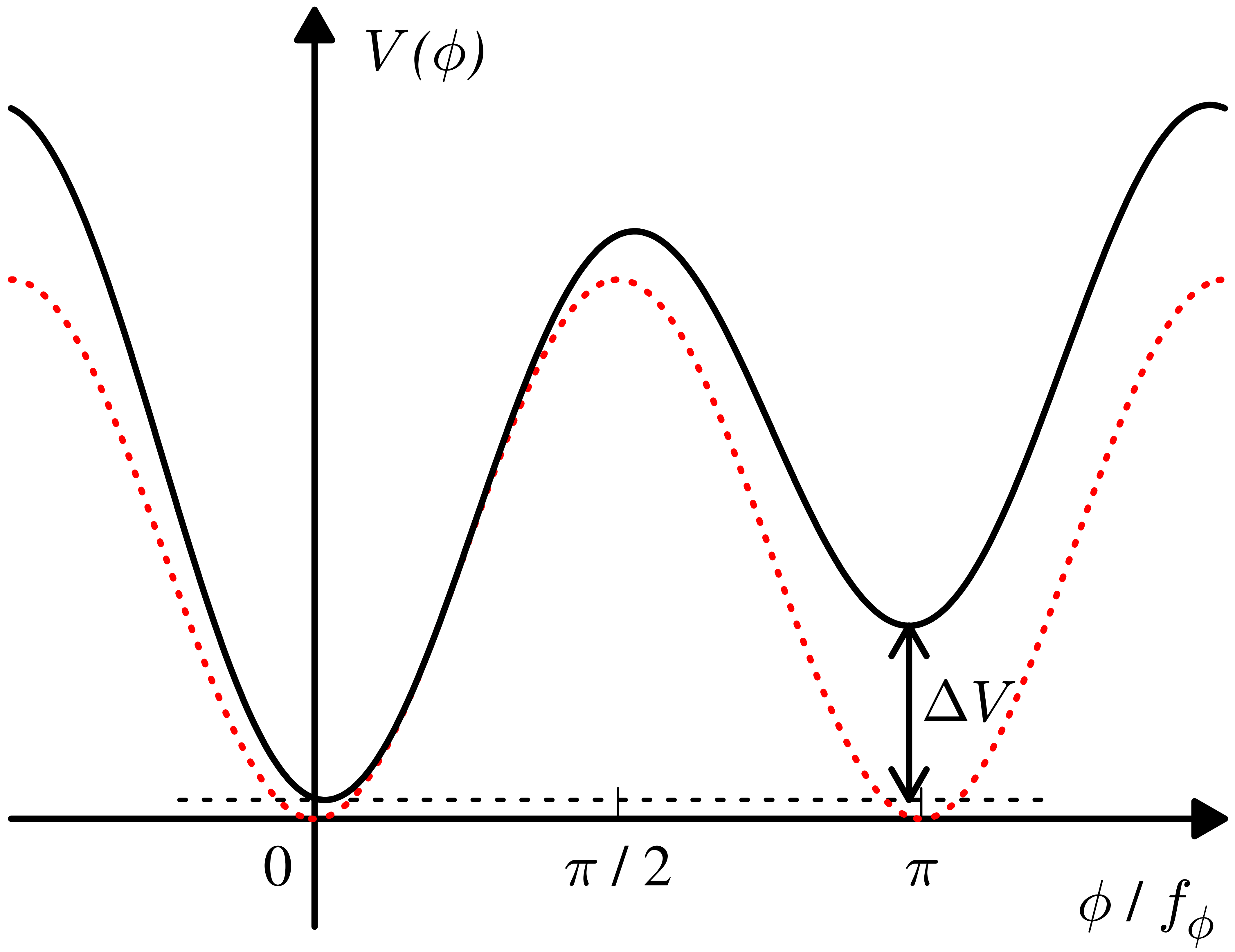
($\sim U(1)_{\text{PQ}}$ when apply to QCD axion)



Q: Is it possible to treat this as a single axion model at the low energy?

Potential bias

Domain walls generally collapse when the degeneracy of vacua is resolved.



Additional potential

Case of $n_1 = 2, n_2 = 3, n_1' = 1, n_2' = 2$

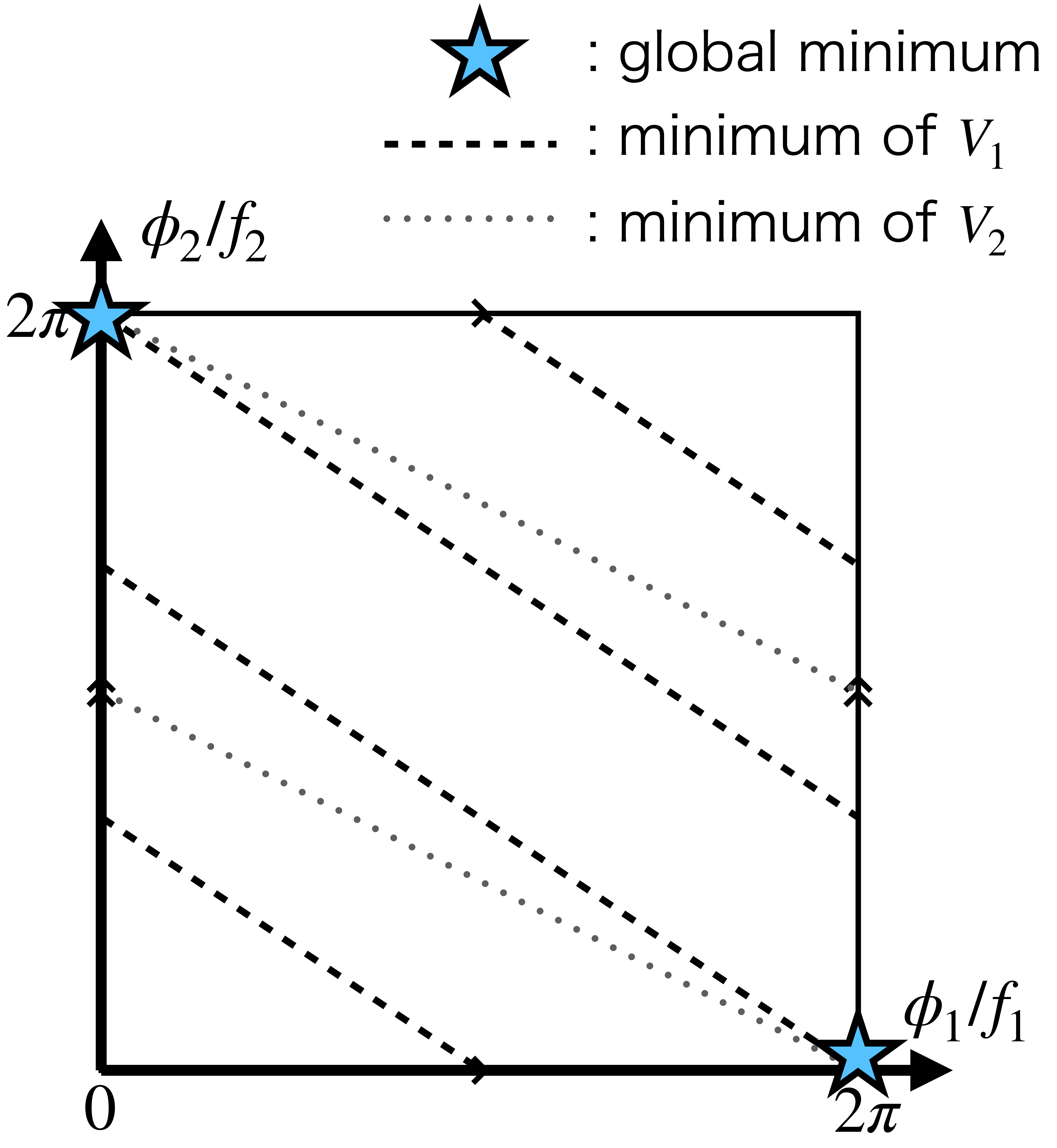
There is a unique global minimum.

$$V(\phi_1, \phi_2) = V_1(\phi_1, \phi_2) + V_2(\phi_1, \phi_2)\,,$$

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1\frac{\phi_1}{f_1} + n_2\frac{\phi_2}{f_2}\right)\right]\,,$$

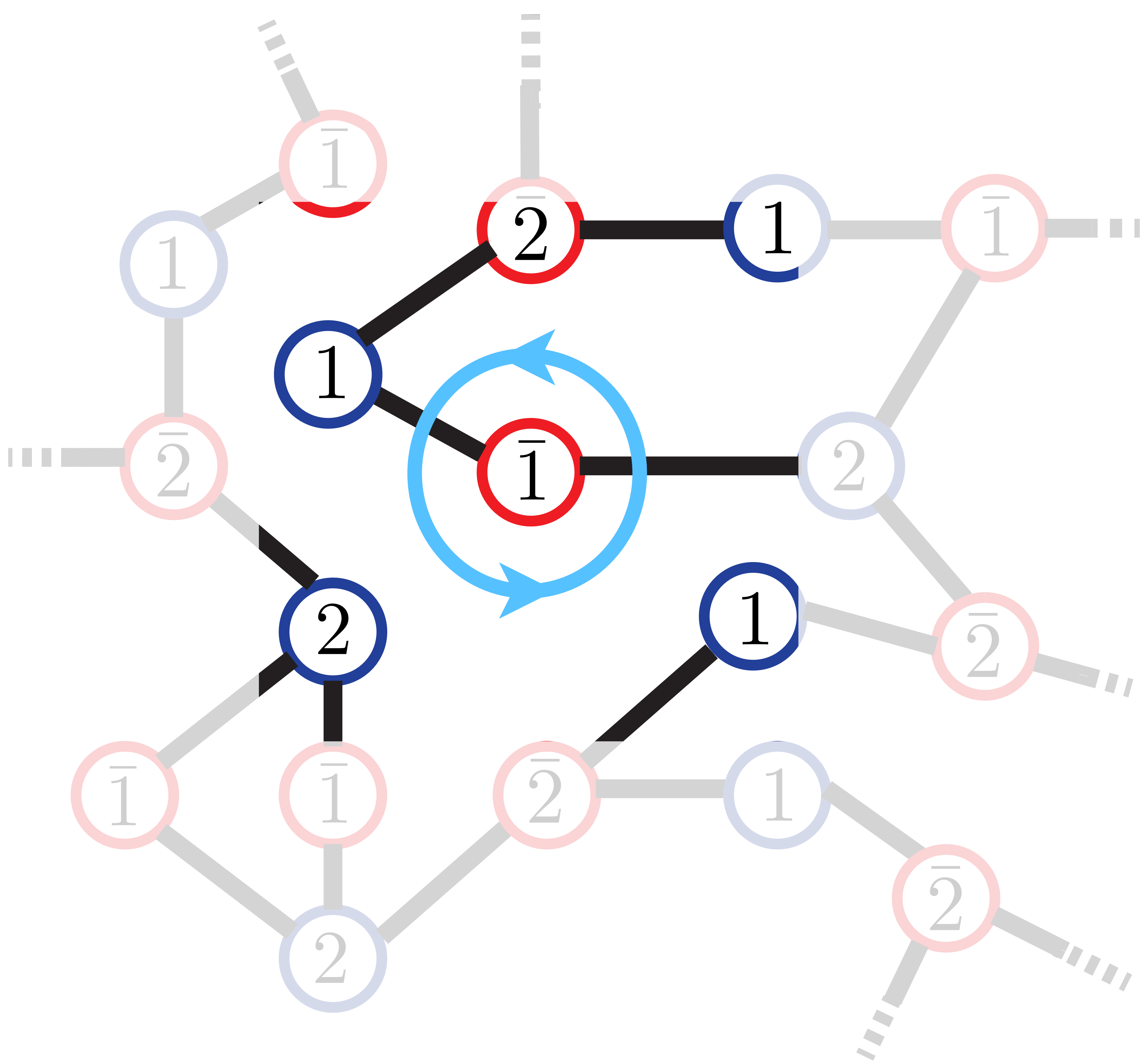
$$V_2(\phi_1, \phi_2) = \Lambda'^4 \left[1 - \cos\left(n_1'\frac{\phi_1}{f_1} + n_2'\frac{\phi_2}{f_2}\right)\right]\,.$$

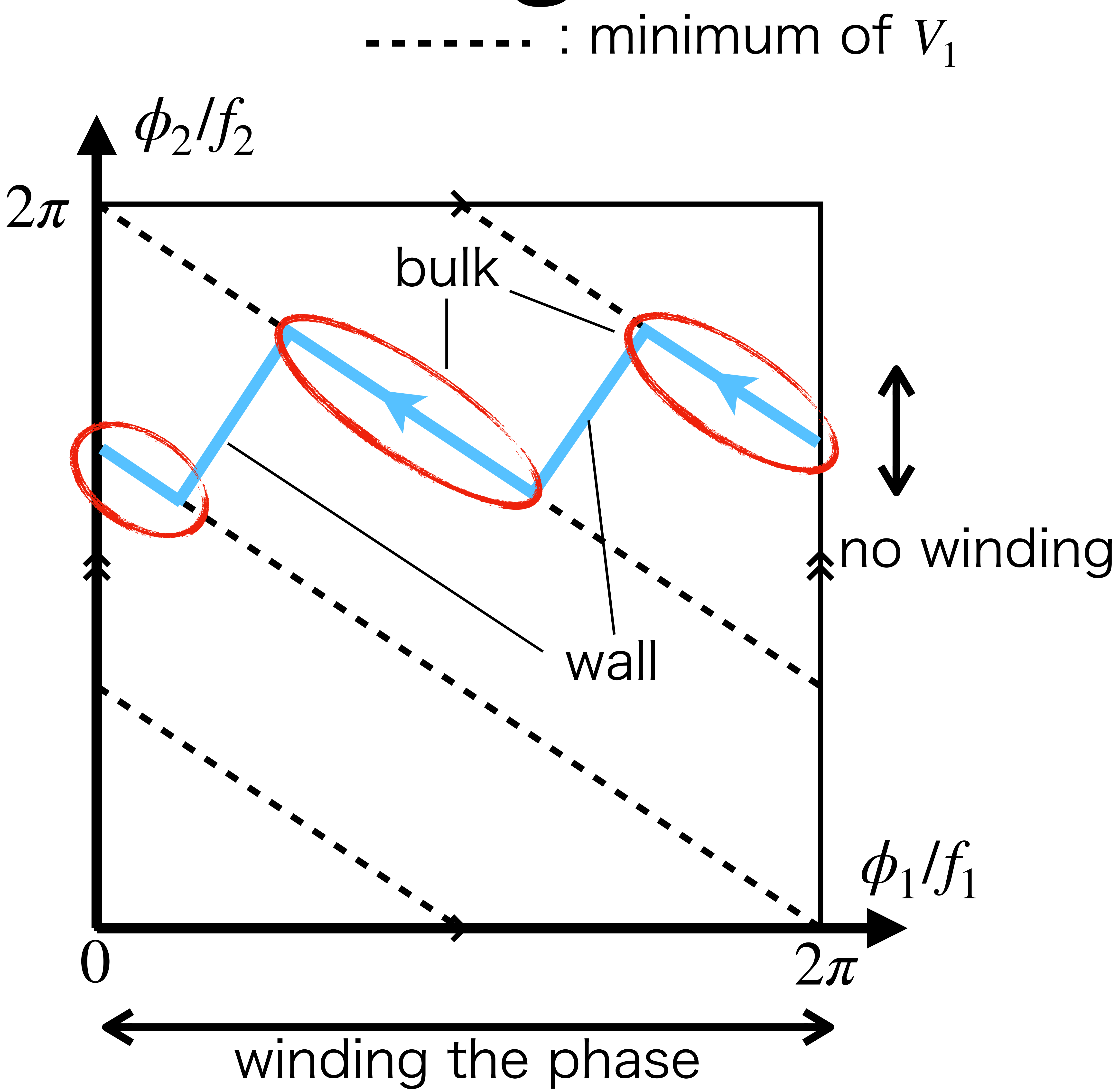
$$\Lambda \gg \Lambda', \qquad n_1 n_2' - n_2 n_1' \neq 0\,.$$



Configuration around the string

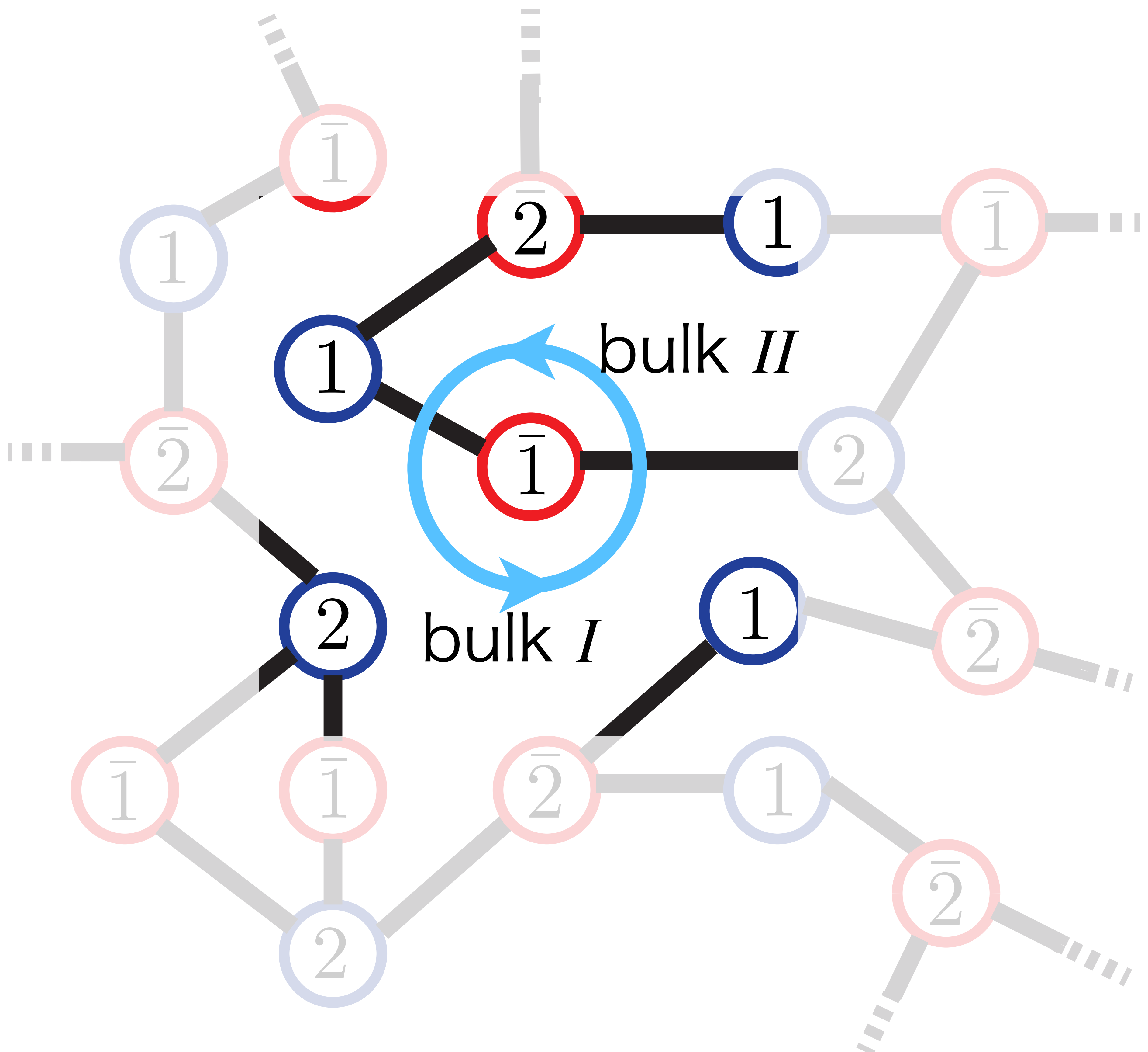
Case of $n_1 = 2, n_2 = 3, n_1' = 1, n_2' = 2$

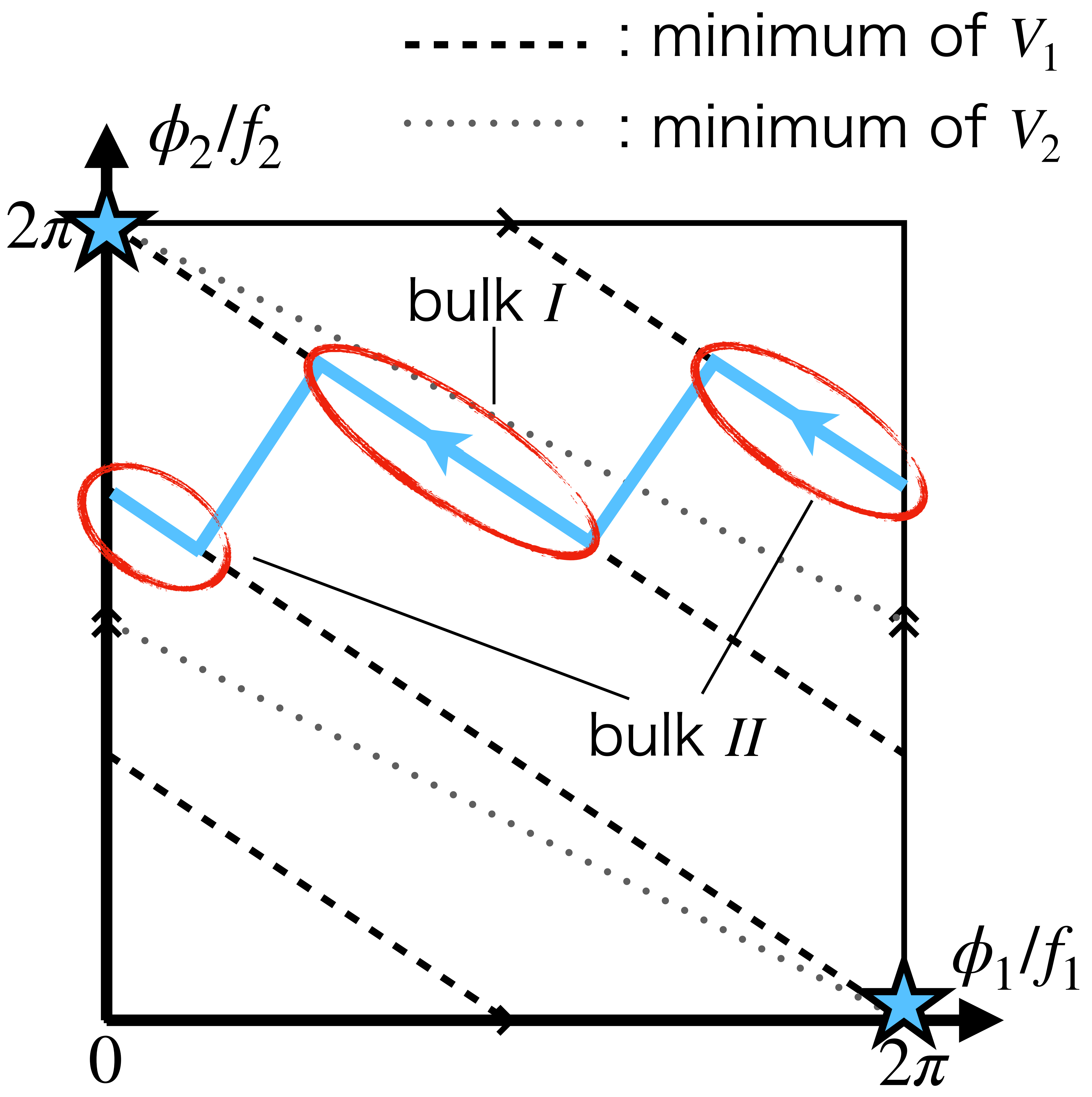




Bulk potential difference

Case of $n_1 = 2, n_2 = 3, n_1' = 1, n_2' = 2$

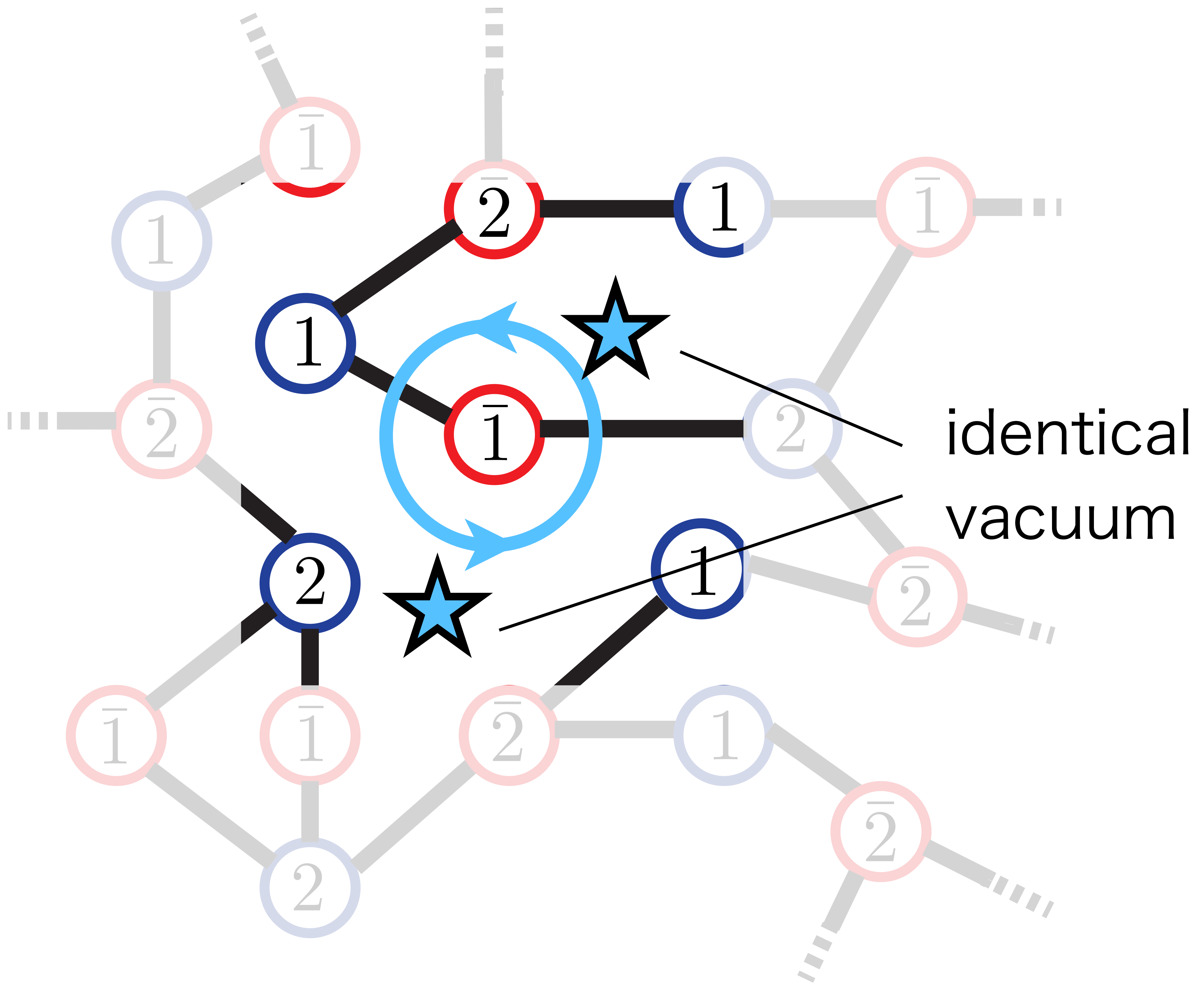


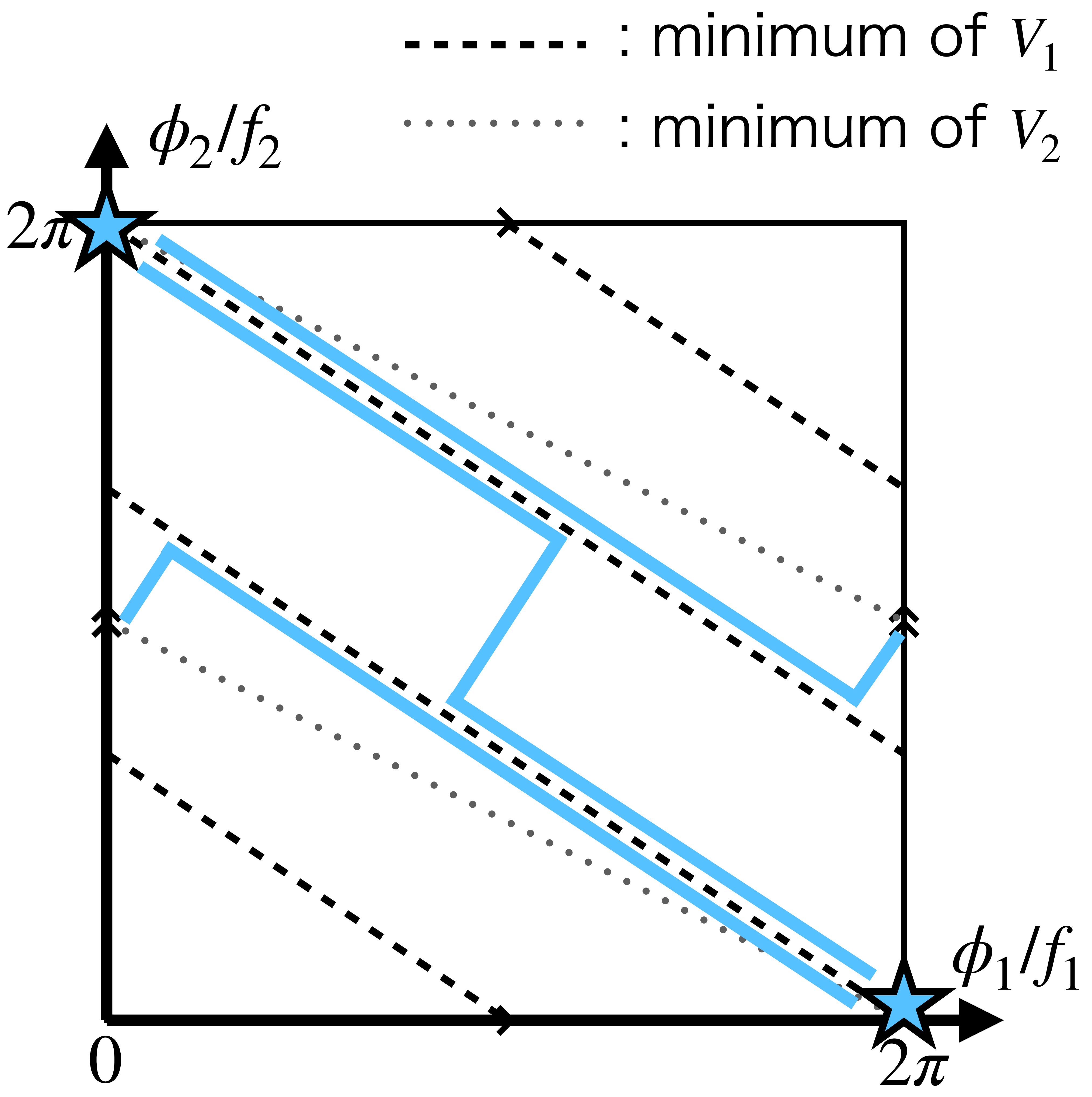


$V_{\text{bulk}I} < V_{\text{bulk}II}$ Domain walls seem to feel potential bias with V_2 .

Domain walls remain stable

Case of $n_1 = 2, n_2 = 3, n_1' = 1, n_2' = 2$





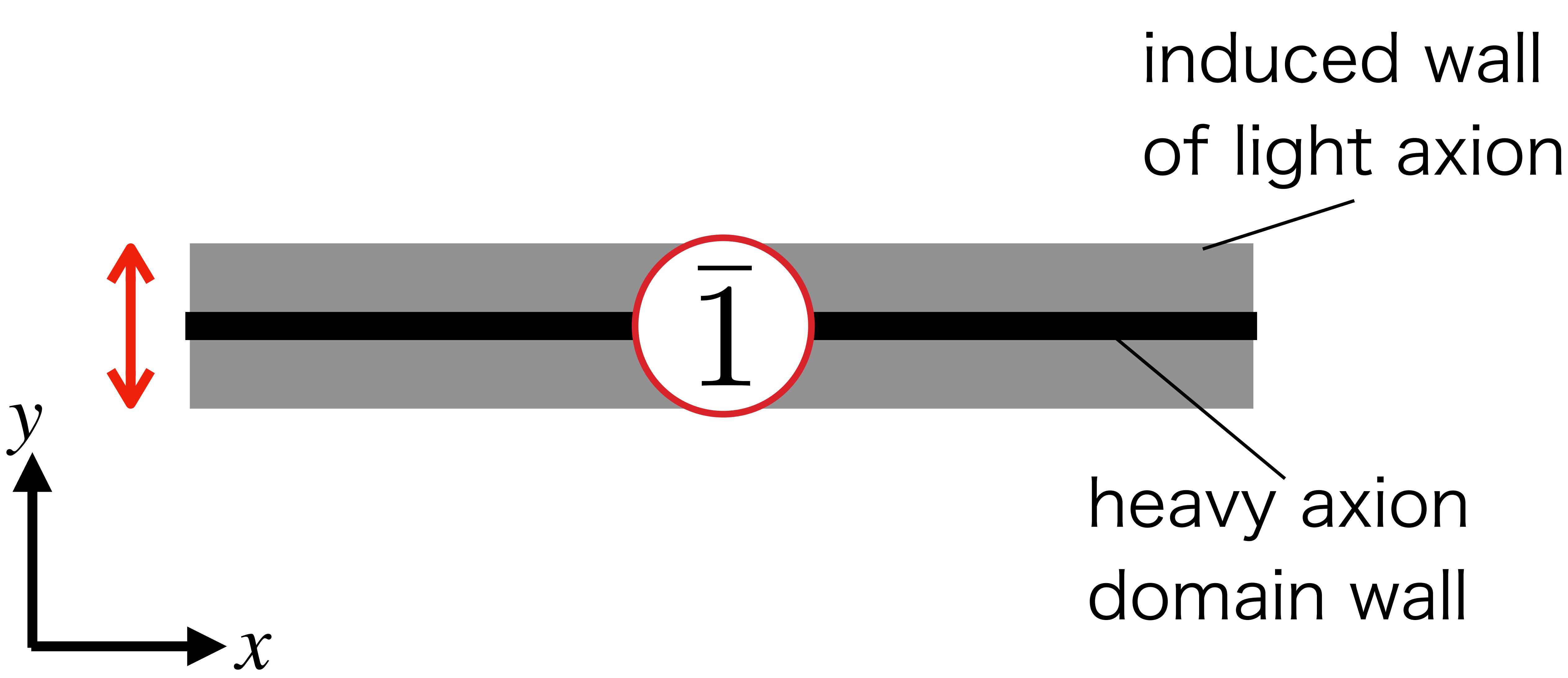
V_1 and V_2 can be minimized in every bulk and there is no bias.

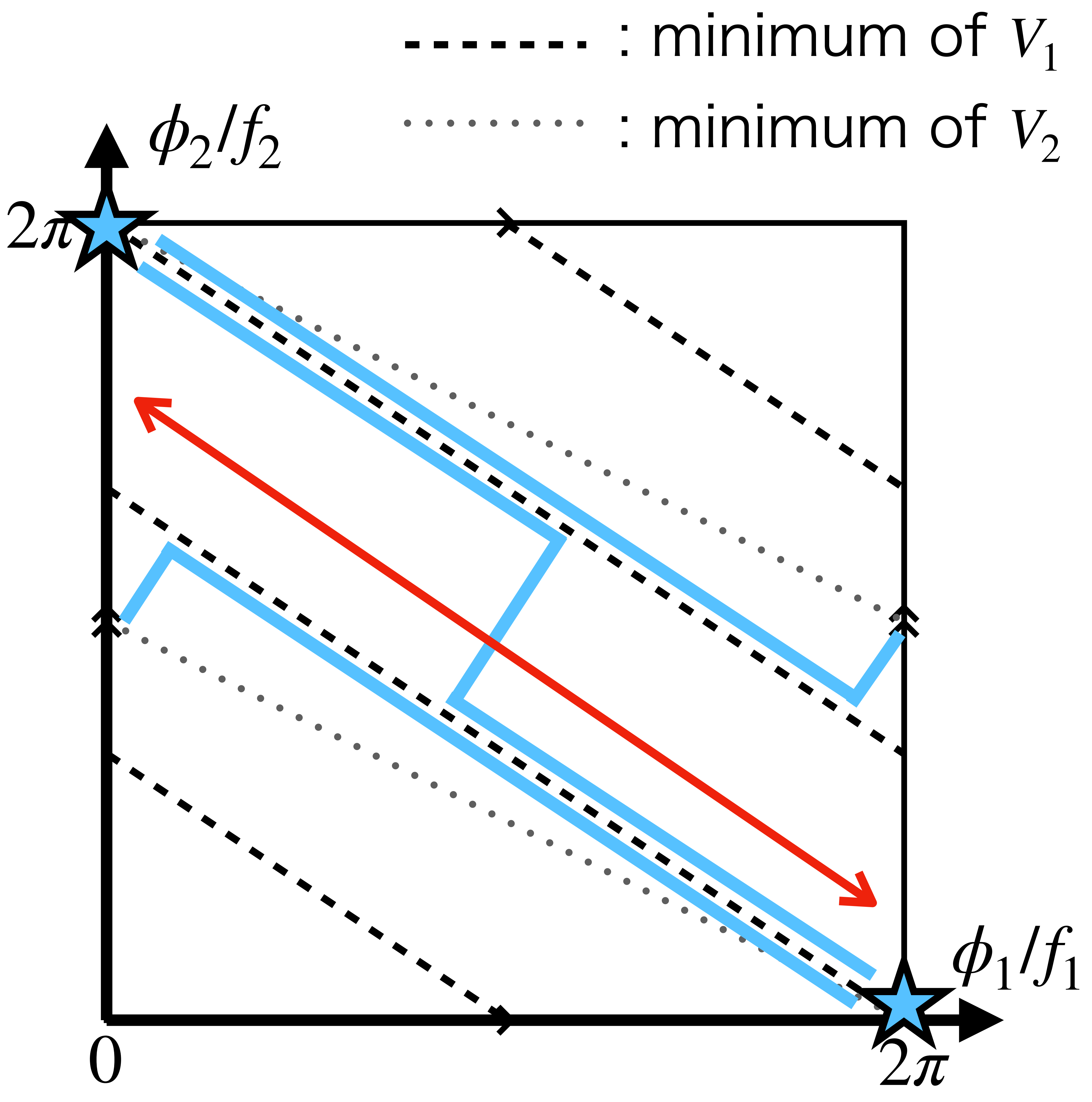
Induced domain wall

JL, Murai, Takahashi, Yin 2407.09478

A wall-like structure of the light axion is induced around the domain walls of the heavy axion.

“Induced domain wall”

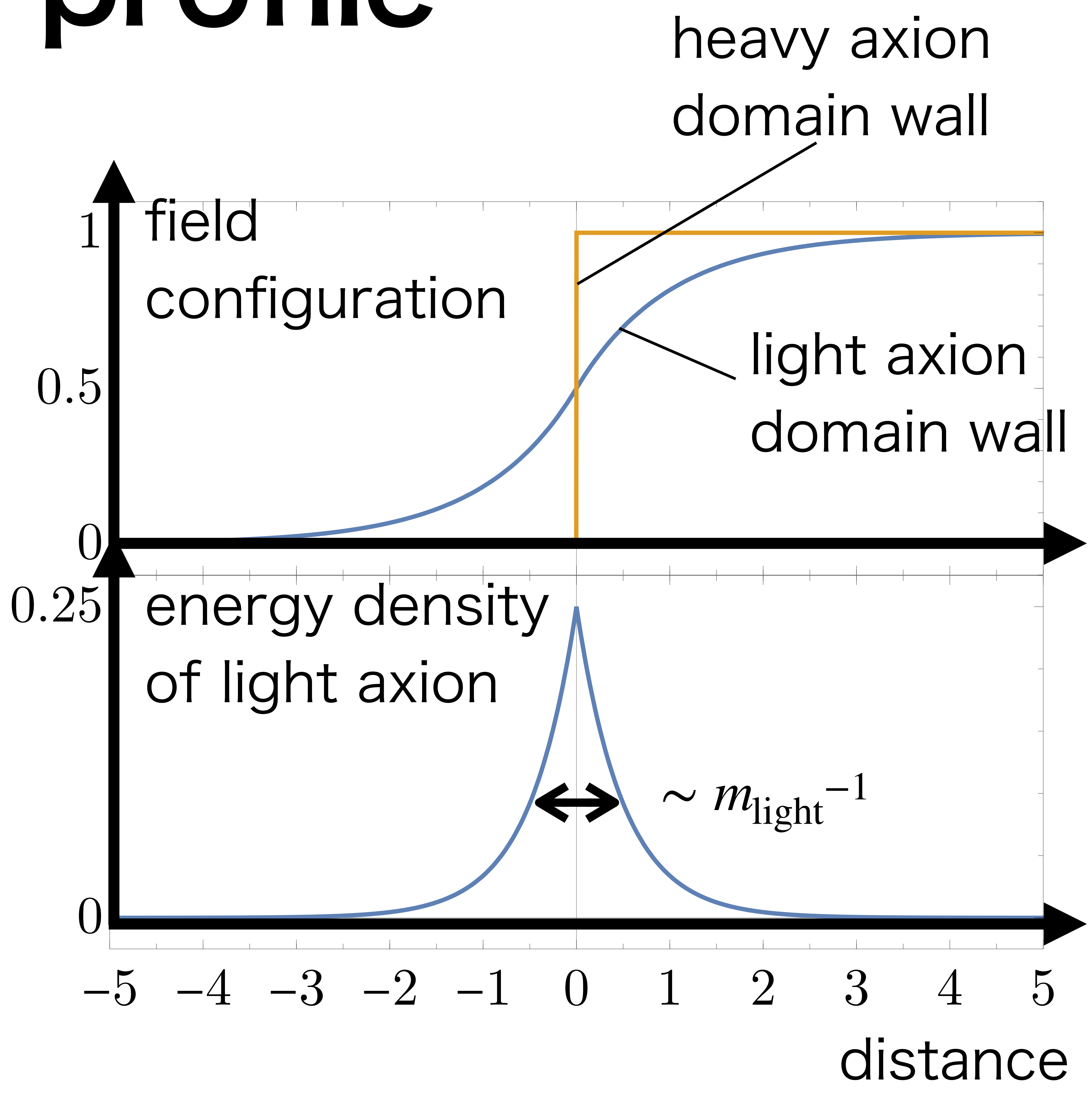




Induced domain wall profile

The width of induced domain walls, $\sim m_{\rm light}{}^{-1}$ ($\sim f/\Lambda'^2$), is much larger than that of the heavy axion domain walls, $\sim m_{\rm heavy}{}^{-1}$ ($\sim f/\Lambda^2$).

The tension of induced walls is approximately $\propto \Lambda'^4/m_{\rm light}$ ($\sim \Lambda'^2 f$), and thus the induced wall's evolution is dominated by heavy axion walls.



$(f \sim f_1 \sim f_2)$

population bias arise at wall formation

When the potential does not change in time,

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$$\frac{p_{\rm false}}{p_{\rm true}} = \exp\left(-\frac{\Delta V}{V_{\rm hill}}\right) ,$$

where $p_{\rm true(false)}$ is the probability for the field value to fall into the true(false) minimum.

This is exponentially suppressed and so is negligible compared to the effect of potential bias.