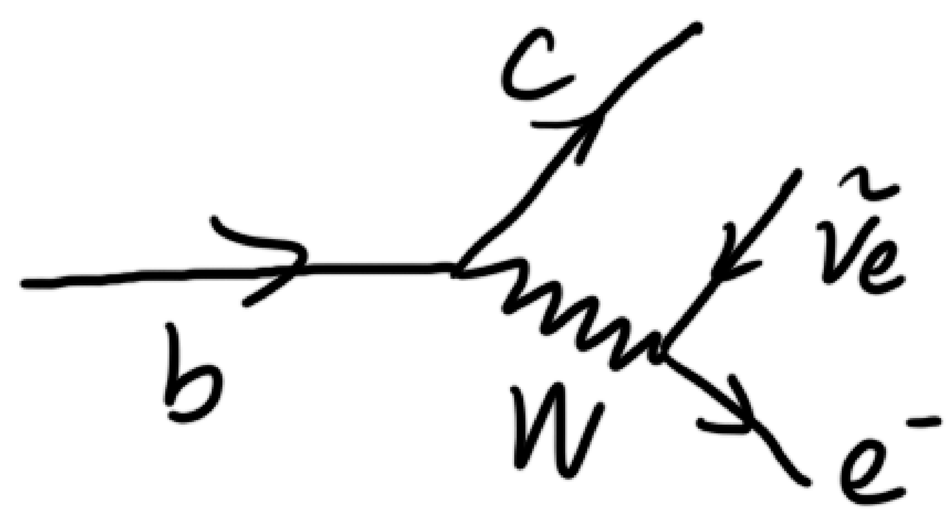
四费米子有效理论

$b \to c e^- \tilde{\nu}_e$



$$iM = \bar{u}(p_c)\frac{ig}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)u(p_b) \times \bar{u}(p_e)\frac{ig}{2\sqrt{2}}\gamma_\nu(1-\gamma_5)v(p_{\tilde{\nu}_e}) \times \frac{-ig^{\mu\nu}}{k^2-m_W^2}$$

$k = p_c - p_b$ $\qquad m_c \approx 1.5\,\text{GeV}$ $\qquad m_b = 4.8\,\text{GeV}$

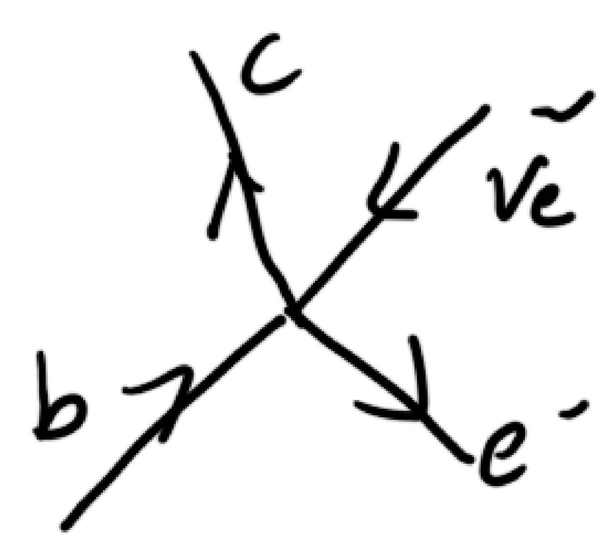
$m_W = 81.4\,\text{GeV}$

$k^2 \ll m_W^2$ $\qquad iM = \frac{-ig^2}{8m_W^2}\bar{u}(p_c)\gamma_\mu(1-\gamma_5)u(p_b)\bar{u}(p_e)\gamma^\mu(1-\gamma_5)v(p_{\tilde{\nu}})$

$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$ $\qquad \mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\bar{c}\gamma_\mu(1-\gamma_5)b\,\bar{e}\gamma^\mu(1-\gamma_5)\tilde{\nu}_e$

$O(G_F)$

$$\langle c e^- \tilde{\nu}_e | -i\int d^4x\, \mathcal{H}_{eff}(x) | b\rangle = iM \cdot (2\pi)^4\delta^{(4)}(p_b - p_c - p_e - p_{\tilde{\nu}})$$



$S = T[\exp(i\int dx\,\mathcal{L}_I(x))]$ $\qquad g^2$阶

$$= T\left\{\frac{ig}{2\sqrt{2}}\int d^4x\,\underline{\bar{c}\gamma_\mu(1-\gamma_5)b\,W^{+\mu}} \cdot \frac{ig}{2\sqrt{2}}\int d^4y\,\underline{\bar{e}\gamma_\nu(1-\gamma_5)\tilde{\nu}_e W^{-\nu}(y)}\right.$$

$$T[\overline{W^{+\mu}W^{-\nu}}] = \int\frac{d^4k}{(2\pi)^4}e^{-ik(x-y)} \cdot \frac{-ig^{\mu\nu}}{k^2-m_W^2} \to 1-\frac{k^2}{m_W^2}$$

$$k^2 \ll m_W^2 \qquad \approx \frac{ig^{\mu\nu}}{m_W^2}\delta^{(4)}(x-y)$$

$$S \approx \frac{-ig^2}{8m_W^2}T\left[\int d^4x\,\underline{\bar{c}\gamma_\mu(1-\gamma_5)b\,\bar{e}\gamma^\mu(1-\gamma_5)\tilde{\nu}_e(x)}\right]$$

$$\underline{O_1(x)}\ \underline{O_2(y)} = \sum_i \boxed{C_i(y-x)}\ \underline{O_i(x)}$$

复合算符

$\hookrightarrow$ Wilson系数 $\quad \dfrac{g^2}{8m_W^2}$

Path Integral

$$Z_W = \int [dW^+][dW^-] \exp[i\int d^4x\, \mathcal{L}_W]$$

$$\mathcal{L}_W = -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) + m_W^2 W_\mu^+ W^{-\mu} + \frac{g}{2\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})$$

$$Z_W = \int [dW^+][dW^-] \exp\{i\int d^4x\, d^4y\, W_\mu^+(x)\underline{[g^{\mu\nu}(\partial^2+m_W^2) - \partial^\mu\partial^\nu]} W_\nu^-(y) \times \delta^{(4)}(x-y) + i\int d^4x\, \frac{g}{2\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})\}$$

$$k^{\mu\nu} = g^{\mu\nu}(\partial^2+m_W^2) - \partial^\mu\partial^\nu$$

$$\Delta^{\mu\nu} = \frac{-1}{k^2 - m_W^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)\delta^{(4)}(x-y) \qquad \boxed{k^2 \ll m_W^2}$$

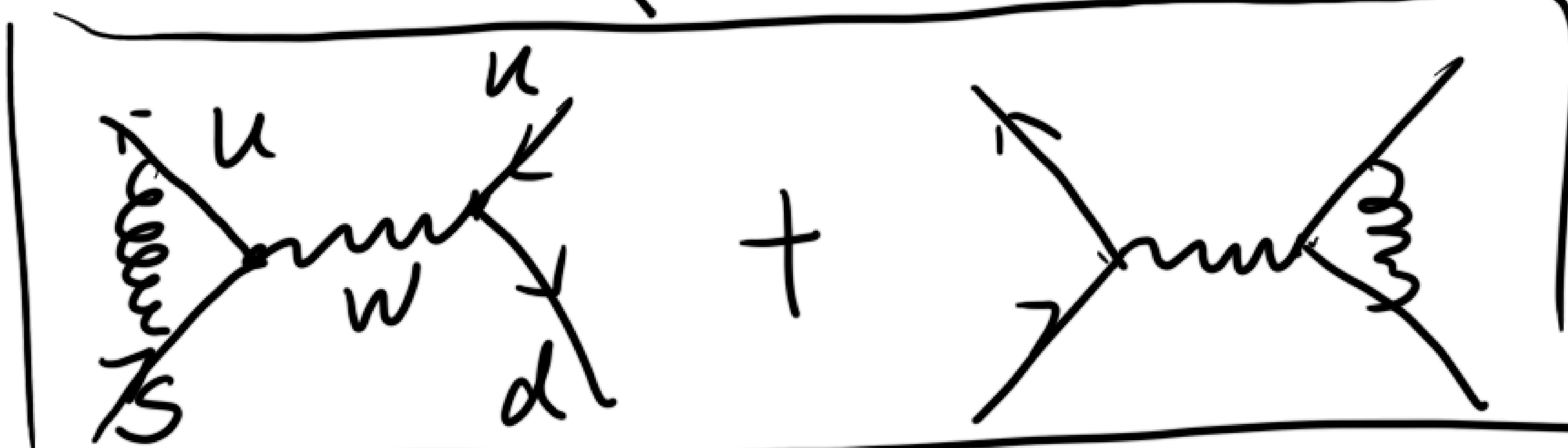
$$\Delta^{\mu\nu} = \frac{g_{\mu\nu}}{m_W^2}\delta^{(4)}(x-y)$$

$$Z_W \sim \exp[-i\frac{G_F}{\sqrt{2}}\int d^4x\ J_\mu^+(x) J^{-\mu}(x)]$$

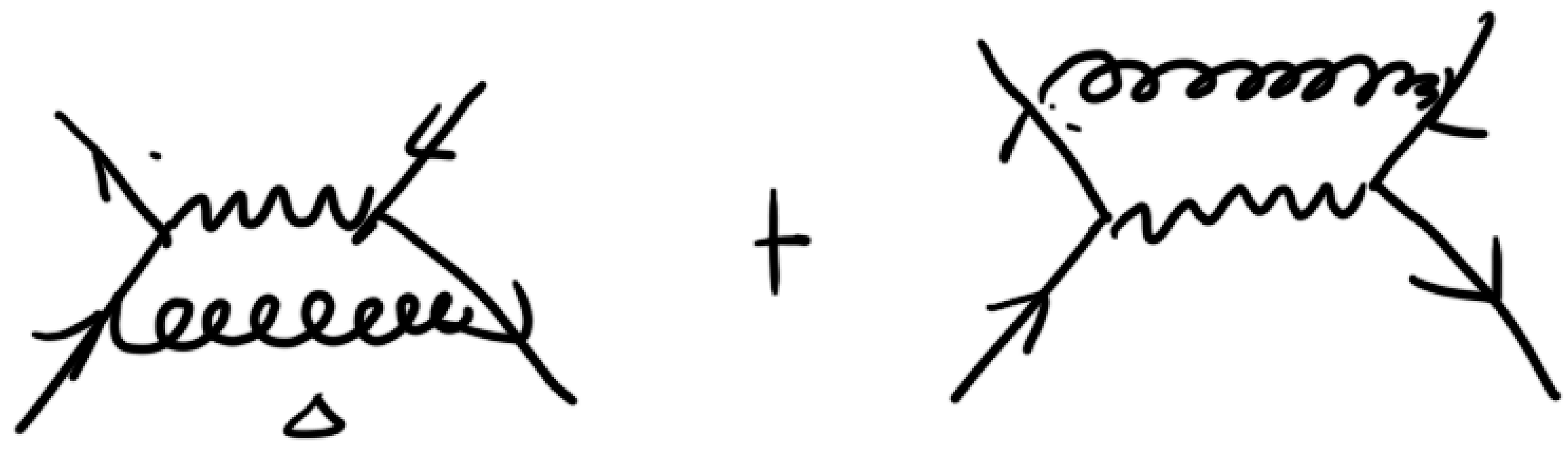
$$\bar{\psi}\gamma_\mu(1-\gamma_5)\psi$$

$s \to u \bar{u} d$

$$H_{eff} = \frac{G_F}{\sqrt{2}} (\bar{u} \gamma_\mu (1-\gamma_5) s)(\bar{d} \gamma^\mu (1-\gamma_5) u)$$



$+$ self-energy



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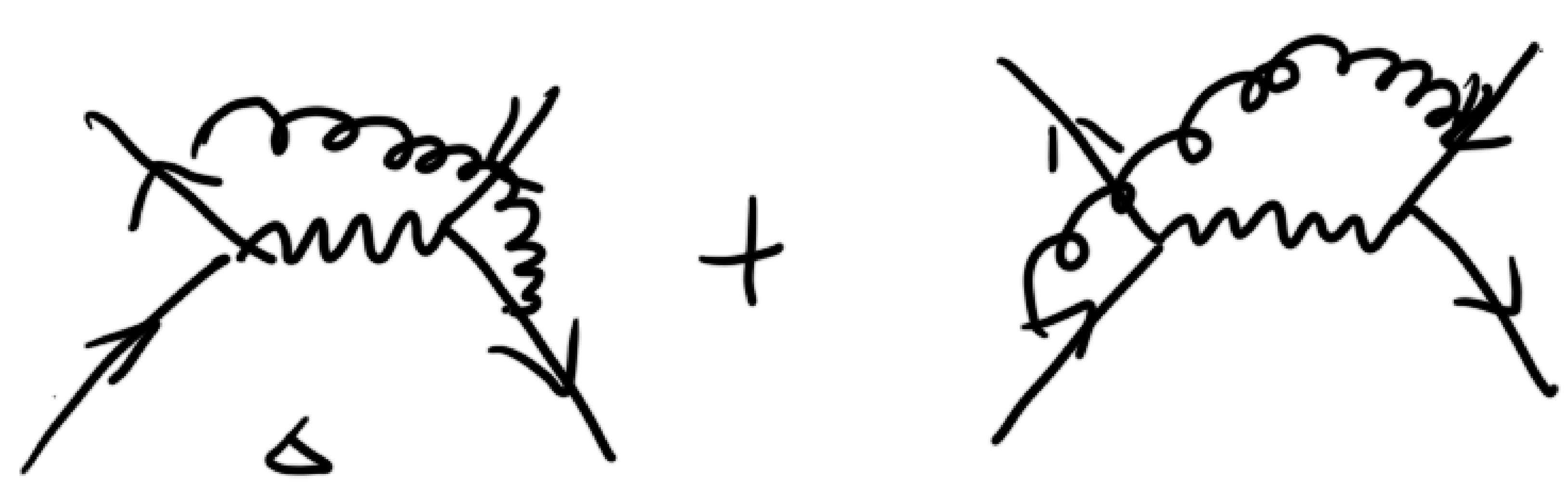

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只考虑发散项

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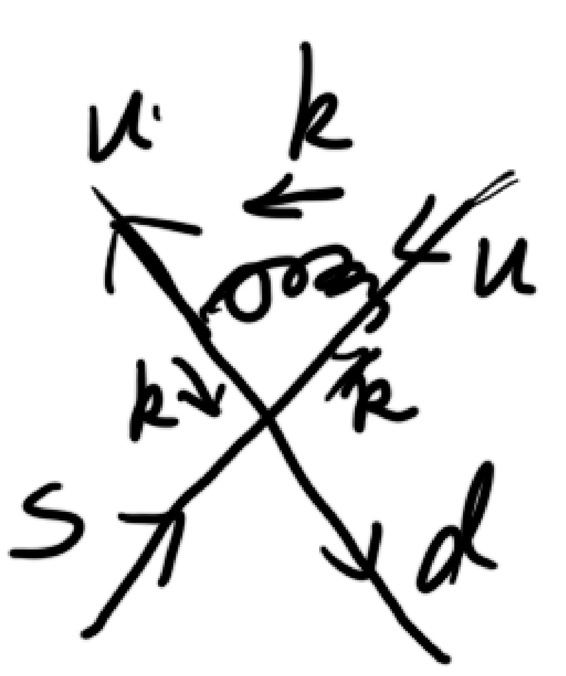
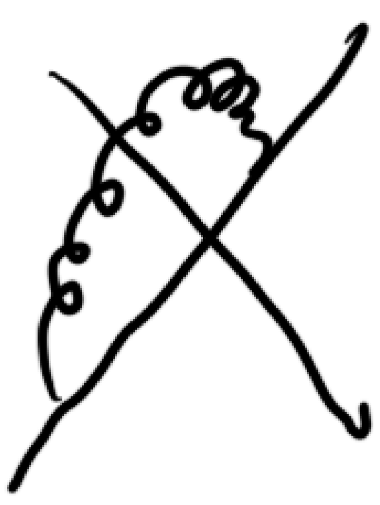


$$= \int \frac{d^d k}{(2\pi)^4} \left(\frac{ig}{2}\right)^2 \frac{-i}{k^2} \left(\bar{d} \gamma^\nu \underline{t^a} \frac{i \not{k}}{k^2} \gamma^\mu (1-\gamma_5) u\right)$$
$$\times \left(\bar{u} \gamma_\nu \underline{t^a} \frac{-i \not{k}}{k^2} \gamma_\mu (1-\gamma_5) s\right)$$

$$(t^a)_{ij} (t^a)_{kl} = \frac{1}{2} \left(\delta_{il} \delta_{kj} - \underline{\underline{\frac{1}{3} \delta_{ij} \delta_{kl}}}\right)$$

$$= -g^2 \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2} \Big[\frac{1}{2} \bar{u}\gamma^\mu(1-\gamma_5) u \bar{d}\gamma_\mu(1-\gamma_5)s - \frac{1}{6}\bar{d}\gamma^\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)s \Big]$$



$$= \int \frac{d^dk}{(2\pi)^d} \left(\frac{ig}{2\sqrt{2}}\right)^2 \frac{-i}{k^2} \left(\bar{d}\gamma^\nu t^a \frac{i\not{k}}{k^2}\gamma^\mu(1-\gamma_5)u\right) \times \left(\bar{u}\gamma_\mu t^a \frac{i\not{k}}{k^2}\gamma_\nu(1-\gamma_5)s\right)$$

$$= \frac{g^2}{4} \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2} \Big(\frac{1}{2} \bar{u}\gamma^\mu(1-\gamma_5) u \bar{d}\gamma_\mu(1-\gamma_5)s - \frac{1}{6}\bar{d}\gamma^\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)s \Big]$$

$O_2 = \bar{u}\gamma^\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)u$ $\qquad$ $O_1 = \bar{u}\gamma^\mu(1-\gamma_5)u\bar{d}\gamma_\mu(1-\gamma_5)s$

$\langle O_1 \rangle$ 矩阵元 $\qquad$ $\langle O_1^{tree}\rangle + \langle O_1^{1\text{-loop}}\rangle + c.t = $ 有限值

ct 复合算符重整化

$\langle O_1^{tree}\rangle$ 有限值

$\langle O_1^{1\text{-loop}}\rangle + ct = $ 有限值

$$\langle O_1^{1\text{-loop}}\rangle = -\frac{3}{4} g^2 \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2} \Big(\frac{1}{2} \bar{u}\gamma^\mu(1-\gamma_5) u \bar{d}\gamma_\mu(1-\gamma_5)s - \frac{1}{6}\bar{d}\gamma^\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)s \Big]$$

$$ct = \delta_{11} O_1 + \delta_{12} O_2$$

$$\delta_{11} = -\frac{1}{4} g^2 \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2} \qquad \delta_{12} = \frac{3}{4} g^2 \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}$$

$$\mathcal{O}_1^0 = \mathcal{O}_1 - \underline{\delta_{11} \mathcal{O}_1 - \delta_{12} \mathcal{O}_{12}}$$

$$\gamma = \mu \frac{\partial}{\partial \mu} \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

$$\mathcal{O}_2^0 = \mathcal{O}_2 - \delta_{21} \mathcal{O}_1 - \delta_{22} \mathcal{O}_{22}$$