Meson DA and BL equation.

① Defination.

② Tree level and 1 loop.

③ BL evolution equation.

④ Gegenbauer moments.

①. π介子DA: $\langle \pi(p)|u\gamma|0\rangle = i f_\pi \int$

$$\langle 0|\bar{u} W_c^+(z) \not{n}_+ \gamma_5 W_c d(0)|\pi(p)\rangle = -i f_\pi \int_0^1 dx\, e^{-ixPz}\, \Phi(x,\mu) \quad ①.$$

π 外态 $|\pi(p)\rangle = |\bar{u}(x_0 p)\, d(\bar{x}_0 p)\rangle \qquad \bar{x}_0 = 1-x$

$$D_\mu = \partial_\mu - i g_s T^a A_\mu^a$$

Wilson line $W_c(z) = P\exp\left(i g_s \int_0^\infty ds\, n_+\cdot A^a(z+sn_+) T^a\right)$.

$n_+^\mu = \frac{1}{\sqrt{2}}(1,0,0,1) \qquad n_-^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1) \qquad n_+\cdot n_- = 1. \qquad n_+\cdot n_+ = 0.$

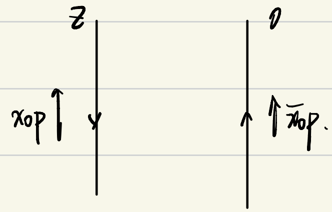
①做 Fourier transfrom

$$-i f_\pi \underset{\Delta}{\Phi}(x,\mu) = \int \frac{dz}{2\pi}\, e^{i\underset{\Delta}{x}Pz} \langle 0|\bar{u} W_c^+(z) \not{n}_+ \gamma_5 W_c d(0)|\bar{u}(x_0 p)\, d(\bar{x}_0 p)\rangle.$$

② ⓐ Tree level.

$$-i f_\pi \Phi^{(0)}(x,\mu) = \int \frac{dz^-}{2\pi} e^{ixpz} \langle 0|\, \bar u(z)\, \not n_+ \gamma_5\, d(0)\, |\, \bar u(x_0 p)\, d(\bar x_0 p) \rangle .$$

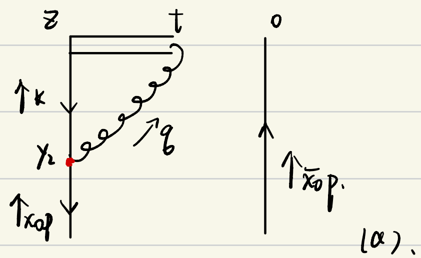
$$= \int \frac{dz^-}{2\pi} \underbrace{e^{ixpz} e^{-ix_0 p z}}_{\delta(p^+(x-x_0)) \;\to\; \frac{1}{p^+}\delta(x-x_0)} \bar u(x_0 p) \not n_+ \gamma_5\, d(\bar x_0 p)\, e^{i\bar x_0 p\cdot 0}$$



$$= \frac{1}{p^+} \bar u(x_0 p) \not n_+ \gamma_5\, d(\bar x_0 p)\, \delta(x-x_0).$$

$$\equiv \langle \Phi^0 \rangle.$$

ⓑ 1-Loop.



$x_0 p = k + q.$

$$-i\hat f_\pi \Phi^{(1,a)}(x,\mu) = \int \frac{dz^-}{2\pi} e^{ixpz} \langle 0|\, (ig_s \int d^4y_2\, \bar\psi \not A^a T^a \psi)\, \bar u(z)\, \overbrace{(-ig_s \int_0^\infty dt\, n_+ A^c(z+tn_+) T^c)}^{W_c^\dagger} (\not n_+ \gamma_5)\, d(0)\, |\, \bar u(x_0 p)\, d(\bar x_0 p) \rangle$$

$$= g_s^2 C_F \int \frac{dz^-}{2\pi} \int d^4y_2 \int_0^\infty dt \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{ixpz}\, \bar u(x_0 p)\, e^{-ix_0 p y_2}\, \gamma^\mu n_+^\nu \frac{-ig_{\mu\nu}}{q^2} e^{-i(z+tn_+-y_2)q} \cdot \frac{i\not k}{k^2} e^{-i(z-y_2)k}$$

$$\times (\not n_+ \gamma_5)\, d(\bar x_0 p).$$

$$\int_0^\infty dt\, e^{-itn_+ q - i\epsilon} \to \frac{-i}{q^+}$$

$$\not n_+ \not k + \not k \not n_+ = 2 n_+ \cdot k .$$

$$= g_s C_F \underbrace{\int_0^\infty dt\, e^{-itn_+ q}}_{=\,\frac{-i}{q^+}} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \int \frac{dz^-}{2\pi} \int d^4y_2\, e^{iz(xp-q-k)}\, e^{iy_2(-x_0 p+q+k)} \frac{1}{k^2} \frac{1}{q^2} \bar u(x_0 p) \underbrace{(\not n_+ \not k)}_{2n_+\cdot k} (\not n_+ \gamma_5)\, d(\bar x_0 p)$$

$$0 = \not n_+ \not n_+ c$$

$$\downarrow\; k = x_0 p - q$$

$$= g_s^2 C_F\, \delta(x-x_0) \int \frac{d^4q}{(2\pi)^4} \frac{-i}{q^+} \frac{1}{q^2} \frac{1}{(x_0 p - q)^2} [2n_+ \cdot (x_0 p - q)]\, \bar u(x_0 p) (\not n_+ \gamma_5)\, d(\bar x_0 p).$$

$\downarrow q^2=0$

$$= -i g_s C_F \,\delta(x-x_0)\, \underline{\bar{\pi}(x_0 p)\, \not{n}_+ \gamma_5\, d(\bar{x}_0 p)} \int \frac{dq^+ dq^- d^2q_\perp}{(2\pi)^4}\, \frac{1}{q^+}\, \frac{1}{2q^+q^- - q_\perp^2}\, \frac{2(x_0p^+ - q^+)}{-2(x_0p^+-q^+)q^- - q_\perp^2}.$$

$$= -\frac{\alpha_s C_F}{2\pi}\langle\Phi^0\rangle\, \frac{1}{\hat\varepsilon}\int_0^{x_0} dy\, \frac{x_0-y}{y}.$$

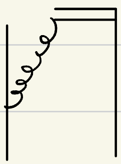
$\alpha_s = \frac{g_s^2}{4\pi}.$

$\underline{\int \frac{d^2q_\perp}{q_\perp^2} = -\frac{\pi}{\hat\varepsilon}}$, $\quad y=\frac{q^+}{p^+}.$

$\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}$

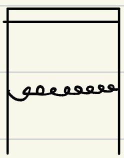
$= \frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}$

类似地:



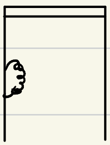
b

$$\langle\Phi^{(1,b)}\rangle = \frac{\alpha_s C_F}{2\pi}\langle\Phi^{(0)}\rangle\frac{1}{\hat\varepsilon}\frac{x}{(x_0-x)x_0}\theta(x_0-x).$$



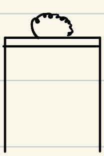
c

$$\langle\Phi^{(1,c)}\rangle = \frac{\alpha_s C_F}{2\pi}\langle\Phi^{(0)}\rangle\frac{1}{\hat\varepsilon}\left[\frac{1-x}{1-x_0}\theta(x-x_0)+\frac{x}{x_0}\theta(x_0-x)\right]$$



d

$$(\sqrt{Z_\xi}-1)\langle\Phi^0\rangle = -\frac{\alpha_s C_F}{4\pi}\frac{1}{\hat\varepsilon}\delta(x-x_0)\langle\Phi^{(0)}\rangle.$$



$=0$

$n_+\cdot n_+ = 0.$

Plus: $[f(x)]_+ = f(x) - \delta(x-1)\int_0^1 dy\, f(y)$

$$\int_0^1 dx\,[f(x)]_+ g(x) \equiv \int_0^1 dx\, f(x)\,[g(x)-g(x_0)].$$

a+b : $\frac{\alpha_s C_F}{2\pi}\langle\Phi^{(0)}\rangle\frac{1}{\hat\varepsilon}\left[\frac{x}{(x_0-x)x_0}\theta(x_0-x)\right]_+$

a+b 镜像 $x_0\to\bar{x}_0$

c+d $\frac{\alpha_s C_F}{2\pi}\langle\Phi^{(0)}\rangle\frac{1}{\hat\varepsilon}\left[\frac{\bar{x}}{\bar{x}_0}\theta(x-x_0)+\frac{x}{x_0}(x_0-x)\right]_+$

$$\frac{1}{\varepsilon_{UV}}-\frac{1}{\varepsilon_{IR}} \Leftarrow \int \frac{d^2 q_\perp}{q_\perp^2}$$

1-loop $\langle \Phi^{(1)} \rangle = \frac{\alpha_s^R C_F}{2\pi} \langle \Phi^0 \rangle \frac{1}{\hat{\varepsilon}} \int_0^1 dy \left[\frac{1-x}{1-y}\left(1+\frac{1}{x-y}\right)\theta(x-y)+\frac{x}{y}\left(1+\frac{1}{y-x}\right)\theta(y-x)\right]_+ \Phi^{(0)}(y,\mu)$.

其中 $\Phi^{(0)}(y,\mu)=\delta(x_0-y)$. α_s^R 满足 $\underline{\alpha_s^B = Z_\alpha \mu^{2\varepsilon} \alpha_s^R(\mu)}$

$\frac{1+2\varepsilon\ln\mu}{\varepsilon}$ $\quad \cancel{\frac{1}{\varepsilon_{UV}}} - \frac{1}{\varepsilon_{IR}} + \underset{\Delta}{\ln\mu_{UV}^2} - \ln\mu_{IR}^2$

③. B-L

$$\frac{d\alpha_s^B}{d\ln\mu}=0 .= 2\varepsilon Z_\alpha \mu^{2\varepsilon}\alpha_s^R + \frac{dZ_\alpha}{d\alpha_s^R}\frac{d\alpha_s^R}{d\ln\mu}\alpha_s^R\mu^{2\varepsilon} + Z_\alpha \frac{d\alpha_s^R}{d\ln\mu}\mu^{2\varepsilon}$$

$$\frac{d\alpha_s^R}{d\ln\mu} = -2\varepsilon\alpha_s^R + \cancel{\beta(\alpha_s^R)}$$

$Z = -\Phi^{(1)}$ $\quad \alpha_s(\mu)$

$$\frac{d\Phi(x,\mu)}{d\ln\mu} = \int_0^1 dy \underset{\Delta}{V_0(x,y)}\Phi(y,\mu) \quad ② \Rightarrow \quad P. \quad V_0 = \frac{d}{d\ln\mu} Z$$

$$V_0 = \frac{d}{d\ln\mu} Z = 2\varepsilon\Phi^{(1)} = \frac{\alpha_s}{\pi}\left[\frac{1-x}{1-y}\left(1+\frac{1}{x-y}\right)\theta(x-y)+\frac{x}{y}\left(1+\frac{1}{y-x}\right)\theta(y-x)\right]_+$$

④. Gegenbauer 展开.

$$\frac{d\Phi}{d\ln\mu} = \int_0^1 dy\, V(x,y)\Phi(y,\mu)$$

$C_n^{3/2}(2x-1)$.

$x\to 0,\ x\to 1$ 渐近 $x(1-x)$.

$C_n^{(\lambda)}(\xi) \quad \xi\in[-1,1] \qquad x=\frac{\xi+1}{2}$

$$\int_{-1}^1 d\xi\,(1-\xi^2)^{\lambda-\frac12} C_n^{(\lambda)}(\xi)\, C_m^{(\lambda)}(\xi) = 0 \quad (n\neq m) \quad (\text{正交性}) \quad 1-\xi^2 \propto x(1-x)$$

$\lambda=\frac{3}{2}$. $\qquad 1-\xi^2 \to 1-(2x-1)^2 \propto x(1-x)$

$$\underbrace{\int_0^1 dx\, C_n^{3/2}(2x-1)}\frac{d\Phi}{d\ln\mu} = \underline{\underline{\int_0^1 dx}} \int_0^1 dy\, \underline{\underline{C_n^{3/2}(2x-1)V(x,y)}}\phi(y,\mu)$$

$$\lambda_n C_n^{3/2}(2y-1).$$

$$\Downarrow$$

$$\int_0^1 dx\, C_n^{3/2}(2x-1)\,\Phi = a_n$$

$$\frac{d a_n}{d\ln\mu} = \lambda_n a_n \quad \checkmark \quad = \lambda_n \int_0^1 dy\, C_n^{3/2}(2y-1)\,\phi(y,\mu)$$

$$C_n^{3/2}(2x-1) = \lambda_n \int dy\, V_0(x,y)\, C_n^{3/2}(2y-1)$$

$$\checkmark \qquad \Delta \qquad \Delta \qquad \checkmark$$

$$\phi(x,\mu) = 6x(1-x)\left[1+\sum_n \underline{\underline{a_n}}\, C_n^{(3/2)}(2x-1)\right].$$

C_n高阶系数

附录：1 loop level result

A. Expand the wilson line 1 and interaction term to g_s-order

$$-i f_\pi \Phi^{(1.a)} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{-ixpz}$$

$$\langle u(x_0p)\bar{d}(\bar{x}_0p)|\bar{u}(z)\left(-ig_s\int d^4y_2\, \bar{\psi} A^a T^a \psi(y_2)\right)\left(ig_s\int dt\, n_+\cdot A^b(z+tn_+)T^b\right)(n_+\cdot\Gamma)\, d(0)|0\rangle$$

$$= \frac{1}{2} g_s^2 C_F \int \frac{dz^-}{2\pi}\int dy_2 \int_0^\infty dt \int\frac{d^4k}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4} e^{-ixpz}\, \bar{u}(xp) e^{ix_0p y_2}\gamma^\mu n_+^\nu \frac{i\not{k}}{k^2} e^{-i(y_2-z)k}(n_+\cdot\Gamma)\frac{-ig_{\mu\nu}}{q^2} e^{-i(y_2-z-tn_+)q}\, d(\bar{x}_0p)$$

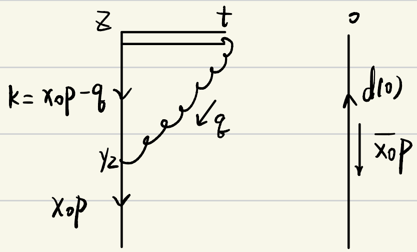
$$= \frac{g_s^2 C_F}{2}\int\frac{dz^-}{2\pi}\int dy_2\, e^{i(x_0p-q-k)y_2}\int\frac{d^4k}{(2\pi)^4}e^{i(k-xp+q)z}\int_0^\infty dt\, e^{itn_+q}\int\frac{d^4q}{(2\pi)^4}\frac{1}{k^2}\frac{1}{q^2}\bar u(x_0p)\not n_+\not k(n_+\cdot\Gamma)d(\bar x_0 p)$$

$$\left(\int_0^\infty dt\, e^{itn_+q} = \int_0^\infty dt\, e^{itq^+-i\varepsilon} = \frac{-i}{q^+}\right)$$

$$\begin{cases}\not n_+\not k+\not k\not n_+ = 2n_+\cdot k\\ n_+\cdot n_+=0\end{cases}$$

$$= g_s^2 C_F\frac{\delta(x-x_0)}{2p^+}\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^+}\frac{1}{(x_0p-q)^2}\frac{1}{q^2}\left[2n_+\cdot(x_0p-q)\right]\bar u(x_0p)(n_+\cdot\Gamma)d(\bar x_0p)$$

$$= -i g_s^2 C_F\frac{\delta(x-x_0)}{2p^+}\int\frac{dq^+dq^-d^2q_\perp}{(2\pi)^4}\frac{1}{q^+}\frac{1}{2q^+q^- - q_\perp^2}\frac{2(x_0p^+-q^+)}{-2(x_0p^+-q^+)q^- - q_\perp^2}\bar u(x_0p)(n_+\cdot\Gamma)d(\bar x_0p)$$



1-loop (a)

$$= -ig_s^2C_F\frac{\delta(x-x_0)}{2p^+}\bar u(x_0p)(n_+\cdot\Gamma)d(\bar x_0p)$$

$$\int\frac{dq^+dq^-d^2q_\perp}{(2\pi)^4}\frac{1}{2(q^+)^2}\frac{1}{q^- - \frac{q_\perp^2+i\varepsilon}{2q^+}}\frac{-1}{q^- - \frac{q_\perp^2+i\varepsilon}{2(q^+-x_0p^+)}}$$

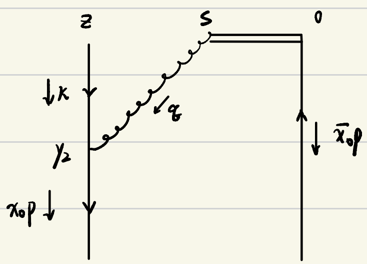
$$\left(0<\frac{q^+}{p^+}<x_0,\quad \frac{q^+}{p^+}\equiv y\right)$$

$$= -ig_s^2C_F\delta(x-x_0)\langle\Phi^{(0)}\rangle\int\frac{dq^+d^2q_\perp}{(2\pi)^4}\frac{1}{2(q^+)^2}\frac{-1}{\frac{q_\perp^2}{2q^+}-\frac{q_\perp^2}{2(q^+-x_0p^+)}}\quad(2\pi i)$$

$$= \frac{2\pi}{(2\pi)^4}g_s^2C_F\delta(x-x_0)\langle\Phi^{(0)}\rangle\int\frac{d^2q_\perp}{q_\perp^2}\int_0^{x_0p^+}dq^+\frac{x_0p^+-q^+}{x_0p^+q^+}$$

$$\boxed{\alpha_s=\frac{g_s^2}{4\pi}}$$

$$= -\frac{\alpha_sC_F}{2\pi}\delta(x-x_0)\langle\Phi^{(0)}\rangle\frac{1}{\hat\varepsilon}\int_0^{x_0}dy\,\frac{x_0-y}{x_0y}\qquad ④$$



1-loop (b)

B. Expand the Wilson line z and the interaction term to g_s-order

$$-if_\pi\Phi^{(1,b)} = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{-ixpz}$$

$$\langle u(x_0p)\bar d(\bar x_0p)|\left(-ig_s\int d^4y_2\,\bar\psi \not A^a T^a\psi\right)\bar u(z)(n_+\cdot\Gamma)\left(-ig_s\int_0^\infty ds\, n_+\cdot A^a(sn_+)T^a\right)d(0)|0\rangle$$

$$= \frac{-g_s^2C_F}{2}\int\frac{dz^-}{2\pi}\int d^4y_2\int_0^\infty ds\, e^{-ixpz}\,\bar u(x_0p)\,e^{ix_0py_2}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\gamma^\mu\frac{i\not k}{k^2}e^{-i(y_2-z)k}$$

$$(n_+\cdot\Gamma)\,n_+^\nu\,\frac{-ig_{\mu\nu}}{q^2}e^{-i(y_2-sn_+)q}\,d(\bar x_0p)$$

$$= \frac{-g_s^2C_F}{2}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\int\frac{dz^-}{2\pi}e^{-i(xp-k)z}\int d^4y_2\,e^{-i(k+q-x_0p)y_2}\int_0^\infty ds\,e^{isn_+q}\,\frac{1}{k^2}\frac{1}{q^2}\,\bar u(x_0p)\overbrace{\not n_+\not k}^{(2n_+\cdot k)}(n_+\cdot\Gamma)d(\bar x_0p)$$

$$= \frac{ig_s^2C_F}{2}\int\frac{d^4q}{(2\pi)^4}\delta((x-x_0)p^++q^+)\frac{-i}{q^+}\frac{1}{q^2}\frac{1}{(x_0p-q)^2}\bar u(x_0p)[2n_+\cdot(x_0p-q)](n_+\cdot\Gamma)d(\bar x_0p)$$

$$= i\frac{g_s^2C_F}{2}\int\frac{dq^+dq^-d^2q_\perp}{(2\pi)^4}\delta((x-x_0)p^++q^+)\frac{1}{q^+}\frac{1}{2q^+q^--q_\perp^2}\frac{2(x_0p^+-q^+)}{-2(x_0p^+-q^+)q^--q_\perp^2}\bar u(x_0p)(n_+\cdot\Gamma)d(\bar x_0p)$$

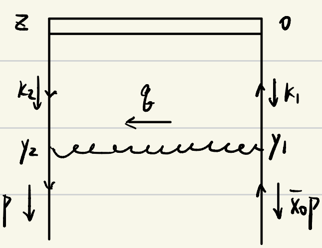
$$= i\frac{g_s^2C_F}{2}\bar u(x_0p)(n_+\cdot\Gamma)d(\bar x_0p)\int\frac{dq^-d^2q_\perp}{(2\pi)^4}\frac{1}{(x_0-x)p^+}\frac{1}{2(x_0-x)p^+q^--q_\perp^2}\frac{-1}{q^-+\frac{q_\perp^2}{2xp^+}}$$

$$= -ig_s^2C_F\frac{1}{2p^+}\bar u(x_0p)(n_+\cdot\Gamma)d(\bar x_0p)\int d^2q_\perp\int\frac{dq^-}{(2\pi)^4}\frac{1}{2(x_0-x)^2p^+}\frac{1}{q^--\frac{q_\perp^2+i\epsilon}{2(x_0-x)p^+}}\frac{1}{q^--\frac{-q_\perp^2+i\epsilon}{2xp^+}}\quad(x_0>x>0).$$

$$= -ig_s^2C_F\langle\Phi^{(0)}\rangle\int\frac{d^2q_\perp}{q_\perp^2}\frac{1}{(2\pi)^4}\theta(x_0-x)\frac{1}{2(x_0-x)^2}\frac{1}{\frac{1}{2(x_0-x)}+\frac{1}{2x}}(-2\pi i)$$

$$= \frac{-g_s^2C_F}{(2\pi)^3}\langle\Phi^{(0)}\rangle\frac{-\pi}{\hat\epsilon}\frac{x}{(x_0-x)x_0}\theta(x_0-x)$$

$$= \frac{\alpha_sC_F}{2\pi}\langle\Phi^{(0)}\rangle\frac{1}{\hat\epsilon}\frac{x}{(x_0-x)x_0}\theta(x_0-x) \qquad (5)$$



1-loop. (c)

C. Expand the interaction term to g_s^2-order

$$-if_\pi \Phi^{(1,C)} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{-ixpz}$$

$$\langle u(x_0p)\,\bar d(\bar x_0 p)|\,(-ig_s\int d^4y_2\, \bar\psi_u A^a T^a \psi_u)\,\bar u(z)\,(n_+\cdot\Gamma)\,(-ig_s\int d^4y_1\, \bar\psi_d A^b T^b\psi_d)\, d(0)\,|0\rangle$$

$$=-\frac{1}{2} g_s^2 C_F\int\frac{dz^-}{2\pi}e^{-ixpz}\int d^4y_2\int d^4y_1\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}$$

$$\bar u(x_0p)\,e^{ix_0py_2}\,\gamma^\mu\frac{i\not k_2}{k_2^2}\,e^{-i(y_2-z)k_2}\,(n_+\cdot\Gamma)\,\frac{i\not k_1}{k_1^2}\,e^{-iy_1k_1}\,\gamma^\nu\,\frac{-ig_{\mu\nu}}{q^2}\,e^{-i(y_2-y_1)q}\,d(\bar x_0p)\,e^{i\bar x_0py_1}$$

$k_2 = x_0p-q$ $\quad$ $k_1=\bar x_0p+q$

$$=\frac{-ig_s^2C_F}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\int\frac{dz^-}{2\pi}e^{-i(xp-k_2)z}\int d^4y_2\, e^{i(x_0p-k_2-q)y_2}\int d^4y_1\, e^{i(\bar x_0p+q-k_1)y_1}\,\frac{1}{k_2^2}\,\frac{1}{k_1^2}\,\frac{1}{q^2}$$

$$\bar u(x_0p)\,\gamma^\mu\not k_2\,(n_+\cdot\Gamma)\not k_1\,\gamma_\mu\, d(\bar x_0p)$$

$$=\frac{-ig_s^2C_F}{2}\int\frac{d^4q}{(2\pi)^4}\,\delta(xp^+-x_0p^++q^+)\,\frac{1}{(x_0p-q)^2}\,\frac{1}{(\bar x_0p+q)^2}\,\frac{1}{q^2}\,\bar u(x_0p)\,\gamma^\mu(x_0\not p-\not q)\,(n_+\cdot\Gamma)\,(\bar x_0\not p+\not q)\,\gamma_\mu\, d(\bar x_0p)$$

其中旋量结构为：

$$\bar u(x_0p)\,\gamma^\mu\not k_2\,(n_+\cdot\Gamma)\not k_1\,\gamma_\mu\, d(\bar x_0p)$$

$$=\bar u(x_0p)\,[-2\not k_1(n_+\Gamma)\not k_2+(4-D)\not k_2(n_+\Gamma)\not k_1]\,d(\bar x_0p)$$

$$\begin{cases}\not p\, d=0\\ \bar u\not p=0\end{cases}$$

$$=\bar u(x_0p)\,[-2\not q\,(n_+\Gamma)(-\not q)+(4-D)(-\not q)(n_+\Gamma)\not q\,]\,d(\bar x_0p)$$

$$=(D-2)\,\bar u(x_0p)\not q\,(n_+\Gamma)\not q\, d(\bar x_0p)$$

$$=-(D-2)\,q_\perp^2\,\bar u(x_0p)\,(n_+\cdot\Gamma)\,d(\bar x_0p)$$

于是上式可以写成：

$$(D-2)\frac{ig_s^2C_F}{2}\,\bar u(x_0p)\,(n_+\cdot\Gamma)\,d(\bar x_0p)\int\frac{d^4q}{(2\pi)^4}\,\delta((x-x_0)p^++q^+)\,\frac{1}{(x_0p-q)^2}\,\frac{1}{(\bar x_0p+q)^2}\,\frac{1}{q^2}\,q_\perp^2$$

$$=\frac{ig_s^2C_F}{2}\,\bar u(x_0p)\,(n_+\cdot\Gamma)\,d(\bar x_0p)\int\frac{dq^+dq^-d^{D-2}q_\perp}{(2\pi)^D}(D-2)\,\delta((x-x_0)p^++q^+)\,\frac{q_\perp^2}{2q^+q^--q_\perp^2}\,\frac{1}{2(q^+-x_0p^+)q^--q_\perp^2}\,\frac{1}{2(q^++\bar x_0p^+)q^--q_\perp^2}$$

$$=\frac{ig_s^2C_F}{2}\,\bar u(x_0p)\,(n_+\cdot\Gamma)\,d(\bar x_0p)\int\frac{dq^-d^{D-2}q_\perp}{(2\pi)^D}(D-2)\,\frac{q_\perp^2}{2(x_0-x)p^+q^--q_\perp^2+i\varepsilon}\,\frac{1}{-xp^+q^--q_\perp^2+i\varepsilon}\,\frac{1}{2\bar xp^+q^--q_\perp^2+i\varepsilon}$$

There are 3 poles : $q_1^- = \dfrac{q_\perp^2 + i\varepsilon}{2(x_0 - x)p^+}$ $\quad q_2^- = \dfrac{-q_\perp^2 - i\varepsilon}{2xp^+}$ $\quad q_3^- = \dfrac{q_\perp^2 + i\varepsilon}{2\bar{x}p^+}$.

① $\theta(x - x_0)$: 代入 q_3^-

$$\int dq^- [\cdots] = \frac{1-x}{1-x_0} (\pi i)$$

② $\theta(x_0 - x)$: 代入 q_2^-

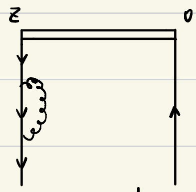
$$\int dq^- [\cdots] = \frac{x}{x_0} (\pi i)$$

$$-if_\pi \Phi^{(1,c)} = -\pi g_s^2 C_F \frac{\bar{u}(x_0 p)(n_+ \cdot \Gamma) d(\bar{x}_0 p)}{2p^+} \int \frac{d^{D-2} q_\perp}{q_\perp^2} (D-2) \left[\frac{1-x}{1-x_0} \theta(x - x_0) + \frac{x}{x_0} \theta(x_0 - x) \right]$$

$D \to 4$

$$= \frac{\alpha_s C_F}{2\pi} \langle \Phi^{(0)} \rangle \frac{1}{\hat{\varepsilon}} \left[\frac{1-x}{1-x_0} \theta(x - x_0) + \frac{x}{x_0} \theta(x_0 - x) \right] \quad ⑥$$

D. Self-Energy of Fermion line

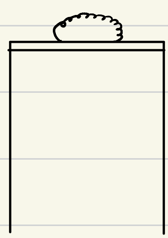


1-loop (d)

$$\Phi^{(1,d)} = \left[(\sqrt{Z_2})^2 - 1 \right] \Phi^{(0)}$$

$$= -\frac{\alpha_s C_F}{4\pi} \frac{1}{\hat{\varepsilon}} \delta(x - x_0) \langle \Phi^{(0)} \rangle \quad ⑦$$

E. Self-Energy of Wilson line


1-loop (e)

There is term of $n_+ \cdot n_+ = 0$, then its result is 0.

Finally, we have $\Phi^{(1,a)} = -\dfrac{\alpha_s C_F}{2\pi} \delta(x - x_0) \langle \Phi^{(0)} \rangle \dfrac{1}{\hat{\varepsilon}} \displaystyle\int_0^{x_0} dy \, \frac{x_0 - y}{x_0 y}$

$$\Phi^{(1,b)} = \frac{\alpha_s C_F}{2\pi} \langle \Phi^{(0)} \rangle \frac{1}{\hat{\varepsilon}} \frac{x}{(x_0 - x) x_0} \theta(x_0 - x)$$

$$\Phi^{(1,c)} = \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{1-x}{1-x_0}\theta(x-x_0) + \frac{x}{x_0}\theta(x_0-x)\right]$$

$$\Phi^{(1,d)} = -\frac{\alpha_s C_F}{4\pi}\frac{1}{\hat{\varepsilon}}\delta(x-x_0)\langle \Phi^{(0)}\rangle$$

①

$$\Phi^{(1,a)} + \Phi^{(1,b)} = \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{x}{(x_0-x)x_0}\theta(x_0-x) - \delta(x-x_0)\int_0^{x_0} dy\,\frac{x_0-y}{x_0 y}\right]$$

换元 $y \to x_0-y$.

$$= \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{x}{(x_0-x)x_0}\theta(x_0-x) - \delta(x-x_0)\int_0^1 dy\,\frac{y}{(x_0-y)x_0}\theta(x_0-y)\right]$$

$$= \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{x}{(x_0-x)x_0}\theta(x_0-x)\right]_+$$

②

与(a)、(b)图对称的图的结果为:

$$\Phi^{(1,a')} + \Phi^{(1,b')} = \frac{\alpha_s C_F}{2\pi}\langle \Phi^{0}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{\bar{x}}{(\bar{x}_0-\bar{x})\bar{x}_0}\theta(\bar{x}_0-\bar{x})\right]_+$$

$$= \frac{\alpha_s C_F}{2\pi}\langle \Phi^{0}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{1-x}{(x-x_0)(1-x_0)}\theta(x-x_0)\right]_+$$

③ $$\Phi^{(1,c)} + \Phi^{(1,d)} = \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{1-x}{1-x_0}\theta(x-x_0) + \frac{x}{x_0}\theta(x_0-x) - \frac{1}{2}\delta(x-x_0)\right]$$

可以证明: $\int_0^1 dy\left[\frac{1-x}{1-x_0}\theta(x-x_0) + \frac{x}{x_0}\theta(x_0-x)\right] = \frac{1}{2}$

因此上式 $= \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\left[\frac{1-x}{1-x_0}\theta(x-x_0) + \frac{x}{x_0}\theta(x_0-x)\right]_+$

将 1-loop 结果相加:

$$\langle \Phi^{(1)}\rangle = \frac{\alpha_s C_F}{2\pi}\langle \Phi^{(0)}\rangle \frac{1}{\hat{\varepsilon}}\int_0^1 dy\, V_0(x,y)\,\Phi^{(0)}(y,\mu)$$

其中 $\Phi^{(0)}(y,\mu) = \delta(x_0-y)$.

$$V_0(x,y) = \left[\frac{1-x}{1-y}\left(1+\frac{1}{x-y}\right)\theta(x-y) + \frac{x}{y}\left(1+\frac{1}{y-x}\right)\theta(y-x)\right]_+$$