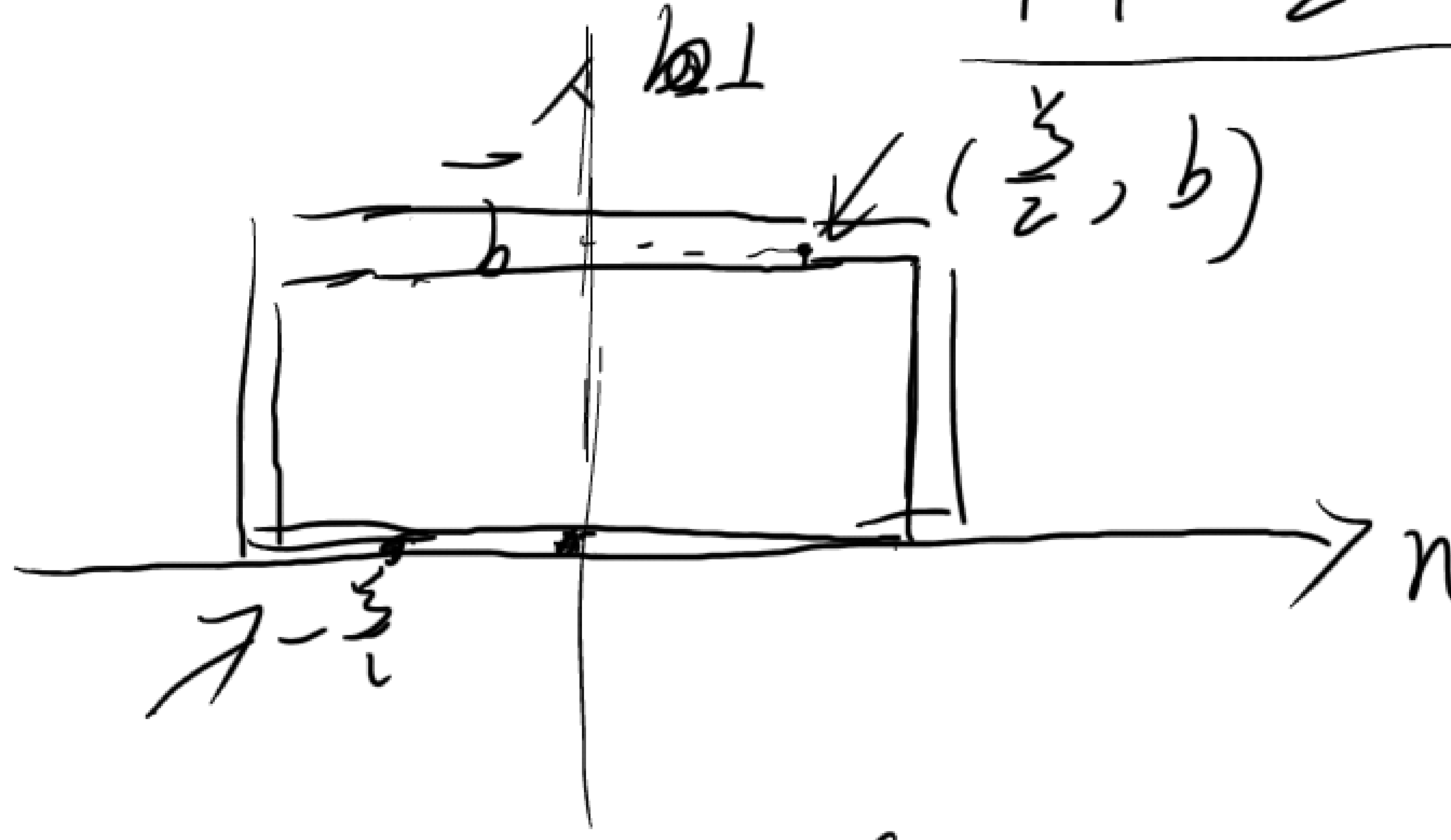
TMDWF 遍举B介子

IR UV rapidity divergence JHEP 09 (2022) 046

TMDWF 定义

$$\psi(x,b_\perp,\mu) = -\frac{1}{if_\pi P^+}\int\frac{d(\xi P^+)}{2\pi}e^{-i(x-\frac{1}{2})\xi P^+}$$

$$\times \langle 0|\bar\psi(\tfrac{\xi}{2}\hat n+\vec b)\gamma^+\gamma^5\psi(-\tfrac{\xi}{2}\hat n)|P\rangle$$



$\downarrow$ wilson line

rapidity divergence

$$W = Pe^{ig\int_0^{\pm\infty}ds\cdot n A(\xi+s\hat n)}$$

$$\Downarrow \delta^-$$

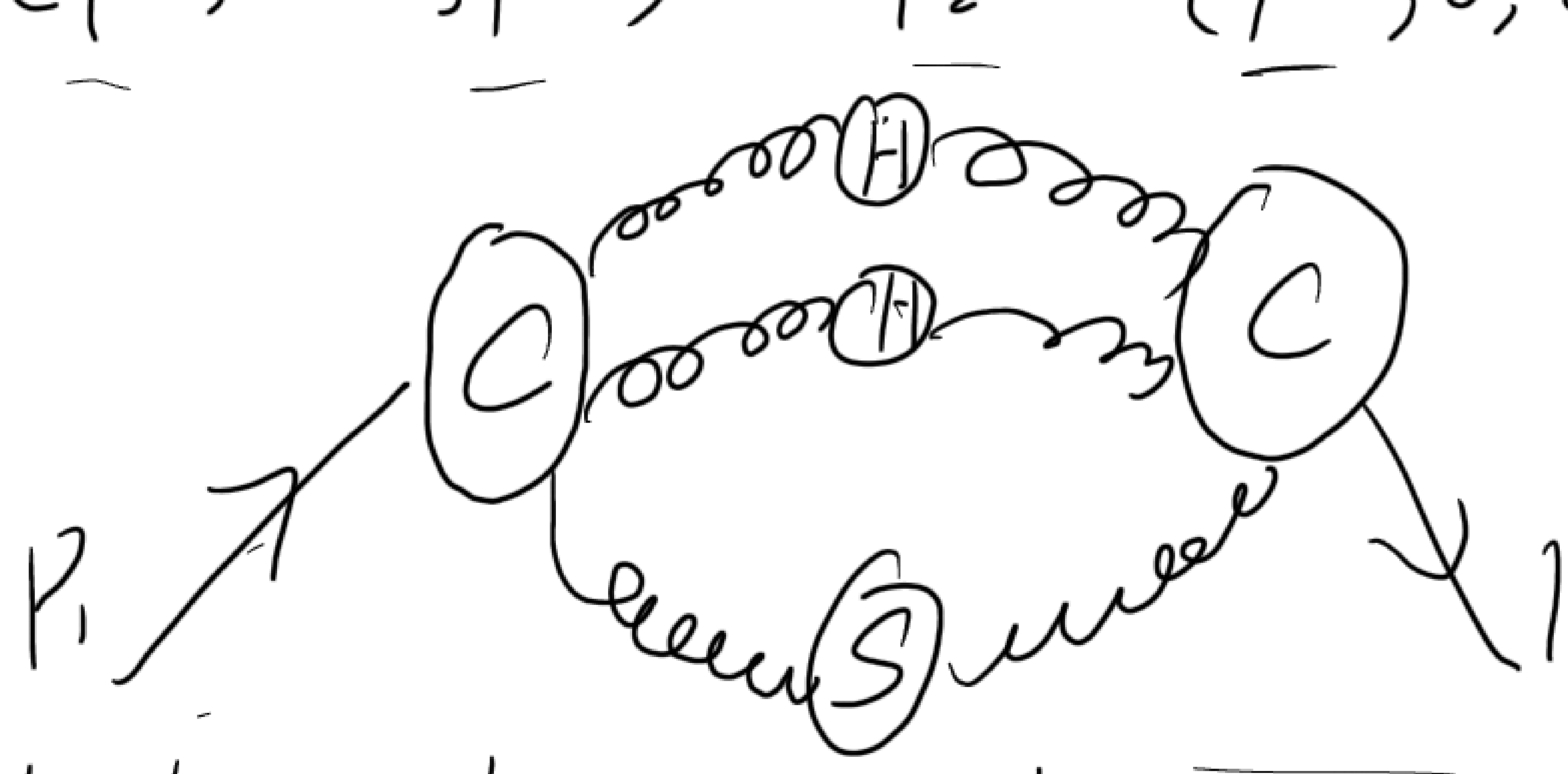
$$W = Pe^{ig\int_0^{\pm\infty}ds\cdot nA(\xi+s\hat n)-\frac{\delta^-}{2}|s|}$$

从形状因子出发

$$F(b_\perp,P_1,P_2,\mu) = \frac{\langle P_2|(\bar\psi_a\Gamma\psi_b)(b_\perp)(\bar\psi_c\Gamma'\psi_d)(0)|P_1\rangle}{f_\pi^2 P_1\cdot P_2}$$

Λ_{QCD}

$P_1^\mu = (P^z,0,0,P^z)$ $\quad P_2^\mu = (P^z,0,0,-P^z)$



Hard $q\sim(1,1,1)P^z$

Collinear $q\sim(P^z,\Lambda,\frac{\Lambda^2}{P^z})$

Soft $q\sim(\Lambda,\Lambda,\Lambda)$

Tree level ; 1-loop level

$$F(b_\perp,P_1,P_2,\mu) = \int dx_1 dx_2\, H_F(Q^2,\bar Q^2,\mu^2)\times\left[\frac{\psi^{\mp}_{\bar qq}(x_2,b_\perp,\mu,\delta'^+)}{\sqrt{S^\pm(b_\perp,\mu,\delta'^+,\delta^-)}}\right]^+$$

$$\times\left[\frac{\psi^{\pm}_{\bar qq}(x_1,b_\perp,\mu,\delta'^-)}{\sqrt{S^\pm(b_\perp,\mu,\delta^+,\delta'^-)}}\right]\times\frac{S^\pm(b_\perp,\mu,\delta^+,\delta^-)}{\sqrt{S^\pm(b_\perp,\mu,\delta'^+,\delta^-)S^\pm(b_\perp,\mu,\delta^+,\delta'^-)}}$$

four quark form factor $\quad |P\rangle$ hadron $\xrightarrow{b_\perp}$ parton state

$$F(b_\perp, P_1, P_2, \mu) = \frac{\langle \bar q_a(\bar x_2 P_2) q_d(x_2 P_2)|(\bar\psi_a \Gamma \psi_b)(b)(\bar\psi_c \Gamma \psi_d)(0)|q_b(x_1 P_1)\bar q_c(\bar x_1 P_1)\rangle}{4 P_1\cdot P_2}$$

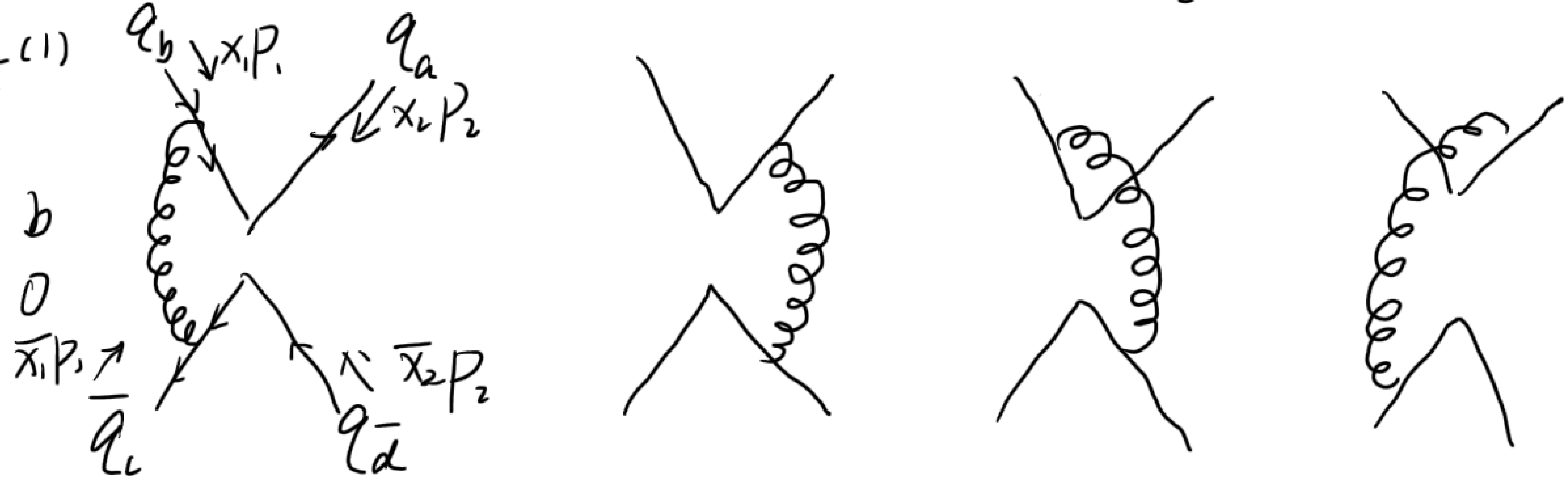
$$\langle 0|\bar\psi_c \gamma^\mu\gamma^5 \psi_b|q_b(x_1P_1)\bar q_c(\bar x_1 P_1)\rangle\Big|_{tree} = 2P_1^\mu$$

$$\langle \bar q_d(\bar x_2 P) q_a(x_2 P)|\bar\psi_a \gamma_\mu\gamma^5\psi_d|0\rangle\Big|_{tree} = 2P_{2\mu}$$

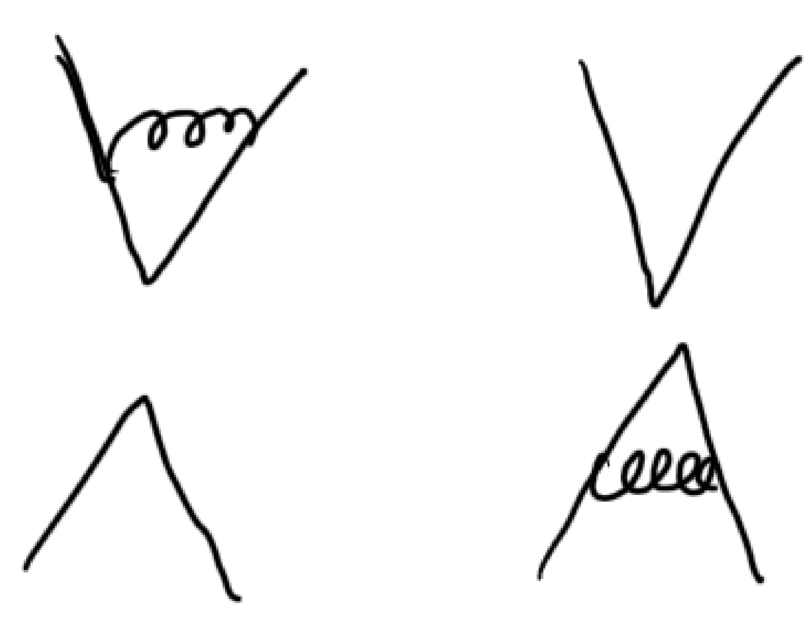
$$F^{(0)} = -\frac{1}{4N_c P_1\cdot P_2}\bar u_a(x_2P_2)e^{ix_2P_2\cdot b}\Gamma u_b(x_1P_1)e^{-ix_1P_1 b}\,\bar v_c(\bar x_1 P_1)\Gamma' v_d(\bar x_2 P_2)$$

$$= \frac{1}{16 N_c P_1\cdot P_2}\mathrm{tr}[\gamma_5 \not{P}_2 \Gamma \not{P}_1 \gamma_5 \Gamma] = \begin{cases} \frac{1}{4N_c} & \Gamma = I \\ -\frac{1}{4N_c} & \Gamma = \gamma_5, \gamma_\perp \text{ or } \gamma_\perp\gamma_5 \end{cases}$$

$F^{(1)}$



$0 \to b$



$e^{iq\cdot b}$ $\qquad L_b = \ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}$

UV

$\frac{1}{b_\perp}$

结论

① 对于不同Γ结果不同:

$\Gamma = I, \gamma_5$

$$F^{(1)}(b_\perp, P_1, P_2, \mu) = F^{(0)}\Big\{1 - \frac{\alpha_s C_F}{2\pi}\Big[L_b^2 + L_b\Big(\ln\frac{4Q^2\bar Q^2}{\mu^2} - 3\Big) + \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2\bar Q^2}{\mu^2} + 1 - \frac{3}{\varepsilon_{UV}}\Big]\Big\}$$

$\Gamma = \gamma_\perp, \gamma_\perp\gamma_5$

$$F^{(1)}(b_\perp, P_1, P_2, \mu) = F^{(0)}\Big\{1 - \frac{\alpha_s C_F}{2\pi}\Big[L_b^2 + L_b\Big(\ln\frac{4Q^2\bar Q^2}{\mu^2} - 3\Big) - \frac{3}{2}\ln\frac{4Q^2\bar Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2\bar Q^2}{\mu^2} + 7\Big]\Big\}$$

② $Z_s = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\varepsilon_{UV}}$ 消除 $\Gamma = I, \gamma_5$ 的UV发散

② IR 在所有图相加后抵消。

TMDWF、Soft function, hard kernel

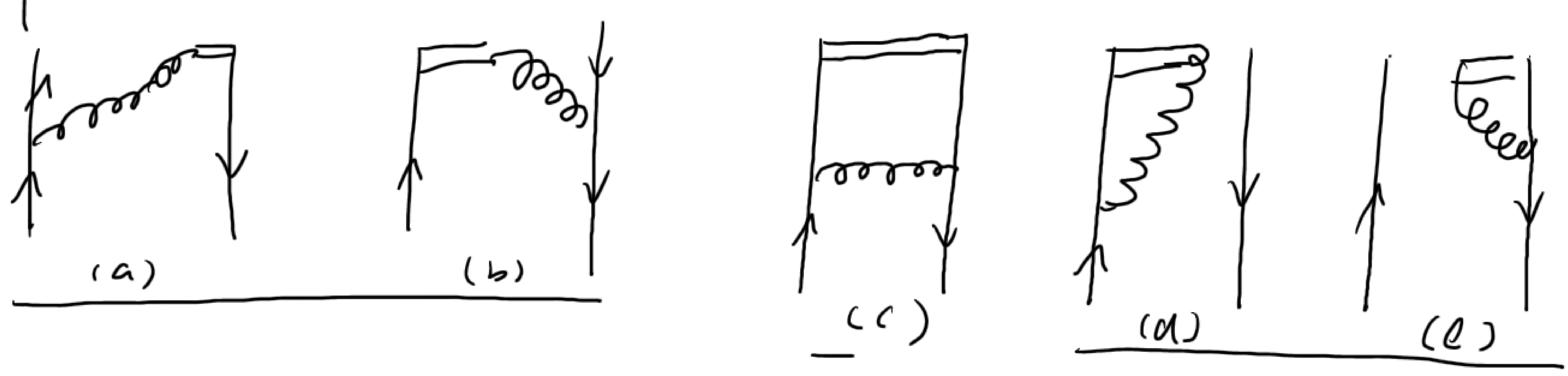
A. TMDWF

$$\psi^{\pm}_{\bar q q}(x, b_\perp, \mu, \delta^-) = \frac{1}{2P^+}\int \frac{d(\xi P^+)}{2\pi} e^{-i(x-\frac{1}{2})P^+\xi}$$

$$\times \langle 0|\bar\psi^{\pm}(\tfrac{\xi}{2}\hat n + \vec b)\gamma^+\gamma^5\psi^{\pm}(-\tfrac{\xi}{2}\hat n)|q\bar q\rangle\Big|_{\delta^-}$$

树图阶 $\psi^{\pm(0)}_{\bar q q} = \delta(x-x_0)$

1-loop



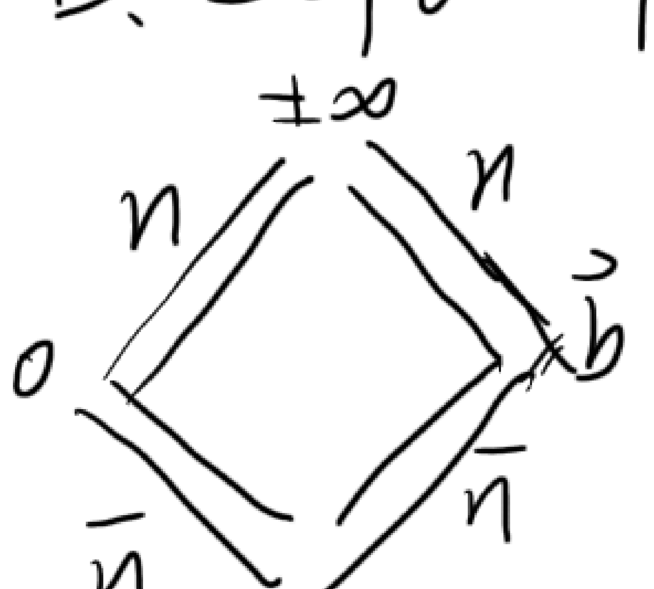
$$\psi^{(a,b)}_{\bar q q} = \frac{\alpha_s C_F}{2\pi}\frac{\theta(x_0-x)\,x}{x_0(x-x_0\pm i\frac{\delta^-}{2P^+})}\cdot\left(\frac{1}{\varepsilon_{IR}}+L_b\right)$$

$$\psi^{(d,e)}_{\bar q q} = \delta(x-x_0)\frac{\alpha_s C_F}{2\pi}\int_0^{x_0} dy\,\frac{\theta(x_0-y)\,y}{x_0(y-x_0\pm i\frac{\delta^-}{2P^+})}\left(\frac{1}{\varepsilon_{UV}}-\frac{1}{\varepsilon_{IR}}\right)$$

$$\psi^{(a+d,\,b+e)}_{\bar q q} = \frac{\alpha_s C_F}{2\pi}\left\{\left[\frac{x\,\theta(x_0-x)}{x_0(x-x_0)}\left(\frac{1}{\varepsilon_{IR}}+L_b\right)\right]_+ + \delta(x-x_0)\left(1+\frac{1}{2}\ln\frac{-\delta^{-2}\mp i0}{4\pi^2P^{+2}}\right)\times\left(\frac{1}{\varepsilon_{UV}}+L_b\right)\right\}$$

$$\psi^{(c)}_{\bar q q} = \frac{\alpha_s C_F}{2\pi}\left\{\left[-\left(\frac{\bar x}{\bar x_0}\theta(x-x_0)+\frac{x}{x_0}\theta(x_0-x)\right)\left(\frac{1}{\varepsilon_{IR}}+L_b-1\right)\right]_+ - \delta(x-x_0)\frac{1}{2}\left(\frac{1}{\varepsilon_{IR}}+L_b-1\right)\right\}$$

B. Soft function



$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = \frac{1}{N_c}\mathrm{tr}\langle 0|T\, W_{\bar n}^{\dagger}(b_\perp)\big|_{\delta^+} W_n(b_\perp)\big|_{\delta^-} W_n^{\dagger}(0)\big|_{\delta^-} W_{\bar n}(0)\big|_{\delta^+}|0\rangle$$

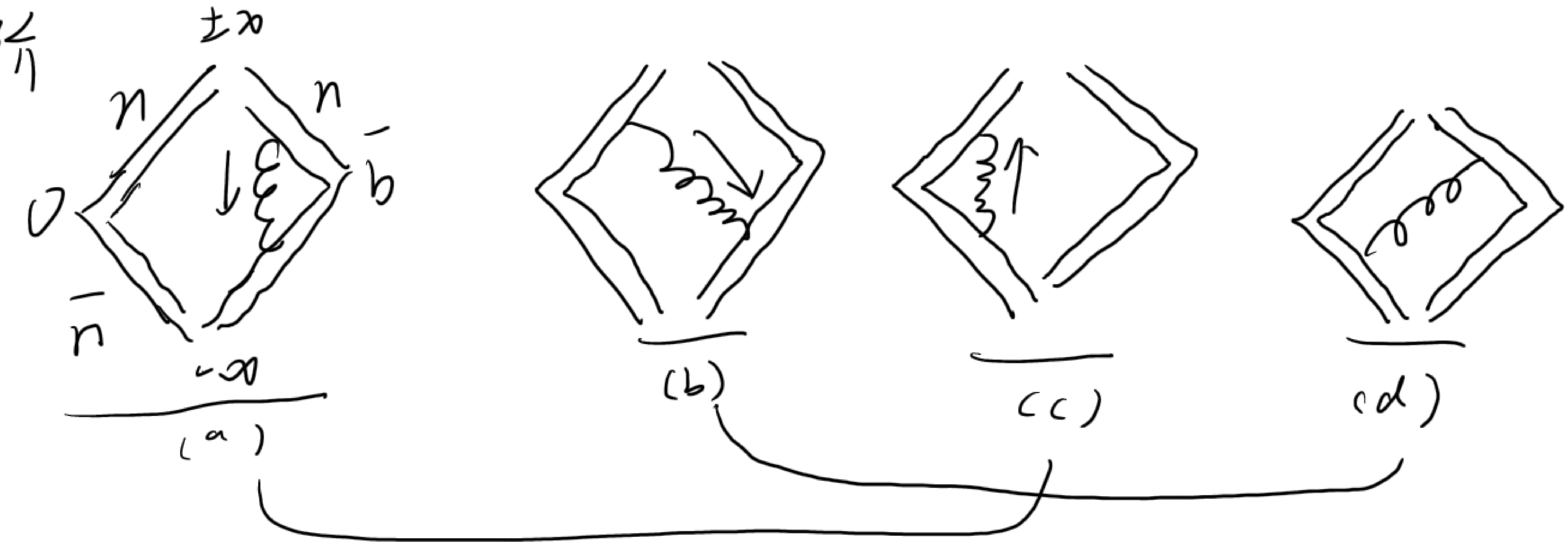
$$\psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\zeta) = \lim_{\delta^-\to 0}\frac{\psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\delta^-)}{\sqrt{S^{\pm}(b_\perp,\mu,\delta^- e^{2y_n},\delta^-)}}$$

$\psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\delta^-)$ → bare TMDWF

"physical" TMDWF

树图阶 $S^{(0)} = 1$

单圈阶



$$S^{(a,c)} = \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}}\ln\frac{\mp\delta^-\delta^+ + i0}{2\mu^2} - \ln^2\frac{\mp\delta^-\delta^+ + i0}{2\mu^2} - \frac{\pi^2}{2}\right]$$

$$\left(q^+\to\infty, q^\perp\to\infty \quad \frac{1}{\varepsilon_{UV}} \quad ; \quad q^-\to\infty, q^\perp\to\infty \quad \frac{1}{\varepsilon_{UV}}\right) \quad \frac{1}{\varepsilon^2}$$

$$S^{(b,d)} = \frac{\alpha_s C_F}{4\pi}\left[L_b^2 + 2L_b\ln\frac{\mp\delta^-\delta^+ + i0}{2\mu^2} + \ln^2\frac{\mp\delta^-\delta^+ + i0}{2\mu^2} + \frac{2\pi^2}{3}\right]$$

δ^+, δ^- 红外正规子

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = 1 + \frac{\alpha_s C_F}{2\pi}\left(L_b^2 + 2L_b\ln\frac{\mp\delta^-\delta^+ + i0}{2\mu^2} + \frac{\pi^2}{6}\right) \qquad \underline{\Psi = \frac{\Psi}{\sqrt{S}}}$$

$$\Psi^{\pm}_{q\bar{q}} = \delta(x-x_0) + \frac{\alpha_s C_F}{2\pi}\left\{\left[\left(\frac{x}{x_0(x-x_0)} - \frac{x}{x_0}\right)\left(\frac{1}{\varepsilon_{IR}} + L_b\right) + \frac{x}{x_0}\right]\theta(x_0-x) + \{x\to 1-x, x_0\to 1-x_0\}\right\}_+$$
$$+ \frac{\alpha_s C_F}{2\pi}\delta(x-x_0)\left\{\frac{L_b^2}{2} + L_b\left(\frac{3}{2} + \ln\frac{\mu^2}{\pm\sqrt{\zeta\bar{\zeta}} - i0}\right) + \frac{1}{2} - \frac{\pi^2}{12}\right\}$$

$\downarrow$

RGE of ζ

$$2\zeta\frac{d}{d\zeta}\ln\Psi^{\pm}_{q\bar{q}}(x,b_\perp,\mu,\zeta) = -\frac{\alpha_s C_F}{\pi}L_b = K_1(b_\perp,\mu)$$

快度演化：CS-kernel $\quad$ Collins-Soper kernel

C. Hard kernel

$$\underline{F(b_\perp,P_1,P_2,\mu)} = \int dx_1 dx_2\, H_F(Q^2,\bar{Q}^2,\mu^2)\,\bar{\Psi}^{+}_{q\bar{q}}(x_2,b_\perp,\mu) \times \Psi_{q\bar{q}}(x_1,b_\perp,\mu) \times \frac{S(b_\perp,\mu,\delta^+,\delta^-)}{\sqrt{S(b_\perp,\mu,\delta'^+,\delta^-)S(b_\perp,\mu,\delta^+,\delta'^-)}}$$

$$H_F^{(0)} = \begin{cases} \frac{1}{4N_c} & \Gamma = I \\ -\frac{1}{4N_c} & \Gamma = \gamma_5, \gamma_\perp\gamma_5 \text{ or } \gamma_\perp \end{cases}$$

1-loop

$$H^{(1)}(Q^2, \bar{Q}^2) = \underset{P^z}{F^{(1)}(b_\perp, P_1, P_2)} - H_F^{(0)}(Q^2, \bar{Q}^2) \times \frac{\alpha_s C_F}{2\pi}\left[-L_b^2 + L_b\left(3 - \ln\frac{4Q^2\bar{Q}^2}{\mu^4}\right) + 1 - \frac{\pi^2}{6}\right]$$

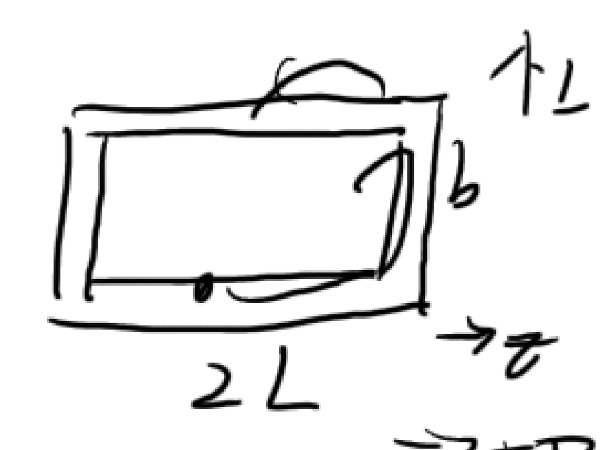
$\boxed{e^{iq_\perp b_\perp}}$ $\downarrow$ 振荡

$\underline{\text{hard} \sim (1,1,1)P^z}$ $\quad \Lambda_{QCD} \ll \frac{1}{b_\perp} \ll P^z$

<u>TMDWF</u> H Form factor

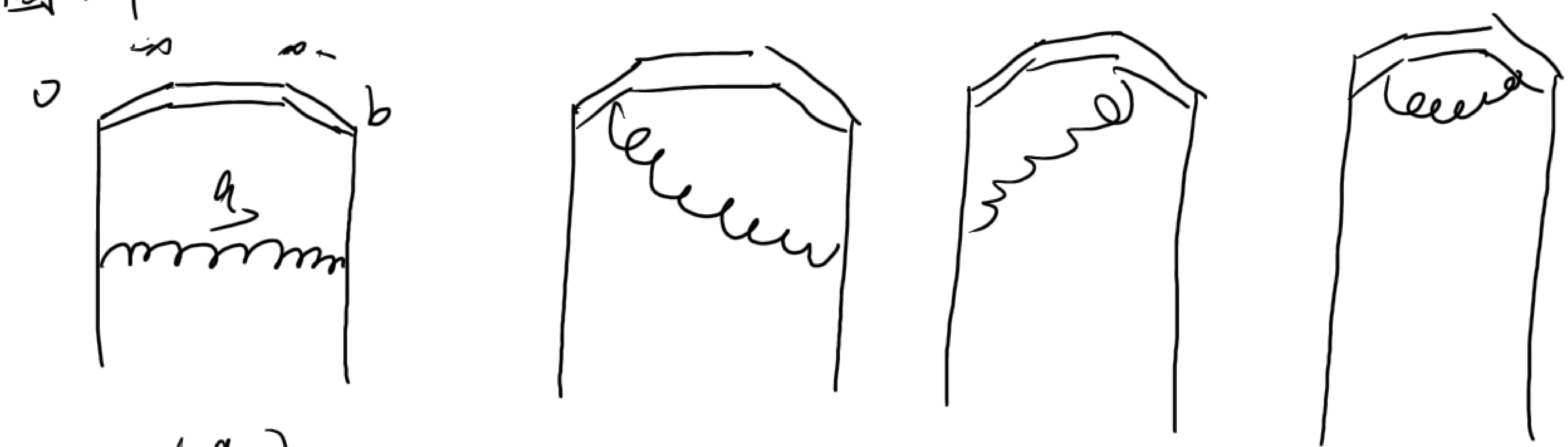
quasi-TMDWF $\quad \underset{TMD}{\xrightarrow{n}} \Rightarrow \underset{\text{quasi-TMD}}{\xrightarrow{z}}$

$$\Psi^\pm(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \frac{1}{-if_\pi} \int \frac{dz}{2\pi} e^{-i(x-\frac{1}{2})(-P^z)z} \frac{\times \langle 0|\bar{\psi}_{n_z}(\underline{\tfrac{z}{2}\hat{z}+b})\gamma^z\gamma^5\psi_{n_z}(-\tfrac{z}{2}\hat{z})|P\rangle}{\sqrt{\underline{Z_E(2L, b_\perp, \mu)}}}$$



树图阶: $\delta(x-x_0)$

单圈阶



(a)

$$\tilde{\psi}_{q\bar{q}}(x, b_\perp, \mu, \boxed{\zeta^z}) = \delta(x-x_0) + \frac{\alpha_s C_F}{2\pi}\left\{\left[\left(\frac{x}{x_0(x-x_0)} - \frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{IR}} + L_b\right) + \frac{x}{x_0}\right]\theta(x-x_0) + \{x \to \bar{x}, x_0 \to \bar{x}_0\}\right\}_+$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(x-x_0)\left(-\frac{L_b^2}{2} + \frac{5}{2}L_b - \frac{3}{2} - \frac{\pi^2}{2} + \left[-\frac{1}{4}\ln\frac{-\zeta^z+i0}{\mu^2} + \frac{1}{2}(1-L_b)\right] \times \ln\frac{-\zeta^z+i0}{\mu^2} + (\zeta^z - \bar{\zeta}^z)\right)$$

$\langle P|\psi\Gamma\psi(b)\,\psi\Gamma\psi(0)|P'\rangle$

$\tilde{F} \to \tilde{\psi}$ quasi-TMDWF 因子化

$$\tilde{\psi}^{\pm}_{qq}(x, b_\perp, \mu, \zeta^z)\, S_r^{\frac{1}{2}}(b_\perp, \mu) = H_1^{\pm}(\zeta^z, \bar{\zeta}^z, \mu)\, e^{\frac{1}{2}\ln\frac{\mp\zeta^z + i0}{\zeta} K(b_\perp, \mu)}\, \psi^{\pm}_{qq}(x, b_\perp, \mu, \zeta) \left(\frac{1}{S_r^{\frac{1}{2}}(b_\perp, \mu)}\right)$$

卷积因子化 → 乘积、因子化

quasi-TMDWF Expansion by region

① hard $q \sim (1,1,1)P^z$，$e^{iq_\perp b_\perp}$ 在 $\frac{1}{b_\perp} \ll P^z$ 时，震荡 ↓

② Collinear $q \sim (Q, \Lambda, \Lambda^2/Q)$，振幅 $\sim \mathcal{O}(Q)$ $\xrightarrow{\text{约化}}$ TMDWF

③ soft $q \sim (\Lambda, \Lambda, \Lambda)$，振幅 $\sim Q\left(\frac{\Lambda^3}{Q}\right)$，↓