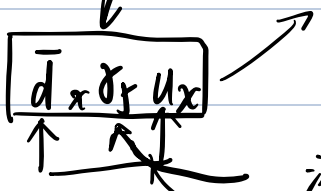
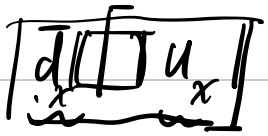
2pt → eff mass jackknife bootstrap

2pt → 介子算符 π^+ → O_{π^+} 湮灭 $\bar{O}_{\pi^+}$ 产生





$J^P = 0^-$ 赝

γ_5 洛仑兹

$\gamma_4\gamma_5$

$I \to 1^-$

$\gamma_4\gamma_5$

$\gamma_i\gamma_5$ 轴矢量 1^+

Tensor $\sigma_{\nu\mu}$ $\gamma_i\gamma_j$?

$\langle O_{\pi^+}(\underset{\text{sink}}{t,x})\ \bar{O}_{\pi^+}(\underset{\text{source}}{t_0,y})\rangle$

以 0 动量 π^+ 为例

$$C_0^{\pi^+}(t,t_0) = \langle O_{\pi^+}(t,x)\,\bar{O}_{\pi^+}(t_0,y)\rangle \qquad \alpha,\beta \text{ 旋 } a \text{ 颜色}$$

$$\propto \sum_{xy} \langle \bar{d}(t,x)^\alpha_a (\gamma_5)_{\alpha\beta} u(t,x)^\beta_a \bar{u}(t_0,y)^{\beta'}_{a'} (\gamma_5)_{\beta'\alpha'} d(t_0,y)^{\alpha'}_{a'} \rangle$$

$$= \sum_{x,y} \langle (\gamma_5)_{\alpha\beta} (\gamma_5)_{\beta'\alpha'} \bar{d}(t,x)^\alpha_a \; u(t,x)^\beta_a \; \bar{u}(t_0,y)^{\beta'}_{a'} \; d(t_0,y)^{\alpha'}_{a'} \rangle$$

$$= \sum_{x,y} \langle (\gamma_5)_{\alpha\beta} (\gamma_5)_{\beta'\alpha'} G^d(t_0,y;t,x)^{\alpha\alpha}_{a'a} G^u(t,x;t_0,y)^{\beta\beta'}_{aa'} \rangle .$$

$$= \sum_{x,y} \mathrm{tr}\left[\gamma_5 G^u(t,x;t_0,y)\,\gamma_5 G^d(t_0,y;t,x)\right]$$

$C_0^{\pi^+}(\tau) = \langle O(t,0)\,\bar{O}(t_0,0)\rangle$ $\quad t-t_0=\tau$ $\quad t_0=0$ $\quad \langle f|i\rangle$

$\propto \langle 0|e^{-iHt}\,O(0,0)\,e^{iHt}\,\bar{O}(0,0)|0\rangle$ $\quad \langle 0|O\ \bar{O}\ |0\rangle$

$\sum_n \langle 0|e^{-iHt}\,O(0,0)\,e^{iHt}|n\rangle\langle n|\bar{O}(0,0)|0\rangle$

$e^{-iHt}|0\rangle = |0\rangle$ $\quad e^{iHt}|n\rangle = e^{iE_n t}$

$= \sum_n e^{iE_n t}\,|\langle 0|O|n\rangle|^2$

$t = i\tau$

$C(\tau) \propto \sum_n e^{-E_n\tau}\,|\langle 0|O|n\rangle|^2$ $\quad \tau$ 较大

E_1 基态 → 0动量 → meff

① $\tau\to\infty$ $\quad \ln\frac{C(\tau)}{C(\tau+1)} = E_1 = m_{eff}$ ...

② τ 较大 → 单态 $C(\tau) = C_0 e^{-E_1\tau}$

→ 双态 $C(\tau) = C_0 e^{-E_1\tau} + C_1 e^{-E'\tau}$

$= C_0 e^{-E_1\tau}\,(1 + \Delta C\, e^{-\Delta E\,\tau})$

jackknife $\quad$ bootstrap ↑

平均值的误差

N个数 → $\bar{n}$

$\langle O\bar{O}\rangle$ $\quad$ 2pt $\quad\quad \langle 0|O\bar{O}|0\rangle$

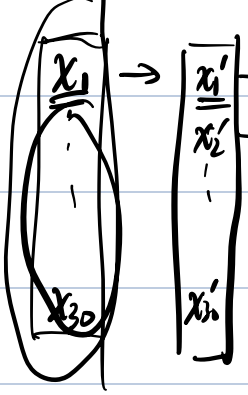
30 $\quad \frac{1}{\sqrt{N-1}}$

std. $\quad \frac{\sum_i^N (n_i-\bar{n})^2}{N(N-1)}$

0 5 10 → 方差更大

平均值一样 $\quad$ 3 5 7

jaeklnife 30



$\frac{x_2 + \cdots x_{30}}{29}$

$\frac{x_1 \cdots x_{30}}{29}$

$x_j' = \frac{(\sum^{30} x_i) - x_j}{29}$

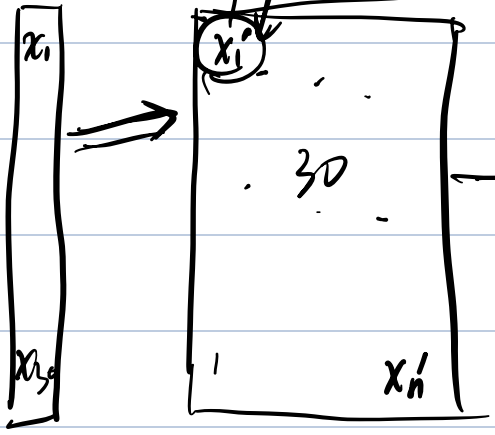
$x_1' \cdots x_{30}'$

$x_1 \cdots x_{30} \rightarrow \bar{x}$

$x_1\ x_1\ x_2\ x_4\ x_1 + x_1 \cdots$ std. $\sqrt{N-1}$ ←

boostrap

30



$x_i' = \frac{x_1 + x_1 \cdots}{30}$

$std_{bs} = std_{eff\ mass}$

$\sqrt{\frac{\Sigma(\ )}{n}}$ ← $\frac{1}{\sqrt{N-1}}$

29↑0 1↑30

29↑ $\frac{30}{29}$ 1 0↑ → $\sqrt{N-1}$

$\sqrt{\frac{\Sigma(\ )}{n(n-1)}}$

eff mass

jackknife 计算法

$\frac{1}{n-1}\sqrt{\frac{\Sigma(\ )}{n}}$

$f(x, y) =$

$f = xy$

$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$

$\sqrt{\sigma_x^2 + \sigma_y^2}$

↑

σ_{eff}

2pt.

$$\text{eff mass} = \ln \frac{2pt(\tau)}{2pt(\tau+1)}$$

$\sigma_{2pt}(\tau)$

$\sigma_{2pt}(\tau+1)$

$\sigma_{\text{eff mass}}$