Standard Model Prediction for Paramagnetic EDM

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Electric Dipole Moments: Experimental and Theoretical Horizons
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Based on 2108.05398, 2202.10524, 2205.11532, and 2207.01679
with M.Pospelov and Y.Ema

Outline

Paramagnetic EDM

Paramagnetic systems are sensitive to leptonic and semileptonic sources of CP-violation:

$$\mathcal{L}_{\not{CP}} = \frac{d_e}{2}\bar{e}\sigma_{\mu\nu}\tilde{F}^{\mu\nu}e + C_S\frac{G_F}{\sqrt{2}}(\bar{e}i\gamma_5 e)(\bar{p}p + \bar{n}n)$$

Paramagnetic EDM experiments measure a linear combination of them:

$$\hbar\omega^{\mathcal{N}\mathcal{E}} = -d_e\mathcal{E}_{\mathbf{eff}} + W_S C_S$$

The result can be inferred as

$$d_e^{eq} = d_e + \mathcal{O}(1)\times C_S \times \ 10^{-20}e\text{cm}$$

Current best constraints on paramagnetic EDMs are:

$$|d_e^{eq}(\text{ThO})| < 1.1\times 10^{-29}e\text{cm (90\% C.L.) (ACME)}$$

$$|d_e^{eq}(\text{HfF}^+)| < 4.1\times 10^{-30}e\text{cm (90\% C.L.) (CU Boulder)}$$

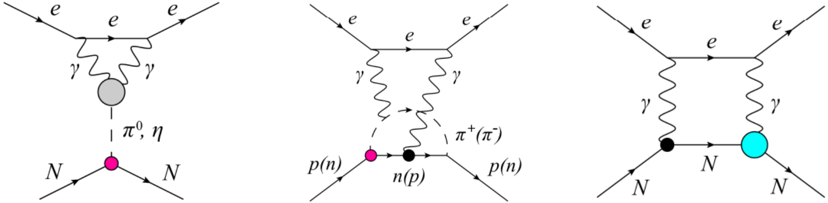
For these systems,

$$d_e^{eq}(\text{ThO}) = d_e + C_S\times 1.5\times 10^{-20}e\text{cm}$$

$$d_e^{eq}(\text{HfF}^+) = d_e + C_S\times 0.9\times 10^{-20}e\text{cm}$$

Previous Predictions: from $\theta_{\rm QCD}$

V. V. Flambaum, M. Pospelov, A. Ritz, and Y. V. Stadnik 2019,
H. Mulder, R. Timmermans, and J. de Vries 2025

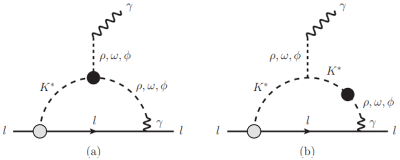


$C_S = 1.63[45] \times 10^{-2}\bar{\theta} \to |\bar{\theta}| < 1.5 \times 10^{-8}$ based on HfF$^+$ experiment

Sub-dominate compared to the constraint from neutron EDM

Previous Predictions: from δ_{KM}

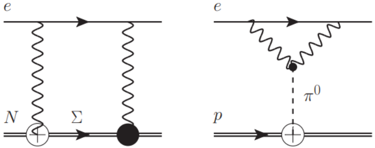
Y. Yamaguchi & N. Yamanaka 2020



$$d_e = 6 \times 10^{-40} e\text{cm}$$

$\sim$ 70% uncertainty

M. Pospelov & A. Ritz 2013

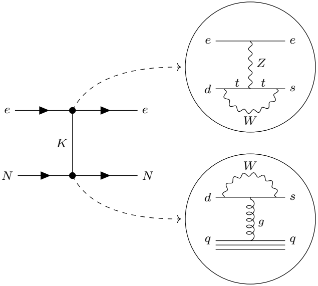


$$C_S \sim 10^{-18} \Rightarrow d_e^{eq} \sim 10^{-38} e\text{cm}$$

$\sim$ an order of magnitude uncertainty

Results were small and uncertain, seem to suggest paramagnetic EDMs have weaker sensitivity to SM CP-violation compared to diamagnetic and neutron EDM experiments.

C_S operator from Kaon exchange: Overview

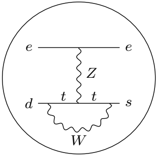


- Our work shows the C_S operator receives the dominate contribution from the Kaon exchange diagram at EW^3 order.
- The result

$$C_S \simeq 6.9 \times 10^{-16}$$
$$\Rightarrow d_e^{eq} \simeq 1.0 \times 10^{-35} e\text{cm}$$

  is 3 orders of magnitude larger than previous result at $\mathrm{EW}^2\mathrm{EM}^2$ order
- The uncertainty is $\sim 30\%$, under better control than previous results.

C_S operator from Kaon exchange: Weak penguin



- Full calculation including penguins and box diagrams was done by T. Inami & C. S. Lim in 1981.
- EW penguin is well understood, leads to
  - Real part of $K_L \to \mu^+\mu^-$
  - $B_{s,d} \to \mu^+\mu^-$
  - $K^+ \to \pi^+ + \nu + \bar{\nu}$
- Result given by

$$\mathcal{L}_{\mathrm{EWP}} = -\mathcal{P}_{\mathrm{EW}} \times \bar{e}\gamma_\mu\gamma_5 e \times \bar{s}\gamma^\mu(1-\gamma_5)d + (h.c.),$$

$$\mathcal{P}_{\mathrm{EW}} = \frac{G_F}{\sqrt{2}} \times V_{ts}^* V_{td} \times \frac{\alpha_{\mathrm{EM}}(m_Z)}{4\pi\sin^2\theta_W} I(x_t),$$

$$I(x_t) = \frac{3}{4}\left(\frac{x_t}{x_t-1}\right)^2 \log x_t + \frac{1}{4}x_t - \frac{3}{4}\frac{x_t}{x_t-1}, \quad x_t = \frac{m_t^2}{m_W^2}.$$

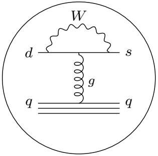
In χPT this gives rise to

$$\mathcal{L}_{Uee} = -\frac{if_0^2}{2}\mathcal{P}_{\mathrm{EW}} \times \bar{e}\gamma_\mu\gamma_5 e \times \mathrm{Tr}\left[h^\dagger\left(\partial^\mu U\right)U^\dagger\right] + (h.c.), \quad h = \delta_{i2}\delta_{j3}.$$

At leading order, this is

$$\mathcal{L}_{Kee} = -2\sqrt{2}f_0 m_e \bar{e}i\gamma_5 e\left(K_S \times \mathrm{Im}\mathcal{P}_{\mathrm{EW}} + K_L \times \mathrm{Re}\mathcal{P}_{\mathrm{EW}}\right).$$

C_S operator from Kaon exchange: Strong penguin



- General $|\Delta S| = 1$ coupling between baryons and mesons at EW[1] order
- Two possible structures transforming as $(8_L, 1_R)$ within χPT:

$$\mathcal{L}_{\rm SP} = -a\mathrm{Tr}(\bar{B}\{\xi^\dagger h\xi, B\}) - b\mathrm{Tr}(\bar{B}[\xi^\dagger h\xi, B]) + (h.c.).$$

- The same structures are responsible for weak non-leptonic hyperon decay

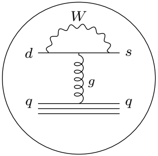
$$A^{(S)}(\Sigma^- \to n\pi^-) = \frac{a-b}{f}, \quad A^{(S)}(\Lambda \to p\pi^-) = -\frac{a+3b}{\sqrt{6}f},$$
$$A^{(S)}(\Sigma^+ \to n\pi^+) = 0, \quad A^{(S)}(\Xi^- \to \Lambda\pi^-) = \frac{3b-a}{\sqrt{6}f}, \cdots$$

- a and b determined by fit to the corresponding experimental s-wave amplitudes (Bijnens, Sonoda and Wise 1985)

$$a = 0.56G_F f_\pi \times [m_{\pi^+}]^2; \quad b = -1.42G_F f_\pi \times [m_{\pi^+}]^2.$$

- $[m_{\pi^+}] = 139.5\mathrm{MeV}$ is understood as a numerical value rather than the physical pion mass.
- Overall sign fixed by performing vacuum factorization for strong penguin.

C_S operator from Kaon exchange: Strong penguin



- Two possible structures transforming as $(8_L, 1_R)$ within χPT:

$$\mathcal{L}_{\rm SP} = -a{\rm Tr}(\bar{B}\{\xi^\dagger h\xi, B\}) - b{\rm Tr}(\bar{B}[\xi^\dagger h\xi, B]) + (h.c.).$$

- Assuming a and b are real gives the K_S vertex:

$$\mathcal{L}_{KNN} = 2^{1/2} f_0^{-1}((b-a)\bar{p}p + 2b\bar{n}n)K_S$$

- Including the sub-dominate K_L coupling gives

$$\mathcal{L}_{KNN} \simeq -\frac{\sqrt{2}G_F \times [m_{\pi^+}]^2 f_\pi}{|V_{ud}V_{us}|f_0} \times 2.84(0.7\bar{p}p + \bar{n}n) \times ({\rm Re}(V_{ud}^* V_{us})K_S + {\rm Im}(V_{ud}^* V_{us})K_L)$$

C_S operator from Kaon exchange: LO result

The diagram gives the result

$$C_S \simeq \mathcal{J} \times \frac{N + 0.7Z}{A} \times \frac{13[m_{\pi^+}]^2 f_\pi m_e G_F}{m_K^2} \times \frac{\alpha_{\rm EM} I(x_t)}{\pi \sin\theta_W^2}$$

$\mathcal{J}$ is the Jarlskog invariant

$$\mathcal{J} = {\rm Im}(V_{ts}^* V_{td} V_{ud}^* V_{us}) \simeq 3.1 \times 10^{-5}$$

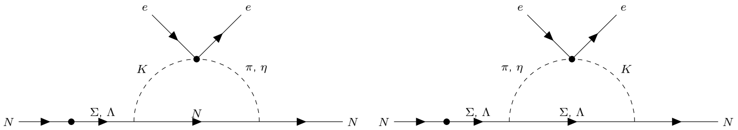
The overall scaling is

$$G_F C_S \propto \mathcal{J} G_F^3 m_t^2 m_e m_s^{-1} \Lambda_{\rm hadr}^2$$

Numerically,

$$C_S({\rm LO}) \simeq 5 \times 10^{-16}.$$

C_S operator from Kaon exchange: NLO contribution



The next-to-leading order contribution comes from baryon pole diagrams, they scale as $(m_s/\Lambda_{\rm hadr})^{1/2}$ relative to the LO diagram, the results are

$$\frac{C_{S,NLO}(p)}{C_{S,LO}(p)} = \frac{m_K^3(0.77D^2 + 2.7DF - 2.3F^2)}{24\pi f_0^2(m_{\Sigma^+} - m_p)} \simeq 30\%$$

$$\frac{C_{S,NLO}(n)}{C_{S,LO}(n)} = \frac{m_K^3}{24\pi f_0^2}\left(\frac{(a/b+3)}{2\sqrt{6}(m_\Lambda - m_n)} \times (-0.44D^2 + 3.2DF + 1.3F^2) + \frac{a/b-1}{2\sqrt{2}(m_{\Sigma^0} - m_n)}(-0.53D^2 - 1.9DF + 1.6F^2)\right) \simeq 40\%$$

C_S operator from Kaon exchange: Final result

- Combining LO and NLO result gives

$$C_S \simeq [5.0(\text{LO}) + 1.9(\text{NLO})] \times 10^{-16}, \quad d_e^{eq}(\text{ThO}) \simeq 1.0 \times 10^{-35} e\text{cm}$$

- 3 orders of magnitude larger than the contribution at $\mathrm{EW}^2\mathrm{EM}^2$ order
- 6 orders of magnitude away from current experimental bound

$$|d_e^{eq}(\text{ThO})| < 1.1 \times 10^{-29} e\text{cm}$$

- C_S contribution dominate over d_e contribution:

$$d_e = 6 \times 10^{-40} e\text{cm}$$

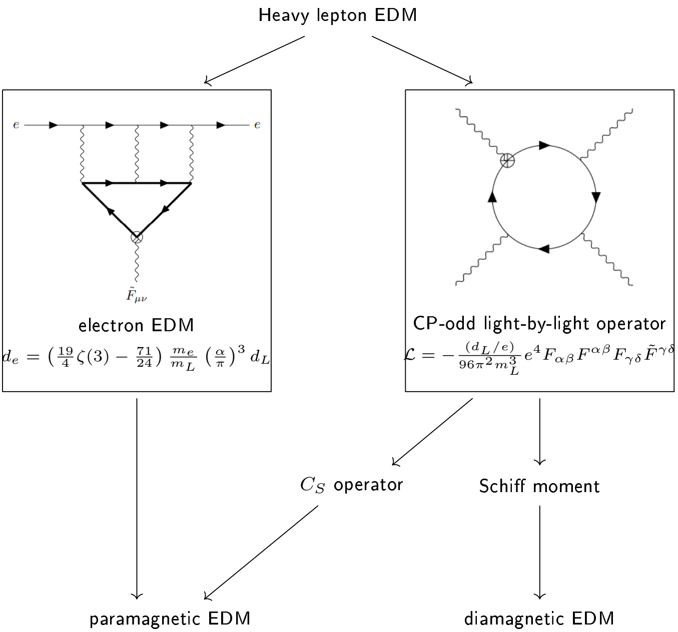
- The relevant hadronic matrix element

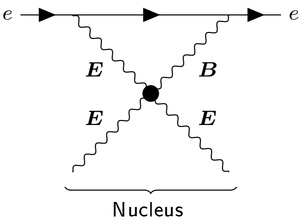
$$\langle N | i(\bar{s}\gamma_\mu(1-\gamma_5)d - \bar{d}\gamma_\mu(1-\gamma_5)s)|N\rangle_{\mathrm{EW}^1} = \frac{f_S}{m_N} i q_\mu \bar{N}N + \frac{f_T}{m_N} q_\nu \bar{N}\sigma_{\mu\nu}\gamma_5 N.$$

  can be attempted on the lattice
- There are proposals to measure d_e^{eq} at similar order of magnitude (Vutha, Horbatsch, Hessels, 1710.08785 and 1806.06774)

Outline

Heavy lepton EDM: Pathways to atomic EDMs





On the electron side,

$$\boldsymbol{E_e} \cdot \boldsymbol{B_e} \to -\frac{3\alpha m_e}{2\pi} \ln\left(\frac{m_\mu}{m_e}\right) \bar{e} i \gamma^5 e$$

On the nucleus side,

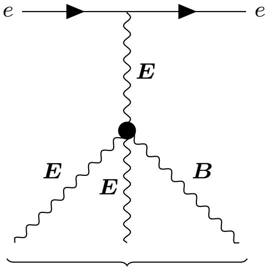
$$(e\boldsymbol{E}_{\text{nucl}})^2 \to \kappa \delta(\boldsymbol{r}) \times \frac{4\pi (Z\alpha)^2}{R_N} \times \frac{6}{5}$$

Fudge factor κ introduced to account for difference between $\bar{N}N$ distribution and $\boldsymbol{E}^2_{\text{nucl}}$ distribution.

$$C_S^{eq} = \kappa \frac{\sqrt{2}}{G_F} \frac{4Z^2\alpha^4}{\pi A} \times \frac{m_e (d_\mu / e)}{m_\mu^3 R_N} \times \ln\left(\frac{m_\mu}{m_e}\right) = 3.1 \times 10^{-10} \left(\frac{d_\mu}{10^{-20} e\text{cm}}\right)$$

- We combine it with the muon EDM contribution to d_e. Based on the ThO experiment, we arrive at the constraint $|d_\mu| < 1.7 \times 10^{-20} e$cm.
- Recent HfF$^+$ experiment refines the result: $|d_\mu| < 8.9 \times 10^{-21} e$cm.
- Factor of $\sim$20 better than previous BNL direct measurement $|d_\mu| < 1.8 \times 10^{-19} e$cm.
- For tau lepton we are getting $|d_\tau| < 4.1 \times 10^{-19} e$cm, better than previous Belle result $\text{Re}(d_\tau) = (1.15 \pm 1.70) \times 10^{-17} e$cm.

Muon EDM: Constraint from Diamagnetic Schiff Moment



The $\boldsymbol{E}^2\boldsymbol{B}$ connected to the nucleus effectively acts as a dipole polarization density $\boldsymbol{P}$, which leads to a Schiff moment

$$H = -4\pi\alpha(\boldsymbol{S}_N/e)\cdot\boldsymbol{\nabla}\delta(\boldsymbol{r})$$

$$\boldsymbol{S} = \frac{1}{6}\int \mathrm{d}^3 r r^2 \boldsymbol{P} - \frac{1}{6}\boldsymbol{d} r_c^2 + \frac{1}{5}\int \mathrm{d}^3 r((\boldsymbol{P}\cdot\boldsymbol{r})\boldsymbol{r} - \frac{1}{3}r^2\boldsymbol{P})$$

Numerically,

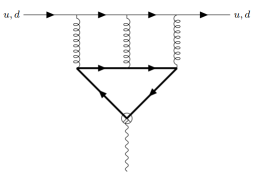
$$S_{^{199}\mathrm{Hg}}/e \simeq (d_\mu/e)\times 4.9\times 10^{-7}\mathrm{fm}^2$$

$$|d_\mu| < 6.4\times 10^{-20} e\mathrm{cm}$$

Factor of ∼3 better than previous BNL direct measurement, provides an independent check for the constraint from paramagnetic EDM

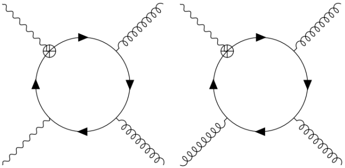
c and b quark EDM: Pathways to atomic and neutron EDMs

c and b quark EDM



light quark EDM

$$d_q = \frac{5(8\zeta(3)-7)}{72}\left(\frac{\alpha_s}{\pi}\right)^3 \frac{m_q}{m_Q} d_Q$$



CP-odd photon-gluon operators

$$\mathcal{L}_{eff,Q} = \frac{eQ_Q d_Q g_s^2}{24\pi^2 m_Q^3} \mathrm{Tr}_c \left[\tilde{F}_{\mu\nu} G^{\mu\nu} F_{\alpha\beta} G^{\alpha\beta} - \tilde{F}_{\mu}^{\nu} G_{\nu}^{\alpha} F_{\alpha}^{\beta} G_{\beta}^{\mu}\right] + \frac{d_Q g_s^3}{48\pi^2 m_Q^3} \mathrm{Tr}_c \left[-\tilde{F}_{\mu\nu} G^{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} + 2\tilde{F}_{\mu}^{\nu} G_{\nu}^{\alpha} G_{\alpha}^{\beta} G_{\beta}^{\mu}\right]$$

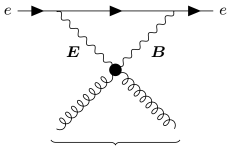
C_S operator

neutron EDM

paramagnetic EDM

c and b quark EDM: indirect constraints

Paramagnetic EDM



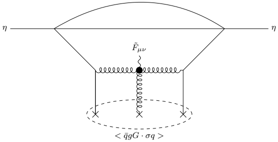
$$C_S \frac{G_F}{\sqrt{2}} = -\frac{4Q_Q \alpha^2}{27} \frac{m_N m_e}{m_Q^3} \times \ln\left(\frac{m_Q}{m_e}\right) \frac{d_Q}{e}$$

$|d_c| < 1.3 \times 10^{-20} e$cm,

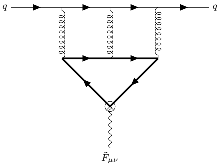
$|d_b| < 7.6 \times 10^{-19} e$cm

Neutron EDM

non-perturbative contribution



perturbative contribution



$$d_n = \frac{5\alpha_s^2 m_n m_0^2 \ln(\frac{M^2}{\Lambda_{IR}^2})}{2^7 3^3 \pi^2 m_Q^3} d_Q + \frac{5(8\zeta(3)-7)}{72} (\frac{\alpha_s}{\pi})^3 \frac{4m_d - m_u}{3m_Q} d_Q$$

$$|d_c| < 6 \times 10^{-22} e\text{cm}, \quad |d_b| < 2 \times 10^{-20} e\text{cm}$$

Summary

- New SM prediction for paramagnetic EDM

$$d_e^{eq} \simeq 1.0 \times 10^{-35} e\text{cm}$$

  - Uncertainty under control.
  - Within the potential reach of future experiments.
- New indirect constraints on muon, tau lepton, and charm and bottom quark EDM

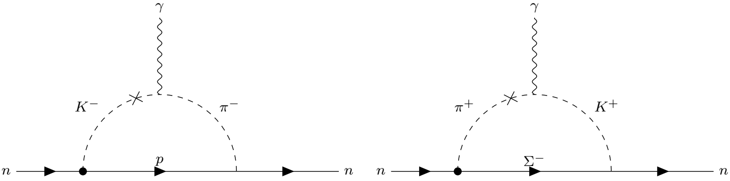
$$|d_\mu| < 8.9 \times 10^{-21} e\text{cm}, \quad |d_\tau| < 4.1 \times 10^{-19} e\text{cm}$$
$$|d_c| < 6 \times 10^{-22} e\text{cm}, \quad |d_b| < 2 \times 10^{-20} e\text{cm}$$

  - Provide important benchmark for future experiments.

Thank you!

Back up slides

Neutron EDM from Electroweak penguin



Cross: $K\pi$ mixing due to EW penguin (A. Pich and A. Rodríguez-Sánchez 2021)

$$d_n \propto m_s^{-1} \log \frac{m_s}{m_u + m_d}$$

numerically

$$d_n = -1.4 \times 10^{-33} e\text{cm}$$

SM predictions and experimental bounds on EDMs

Type of EDM	SM prediction from CKM phase
Paramagnetic	$\|d_e^{eq}(\text{ThO})\| \approx 1.0 \times 10^{-35} e\text{cm}$
Diamagnetic	$\|d(^{199}\text{Hg})\| \approx (0.4 - 2.4) \times 10^{-35} e\text{cm}$ (Dmitriev & Sen'kov 2003)
Neutron	$\|d_n\| \approx (1 - 6) \times 10^{-32} e\text{cm}$ (CY Seng 2015)

Type of EDM	Experimental bound
Paramagnetic	$\|d_e^{eq}(\text{ThO})\| < 1.1 \times 10^{-29} e\text{cm}$ (90% C.L.) (ACME) $\|d_e^{eq}(\text{HfF}^+)\| < 4.1 \times 10^{-30} e\text{cm}$ (90% C.L.) (CU Boulder)
Diamagnetic	$\|d(^{199}\text{Hg})\| < 7.4 \times 10^{-30} e\text{cm}$ (95% C.L.) (UW)
Neutron	$\|d_n\| < 1.8 \times 10^{-26} e\text{cm}$ (90% C.L.) (C. Abel et al. 2020)

SM prediction for muon and tau lepton EDM

Heavy lepton EDMs are also predicted within the SM by Y. Yamaguchi & N. Yamanaka 2020:

$$d_\mu = 1.4 \times 10^{-38} e\text{cm}$$
$$d_\tau = -7.3 \times 10^{-38} e\text{cm}$$

Currently they are ~ 20 orders of magnitude below the experimental limit.