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After $(g-2)_\mu$, what next for $U(1)_{L_\mu - L_\tau}$?

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Based on work with M.W. Li, X.G. He, X. Chu, and TRIDENT collaboration

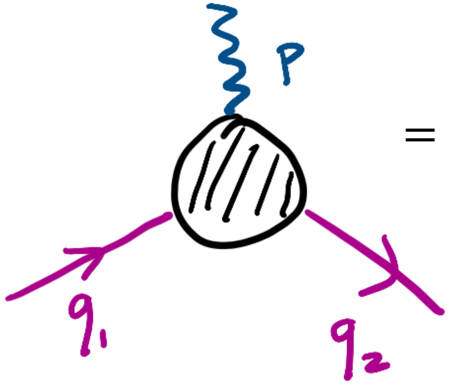
$g - 2$: 80 years of precision

From the Dirac equation one can derive magnetic moment of a Fermion

$$\vec{\mu} = g \frac{q}{2m} \vec{S}$$

Where $g = 2$, q is the electric charge $\vec{S} = \frac{1}{2} \vec{\sigma}$. Experimental hints that $g \neq 2$ in 1947!

Comes from $F_{\mu\nu}\sigma^{\mu\nu}$ term $\Longrightarrow$ quantum corrections will come from loops with this form.



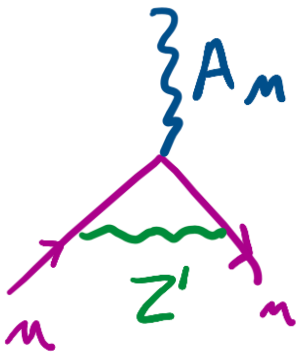
$$= (-ie)\bar{u}(q_2)\left[F_1\left(\frac{p^2}{m^2}\right)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m} p_\nu F_2\left(\frac{p^2}{m^2}\right)\right]u(q_1) \qquad \Longrightarrow g \to 2 + 2\, F_2\left(\frac{p^2}{m^2}\right)$$

1-loop order $F_2(0) = \frac{\alpha}{2\pi}$ (Schwinger 1948, then Feynman and Tomonaga)

Vital for development of QFT!

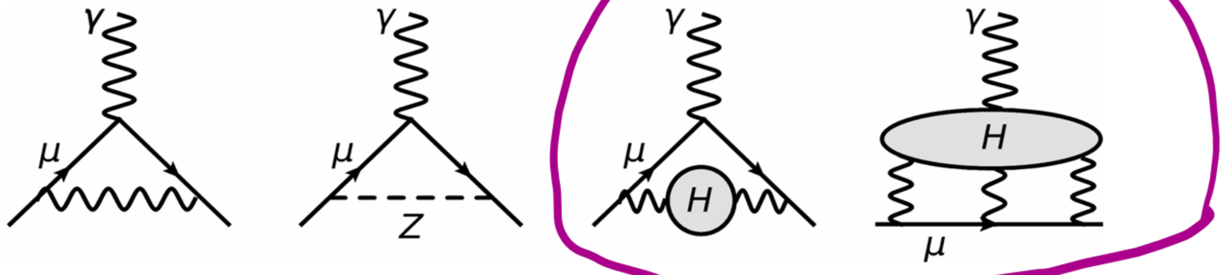
The muon $(g-2)$

Measuring $(g-2)_\mu$ is a clean test of new physics, and contribution is enhanced compared to electron $\Delta a_\mu \propto \left(\frac{m_\mu}{m_{NP}}\right)^2$



$$\rightarrow \Delta a_\mu^{Z'} = \frac{\tilde{g}^2}{8\pi^2} \times \left(\frac{m_\mu}{m_{Z'}}\right)^2 \int_0^1 \frac{2x^2(1-x)dx}{1-x+\left(\frac{m_\mu^2}{m_{Z'}^2}\right)x^2} \xrightarrow{m_{Z'} \gg m_\mu} \frac{\tilde{g}^2}{12\pi^2} \times \left(\frac{m_\mu}{m_{Z'}}\right)^2$$

SM contribution calculated to many loop level

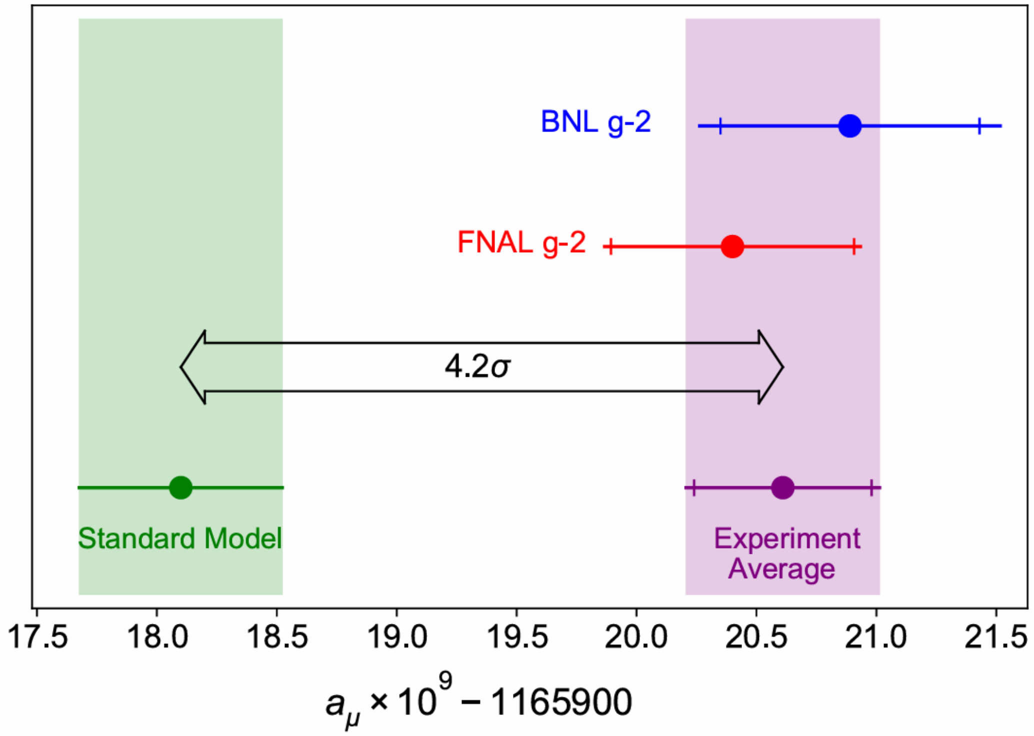


Largest error

Hints of new physics from $(g-2)_\mu$

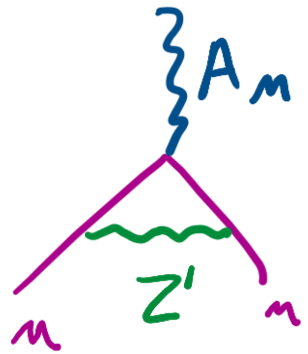
Since 2006, E821 at BNL saw hints of deviations from SM.

In 2021, Fermilab Muon $g-2$ experiment started to confirm this



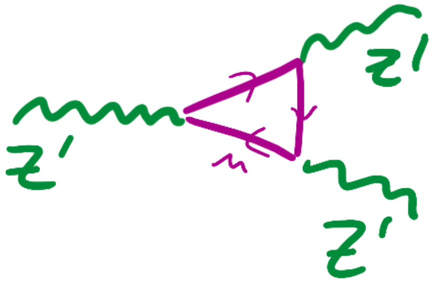
New physics to explain this is varied

The simplest is by introducing only 1 new particle



The $U(1)_{L_\mu - L_\tau}$ extension

Coupling only to the muon is anomalous, i.e.



$\neq 0$

We can either add more particle content

- Solutions lead to flavor changing constraints (see Carena et.al. arXiv:1908.04818)

More simply, look for anomaly-free models

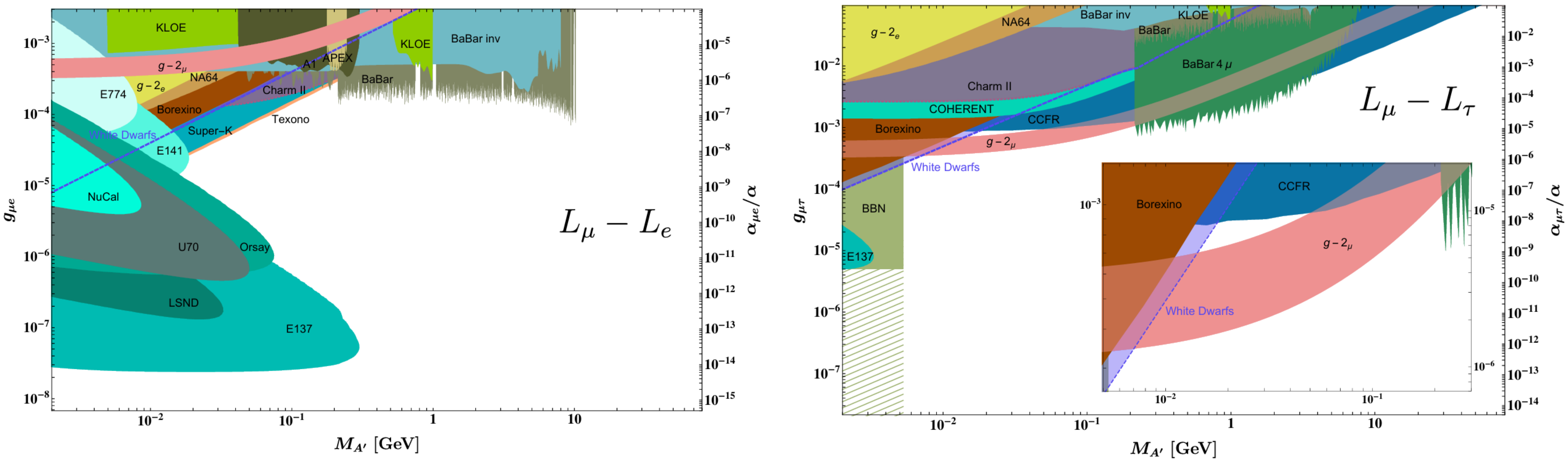
- Only two that involve muon coupling (see XG He et. al. PRD 1991).

$U(1)_{L_\mu - L_\tau}$ or $U(1)_{L_\mu - L_e}$ where the gauge current is

$$j_\alpha^{L_i - L_j} = \overline{L}_i \gamma_\alpha L_i + \overline{\ell}_{i,R} \gamma_\alpha \ell_{i,R} - \overline{L}_j \gamma_\alpha L_j - \overline{\ell}_{j,R} \gamma_\alpha \ell_{j,R} \qquad \text{with } L_i = \begin{pmatrix} \nu_{i,L} \\ \ell_{i,L} \end{pmatrix}$$

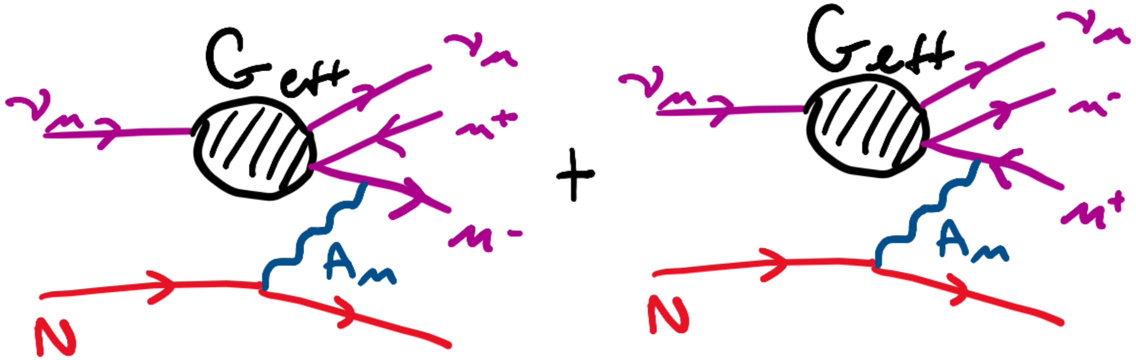
The $U(1)_{L_\mu - L_\tau}$ extension

M. Bauer et. al. [arXiv:1803.05466]



Focus on low Z' mass

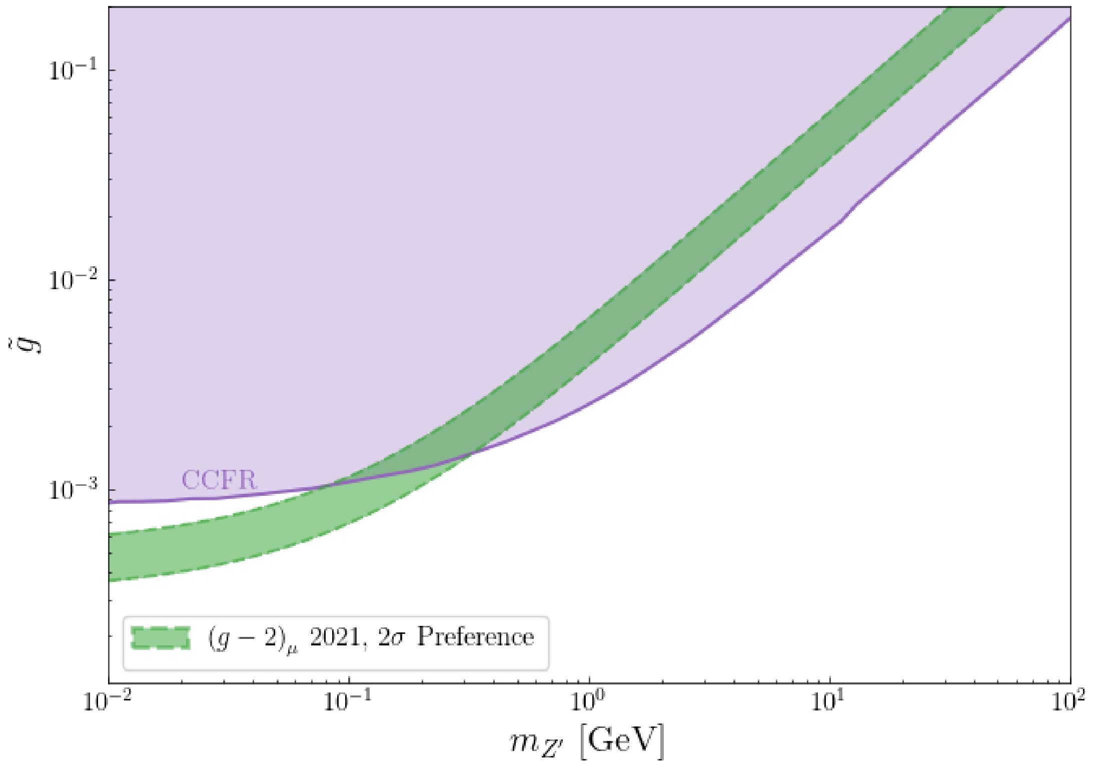
Muon Neutrino Trident (MNT) measurements limit how heavy Z' can be



$$\mathcal{L}^{\text{eff}}_{\text{total}} = -g_V\left(\bar{\nu}_\mu\gamma^\mu P_L\nu_\mu\right)\left(\bar{\mu}\gamma_\mu\mu\right) - g_A\left(\bar{\nu}_\mu\gamma^\mu P_L\nu_\mu\right)\left(\bar{\mu}\gamma_\mu\gamma^5\mu\right)$$

where $g_V = \frac{G_F}{\sqrt{2}}(1+4\sin^2\theta_W) + G_{Z'}$ and $g_A = -\frac{G_F}{\sqrt{2}}$

$$\left.\frac{\sigma_{Z'}}{\sigma_{\text{SM}}}\right|_{\text{trident}} = \frac{(1+4s_W^2+8\tilde{g}^2m_W^2/g^2m_{Z'}^2)^2+1}{1+(1+4s_W^2)^2}$$



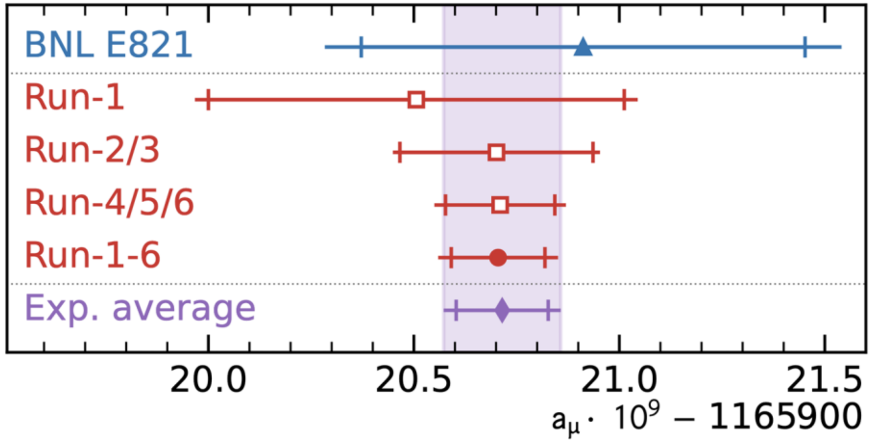
2025:The end of an anomaly

The Muon $g-2$ collaboration has completed all their runs.

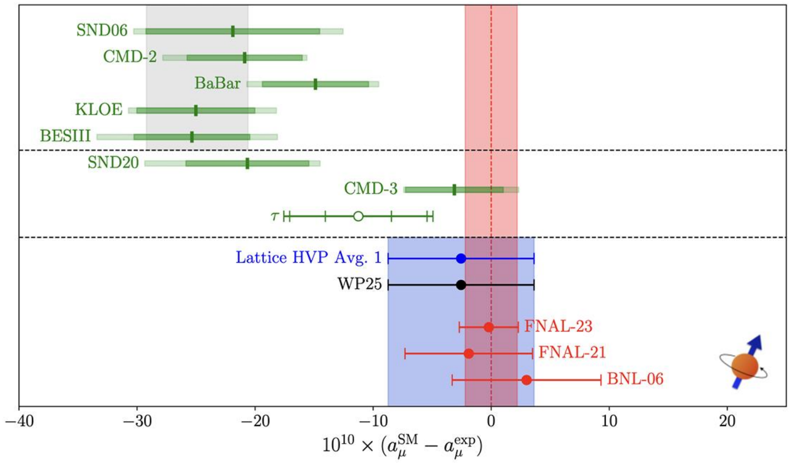
Much more precise, and consistent with 2021!

But the SM theory prediction has changed.

Muon $g-2$ collab. [arXiv:2506.03069]



$g-2$ theory WG. [arXiv:2505.21476]

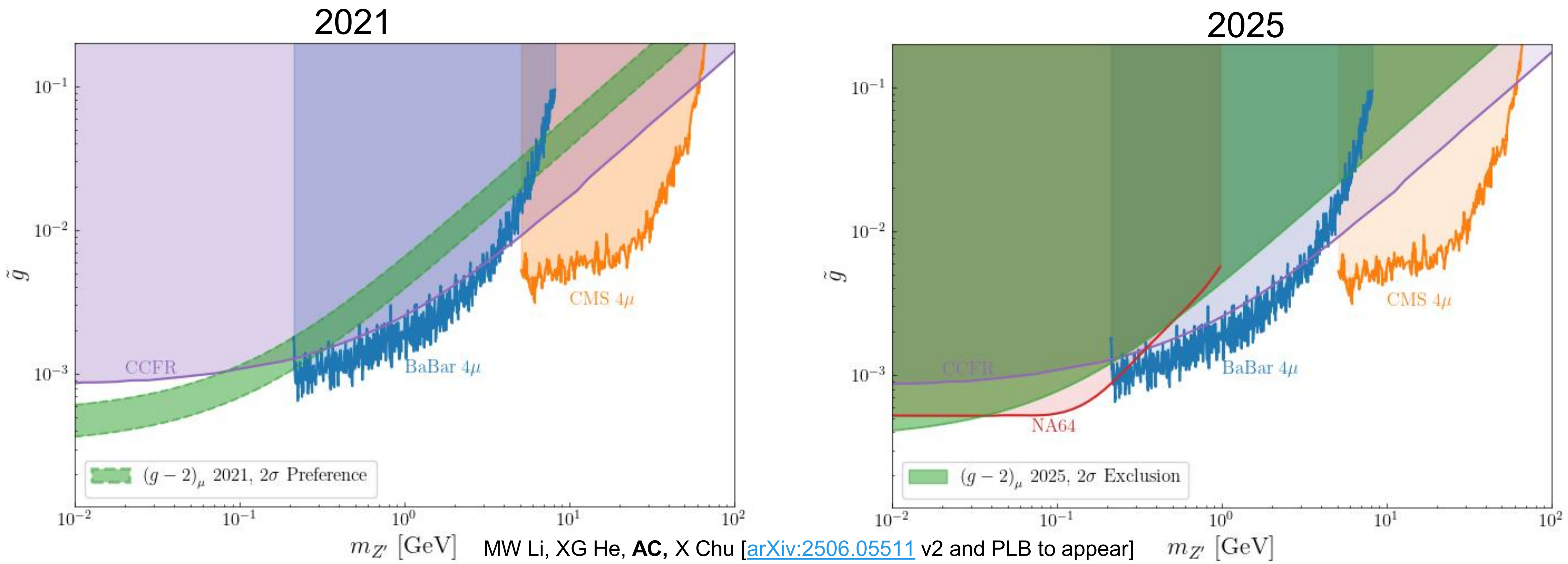


Green: Is dispersive methods for calculating the hadronic vacuum polarization.

CMD-3 measures $e^+e^- \to \pi^+\pi^-$ and shows large discrepancy.

From tension to exclusion

2021 | 2025



MW Li, XG He, **AC,** X Chu [arXiv:2506.05511 v2 and PLB to appear]

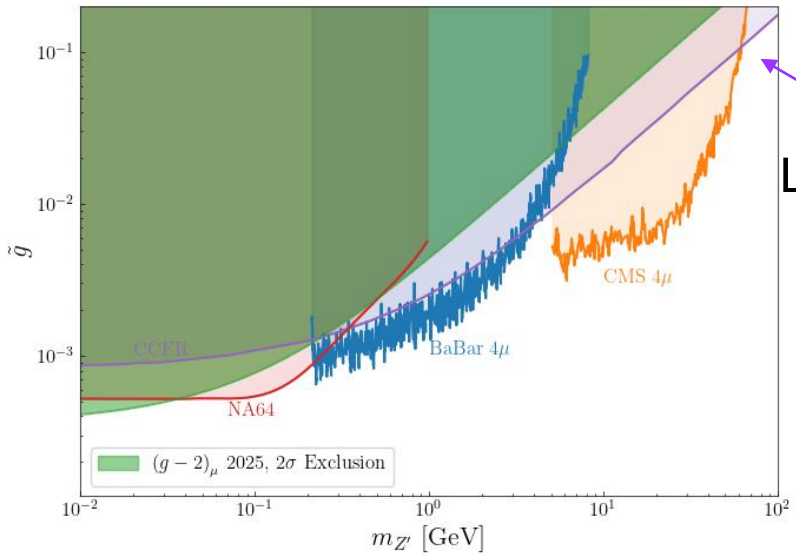
Green preferred!

Green disfavored!

Time to give up on $U(1)_{L_\mu - L_\tau}$?

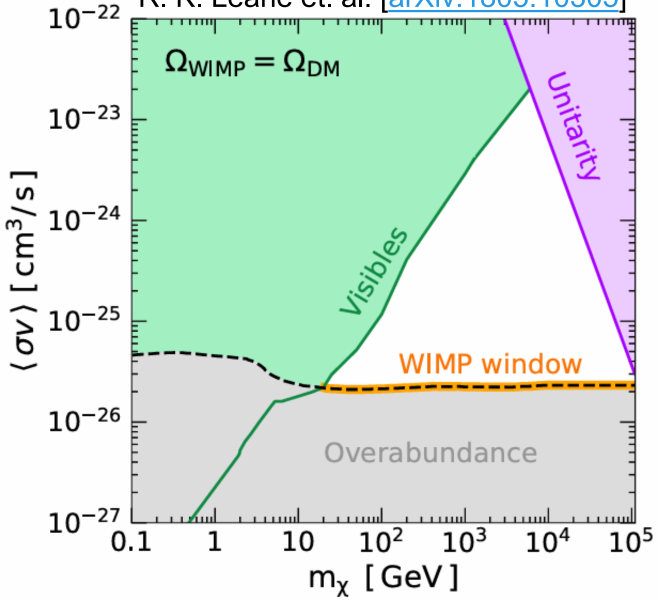
No!

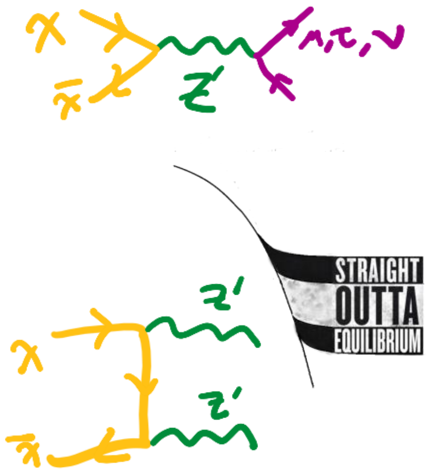
1. As one of the simplest consistent extension to the SM, it should be thoroughly tested.
2. Thermal dark matter is still not ruled out!



Lets look here

R. K. Leane et. al. [arXiv:1805.10305]





$U(1)_{L_\mu - L_\tau}$ at a future muon collider

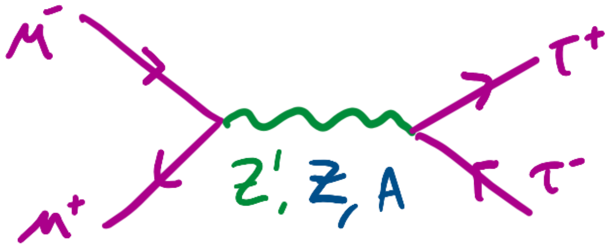
Taking the MNT limit as a benchmark,

CHARM-II: $\sigma_{\text{exp}}/\sigma_{\text{SM}} = 1.58 \pm 0.64$

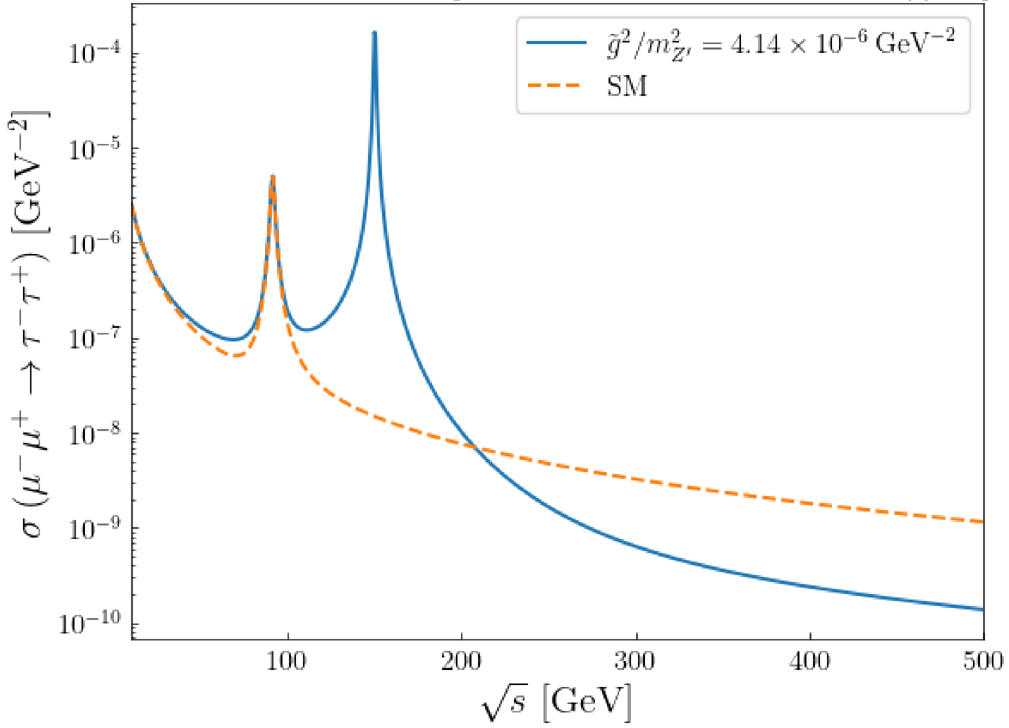
CCFR: $\sigma_{\text{exp}}/\sigma_{\text{SM}} = 0.82 \pm 0.28$

NuTeV: $\sigma_{\text{exp}}/\sigma_{\text{SM}} = 0.72^{+1.73}_{-0.72}$

We would see a large deviation from SM in future muon collider

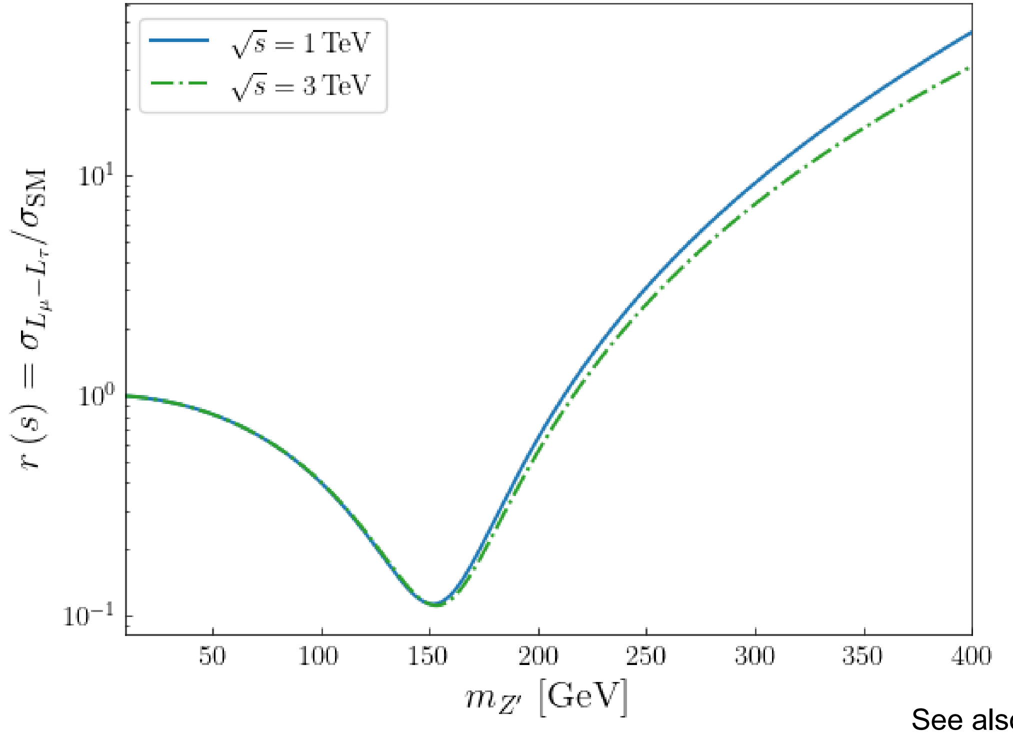


MW Li, XG He, **AC,** X Chu [arXiv:2506.05511 v2 and PLB to appear]

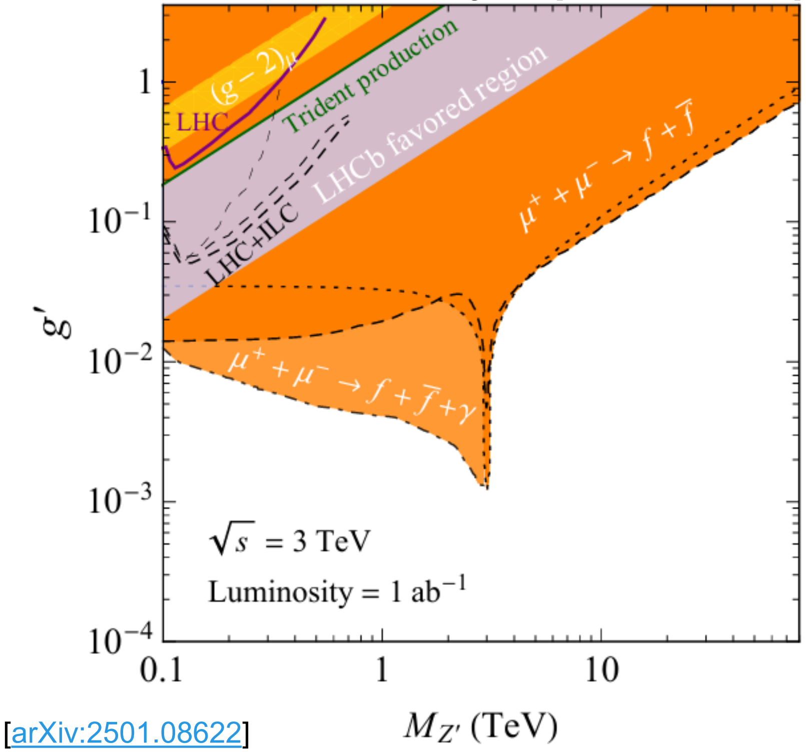


$U(1)_{L_\mu - L_\tau}$ at a future muon collider

MW Li, XG He, **AC,** X Chu [arXiv:2506.05511 v2 and PLB to appear]

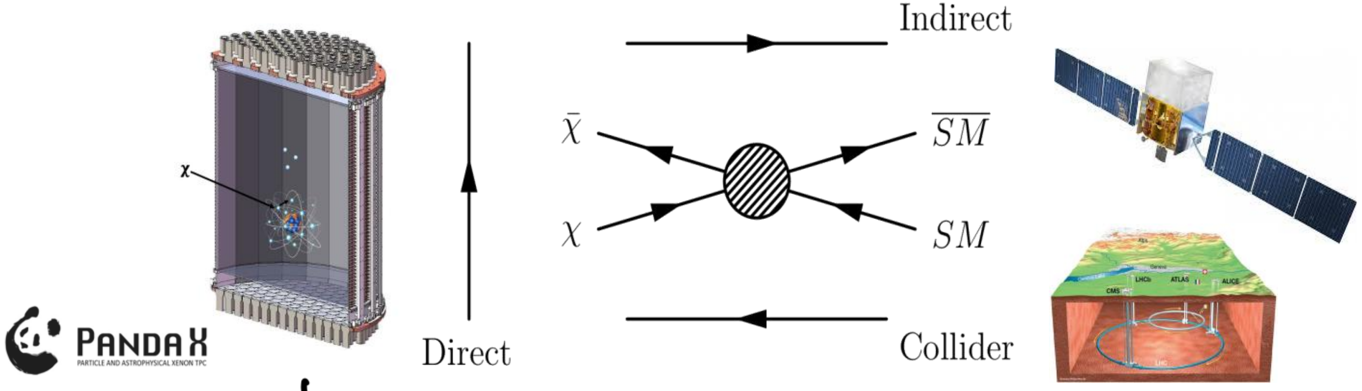


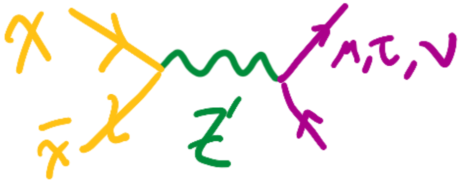
GY Huang et.al [arXiv:2101.04956]

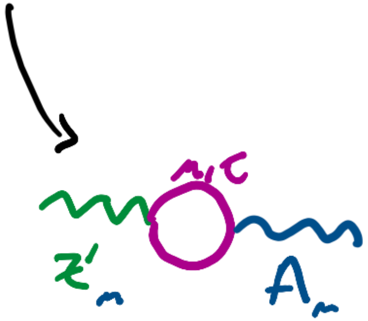


See also ZW Wang et.al [arXiv:2501.08622]

Dark matter and $U(1)_{L_\mu - L_\tau}$



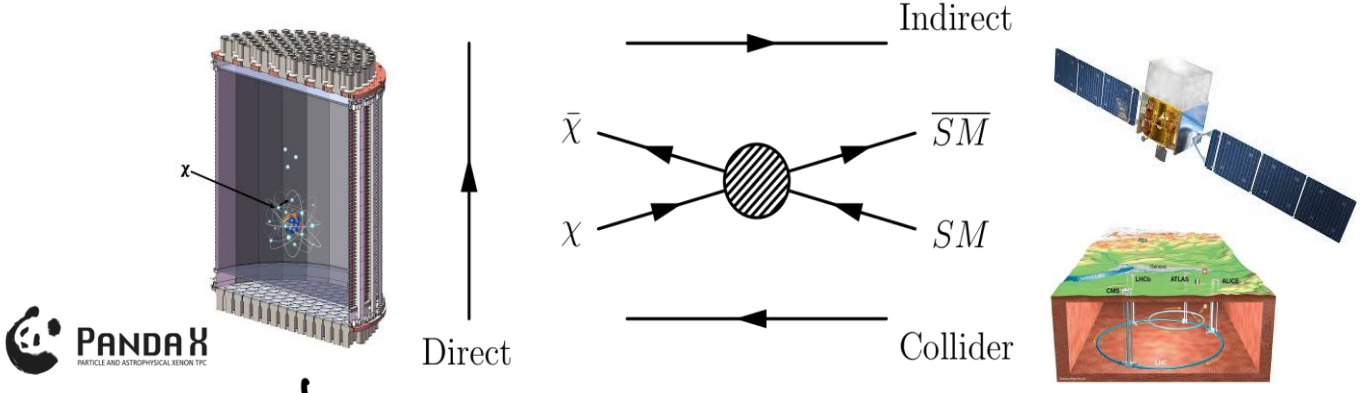




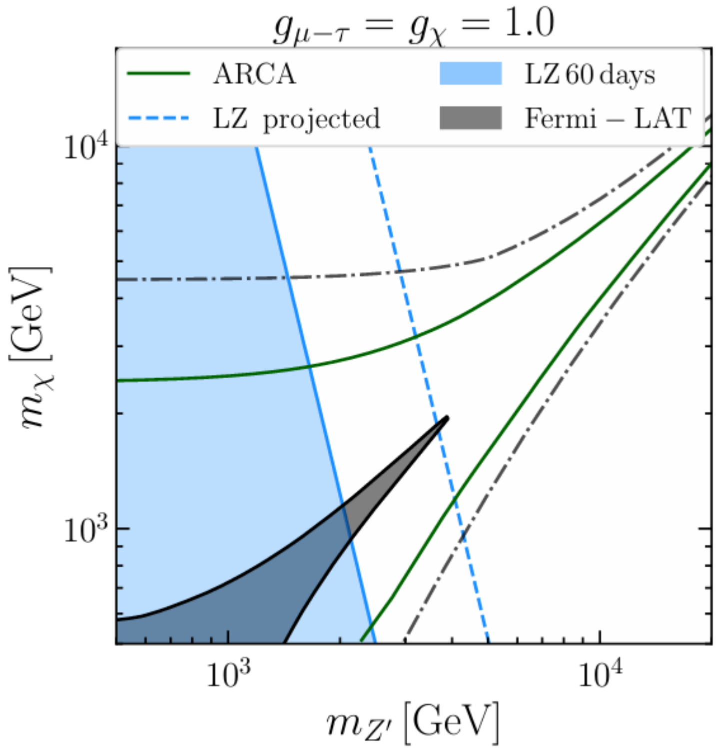
- Charged leptons and neutrinos as final states
- Future gamma-ray telescopes include, CTA, LHAASO, H.E.S.S., etc

Interaction with nucleons come from kinetic mixing

Dark matter and $U(1)_{L_\mu - L_\tau}$

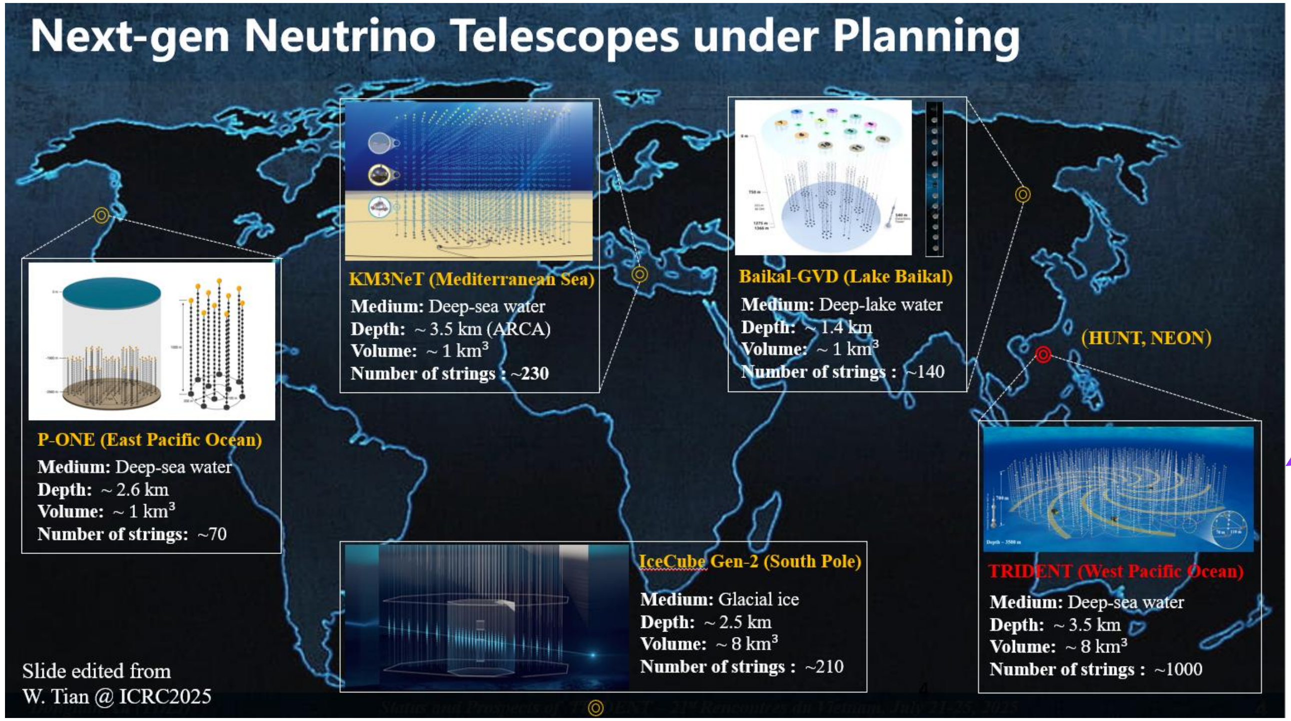


AC + [arXiv:2211.12235]



Interaction with nucleons come from kinetic mixing

Neutrino telescopes



At TDLI! Currently at phase-I (build and deploy 10 strings by 2026).

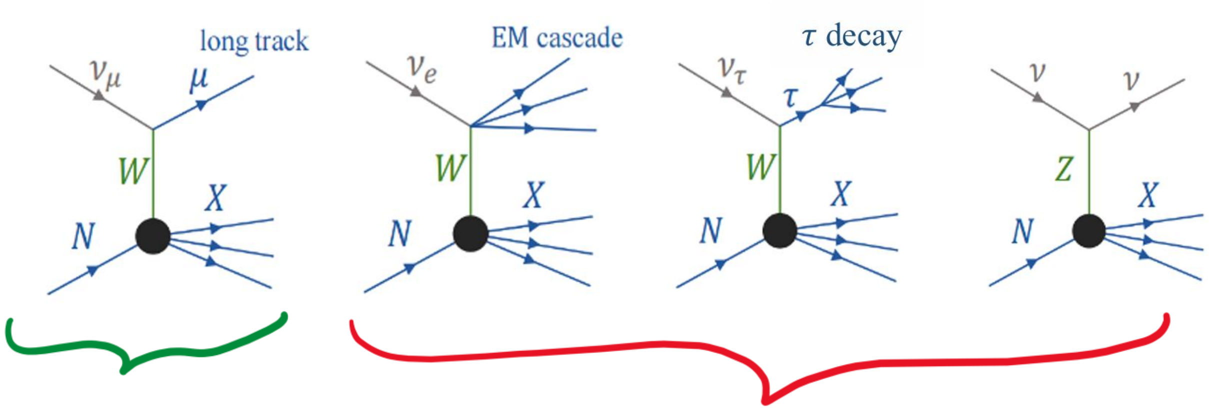
Full conceptual design = 1200 strings.

Trident prospects

TRIDENT will be roughly 10 times as big as KM3NeT.

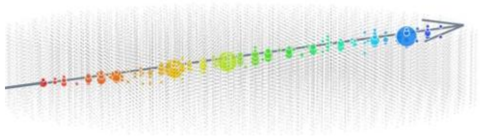
Visibility of galactic center a little less.

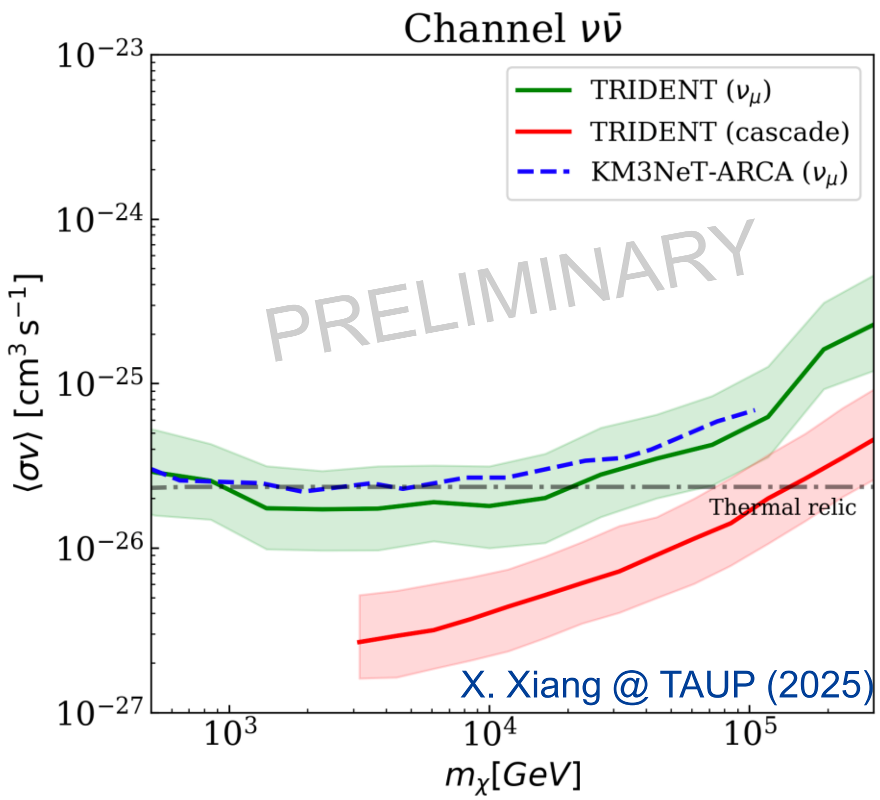
Sea-water detectors are better at observing cascades.



Muon tracks

Cascades





Conclusion

Tension of $(g-2)_\mu$ appears to be gone thanks to updates in SM theory calculations.

Previous emphasis on light Z' no longer required, encourages exploration of high-scale extensions.

We see this as a strong motivation for a muon collider.

One of the last allowed simple thermal DM models is the $U(1)_{L_\mu - L_\tau}$ and will be probed by future neutrino telescopes such as TRIDENT.