Z/W Resonance Schemes @ Lepton Colliders

Shao-Feng Ge
葛韶锋

gesf@sjtu.edu.cn

Ui Min   Zhuoni Qian

上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

SPCS: Precision Higgs Factory Physics
Oct/12, 2025

李政道研究所
Tsung-Dao Lee Institute

Outline

1) **Resonance schemes**

Breit-Wigner vs Theoretical & energy-dependent schemes

2) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries
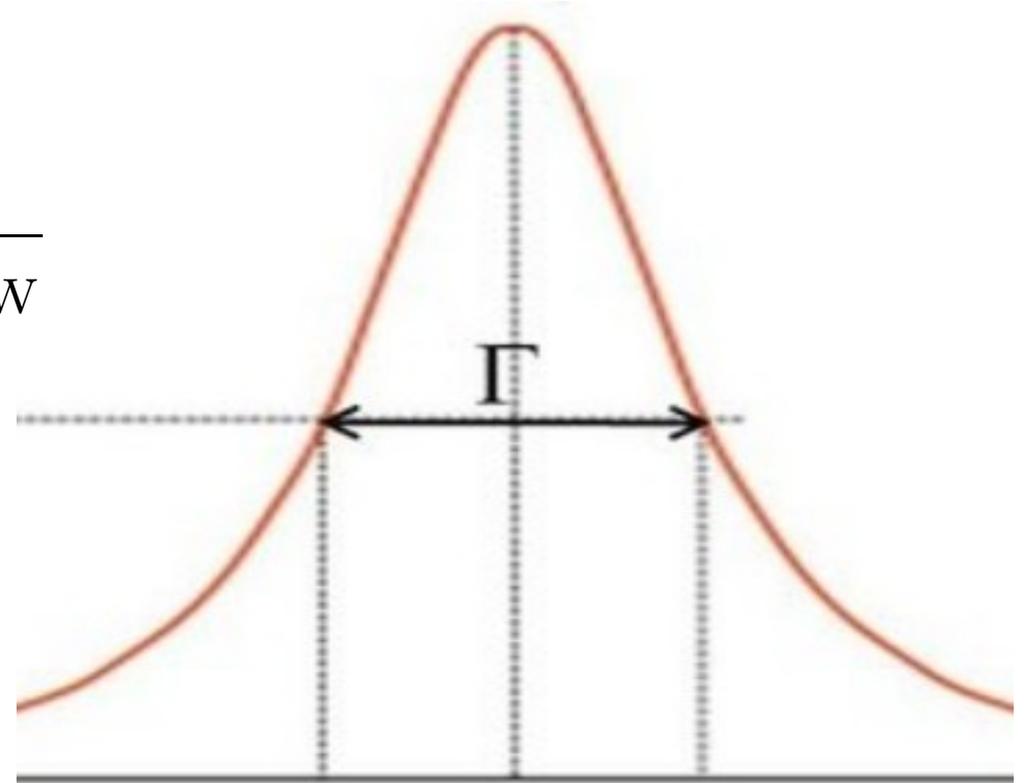
3) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

4) Summary

1. Breit-Wigner Resonance

$$\mathcal{D}_{\mathrm{BW}} \sim \frac{1}{p^2 - m_{\mathrm{BW}}^2 + i m_{\mathrm{BW}} \Gamma_{\mathrm{BW}}}$$

$$|\mathcal{D}_{\mathrm{BW}}|^2 \sim \frac{1}{(p^2 - m_{\mathrm{BW}}^2)^2 + m_{\mathrm{BW}}^2 \Gamma_{\mathrm{BW}}^2}$$

Resonance peak is at exactly the mass m_{BW}

A fundamental particle is described by wave function

$$\psi \xrightarrow{\text{Schodinger Equation}} e^{-iEt}\psi$$

Schodinger Equation

This form is intrinsically for stable particles!

How about unstable particles?

$$E \longrightarrow E - \frac{i}{2}\Gamma$$

Then the wave function decays with time

Willenbrock & Valencia [PLB91]
Willenbrock [2203.11056]

$$\psi(t) \propto e^{-\frac{1}{2}\Gamma t}$$

It is much more convenient to define mass & decay width in the rest frame.

$$m - \frac{i}{2}\Gamma$$

A physical pole can be generally parametrized as

$$\mu \equiv m - \frac{i}{2}\Gamma$$

$$\frac{1}{p^2 - \mu^2} \longrightarrow \frac{1}{p^2 - m^2 + \frac{1}{4}\Gamma^2 + im\Gamma}$$

Willenbrock & Valencia [PLB91]
Willenbrock [2203.11056]

Resonance peak is at $\quad m^2 - \frac{1}{4}\Gamma^2$

Rather than m²!

2. Breit-Wigner vs Theoretical

李政道研究所
Tsung-Dao Lee Institute

$$\mathcal{D}_{\mathrm{BW}} \sim \frac{1}{p^2 - \boxed{m_{\mathrm{BW}}^2} + im_{\mathrm{BW}}\Gamma_{\mathrm{BW}}}$$

Equivalence

$$\frac{m_{\mathrm{th}}}{m_{\mathrm{BW}}} = \frac{\Gamma_{\mathrm{BW}}}{\Gamma_{\mathrm{th}}} \simeq 1 + \frac{\Gamma_Z^2}{8m_Z^2} \equiv R_Z$$

$$9.2 \times 10^{-5}$$

$$\mathcal{D}_{\mathrm{th}} \sim \frac{1}{p^2 - \boxed{m_{\mathrm{th}}^2 + \frac{1}{4}\Gamma_{\mathrm{th}}^2} + im_{\mathrm{th}}\Gamma_{\mathrm{th}}}$$

The equivalence happens at the amplitude level already!

3. Energy-Dependent Scheme

Decay width is defined as

$$\Gamma \equiv \frac{1}{2m} \int |\mathcal{M}|^2 d\Omega \quad \text{where} \quad \int |\mathcal{M}|^2 d\Omega \propto p^2$$

$$m_{\text{parent}} \gg m_{\text{daughters}}$$

On-shell: $p^2 = m^2$ ➡ $\Gamma \propto m$

Decay width scales with virtuality p² !

$$\Gamma(p^2) \equiv \frac{p^2}{m_{\text{ED}}^2} \Gamma_{\text{ED}} \quad \Longrightarrow \quad \mathcal{D}_{\text{ED}} \sim \frac{1}{p^2 - m_{\text{ED}}^2 + i \frac{p^2 \Gamma_{\text{ED}}}{m_{\text{ED}}}}$$

Berends, Burgers, Hollik & van Neerven [PLB88]
Bardin, Leike, Riemann & Sachwitz [PLB88]
Bardin, Bilenky, Mitselmakher, Riemann & Sachwitz [ZPC89]

李政道研究所

Tsung-Dao Lee Institute

$$\mathcal{D}_{\text{ED}} \sim \frac{1}{p^2 - m_{\text{ED}}^2 + i\frac{p^2 \Gamma_{\text{ED}}}{m_{\text{ED}}}}$$

$$m_{\text{BW}} = \sqrt{\mathcal{Z}} \, m_{\text{ED}}$$

$$\Gamma_{\text{BW}} = \sqrt{\mathcal{Z}} \, \Gamma_{\text{ED}}$$

$$\mathcal{Z} \equiv \frac{1}{1 + \frac{\Gamma_Z^2}{m_Z^2}}$$

$$g_{\text{BW}} = \mathcal{Z}^{1/4} g_{\text{ED}}$$

$$1 - \sqrt{\mathcal{Z}} \sim 3.7 \times 10^{-4}$$

For both Z & W

$$|\mathcal{D}_{\text{ED}}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z}m_{\text{ED}}^2)^2 + \mathcal{Z}^2 m_{\text{ED}}^2 \Gamma_{\text{ED}}^2}$$

Equivalent!

$$|\mathcal{D}_{\text{BW}}| \sim \frac{1}{(p^2 - m_{\text{BW}}^2)^2 + m_{\text{BW}}^2 \Gamma_{\text{BW}}^2}$$

How to distinguish them?

Resonance Schemes

李政道研究所
Tsung-Dao Lee Institute

$$\frac{m_{\text{th}}}{m_{\text{BW}}} = \frac{\Gamma_{\text{BW}}}{\Gamma_{\text{th}}} = R_Z$$

Theoretical

$$\frac{1}{p^2 - m_{\text{th}}^2 + \frac{1}{4}\Gamma_{\text{th}}^2 + im_{\text{th}}\Gamma_{\text{th}}}$$

$$R_Z \simeq 1 + \frac{\Gamma_Z^2}{8m_Z^2} \qquad 9.2 \times 10^{-5}$$

$$\mathcal{Z} \approx 1 - \frac{\Gamma_Z^2}{m_Z^2} \qquad 3.7 \times 10^{-4}$$

Breit-Wigner

$$\frac{1}{p^2 - m_{\text{BW}}^2 + im_{\text{BW}}\Gamma_{\text{BW}}}$$

$$m_{\text{BW}} = \sqrt{\mathcal{Z}}\,m_{\text{ED}}$$

$$\Gamma_{\text{BW}} = \sqrt{\mathcal{Z}}\,\Gamma_{\text{ED}}$$

$$g_{\text{BW}} = \mathcal{Z}^{1/4} g_{\text{ED}}$$

Energy-Dependent

$$\frac{1}{p^2 - m_{\text{ED}}^2 + i\frac{p^2\Gamma_{\text{ED}}}{m_{\text{ED}}}}$$

Resonance Schemes

Theoretical ⬌ **Breit-Wigner** ⬌ **Energy-dependent**

$$R_Z \simeq 1 + \frac{\Gamma_Z^2}{8m_Z^2}$$

$$9.2 \times 10^{-5}$$

$$\mathcal{Z} \approx 1 - \frac{\Gamma_Z^2}{m_Z^2}$$

$$3.7 \times 10^{-4}$$

	Δm_Z	$\Delta \Gamma_Z$	Δm_W	$\Delta \Gamma_W$
R_Z	34	0.93	30	0.82
$\mathcal{Z}$	8.4	0.23	7.5	0.21
LEP	2.1	**2.3**	33	**830**
CEPC	0.1	0.025	0.5	2.0
FCC-ee	0.1	0.025	0.4	1.2
ILC	0.2	0.12	2.4	2.0

1) Resonance schemes

Breit-Wigner vs Theoretical & energy-dependent schemes

2) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

3) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

4) Summary

李政道研究所
Tsung-Dao Lee Institute

$$\mathcal{D}_{\mathrm{ED}} \sim \frac{1}{p^2 - m_{\mathrm{ED}}^2 + i\frac{p^2 \Gamma_{\mathrm{ED}}}{m_{\mathrm{ED}}}}$$

$$m_{\mathrm{BW}} = \sqrt{\mathcal{Z}} \, m_{\mathrm{ED}}$$

$$\Gamma_{\mathrm{BW}} = \sqrt{\mathcal{Z}} \, \Gamma_{\mathrm{ED}}$$

$$\mathcal{Z} \equiv \frac{1}{1 + \frac{\Gamma_Z^2}{m_Z^2}}$$

$$g_{\mathrm{BW}} = \mathcal{Z}^{1/4} g_{\mathrm{ED}}$$

$$\boxed{1 - \sqrt{\mathcal{Z}} \sim 3.7 \times 10^{-4}}$$

For both Z & W

$$|\mathcal{D}_{\mathrm{ED}}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z} m_{\mathrm{ED}}^2)^2 + \mathcal{Z}^2 m_{\mathrm{ED}}^2 \Gamma_{\mathrm{ED}}^2}$$

$$|\mathcal{D}_{\mathrm{BW}}| \sim \frac{1}{(p^2 - m_{\mathrm{BW}}^2)^2 + m_{\mathrm{BW}}^2 \Gamma_{\mathrm{BW}}^2}$$

Equivalent!

How to distinguish them?

$$|\mathcal{M}_Z|^2 \sim \ |\mathcal{D}_{\mathrm{ED}}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z}m_{\mathrm{ED}}^2)^2 + \mathcal{Z}^2 m_{\mathrm{ED}}^2 \Gamma_{\mathrm{ED}}^2}$$

Equivalence happens for the Z mediated contribution!

$$(\mathcal{M}_Z\mathcal{M}_\gamma^*)_{\mathrm{BW}} \propto \frac{g_{Z,\mathrm{BW}}^2 e^2 (s - m_{\mathrm{BW}}^2)}{[(s - m_{\mathrm{BW}}^2)^2 + m_{\mathrm{BW}}^2\Gamma_{\mathrm{BW}}^2]\,s}$$

$$(\mathcal{M}_Z\mathcal{M}_\gamma^*)_{\mathrm{ED}} \propto \frac{\mathcal{Z}g_{Z,\mathrm{ED}}^2 e^2 (s - m_{\mathrm{ED}}^2)}{[(s - \mathcal{Z}m_{\mathrm{ED}}^2)^2 + \mathcal{Z}^2 m_{\mathrm{ED}}^2\Gamma_{\mathrm{ED}}^2]\,s}$$

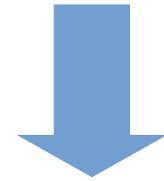**No simultaneous equivalence for the interference term!**

Z-γ Interference Term

$$\sigma_{Z\gamma} = \frac{\alpha Q_e Q_f}{6} \frac{(g_{eL} + g_{eR})(g_{fL} + g_{fR})(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}$$

$$\left.\begin{array}{l} g_{\ell L} = -\dfrac{1}{2} + s_w^2 \\[2em] g_{\ell R} = s_w^2 \end{array}\right\} \quad \Rightarrow \quad g_{\ell L} + g_{\ell R} = -\dfrac{1}{2} + 2 s_w^2 \sim \mathcal{O}(1\%)$$

The interference term is unfortunately suppressed!

Forward-Backward Asymmetry

$$\sigma_{Z\gamma} = \frac{\alpha Q_e Q_f}{6} \frac{(g_{eL} + g_{eR})(g_{fL} + g_{fR})(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}$$
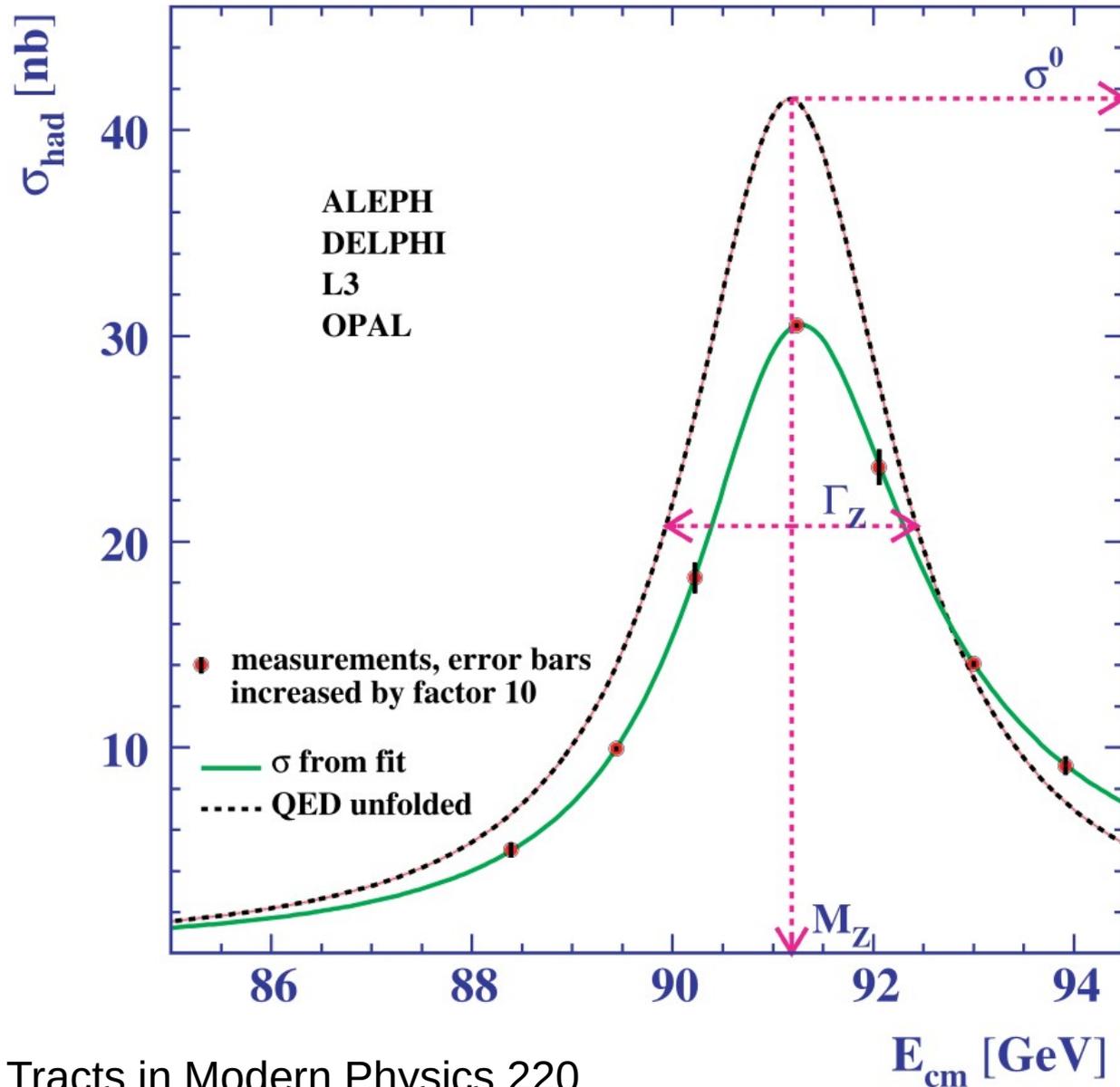
$$\sigma_F - \sigma_B \propto \frac{3s}{64\pi}(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 - g_{fR}^2)$$

$$+ \frac{3 Q_e Q_f \alpha (s - M_Z^2)}{8}\left(g_{eL} - g_{eR}\right)\left(g_{fL} - g_{fR}\right)$$

The interference term is enhanced!

Z Lineshape Scan

ALEPH
DELPHI
L3
OPAL

• measurements, error bars
  increased by factor 10

— σ from fit
···· QED unfolded

Roth, Springer Tracts in Modern Physics 220

Z Lineshape Scan + A$_{FB}$

$$\sigma_{\mathrm{had}} \equiv \sum_q \sigma_F^q + \sigma_B^q \qquad A_{FB}^f \equiv \frac{\sigma_F^f - \sigma_B^f}{\sigma_F^f + \sigma_B^f}$$

Required precision: $\quad \Delta m_Z \lesssim 34\,\mathrm{MeV} \qquad \Delta \Gamma_Z \lesssim 0.9\,\mathrm{MeV}$

Need both of them simultaneously!

$\Delta \chi_{\min}^2$	ALEPH	DELPHI	L3	OPAL	Combined
σ_{had}	0.106	0.0083	0.0127	0.0529	0.183
$\sigma_{\mathrm{had}} + A_{FB}^\mu$	8.34	10.1	5.70	14.1	37.9
$\sigma_{\mathrm{had}} + A_{FB}^{b,c}$	1.19	0.878	0.010*	0.734	2.97
$\sigma_{\mathrm{had}} + A_{FB}^{\mu,b,c}$	8.42	9.67	5.58*	13.3	37.0

CEPC: **490** ($\sigma_{\mathrm{had}} + A^\mu$), **5.1x10^5** ($\sigma_{\mathrm{had}} + A^\mu + A^q$)

Outline

1) Resonance schemes

Breit-Wigner vs Theoretical & energy-dependent schemes

2) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

3) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

4) Summary

Reciprocal Scaling Behaviors

$$\frac{1}{p^2 - m_{\mathrm{BW}}^2 + i m_{\mathrm{BW}} \Gamma_{\mathrm{BW}}} \quad \textbf{vs} \quad \frac{1}{p^2 - m_{\mathrm{th}}^2 + \frac{1}{4}\Gamma_{\mathrm{th}}^2 + i m_{\mathrm{th}} \Gamma_{\mathrm{th}}}$$

Being extracted from data, the imaginary parts should be the same!

$$\mathrm{Data} = m_{\mathrm{BW}} \Gamma_{\mathrm{BW}} = m_{\mathrm{th}} \Gamma_{\mathrm{th}} \qquad \Longrightarrow \qquad \Gamma \propto \frac{1}{m}$$

Different!

Theoretical predictions should be

$$\Gamma_Z \propto m_Z$$

$$\Gamma_Z = \frac{m_Z}{24\pi} \beta_f \left[g_L^2 + g_R^2 - (g_L^2 - 6 g_L g_R + g_R^2)\frac{m_f^2}{m_Z^2} \right]$$

$$\beta_f \equiv \sqrt{1 - 4 m_f^2 / m_Z^2}$$

Constrained Fit

$$\chi_Z^2 \equiv \left(\frac{m_{\mathrm{th}}/R_Z - m_Z}{\Delta m_Z} \right)^2 + \left(\frac{\Gamma_{\mathrm{th}} R_Z - \Gamma_Z}{\Delta \Gamma_Z} \right)^2$$

Required precision: $\Delta m_Z \lesssim 8.4 \, \mathrm{MeV}$ $\Delta \Gamma_Z \lesssim 0.23 \, \mathrm{MeV}$

Need both of them simultaneously!

	Δm_Z (MeV)	$\Delta \Gamma_Z$ (MeV)
LEP	2.1	2.3
CEPC	0.1	0.025
FCC-ee	0.1	0.025
ILC	0.2	0.12

$\chi_{Z,\mathrm{min}}^2 = 326$ (CEPC & FCC-ee) 14.3 (ILC)

Outline

1) Resonance schemes

Breit-Wigner vs Theoretical & energy-dependent schemes

2) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

3) Breit-Wigner vs Theoretical Schemes

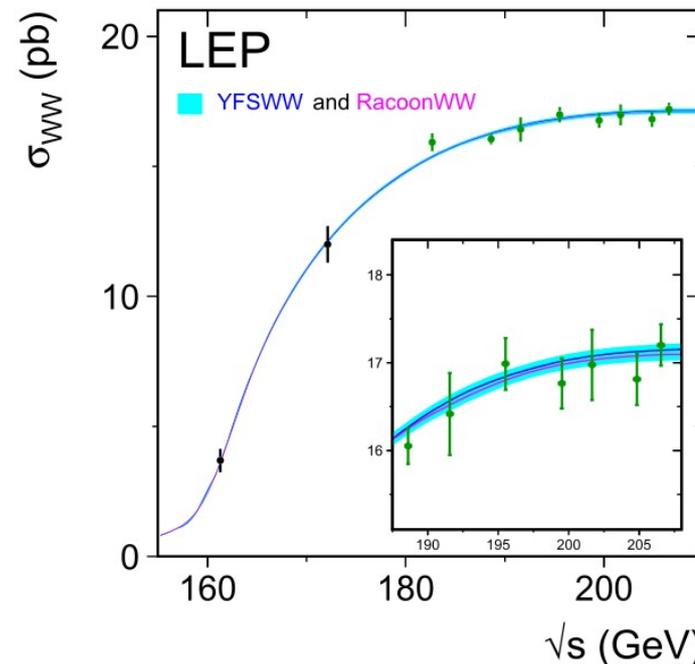Z Lineshape Scan

WW Threshold Scan + Fermi Constant

4) Summary

李政道研究所
Tsung-Dao Lee Institute

$$\chi_W^2 \equiv \left(\frac{m_{W,\mathrm{th}}/R_W - m_W}{\Delta m_W} \right)^2 + \left(\frac{\Gamma_{W,\mathrm{th}} R_W - \Gamma_W}{\Delta \Gamma_W} \right)^2$$

$$\Gamma_W = \frac{m_W}{24\pi} g^2 \beta_{12} \left[1 - \frac{m_1^2 + m_2^2}{2m_W^2} - \frac{(m_1^2 - m_2^2)^2}{2m_W^4} \right]$$

Unfortunately, the mass & decay width precisions are not enough!

	Δm_W (MeV)	$\Delta \Gamma_W$ (MeV)
CEPC	0.5	2.0
FCC-ee	0.4	1.2
ILC	2.4	2.0

1302.3415

$$G_{F,\mathrm{BW}} \approx \frac{g^2}{4\sqrt{2}m_{W,\mathrm{BW}}^2} \left(1 - \frac{\overline{\Gamma}_W^2}{2m_W^2} \right)$$

Momentum transfer in muon decay $p^2 < m_\mu^2$

$$\overline{\Gamma}_W = \frac{p^2}{m_W^2}\frac{2}{5}\Gamma_W(m_W^2) \sim 10^{-7}\Gamma_W(m_W^2)$$

Fermi constant is free of decay width @ tree level

$$G_{F,\mathrm{BW}} \approx \frac{g^2}{4\sqrt{2}m_{W,\mathrm{BW}}^2} \approx \frac{g^2}{4\sqrt{2}m_{W,\mathrm{th}}^2}$$

Fermi constant fixes m$_W$ to the same value!

WW Threshold + Fermi Constant

$$\chi_W^2 \equiv \left(\frac{m_{W,\mathrm{th}}/R_W - m_W}{\Delta m_W} \right)^2 + \left(\frac{\Gamma_{W,\mathrm{th}} R_W - \Gamma_W}{\Delta \Gamma_W} \right)^2$$

$$+ \left(\frac{G_{F,\mathrm{th}} - G_F}{\Delta G_F} \right)^2$$

	Δm_W (MeV)	$\Delta \Gamma_W$ (MeV)
CEPC	0.5	2.0
FCC-ee	0.4	1.2
ILC	2.4	2.0

	$\Delta \chi^2_{\min}$
CEPC	**169**
FCC-ee	**263**
ILC	**7.3**

$$G_F = 1.166\,378\,8\,(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

Summary

Breit-Wigner
$$\frac{1}{p^2 - m_{\mathrm{BW}}^2 + i m_{\mathrm{BW}} \Gamma_{\mathrm{BW}}}$$

m, Γ, g changes

m & Γ changes

Z lineshape + A$_{\mathrm{FB}}$

Z: reciprocal scaling

W: + Fermi constant G$_F$
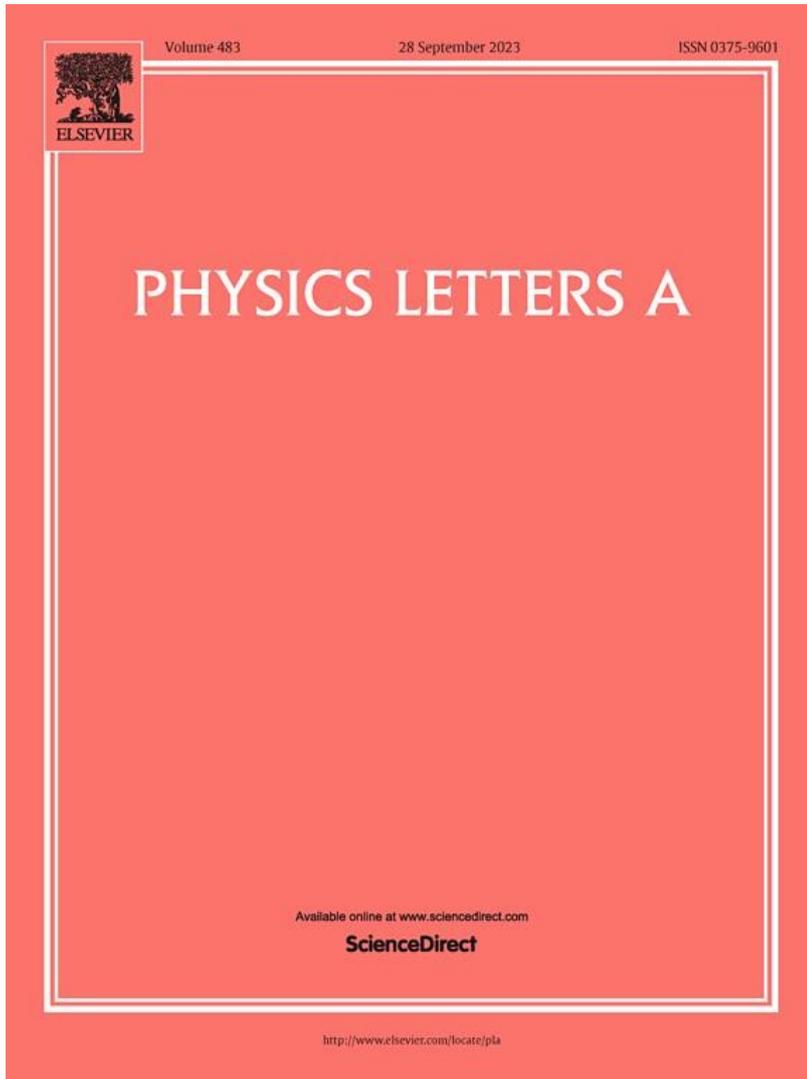
Energy-Dependent
$$\frac{1}{p^2 - m_{\mathrm{ED}}^2 + i \frac{p^2 \Gamma_{\mathrm{ED}}}{m_{\mathrm{ED}}}}$$

Theoretical
$$\frac{1}{p^2 - m_{\mathrm{th}}^2 + \frac{1}{4}\Gamma_{\mathrm{th}}^2 + i m_{\mathrm{th}} \Gamma_{\mathrm{th}}}$$

1. Lepton collider can test one important aspect of quantum & field theories.

2. Resonance scheme can affect all precision measurements.

Thank You

《 Physics Letters A 》

Volume 483   28 September 2023   ISSN 0375-9601

ELSEVIER

PHYSICS LETTERS A

Available online at www.sciencedirect.com

ScienceDirect

http://www.elsevier.com/locate/pla

Aims & Scope

- Nonlinear science,

- Statistical physics,

- Mathematical and computational physics,

- AMO and physics of complex systems,

- Plasma and fluid physics,

- Optical physics,

- General and cross-disciplinary physics,

- Biological physics and nanoscience,

- Astrophysics, Particle physics and Cosmology.

PLA now also accepts submissions on Particle, Astrophysics, and Cosmology! You are welcome to submit.